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Comment on Weakly dissipative dust-ion acoustic wave modulation (*J. Plasma Phys.* 82, 905820104, 2016)

I. Kourakis[1,†] **and I. S. Elkamash**[1,2]

[1]Centre for Plasma Physics, Department of Physics and Astronomy, Queen's University Belfast, Belfast BT7 1NN, UK

[2]Physics Department, Faculty of Science, Mansoura University, 35516 Mansoura, Egypt

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In a recent article (*J. Plasma Phys.*, vol. 82, 2009, 905820104), weakly dissipative dust-ion acoustic wave modulation in dusty plasmas was considered. It is shown in this Comment that the analysis therein involved severe fallacies, and is in fact based on an erroneous plasma fluid model, which fails to satisfy an equilibrium condition, among other shortcomings. The subsequent analysis therefore is dubious and of limited scientific value.

Key words: dusty plasmas, nonlinear phenomena, plasma waves

In the recent article Alinejad, Mahdavi & Shahmansouri (2016) (henceforth to be referred to as Paper 1) an investigation was undertaken of the mechanism of weakly dissipative dust-ion acoustic wave modulation in dusty plasmas. It will be shown in the following that the analysis presented in that paper contains a number of intrinsic flaws, which render the results of doubtful value.

The authors of Paper 1 consider the following fluid model:

$$\left.\begin{aligned} \frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i u_i) &= -\nu_r n_i+\nu_i n_e, \\ \frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}+\frac{\partial \phi}{\partial x} &= -\frac{\sigma}{n_i}\frac{\partial n_i}{\partial x}-\nu_e^{eff}u_i, \\ \frac{\partial^2\phi}{\partial x^2} &= n_e-n_i+1-\mu; \end{aligned}\right\} \tag{1}$$

see equations (2.1)–(3) in Paper 1. According to their formulation, the variables n_i, u_i and ϕ denote the ion number density, the ion fluid speed and the 'electrostatic wave potential' (sic), respectively, while the electrons are taken to be Maxwellian, *viz.* their density reads $n_e=\mu e^{\phi}$. Note that the right-hand side of the first (continuity) equation involves the parameters ν_r and ν_i, which allegedly represent the 'frequency of ion recombination on dust particles' and the 'plasma ionization frequency', respectively, while the right-hand side in the second (momentum) equation involves the parameter

† Email address for correspondence: IoannisKourakisSci@gmail.com

ν_e^{eff} which denotes 'the effective frequency characterizing a loss in the ion momentum due to recombination on dust particles and Coulomb elastic collisions between ions and dust grains' (quoting the authors in Paper 1). The dust component is explicitly assumed to be 'stationary' (i.e. implied to be characterized by constant charge state Z_d and fixed number density n_d), and contributes to the model via the parameter $\mu = n_{e0}/n_{i0} = 1 - Z_d n_{d0}/n_{i0}$.

A number of remarks and comments are in row.

Comment i. Clearly, the total number of ions is not conserved within the aforementioned model: this is reflected by the non-zero right-hand side in the first (ion continuity) equation. Physically speaking, it is implied that both ions and electrons populate the dust grains dynamically, as suggested by the charging rates ν_r and ν_i (defined above). However, if this was true, the charge state Z_d would – obviously – not remain constant, as implied in the model, nor would the total electron density (implicitly assumed to be equal to $n_e = \mu$, i.e. constant).

It is evident that the model (along with the accompanying parameter definitions) has been inspired from a number of previous works (Goree *et al.* 1999; Vladimirov, Ostrikov & Yu 1999; Cramer & Vladimirov 2001; Popel *et al.* 2003), which are explicitly cited in Paper 1. However, it must be emphasized that the models proposed in those articles consider a variable dust charge and dust density, and are thus consistent (in contrast with Paper 1), both physically and mathematically. In particular, the model employed by Vladimirov *et al.* (1999) involves variable electron and ion number density (see equations (1) and (3) in the latter reference), but this is done in conjunction with variable dust number density and dust-charge dynamics (note equations (4) and (5) in the same reference). That model may not be applied gratuitously, if one considers a constant dust charge, stationary dust grains and inertialess electrons, as it the case in Paper 1.

From a fundamental point of view, it appears that Alinejad *et al.* (2016) have taken into account the effects of ion recombination on the dust particles, and that of ionic momentum loss due to recombination on the dust particles and also due to electrostatic collisions between ions and dust grains, but they have neglected the effect of dust charging. The frequency scales characterizing these effects are typically of the same order of magnitude (Mamun & Shukla 2002), and therefore these should have been considered on an equal footing in the model. This assumption can therefore not be justified physically, nor mathematically.

Concluding our first observation, the ion-fluid model introduced in Paper 1 is of limited validity and of questionable physical value, as it fails to preserve the ion number density, entailing a dubious and unclear physical interpretation of the associated results. Although dust charging implies precisely a variation of the electron and ion population density, this is not taken into account properly through the specific fluid model considered in Paper 1 (Alinejad *et al.* 2016).

Comment ii. The authors of Paper 1 consider an equilibrium state, namely $S^{(0)}$ in their equation (3.1), which is later defined as the triad (vector) $S^{(0)} = (n^{(0)}, u^{(0)}, \phi^{(0)}) = (1, 0, 0)$. This equilibrium state is then used as reference state, around which the state variables are expanded in a polynomial series in ϵ ($\ll 1$): cf. equation (3.1) in Paper 1.

A crucial point needs to be made at this stage. It is straightforward to see that the above reference state does not satisfy the system of fluid equations (1)! To see this, one may simply substitute for $(n^{(0)}, u^{(0)}, \phi^{(0)}) = (1, 0, 0)$ into (1), to find that the first of (1) yields a non-zero right-hand side in this case. This is essentially an algebraic manifestation of the physical fact that (as mentioned above) ion continuity is not preserved through this model.

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Interestingly, Alinejad *et al.* overcome the latter issue (i.e. the lack of equilibrium state in their model) by inventing an analytical trick, namely by scaling down the (fictitious) ion and electron annihilation mechanisms to order ϵ^2. It turns out that the linear dispersion characteristics of the model, expressed at first and second order in this perturbation method (Kourakis & Shukla 2005) are thus left unaffected by the erroneous physical mechanism introduced (i.e. ion and electron number variation). As a consequence, expressions (3.3)–(5) in Paper 1, providing the dispersion relation, the first-order amplitude corrections and the group velocity, coincide with earlier results (Kourakis & Shukla 2003, 2004), as the authors of Paper 1 correctly point out, in corroboration of their result.

The procedure outlined in Paper 1 is then pursued in third order, leading to a dissipative nonlinear Schrödinger equation (NLSE) in the form:

$$\mathrm{i}\frac{\partial \phi}{\partial \tau} + P\frac{\partial^2 \phi}{\partial \xi^2} + Q|\phi|^2\phi = -\mathrm{i}R\phi, \tag{2}$$

where ϕ is now redefined as the first-order correction to the electrostatic potential, and ξ and τ are space and time coordinates, consistently defined in the process (Alinejad *et al.* 2016). Not unexpectedly, the (linear) dispersion coefficient P in the NLSE – see (3.12)–(33*a*) in Paper 1 – is left unaffected by the artificial dissipation mechanism discussed above. This is simply due to the fact that P is related to the linear dynamics of the problem. This brings us to a third important comment.

Comment iii. It is known that the dispersion coefficient in the NLSE (2) above must satisfy the relation $P = (\mathrm{d}^2\omega/\mathrm{d}k^2)/2$. This is an explicit by-product of the multiple scale technique adopted in Paper 1; see e.g. in Kako (1972), Kakutani & Sugimoto (1974) or in Kourakis & Shukla (2003, 2004, 2005). One is tempted to test whether this requirement is met, by combining expressions (3.3) and (3.13*a*) in Paper 1, for $\omega(k)$ and $P(k)$, respectively. It turns out, upon simple substitution and some straightforward algebra, that this relation is not satisfied (!), *viz.*, $P \neq (\mathrm{d}^2\omega/\mathrm{d}k^2)/2$ – referring to equations (3.3) and (3.13*a*) in Paper 1, specifically. Equation (3.13*a*) is therefore not correct, and the subsequent analysis is presumably wrong (algebraically speaking, hence physically too).

It should be stated, in passing, that the expression derived for the nonlinearity Q – see (3.13*b*) in Paper 1 – should normally coincide with the earlier result in the 'dissipation-free' case (Kourakis & Shukla 2004). This is implied, but not rigorously shown not discussed in Paper 1.

By assuming that the right-hand side of the ion continuity equation scales as $\sim\epsilon^2$, the algebraic effect of the artificial dissipation mechanism thus introduced is 'boosted' to order ϵ^3, and thus naturally appears (and is limited to) the damping term $-\mathrm{i}R\phi$ appearing in the right-hand side of the NLSE, i.e. (3.12) in Paper 1. This builds up a straightforward and somehow 'legitimate' algebraic model, with a rather unsurprising outcome (a dissipative NLSE), yet with dubious physical interpretation, as discussed above.

Comment iv. It can actually be shown that a dissipative NLSE in the form of (2) above can be obtained by considering any linear combination of the state variables (i.e. terms of the form $\nu_1 n_i$, $\nu_2 u_i$ etc., adopting an *ad hoc* notation here) in the right-hand side of the evolution equation. Such a procedure defines an interesting algebraic procedure, in that it introduces a 'dissipative fluid model' which can be analysed as shown in Paper 1. It remains to be seen how realistic, and physically acceptable, this model is (see our discussion above).

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According to the above considerations, one raises the question of the validity of the results in Paper 1. It may be argued that, focusing on the formal structure of the dissipative NLSE (2) above, and ignoring for a minute the definitions of the coefficients in it, that the analysis of modulational instability presented in section 4 of Paper 1 is interesting and valuable *per se*. A twofold approach may be adopted here. First of all, a crucial point in section 4 of Paper 1 is the derivation of the expression

$$\Delta(\tau) = -Q\phi_0^2 \exp(-2R\tau) \tag{3}$$

for the nonlinear frequency shift (see the discussion following (4.1) in Paper 1), assuming an unperturbed amplitude (argument, in the polar representation) $\phi_0 = \mathrm{Arg}(\phi)$ (recall that ϕ is complex). Formally, this is tantamount to a transformation of the form $Q \to Q\exp(-2R\tau)$, which naturally entails an exponential decay of the critical wavenumber $K_c = (2Q/P)^{1/2}$, *viz.*

$$K_c = (2Q/P)^{1/2} \to (2Q/P)^{1/2} \exp(-R\tau). \tag{4}$$

The authors admit that this procedure is adapted from Xue (2003), who described dust-acoustic wave modulation taking into account dust-charge fluctuations (a different physical problem, nonetheless leading to an equation formally identical to (2) above). It can be pointed out that the above result, as expressed, say, in relations (3) and (4) above, is rather trivial, as it follows directly from a simple transformation of the form

$$\phi \to \phi' \exp(-R\tau). \tag{5}$$

In other words, it may be shown upon simple substitution of (5) into (2), that (2) becomes:

$$\mathrm{i}\frac{\partial \phi'}{\partial \tau} + P\frac{\partial^2 \phi'}{\partial \xi^2} + Q'|\phi'|^2\phi' = 0, \tag{6}$$

where $Q' = Q\exp(-2R\tau)$. The analytical findings in the first part of § 4 in Paper 1, relying on equations in the form of our equations (3) and (4) above in particular (for the exponentially decaying nonlinear frequency shift Δ and for the wavenumber threshold K_c), thus simply follow from the above considerations, combined into the standard definitions for these quantities; see e.g. in Kourakis & Shukla (2003, 2004, 2005). For instance, the monochromatic wave solution of the 'standard' form of the NLSE (6) reads $\phi' = |\phi'|\exp(-Q'|\phi'|^2\tau)$, which immediately yields the frequency shift $\Delta = -Q'|\phi'|^2 = -Q\mathrm{e}^{-2R\tau}|\phi'|^2$, i.e. precisely equation (3) above (upon a trivial change in notation).

It follows that the plots presented in figures 2 and 3 in Paper 1 are founded on the above rationale, while depending on the actual definition of R – which is dubious, as discussed earlier. Regretfully, the graphical results presented in Paper 1 are therefore of doubtful value, as they reply on the (questionable) physical assumptions of the model, as discussed above.

The above suggests that the modulational instability related results in Alinejad *et al.* (2016) (namely, the instability growth and associated critical wavenumber) are not erroneous, but could have been obtained via a simpler analysis than the one adopted – in turn based on Xue (2003).

In conclusion, we have shown that the fluid model presented in Alinejad *et al.* (2016) is intrinsically flawed, as it involves insurmountable errors in its physical interpretation, but also algebraic errors in the perturbative analysis presented therein. The results are therefore of no physical value. Admittedly, the modulational stability

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obtained in that article, based on a generic form of the dissipative NLSE, is legitimate and perhaps interesting, but the outcome is rather trivial (as it can be recovered upon a simple phase transformation from the original equation). Still, one is led to questionable results, once the so called ion-dust and electron-dust collisions are taken into account.

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REFERENCES

ALINEJAD, H., MAHDAVI, M. & SHAHMANSOURI, M. 2016 Weakly dissipative dust-ion acoustic wave modulation. *J. Plasma Phys.* **82**, 905820104.

CRAMER, N. F. & VLADIMIROV, S. V. 2001 Waves in dusty plasma discharges. *Phys. Scr.* T **89**, 122.

GOREE, J., MORFILL, G. E., TSYTOVICH, V. N. & VLADIMIROV, S. V. 1999 Theory of dust voids in plasmas. *Phys. Rev.* E **59**, 7055.

KAKO, M. 1972 Nonlinear wave modulation in cold magnetized plasmas. *J. Phys. Soc. Japan* **33** (6), 1678.

KAKUTANI, T. & SUGIMOTO, N. 1974 Krylov–Bogoliubov–Mitropolsky method for nonlinear wave modulation. *Phys. Fluids* **17** (8), 1617.

KOURAKIS, I. & SHUKLA, P. K. 2003 Modulational instability and localized excitations of dust-ion acoustic waves. *Phys. Plasmas* **10**, 3459.

KOURAKIS, I. & SHUKLA, P. K. 2004 Finite ion temperature effects on oblique modulational stability and envelope excitations of dust-ion acoustic waves. *Eur. Phys. J.* D **28**, 109.

KOURAKIS, I. & SHUKLA, P. K. 2005 Exact theory for localized envelope modulated electrostatic wavepackets in space and dusty plasmas. *Nonlinear Process. Geophys.* **12**, 407.

MAMUN, A. A. & SHUKLA, P. K. 2002 *Introduction to Dusty Plasma Physics*. IOP.

POPEL, S. I., GOLUB, A. P., LOSSEVA, T. V., IVLEV, A. V., KHRAPAK, S. A. & MORFILL, G. 2003 Weakly dissipative dust-ion-acoustic solitons. *Phys. Rev.* E **67**, 056402.

VLADIMIROV, S. V., OSTRIKOV, K. N. & YU, M. Y. 1999 Ion-acoustic waves in a dust-contaminated plasma. *Phys. Rev.* E **60**, 3257.

XUE, J. K. 2003 Modulation of dust acoustic waves with non-adiabatic dust charge fluctuations. *Phys. Lett.* A **320**, 226.