*J. Plasma Physics* (2002), *vol.* 68, *part* 1, *pp.* 1–25. © 2002 Cambridge University Press 1
DOI: 10.1017/S002237780200185X Printed in the United Kingdom

# Impossibility of Zakharov's short-wavelength modulational instability in plasmas with intense Langmuir turbulence

V. I. EROFEEV

Institute of Automation & Electrometry, Russian Academy of Sciences,
1 Kopt'ug Prospect, 630090, Novosibirsk, Russia

(Received 5 November 2001)

**Abstract.** An analysis of two experimental observations of Langmuir wave collapse is performed. The corresponding experimental data are shown as evidence against the occurrence of collapses. The physical reason preventing the collapses is found to be the nonresonant electron diffusion in momentum. In this process, plasma thermal electrons are efficiently heated at the expense of wave energy, therefore intense collisionless wave dissipation takes place. The basic reason for the underestimation of nonresonant electron diffusion in the traditional theory is shown to be the substitution of a real plasma by a plasma probabilistic ensemble.

A study of nonresonant electron diffusion refraining from ensemble substitution is performed. It is shown that its intensity is sufficient for suppression of Zakharov's short-wavelength modulational plasma instability [Zakharov, V. E., *Sov. Phys. JETP* **35**, 908 (1972)]. This explains the nonoccurrence of Zakharov's Langmuir wave collapse in experiments.

## 1. Introduction

The existing plasma kinetic theory was developed as a specific branch of *nonequilibrium statistical physics*. Following the traditions of the latter, theorists substitute real plasmas by *probabilistic plasma ensembles,* either directly or indirectly, and develop evolution laws for the ensemble statistics. The corresponding laws are believed to approximate the physical evolution of the plasma fairly well.

In this paper we look at two series of experiments that demonstrate the inconsistency of this approach in practice. The first of the series was performed by the group of Vyacheslavov (BudkerINP) and the second by Wong and his colleagues (University of California, Los Angeles). These widely reported beam–plasma experiments were believed to confirm the conception of Langmuir wave collapse (Zakharov 1972) but will be shown to constitute evidence against it. The absence of collapses in the given experiments suggests the presence of intense collisionless wave dissipation in the plasmas that precludes the development of the collapse. The intensity of the corresponding dissipation contradicts the knowledge formulated in published plasma kinetic studies. The basic reason for the underestimation of wave dissipation in the traditional theory is the substitution of a single plasma by a plasma ensemble. With such a substitution, one unintentionally smears out the bulk of some physical phenomena.

The main goal of this paper is to show that in a real *single* plasma the collisionless wave dissipation is always intense enough to prevent Langmuir wave collapse.

The paper is organized as follows. In Sec. 2 we consider the above-mentioned series of beam–plasma experiments with strong Langmuir turbulence. In Sec. 3 we present an auxiliary analysis of the problem of nonresonant electron diffusion. (This phenomenon is the main channel of collisionless wave dissipation.) We show there that the known results of traditional plasma kinetics comprise at least two diverse ideas for the intensity of this process. The corresponding ideas result in notably different pictures for the physical evolution of a plasma. To exemplify this position, in Sec. 4 we show that the nonresonant electron diffusion is always sufficient to preclude wave collapse in strong Langmuir turbulence. The difference in conclusions regarding the presence or the absence of collapse in strongly turbulent plasmas shows that ensemble studies cannot help one form an objective picture of the physical evolution of a plasma. Correspondingly, a special kinetic consideration of a 'single' plasma is required. The calculation of wave dissipation caused by nonresonant electron diffusion in a single collisionless plasma is presented in Sec. 5. In Sec. 6 we concisely list the basic points of this study and formulate our final conclusions.

## 2. Experimental observations of strong Langmuir turbulence

In studies of plasma heating by powerful relativistic electron beams, much of the interesting physics was discovered by the group of Vyacheslavov at BudkerINP (Vyacheslavov et al. 1995, 1996, 1998a, 1998b; Burmasov et al. 1997). In the experiments of this team, a beam excites Langmuir turbulence with an extremely high intensity. This turbulence has long been recognized as being rather strong. For instance, in a review on strong plasma turbulence by Robinson (1997, p. 565) one finds, '*. . . There are clearly strong signs of wave collapse and strong turbulence in these experiments . . .*'. In all of the above-cited papers Vyacheslavov and his colleagues discussed various confirmations of the collapse physics. In particular, they claimed once that in their experiment plenty of cavities collapsed from scales of 30 to 800 plasma Debye lengths, $r_{\mathrm{D}}$ (Burmasov et al. 1997). To reinforce their declarations concerning the strongly turbulent regime, note that Vyacheslavov explicitly checked that the level of turbulence observed was 30 times larger than the threshold of plasma instability with respect to long-scale transverse perturbations (Vyacheslavov et al. 1995). We stress that this figure accounts for the effect of the magnetic field: Vyacheslavov et al. used results of the study by Pozzoli and Ryutov who considered *long-wavelength* modulational instability of a plasma in a magnetic field (Pozzoli and Ryutov 1979). It can be shown that this 30-fold excess automatically means that Zakharov's *short-length* density cavities should constantly develop and collapse.

Contrary to previous declarations, in the last report by Vyacheslavov et al. a trial was undertaken to find channels of wave energy dissipation that were alternative to wave collapse (Vyacheslavov et al. 2000). The reason is that the power of the plasma electromagnetic emission at double the plasma frequency does not demonstrate any increase in experiments compared with weakly turbulent estimates. This position is a crucial one for conclusions regarding the presence or absence of collapse in the experiments. Let us specify the increase in emission that should have manifested itself had wave collapse been the main mechanism of turbulence removal from the region of wave pumping.

In the case of homogeneous Langmuir turbulence the power of electromagnetic radiation from a unit plasma volume at frequencies near $2\omega_{pe}$ can be evaluated as

$$P \sim k^3 \left( \frac{ne}{k^3} \frac{Wke^2}{m^2 \omega_{pe}^2} \right)^2 \frac{1}{c^3} \sim nT \left( \frac{W}{nT} \right)^2 \left( \frac{\omega_{pe}}{kc} \right)^2 \left( \frac{v_{Te}}{c} \right)^2 kc. \tag{1}$$

This estimate can be developed from formulae by Tsytovich (1970); the intermediate form of the estimate exposes its structure. Namely, a plasma volume falls into domains with spatial dimensions of the order of the wave correlation length, $k^{-1}$ (where $k$ is the characteristic wavenumber in the turbulence spectrum). In each of the domains, the electrons oscillate synchronously; emission from different domains is uncorrelated. $Wke^2/(m^2\omega_{pe}^2)$ is the typical acceleration of the plasma electron at frequency $2\omega_{pe}$ (where $W$ is the turbulence energy density).

We now develop the corresponding estimate for the case of Langmuir turbulence that is composed of collapsing density cavities, using the idea of supersonic collapse. Then the typical spatial dimension $a$ of the cavity evolves as $a \sim (t_0 - t)^{2/3}$. For constant rate cavity generation the current $I$ through scales $a$ from $a \sim k^{-1}$ to shorter scales is constant, therefore we have

$$I \equiv N(a) \frac{da}{dt} \sim N(a)/\sqrt{a} = \text{constant},$$

where $N(a)\, da$ gives the number of cavities with dimensions from $a$ to $a + da$ in the unit plasma volume. Therefore,

$$N(a) = \frac{N(k^{-1})/k^{-1/2}}{a^{-1/2}} \sim k^4 \sqrt{ka}.$$

(The natural assumption involved here was that on scales of $k^{-1}$ the turbulence is a compactly packed set of cavities with $a \sim k^{-1}$. Therefore one has $N(k^{-1})k^{-1} \sim k^3$.) Correspondingly, the estimate for power emission at $\omega \approx 2\omega_{pe}$ is

$$\begin{aligned} P &\sim \int N(a)\, da \left[ na^3 e \frac{(Wk^{-3}/a^3)a^{-1}e^2}{m^2\omega_{pe}^2} \right]^2 \frac{1}{c^3} \\ &\sim nT \left( \frac{W}{nT} \right)^2 \left( \frac{\omega_{pe}}{kc} \right)^2 \left( \frac{v_{Te}}{c} \right)^2 kc \int \frac{k\, da}{(ka)^{3/2}}. \end{aligned} \tag{2}$$

In this estimate, emissions from independent cavities are not correlated, $Wk^{-3}/a^3$ denotes a typical value of the square of the electric field in the cavity and $na^3$ denotes the number of electrons in the cavity. Electrons in the cavity oscillate synchronously. It can clearly be seen that the power (2) is large compared with the estimate (1). The final stages of collapse are stressed and provide a multiple increase in the radiation intensity. It is worth noting that Vyacheslavov et al. developed a more accurate calculation for the weakly turbulent estimate of a given intensity (using kinetic formulae from Tsytovich (1970) and the measured turbulence spectrum), and still had not recognized any increase in the registered intensity as compared with the estimate. Moreover, in the case of collapse the radiation at $2\omega_{pe}$ should have contained spiky flashes, but no spikes were registered (Vyacheslavov et al. 1998a).

Contrary to the suspicion of Vyacheslavov, in his experiments hot plasma electrons cannot arrest the collapse, neither at its initial stage nor at intermediate stages of cavity compression. The reader can check this using data reported in Vyacheslavov et al. (1995, 2000). (We remind the reader that arrest of the collapse takes place at a scale $a$ if the Landau damping on hot plasma electrons with velocities $v \sim a\omega_{pe}$ is comparable with the calculated rate of increase in electric field in the cavity at the corresponding stage of the collapse. The former can be evaluated as $\omega_{pe} n_{\text{hot}}(a\omega_{pe})/n_0$, where $n_{\text{hot}}(a\omega_{pe})$ is the full density of hot plasma electrons with $v > a\omega_{pe}$. The rate of increase in the electric field can be evaluated as $\gamma_{\text{mod}}(ka)^{-3/2}$, where $\gamma_{\text{mod}}$ is the growth rate of the modulational instability. The growth rate $\gamma_{\text{mod}}$ can be fairly well evaluated as $\gamma_{\text{mod}} \sim k(T_e/M)^{1/2}\left[(T_{\text{eff}}/T_e)^{1/2} - 1\right]$, see Pozzoli and Ryutov (1979).) Besides, it is not correct to speak about an ancillary role for the collapses within a basically hydrodynamic plasma description: once Zakharov's theory is accepted and predicts intense collapse, the collapse should develop. It is also not correct to speak about wave scattering on density inhomogeneities generated by the collapse within Zakharov's paradigm: these density inhomogeneities are the ready seeds for developing wave modulational instability that efficiently transfers energy from waves to cavities. In short, a significant number of collapses should have occurred in the experiments by Vyacheslavov et al., but they were not observed.

Now consider other experimental results that are currently admitted as convincing evidence for Langmuir wave collapse, i.e. the experiments by Wong and his colleagues (Wong and Cheung 1984; McFarland and Wong 2000). In the experiments of this team a fast electron beam is injected into a large, unmagnetized argon plasma, and Langmuir plasma oscillations are excited. Over the course of time, a localized field structure develops at a distance of 23 cm away from the point of beam injection. The evolution of this structure resembles the collapse of Zakharov's density cavity with trapped Langmuir waves. From the very beginning Wong and his colleagues assumed that they had observed nothing other than a true collapse of a single cavity with a trapped Langmuir wave field (see Wong and Cheung 1984). Meanwhile, in some aspects their experimental data contradicts the picture of Zakharov's Langmuir wave collapse. First, it is not typical for Zakharov's plasma hydrodynamics that in a regime with a 'sequential series of collapses' they occurred at the same point with an extremely accurate periodicity (McFarland and Wong 2000). It is known rather well that Zakharov equations are capable of exhibiting stochastically unstable behaviour (see, e.g., Wong et al. 1995). That is, both the location of the second collapse and its time delay from the first one should vary strongly because of inevitable and unpredictable small variations in the plasma dynamics. Secondly, the Langmuir waves that are released from the 'collapsing cavity' after its 'burning out' do not have short wavelengths. Thus, McFarland and Wong reported observations of corresponding Langmuir waves that they called *freely propagating electrostatic waves produced by intense, localized field structures* (McFarland and Wong 2000). The authors have measured the group velocity of these waves and found it to be of the order $v_g \simeq 3v_{Te}^2/v_{\text{beam}}$. But this order of magnitude corresponds to waves nearly resonant with the beam, whereas with shortening of the wavelength (due to the wave collapse) the wave group velocity should have been increased substantially. Therefore, the observed *freely propagating electrostatic waves* are evidence against the collapse. Thirdly, reports by Wong and colleagues contain yet more convincing evidence of the nonoccurrence of Zakharov's Langmuir wave collapse in their experiments. In Wong and Cheung (1984) it was deduced that the saturated intensity of Langmuir waves in their experiment was of the order of $0.3nT_e \gg (m_e/m_i)nT_e$, and therefore the wave packet was expected to collapse supersonically. Note that the characteristic time of the collapse $t_0$ should be less than the inverse growth rate of the plasma short-scale modulational instability. Using Zakharov's original formula for the corresponding growth rate $\gamma_{\max} = 1/\sqrt{3}\omega_{pe}\sqrt{m_e/m_i W/nT_e} \simeq 1/\sqrt{3}\omega_{pi}\sqrt{W/nT_e}$, with this, the time of collapse development $t_0$ should be less than $1/\omega_{pi}\sqrt{3(W/nT_e)^{-1}} \approx 3/\omega_{pi}$. The actually observed value of $t_0$ is 20 times greater.

Finally, recall that the picture of the processes was quite reproducible from shot to shot (the 'collapses' occurred at the same point of the installation and after the same period since turning on the beam, see Wong and Cheung (1984)). In view of this it is most natural to assume the following idea. The observed data represents the complicated physics of the beam–plasma interaction, currently insufficiently well explored, rather than the physics of Langmuir wave collapse. Basically, this beam–plasma interaction has a *hydrodynamic* character: the beam can be considered as a jet with spatially and temporally varying longitudinal velocity of electrons. (The hydrodynamic character of beam–plasma interaction is the true reason that leads to the reproducibility of the experimental data.) This jet travels in a cold plasma; the observed beam-excited Langmuir waves move along the beam with a negligibly small group velocity (i.e. form a standing pattern). At the same time, the *phase* velocities of excited Langmuir harmonics are only a little less than the beam velocity.

In the one-dimensional approximation, the corresponding hydrodynamic beam–plasma interaction is governed by the following simultaneous equations:

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{pe}^2\right)\Delta\varphi = 4\pi e\frac{\partial^2 n_b}{\partial t^2}, \tag{3}$$

$$\frac{\partial v_b}{\partial t} + v_b\frac{\partial v_b}{\partial x} = \frac{e}{m_e}\nabla\varphi, \tag{4}$$

$$\frac{\partial n_b}{\partial t} + \mathrm{div}(n_b v_b) = 0. \tag{5}$$

Here $\varphi$ is the electrostatic potential of the Langmuir waves, $n_b$ is the beam electron density and $e$ is an absolute value of electron charge. The first of these equations describes wave excitation, the second is the beam electron motion equation and the third is the beam electron continuity equation.

Note that on the boundary $x = 0$, i.e. at the place of beam injection, we have $n_b =$ constant over the entire period when the beam is turned on, i.e. here any excitation of Langmuir oscillations takes place only at the moment of beam turning on and off (see (3)). Therefore, in the hydrodynamic beam–plasma interaction the most interesting processes takes place at some distance from the beam entry. At the location of the corresponding field structure, a constant energy exchange between transiting beam electrons and plasma waves takes place, with periodic changes in the direction of energy transfer. The corresponding changes in the field structure resemble wave collapse.

It is noteworthy that the distance of 23 cm correlates well with the half-length of the beam longitudinal flight for a period of electron bounce oscillation within the potential trough of the accompanying wave. The corresponding evaluation of the electron flight length can be performed in the following way. Looking at Fig. 3 of

Wong and Cheung (1984), one finds that the typical amplitude of electron bounce velocity oscillations around a mean value of $1.55 \times 10^9\ \mathrm{cm\,s^{-1}}$ is $10^8\ \mathrm{cm\,s^{-1}}$. (Note that this value corresponds well to ideas of linear two-stream instability where the phase velocity of a beam-excited Langmuir wave $v_{\mathrm{ph}}$ is less than the beam velocity by a value of the order of $v_b(n_b/n_0)^{1/3}$.) The amplitude $10^8\ \mathrm{cm\,s^{-1}}$ of electron bounce velocity oscillations defines the energy of these oscillations and hence the corresponding amplitude of the electric field in mode $\tilde{E}$, namely $\tilde{E} \sim \sqrt{8\pi n_b m_e (\Delta v_b)^2/2}$. The longitudinal wavelength in this mode is the resonant one $2\pi v_b/\omega_{pe}$. In view of this, the upper bound for the bounce frequency of the beam electron in this wave in a frame that moves with the wave is $\sqrt{e/m_e \tilde{E}\omega_{pe}/(2\pi v_b)}$. Dividing the beam velocity by this frequency and multiplying by $2\pi$, we get 51 cm, and half of this length is 25.5 cm, i.e. only 10% larger than 23 cm.

The above ideas agree with many of results by Wong and colleagues. They explain the good periodicity of the ‘collapses’, the absence of short free Langmuir waves after the ‘collapse’ and the periodicity of the changes in the distribution of beam electrons. They also explain the reason for the good reproducibility of the experiments. In such a way, we advanced an idea of physics that corresponds to the data observed in the given experiments to a greater degree than the collapse physics.

An extended study of beam–plasma interaction within the suggested physical picture is a problem worthy of an independent paper. For our purposes it is sufficient to understand that the very first and the most commonly recognized reports on observations of ‘true Zakharov Langmuir wave collapse’ are no more than a simple misinterpretations of the experiments.

In such a way, two important series of ‘strong plasma turbulence’ experiments do not exhibit Langmuir wave collapses. It is worth noting that these laboratory experiments were well equipped with various plasma diagnostics, thus the reported data, and hence our conclusions too, are extremely reliable. Later on we will show that the nonoccurrence of Langmuir wave collapse in the given experiments exemplifies in a most instructive way the *impossibility* of the corresponding physical phenomenon in real turbulent plasmas.

As a matter of fact, the collapse of Langmuir waves is a mere theoretical abstraction that cannot take place in nature. The idea of this phenomenon was advanced within plasma hydrodynamics that oversimplifies plasma processes and gives an inadequate picture of plasma physics. Zakharov assumed the model of Langmuir turbulence with electrons oscillating synchronously at each point of the plasma volume; that is, he disengaged himself from a picture of interpenetrating electron jets in the plasma. With this, no place was left for electron diffusion in momentum. This *kinetic* effect leads to rather quick dissipation of Langmuir waves: they spend their energy on heating of plasma thermal electrons. Note that prior to our study the given phenomenon has not received fair consideration. After Tsytovich, the plasma community assumed that nonresonant electron diffusion in the plasma conserved the total number of ‘Langmuir quanta’ and could not lead to noticeable wave decay (see Tsytovich 1972, 1977). In particular, *this idea was a cornerstone that Zakharov had laid into the basement of his hydrodynamics.* In reality, Tsytovich substantially underestimated the intensity of nonresonant electron diffusion. Let us thoroughly consider the problem of this diffusion.

## 3. Nonresonant electron diffusion in traditional plasma kinetics

Genetically, Tsytovich's conclusions regarding the physics of nonresonant electron diffusion were based on a rigorous perturbation expansion, and his approaches corresponded to the commonly accepted paradigm of plasma kinetics. The absence of collapses in the above analysed experiments indicates problems with the base of this paradigm. The main false cornerstone in traditional plasma kinetics is the method of the plasma probabilistic ensemble. The reader is reminded that originally the method of the probabilistic ensemble was introduced into *equilibrium thermodynamics* by Gibbs merely to simplify analytic studies (see Gibbs 1902). In corresponding considerations the content of the ensemble is dictated by the problem that the researcher considers, and ideas underlying one or other choice of the ensemble (either the microcanonical or the big canonical ensemble) are rather meaningful. In contrast, in studies of evolving nonequilibrium systems one cannot advance reasons for the preference of one or other system ensemble. Varying the content of the ensemble, one develops diverse pictures for the macroscopic evolution of the ensemble statistics and therefore arrives at plenty of laws for the system evolution. Generally, they contradict each other. In particular, some recognized results of weak plasma turbulence theory comprise a consideration that gives a substantial increase in the intensity of nonresonant electron diffusion in momentum compared with Tsytovich's rigorous calculations. Let us visualize the corresponding consideration.

Nonresonant electron diffusion was discussed for the first time by Bass et al. (1965). In particular, Bass et al. considered the case of a Langmuir wave with time decay of the correlation function $\Phi(t) = \langle E(t+T)E^*(T)\rangle$ following $\exp(-i\omega_{\mathbf{k}}t-|t|/\tau)$. The frequency spectrum of this correlation function contains a Lorentzian structure with frequency width $1/\tau$,

$$\Phi_\omega = \frac{1}{\pi}\frac{\tau}{1+(\omega-\omega_{\mathbf{k}})^2\tau^2} \sim \frac{1/\tau}{(\omega-\omega_{\mathbf{k}})^2+(1/\tau)^2}. \tag{6}$$

The corresponding rate of the collisionless wave decay is $\nu = \omega_{pe}^2\tau/(1+\omega_{\mathbf{k}}^2\tau^2)$ (see the last but one formula in Bass et al. (1965)), which in the typical case of $\tau \gg \omega_{\mathbf{k}}^{-1} \approx \omega_{pe}^{-1}$ becomes $\nu = 1/\tau$. Note that Bass et al. have not advanced any arguments in favour of the Lorentzian profile of a wave line. (Tsytovich's related calculations indicated a deformation of this profile, with the effect of substantially suppressing the diffusion (Tsytovich 1972, 1977).)

A conclusion concerning the correspondence of the genuine wave line shape to the Lorentzian profile and relevant estimates for the time $\tau$ can be inferred from papers by Malkin (1982a, 1982b, 1982c, 1984). He adopted the Wyld diagram technique (Wyld 1961; Zakharov and L'vov 1975) to study Langmuir turbulence. As a starting point for his considerations he used Zakharov's plasma hydrodynamics (Zakharov 1972). The main distinctive feature of Malkin's calculations is that he uses predominantly a two-time representation for the wave correlation function $\Phi_{\mathbf{k}}(t,t')$. This function obeys an evolution equation. In the case of weak turbulence the corresponding equation can be solved by iteration with successive direct integration over time $t$ of the equation to obtain sequential approximations of $\Phi_{\mathbf{k}}(t,t')$. It is noteworthy that the leading order of the two-time correlation function $\Phi_{\mathbf{k}}(t,t')$ just corresponds to the assumption $\Phi_{\mathbf{k}}(t,t') \sim \exp(-i\omega_{\mathbf{k}}t-|t|/\tau)$, and hence really possesses a true Lorentzian frequency profile. This can easily be checked for the case of stationary weak Langmuir turbulence. For *evolving* Langmuir turbulence, the structure of the two-time correlation function is explicitly described in Malkin (1982b) by equations (7) and (10), and it is the following:

$$\Phi_{\mathbf{k}}(t,t') = \begin{cases} n_{\mathbf{k}}(t')\exp\left[-i\int_{t'}^{t}\omega_{\mathbf{k}}(t_1)\,dt_1\right], & t>t', \\ n_{\mathbf{k}}(t)\exp\left[i\int_{t}^{t'}\omega^*_{\mathbf{k}}(t_1)\,dt_1\right], & t<t'. \end{cases} \tag{7}$$

Here $\omega_{\mathbf{k}}(t) \equiv \mathrm{Re}\,\omega_{\mathbf{k}}(t) - i\gamma^{\mathrm{nl}}_{\mathbf{k}}(t)$ is the complex renormalized natural wave frequency. $\gamma^{\mathrm{nl}}_{\mathbf{k}}$ is the sign-inverted imaginary part of the renormalized wave natural frequency. It measures the intensity of nonlinear interactions in the plasma. Following the term advanced in Wyld's diagram technique (Wyld 1961; Zakharov and L'vov 1975) we call this feature the *nonlinear wave damping rate*.

In the case of stationary turbulence neither the wave spectral density $n_{\mathbf{k}}$ nor the wave natural frequency $\omega_{\mathbf{k}}$ depend on time, and expression (7) becomes

$$\Phi_{\mathbf{k}}(t,t') = n_{\mathbf{k}}\exp(-i\mathrm{Re}\,\omega_{\mathbf{k}}(t-t') - \gamma^{\mathrm{nl}}_{\mathbf{k}}|t-t'|). \tag{8}$$

This form coincides with one that was considered by Bass et al. Correspondingly, $\gamma^{\mathrm{nl}}_{\mathbf{k}}$ denotes the inverse correlation time $\tau^{-1}$. To put it another way, the rate of collisionless wave dissipation due to nonresonant electron diffusion $\nu$ is equal quantitatively to $\gamma^{\mathrm{nl}}_{\mathbf{k}}$. Depending on the turbulence energy density, it may be rather high.

In such a way, the modern theory of weak Langmuir turbulence comprises at least two different opinions regarding the intensity of nonresonant electron diffusion. It is noteworthy that both lines of the process study (following Tsytovich and following Malkin's ideas) were based on the ensemble method. Tsytovich developed some relations on frequency harmonics of the wave field and then averaged them over the plasma ensemble. In Malkin's considerations the plasma ensemble entered once through Zakharov's hydrodynamic approximation to the plasma description† and a second time through averaging over an infinitesimal external random force that he used for developing the diagrammatic technique.

The difference in the two shapes for the wave frequency spectrum means that corresponding calculations implied essentially different properties for the plasma ensemble. This confirms our thesis that by varying the content of the plasma probabilistic ensemble one may advance diverse opinions regarding some physical phenomena. In the next section we reinforce this statement by the demonstration of the suppression of short-scale plasma modulational instability by collisionless wave dissipation following Malkin's idea of wave line shape. (In the tradition of plasma turbulence theory this instability takes place in a strongly turbulent plasma just due to the inefficiency of Tsytovich's nonresonant electron diffusion.)

The controversy between the two opinions regarding the physics of nonresonant electron diffusion illustrates rather well the *uselessness* of the ensemble method for plasma kinetic studies. Let us expand this statement to some extent.

Studying the evolution of some plasma ensemble, one can never be sure that

† In the hydrodynamic plasma description, the plasma ensemble is usually not directly referred to but just implied, since here any real plasma cannot be distinguished from many others with slightly different positions and momenta of individual plasma particles and hence from many plasma ensembles.

the evolution of this ensemble repeats the macroscopic evolution of a given single plasma. Naturally, for each single plasma one can compose plenty of ensembles that follow the macroscopic behaviour of a given plasma in terms of the dynamics of their statistics, but many others demonstrate quite different evolution of their statistics. To have the possibility of selecting a 'proper' ensemble, one should perform a preliminary study of the macroscopic evolution of the single plasma itself, because otherwise one cannot judge whether or not a certain ensemble adequately models the behaviour of the plasma. From the viewpoint of common sense, *after* learning the physics of a single plasma the search for the 'proper ensemble' becomes a senseless enterprise. (The reader is reminded again that substitution of a physical system by a system ensemble was proposed by Gibbs exclusively for the technical simplification of the physical system study.)

In view of the last idea, the true physics of plasma evolution cannot be learned without a direct study of a single evolving plasma. A kinetic approach fit for such a study was created by the author and thoroughly reported in Erofeev (1997, 1998).† Using it, we consider in Sec. 5 the real physics of nonresonant electron diffusion.

## 4. Suppression of Zakharov's short-scale plasma modulational instability

One of the most challenging problems of weak plasma turbulence theory was the problem of the Langmuir condensate. This structure was believed to form in the long-wavelength part of the turbulence spectrum as a result of wave scattering on plasma particles and the inefficiency of the energy dissipation. There was a question of finding a mechanism for the dissipation of its energy within the theory. The currently recognized solution to this question was proposed in Zakharov (1972). In the corresponding hydrodynamic plasma description, the Langmuir condensate is unstable with respect to excitation of low-frequency short-wavelength perturbations in plasma density. In view of this, Zakharov assumed that regions of high field intensity and low ion density form constantly in the plasma with intense Langmuir turbulence, and that they have a tendency to collapse. These regions were called 'cavitons', and they were assumed to collapse down to sizes of a few Debye lengths where Landau damping becomes effective in transferring the energy of Langmuir oscillations to epithermal plasma electrons.

In this section we intend to show that collisionless wave dissipation suppresses Zakharov's short-wavelength plasma modulational instability and hence precludes Langmuir wave collapse. This will be organized as follows. In Sec. 4.1 we estimate the rate of wave collisionless decay due to nonresonant electron diffusion. In Sec. 4.2 we present Zakharov's data for the short-scale instability growth rate. In Sec. 4.3 we show that the corresponding growth rates are always small compared with the collisionless wave damping rate.

† The reader should not be confused by the conclusions advanced in Erofeev (1997). At the last stage of the reported analytical calculations the author missed a mistake. As a result, he deduced a substantial increase in intensity of the three-wave process in a plasma. The given mistake was corrected in Erofeev (1998): the corrected data for the process intensity coincides basically with those of traditional calculations given in Kadomtsev (1965), Davidson (1972) and Rogister and Oberman (1969).

https://doi.org/10.1017/S002237780200185X Published online by Cambridge University Press

### 4.1. Estimates of wave dissipation due to nonresonant electron diffusion

Following Malkin's idea of wave line shape, the rate of turbulence dissipation due to nonresonant electron diffusion $\nu$ is equal quantitatively to some mean $\gamma^{\rm nl}$ of the nonlinear wave damping rate $\gamma^{\rm nl}_{\mathbf{k}}$. Papers by Malkin contain all the necessary estimates for $\gamma^{\rm nl}$. In Malkin (1982a), a kinetic equation is derived that governs weak Langmuir turbulence, taking account for nonlinear phenomena up to four-wave interactions. Here the author has shown that the structure of the four-wave collision integral differs substantially from that usually assumed on the basis of a quantum-mechanical analogy (see, e.g., Kovrizhnykh 1965a, 1965b). Namely, he has shown that part of the four-wave interaction (wave scattering by forced density inhomogeneities) may be far more intense than wave scattering induced by plasma ions. (Note that prior to Malkin's work the common belief was the opposite.) Recall Malkin's estimates for characteristic inverse times of the corresponding nonlinear processes.

First, one should distinguish cases with $\Delta k \gg k_* = 1/3\sqrt{m/M}r_{\rm D}^{-1}$, where $\Delta k$ is the typical width of the turbulence spectrum in wavevectors and with $\Delta k \lesssim k_*$. In the former Langmuir waves can irradiate ion sound, whereas in the latter they cannot. The estimate developed in Malkin (1982a) corresponds to the former case. The inverse time of wave scattering induced by forced density inhomogeneities is

$$\gamma_{\rm four} \sim \omega_{pe}\left(\frac{W}{nT}\right)^2 \frac{\omega_{pe}(\Delta k c_s)}{(\Delta\omega)^2}\frac{\Delta k c_s}{\gamma_s}. \tag{9}$$

In this estimate, $(\Delta k c_s)$ represents the characteristic natural frequency of the ion sound, with $\gamma_s$ being the corresponding damping rate of the ion sound and $\Delta\omega$ is the width of Langmuir turbulence in natural frequencies.

The inverse time of wave scattering induced by the plasma ions is

$$\gamma_{\rm ion} \sim \omega_{pe}\frac{W}{nT}\left(\frac{\Delta k c_s}{\Delta\omega}\right)^2. \tag{10}$$

In the case where $\Delta k \lesssim k_*$ the corresponding inverse times become

$$\gamma_{\rm four} \sim \omega_{pe}\left(\frac{W}{nT}\right)^2 \frac{1}{(\Delta k r_{\rm D})^2}, \tag{11}$$

(in a written form the reader may find this estimate in Malkin (1984), formula (9)) and

$$\gamma_{\rm ion} \sim \omega_{pe}\frac{W}{nT}, \tag{12}$$

(this estimate can also be found in Tsytovich's papers; for instance, in Tsytovich (1970), formula (8.27)).

The given inverse times can be accepted as ready estimates of contributions to the nonlinear wave damping rate that are due to corresponding nonlinear phenomena. Note that it was just a time decay of the two-time correlation function $\Phi_{\mathbf{k}}(t,t')$ with an increase of $|t-t'|$ that Malkin used to obtain the corresponding estimates (a track along this line can be recognized in above formulae (7) and (10) of Malkin (1982b)).

### 4.2. Dispersion equation for Zakharov's modulational instability

Bearing in mind the problem of the Langmuir condensate (a long-wavelength part of the Langmuir wave spectrum), Zakharov approximated the condensate by a single monochromatic Langmuir wave with $\mathbf{k} = 0$. Therefore he considered the hydrodynamic stability of a plasma with an intense plane monochromatic wave, generally having $\mathbf{k} \neq 0$. Recall his calculations. The complex amplitude $\psi$ of the electrostatic potential $\phi = \frac{1}{2}[\psi \exp(-i\omega_{pe}t) + \psi^* \exp(i\omega_{pe}t)]$ satisfies the following equation in Zakharov's plasma hydrodynamics:

$$\Delta \left( i \frac{\partial \psi}{\partial t} + \tfrac{3}{2}\omega_{pe} r_{\mathrm{D}}^2 \Delta\psi \right) = \frac{\omega_{pe}}{2n_0} \left[ \boldsymbol{\nabla} \cdot (\delta n \boldsymbol{\nabla}\psi) \right]. \tag{13}$$

This equation was complemented by the relation between the low-frequency perturbations in plasma density $\delta n$ and in the turbulence parameters. The motive force for low-frequency perturbations is the ponderomotive force that acts on plasma electrons (Miller force): the electrons are pushed out of regions with intense Langmuir oscillations. The electrons drag ions, and a density perturbation arises. This effect can be described in terms of a high-frequency potential $U = e^2|\psi|^2/(4m\omega_{pe}^2)$ and the Green function for density perturbations $\hat{G}$. In a $\mathbf{k}$–$\omega$ representation

$$\delta n_{\mathbf{k}\omega} = G_{\mathbf{k}\omega} U_{\mathbf{k}\omega}. \tag{14}$$

The Green function $G_{\mathbf{k}\omega}$ is a functional of the electron distribution in momentum:

$$G_{\mathbf{k}\omega} = \frac{L_{\mathbf{k}\omega}}{M - 4\pi e^2 r_{\mathrm{D}}^2 L_{\mathbf{k}\omega}},$$

$$L_{\mathbf{k}\omega} = \int \frac{[\mathbf{k} \cdot (\partial f/\partial \mathbf{v})]}{\mathbf{kv} - \omega}\, d^3\mathbf{v}.$$

(Here $M$ is the ion mass.)

It is accepted practice to distinguish two different cases: that of a static approximation to describe ion motions ($\tau^{-1} \ll kv_{Ti}$, where $v_{Ti}$ is the ion thermal velocity and $\tau$ is the characteristic time of a nonlinear process) and that of a hydrodynamic approximation ($\tau^{-1} \gg kv_{Ti}$). In the static case

$$G_{\mathbf{k}\omega} = -n_0/(T_i + T_e), \tag{15}$$

and in the hydrodynamic case

$$G_{\mathbf{k}\omega} = \frac{n_0}{M} \frac{k^2}{\omega^2 + 2i\gamma_s(\mathbf{k})\omega - c_s^2 k^2}. \tag{16}$$

In the latter formula $\gamma_s(\mathbf{k})$ is the damping rate of the ion sound.

For the monochromatic wave with given $\mathbf{k}$, the unperturbed potential $\psi$ is

$$\psi = \frac{E}{k} \exp[-i\omega_k t + i(\mathbf{k} \cdot \mathbf{r})], \tag{17}$$

where

$$\omega_k = \tfrac{3}{2}\omega_{pe} k^2 r_{\mathrm{D}}^2.$$

The potential (17) is a solution to (13) for $\delta n = 0$.

Linearizing equations (13) and (14) with the background of the above solution, Zakharov took perturbations of $\delta n$ and $\psi$ in the form

$$\delta n \sim \exp[-i\Omega t + i(\boldsymbol{\kappa} \cdot \mathbf{r})],$$

$$\delta\psi \sim \exp\{-i\Omega t - i\omega_k t + i[(\mathbf{k} + \boldsymbol{\kappa}) \cdot \mathbf{r}]\},$$
$$\delta\psi^* \sim \exp\{-i\Omega t + i\omega_k t + i[(\boldsymbol{\kappa} - \mathbf{k}) \cdot \mathbf{r}]\}.$$

https://doi.org/10.1017/S002237780200185X Published online by Cambridge University Press

Conceptually, this means that he considered a coordinated evolution of the plasma density perturbation and two side bands of the original wave.

The easy algebra that Zakharov omitted yields for $\Omega$ the following dispersion equation:

$$1+\frac{\omega_{pe}}{16}\frac{W}{n_0^2}G_{\kappa\Omega}\left\{\frac{[\mathbf{k}\cdot(\mathbf{k}+\boldsymbol{\kappa})]^2}{k^2(\mathbf{k}+\boldsymbol{\kappa})^2}\frac{1}{-\Omega+\omega_{|\mathbf{k}+\boldsymbol{\kappa}|}-\omega_k}\right.$$
$$\left.+\frac{[\mathbf{k}\cdot(\mathbf{k}-\boldsymbol{\kappa})]^2}{k^2(\mathbf{k}-\boldsymbol{\kappa})^2}\frac{1}{\Omega+\omega_{|\mathbf{k}-\boldsymbol{\kappa}|}-\omega_k}\right\}=0. \quad (18)$$

Here $W=|E|^2/(2\pi)$ is the wave (condensate) energy density.

Setting $\mathbf{k}=0$, one returns to the study of 'condensate' stability. Then the dispersion equation (18) becomes

$$\Omega^2-\omega_\kappa^2=\tfrac{1}{8}\omega_{pe}Wn_0^{-2}\omega_\kappa G_{\boldsymbol{\kappa}\Omega}\cos^2(\alpha). \quad (19)$$

In this equation $\alpha$ is an angle between the direction of $\mathbf{E}$ in the original oscillation and $\boldsymbol{\kappa}$.

The dispersion equations (18) and (19) were developed with the Langmuir condensate approximated by a plane wave. In reality, the Langmuir condensate is not a plane wave, and only on spatial dimensions that are small compared with the inverse width of the condensate in wavenumbers $(\Delta k)^{-1}$, can it be approximated by a plane wave. It should be stressed that at intermediate stages of the derivation of (18), the coherence of the background Langmuir wave was essentially used, although the final equations do not contain any hint as to this coherence. Therefore, dispersion equations (18) and (19) are only valid for wavevectors $\boldsymbol{\kappa}$ with absolute values that are large compared with $\Delta k$: perturbations of this type only 'feel' the Langmuir condensate as a plane wave. Had one tried to extend the above considerations to smaller values of $\kappa$, one would have to take into account that a large number of plane waves contribute to the wave field at typical perturbation lengths, and interference from perturbations in $\delta\psi$ and $\delta\psi^*$ owing to different waves would have smoothed out the picture of the instability.

Thus, it is stated that the dispersion equations (18) and (19) are only applicable for perturbations with $\kappa \gg \Delta k$. Note that the opposite case of perturbations with $\kappa \ll \Delta k$ corresponds conceptually to the area of Vedenov–Rudakov analysis (Vedenov and Rudakov 1964). Note that our collisionless wave dissipation is sufficient for suppression of the Vedenov–Rudakov modulational instability in all area where $\kappa \ll \Delta k$ (given without proof in this paper). Below we show the same for Zakharov's short-wavelength modulational instability, in all domain $\kappa \gg \Delta k$. For brevity, we call the corresponding instability a condensate instability.

We present formulae for the growth rates of the condensate instability $\Gamma$. In the static case

$$\Gamma=\sqrt{\tfrac{1}{8}\omega_{\boldsymbol{\kappa}}\omega_{pe}\cos^2(\alpha)\frac{W}{n_0(T_e+T_i)}-\omega_{\boldsymbol{\kappa}}^2}, \quad (20)$$

and in the hydrodynamic case

$$\Gamma=\sqrt{\sqrt{\left(\frac{\omega_{\boldsymbol{\kappa}}^2+c_s^2\kappa^2}{2}\right)^2+\tfrac{1}{8}\omega_{\boldsymbol{\kappa}}\omega_{pe}\cos^2(\alpha)c_s^2\kappa^2\frac{W}{nT_e}-\omega_{\boldsymbol{\kappa}}^2c_s^2\kappa^2}-\frac{\omega_{\boldsymbol{\kappa}}^2+c_s^2\kappa^2}{2}}. \quad (21)$$

https://doi.org/10.1017/S002237780200185X Published online by Cambridge University Press

### 4.3. *Plasma stability*

In this subsection we compare growth rates (20) and (21) with real collisionless wave dissipation, to check the plasma stability. The case of the hydrodynamic approximation to the description of ion motions is considered in Sec. 4.3.1, and the case of the static approximation in Sec. 4.3.2.

*4.3.1. Plasma stability in a hydrodynamic approximation.* For the condensate instability to be possible, the growth rate $\Gamma$ following (21) should exceed the characteristic rate of wave energetic decay $\nu$ (a mean of the nonlinear wave damping rate). Therefore we can write $\Gamma^2 > \nu^2$, i.e.

$$\sqrt{\left(\frac{\omega_{\boldsymbol{\kappa}}^2 + c_s^2\kappa^2}{2}\right)^2 + \tfrac{1}{8}\omega_{\boldsymbol{\kappa}}\omega_{pe}\cos^2(\alpha)c_s^2\kappa^2\frac{W}{nT_e} - \omega_{\boldsymbol{\kappa}}^2 c_s^2\kappa^2} - \frac{\omega_{\boldsymbol{\kappa}}^2 + c_s^2\kappa^2}{2} > \nu^2. \tag{22}$$

Transfer $(\omega_{\boldsymbol{\kappa}}^2 + c_s^2\kappa^2)/2$ to the right and then square; omitting equal terms on both sides, we obtain

$$\tfrac{1}{8}\omega_{\boldsymbol{\kappa}}\omega_{pe}\cos^2(\alpha)c_s^2\kappa^2\frac{W}{nT_e} - \omega_{\boldsymbol{\kappa}}^2 c_s^2\kappa^2 > \nu^2(\omega_{\boldsymbol{\kappa}}^2 + c_s^2\kappa^2) + \nu^4. \tag{23}$$

Let us soften the given requirement: skip the $\nu^4$ term on the right-hand side, and also the $\nu^2\omega_{\boldsymbol{\kappa}}^2$ term. Then, after simple manipulations, the *necessary* condition for instability takes the form

$$\frac{W}{nT} > 12(\kappa r_{\mathrm{D}})^2 + \frac{16}{3}\frac{\nu^2}{\omega_{pe}^2}\frac{1}{(\kappa r_{\mathrm{D}})^2}. \tag{24}$$

In this inequality, the right-hand side as a function of $(\kappa r_{\mathrm{D}})^2$ takes its minimum value at $(\kappa r_{\mathrm{D}})^2 = \frac{2}{3}(\nu/\omega_{pe})$, and the corresponding minimum value is $16\nu/\omega_{pe}$. Let us check whether $W/nT$ can exceed this minimum or not. Take $\nu \gtrsim \gamma_{\text{four}}$. Then in the case, $\Delta k \gg k_*$, the requirement $W/nT > 16\nu/\omega_{pe}$ reduces to

$$\frac{W}{nT} < \frac{9}{64\sqrt{3}}(\Delta k r_{\mathrm{D}})^3,$$

(see estimate (9)). But this condition contradicts the requirement that $W$ should be large compared with $12nT(\Delta k r_{\mathrm{D}})^2$ (see once again condition (24) where the lower bound $\Delta k$ was set for wavenumber $\kappa$).

In the case $\Delta k \lesssim k_*$ the requirement $W > 16nT\nu/\omega_{pe}$ becomes

$$\frac{W}{nT} < \frac{(\Delta k r_{\mathrm{D}})^2}{16},$$

(see estimate (11)), which again contradicts the requirement $W > 12nT(\Delta k r_{\mathrm{D}})^2$.

*4.3.2. Plasma stability in a static approximation.* According to (20), with respect to excitation of perturbations with wavevector $\boldsymbol{\kappa}$ the plasma is unstable at $W/nT > 12(\kappa r_{\mathrm{D}})^2$. Recall that our analysis is valid for $\kappa \gg \Delta k$ only. Taking $\kappa \simeq \Delta k$, one obtains a *necessary* condition for the condensate instability

$$W > 12nT(\Delta k r_{\mathrm{D}})^2/q. \tag{25}$$

Here $q = T_e/(T_e + T_i)$, $T \equiv T_e$.

https://doi.org/10.1017/S002237780200185X Published online by Cambridge University Press

As a function of $(\kappa r_{\mathrm{D}})^2$, the 'instability growth rate' (20) takes a maximum value at $\cos^2(\alpha) = 1$, $(\kappa r_{\mathrm{D}})^2 = \frac{1}{24} W/nT$. It is

$$\Gamma_{\max} = \omega_{pe} \frac{q}{16} \frac{W}{nT}. \tag{26}$$

Let us check whether it can exceed the wave damping rate $\nu$ or not. The easiest consideration is in the case of the 'condensate' with $\Delta k \lesssim k_*$. For this case the lower bound for dissipation $\nu$ is given by the estimate (11). Correspondingly, the growth rate (26) is greater than $\nu$ only when the wave energy density does not exceed the top bound $nT(\Delta k r_{\mathrm{D}})^2 q/16$. But this requirement contradicts condition (25).

In the case $\Delta k \gtrsim k_*$, the value $\nu$ corresponds to estimate (9), with $\gamma_s \simeq \Delta k c_s$. (The static approximation to the description of ion motion makes sense mainly in isothermal plasmas, and there the 'damping rate of ion sound' is of the order of its 'natural frequency'. This gives a lower bound for the rate of wave scattering on forced density inhomogeneities that equally well could be applied in the case of 'supersonic' ion motions with $\partial/\partial t \gg (\Delta k)c_s$.) Correspondingly, the growth rate (26) is greater than $\nu$ when

$$\frac{W}{nT} < \frac{3q}{64} \frac{\Delta k}{k_*} (\Delta k r_{\mathrm{D}})^2,$$

where $q$ is of the order of $\frac{1}{2}$. This condition is compatible with condition (25) only when $\Delta k \gg 256 k_*$, which is unrealistic. For example, for a fully ionized plasma of $\mathrm{He}^4$ the requirement $\Delta k \gg 256 k_*$ means $\Delta k \gg 1/2 r_{\mathrm{D}}^{-1}$, therefore even the linear Landau damping of the corresponding spectrum is intense enough to suppress the instability. For a hydrogen plasma the situation is even more dramatic.

The controversy between Tsytovich's and Malkin's ideas of wave line shapes and the corresponding conclusions concerning the possibility/impossibility of short-length plasma modulational instability substantiates the basic idea postulated in the introduction: with substitution of a real plasma by a plasma ensemble one may unintentionally smear out the bulk of some physical phenomenon under consideration. Since the ensemble implied in Malkin's theory leads to a lighter suppression of the process intensity than Tsytovich's ensemble, new estimates of the collisionless wave dissipation provide us with a better picture of the wave energy dissipation. For this reason, the conclusion of the impossibility of short-wavelength plasma modulational instability is more reliable than the traditional deduction of its occurrence in strongly turbulent plasmas.

At the same time, note that Malkin's perturbations were based on a hydrodynamic description of electron motion that assumed the absence of collisionless wave dissipation; i.e. the corresponding theory of Langmuir turbulence is controversial. Assuming, following Malkin and Zakharov after Tsytovich's conclusions, that nonresonant electron diffusion is not essential and therefore the hydrodynamic plasma description is applicable, we arrived at rather large losses for the wave energy just on heating of plasma electrons, because of their nonresonant diffusion. This means that the idea of using electron hydrodynamics for a description of turbulence kinetics is erroneous. Correspondingly, the conclusions developed on the basis of Malkin's estimates should be substantiated independently using a kinetic electron description. This emphasizes once again the importance of the forthcoming section.

## 5. Nonresonant electron diffusion in a single collisionless plasma

Physically, the classical ionized plasma is a mixture of individual charged particles. The full study of its evolution implies the simultaneous integration of motion equations for all individual plasma particles. On a kinetic level of description this corresponds to considering Klimontovich–Duprees distributions of particles in six-dimensional phase space of spatial positions–momenta $N_{e,i}(\mathbf{r},\mathbf{p},t)$ (Dupree 1963; Klimontovich 1967),

$$N_{e,i}(\mathbf{r},\mathbf{p},t) = \sum_n \delta^3(\mathbf{r}-\mathbf{r}_n(t))\delta^3(\mathbf{p}-\mathbf{p}_n(t)). \tag{27}$$

Here the subscript '$n$' signifies particles of the given species, functions $\mathbf{r}_n(t)$ and $\mathbf{p}_n(t)$ describe trajectories of individual plasma particles. We call this distribution function a *microdistribution*.

The microdistribution $N_\alpha(\mathbf{r},\mathbf{p},t)$ evolves according to the following partial differential equation:

$$\frac{\partial N_\alpha}{\partial t} + (\mathbf{v}\cdot\nabla)N_\alpha + e_\alpha\left[\left(\mathbf{E}' + \frac{1}{c}[\mathbf{v}\times\mathbf{B}']\right)\cdot\frac{\partial N_\alpha}{\partial \mathbf{p}}\right] = 0. \tag{28}$$

The trajectories of the plasma particles are characteristics of this partial differential equation; $\mathbf{E}'$ and $\mathbf{B}'$ are the electric and magnetic fields acting on a particle. One can substitute the total electric and magnetic fields for these fields, since under any consistent assumptions of particle structure the contribution of a particle to the total field cannot influence the particle motion (except for the effect of the radiation reaction).

Note that conceptually (28) is a continuity equation for particles of the given species.

The total electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields are advanced in time by microdistributions $N_\alpha(\mathbf{r},\mathbf{p},t)$. Evolution equations of these fields are two of Maxwell's equations where the current and charge densities are associated with corresponding integrals of the microdistributions. In a tensor description of the electromagnetic field, $F_{ij} = \partial A_i(\mathbf{r},t)/\partial r^j - \partial A_j(\mathbf{r},t)/\partial r^i$ (here $A_j = (-\varphi(\mathbf{r},t), \mathbf{A}(\mathbf{r},t))$ are the contravariant components of the four-dimensional vector potential), these Maxwell equations are

$$\frac{1}{c}\frac{\partial}{\partial t}F_{\beta\gamma} = -\frac{\partial F_{\gamma 0}}{\partial r^\beta} + \frac{\partial F_{\beta 0}}{\partial r^\gamma}, \tag{29}$$

$$\frac{1}{c}\frac{\partial}{\partial t}F^{\beta 0} = -\frac{4\pi}{c}j^\beta - \frac{\partial}{\partial r^\gamma}F^{\beta\gamma}. \tag{30}$$

The first of these equations governs the evolution of the magnetic field and the second governs the electric field.

Because of the large number of individual particles, full integration of the Klimontovich–Dupree–Maxwell equations is technically infeasible. That is, the constructive description of plasma macroscopic evolution cannot be based on the microdistribution. Some other function should be invented for objective characterization of the plasma macroscopic state and its evolution. We suggest taking a mean density of plasma particles within large volumes of phase space $(\mathbf{r},\mathbf{p})$ for such a function. To put it in another way, we average the microdistribution in phase space. The mode of averaging should depend on the problem under study. For instance, Erofeev (1997, 1998) were motivated by a desire to describe the effect of drift turbulence on plasma diffusion across a leading magnetic field in a slab plasma geometry. Within this problem it is most natural to average along level surfaces of the plasma density, i.e. along planes normal to the density gradient. In a study of nonresonant electron diffusion we are interested in the mean rate of electron heating in a unit plasma volume. In view of this, it is convenient to average the microdistribution over the plasma volume. Other problems may dictate another mode of phase space averaging. Only one position is important here: to obtain a consistent statistic via the averaging, a statistic that objectively describes the distribution of plasma particles in phase space while accounting for important physical details of the problem under study. Below we disengage ourselves temporarily from the particular problem at hand and discuss some general principles of the corresponding phase space averaging and the subsequent construction of a plasma kinetic description. In this consideration we assume that the problem is homogeneous along at least one spatial axis.

Take any point $(\mathbf{r}, \mathbf{p})$ of phase space and surround it by a six-dimensional parallelepiped, with centre at $(\mathbf{r}, \mathbf{p})$. Having averaged the microdistribution $N_\alpha$ over the parallelepiped, we choose the average as the current value of the distribution function at a given point $f_\alpha(\mathbf{r}, \mathbf{p}, t)$.

For the most important part of the phase space, where the bulk of plasma particles are located (outside the high energetic tail of the distribution), the corresponding parallelepipeds contain very large numbers of particles provided that the volume of the parallelepipeds is sufficiently large ($V \gg (m_\alpha v_{T\alpha})^3/n$; the parallelepipeds are taken to be uniform). Therefore, if we wander in space from point to point, the variation of $N$, the total number of particles in parallelepipeds corresponding to different points of the plasma volume, is negligibly small compared with $N$. This variation can be neglected, and therefore the function $f_\alpha$ provides a statistically reliable description of the plasma particle distributions in phase space. The evolution of this statistic is strictly specified for each single plasma. With this, the plasma study reduces to developing a good approximation for the corresponding evolution law.

In a homogeneous plasma, we do not need small gradations of the distribution with respect to spatial variables $x$, $y$ and $z$. In the long run, we are interested in developing the mean rate of electron heating in the unit plasma volume. Consequently, the spatial dimensions of a parallelepiped can be taken to be as large as required, up to the corresponding linear dimensions of the plasma volume. At the expense of these large dimensions, the momentum dimensions of the parallelepiped can be chosen to be small compared with the thermal velocity.

At this point some extra comments are in order. Hearing about Langmuir turbulence, the reader with a traditional education may recall the notion of the wave, the plasma natural oscillation, and ask for its place within the theory. Equally, for the place of other structural elements of the turbulence, e.g. solitons (Beresin and Karpman 1964, 1967; Krall 1969), collapsing density cavities with trapped Langmuir waves (Zakharov 1972), etc. Within a traditional approach, these structures are revealed by scrupulous analysis of the dynamics of small-scale spatial gradations in the particle distributions. This implies the availability of the corresponding gradations that are embodied in the concept of a distribution function of Vlasov type, $f_\alpha(\mathbf{r}, \mathbf{p}, t)$. In our theory, such gradations are not available. The minimal scale of spatial gradations in the distribution is the same spatial dimension as the auxiliary six-dimensional parallelepiped, and attempts to lessen it without an undesirable increase in scale of the momentum gradations leads to a loss of reliability of our statistic $f_\alpha$. Moreover, we deliberately choose spatial scales of averaging that are large compared with the characteristic wavelength in the Langmuir turbulence spectrum (the correlation length of the turbulence). With this, our statistic $f_\alpha$ does not demonstrate oscillations with plasma frequency, and all of its time evolution runs on a rather large time scale of nonresonant electron diffusion.

The impossibility of the simultaneous achievement of small gradations in the distribution along all axes of phase space should not be regarded as a disadvantage of our approach. Following common sense, the primary objects of the plasma turbulence are plasma particles, whereas waves, collapsing cavities and other objects are secondary objects that were discerned in studies of particle kinetics. In the traditional paradigm, these structures are regarded as independent physical realities, and much effort has been spent on their study. But from the practical point of view, there is no sense in viewing the interplay of tiny structures in the turbulent plasma. Really, the plasma macrophysics depend only on correlation properties of the turbulent field, i.e. on correlation functions freed from tiny features of the above structures and their complicated interaction. Correspondingly, it is natural to take the correlation function as a basic object of the theory, and in the forthcoming theory it properly accounts for the integral effect of the mentioned tiny structures.

As a matter of fact, our basic problem is to develop an evolution equation for the well-defined statistic $f_\alpha$, the averaged microdistribution. This problem itself dictates strictly the forthcoming logic of the process used to solve it. One should not be anxious from the very beginning about waves and other structures here: their effect is correctly described by the final equations. The reader should understand that our insight into plasma problems dictates the corresponding schemes for their solution, and attempts to implement the ideas of the traditional paradigm into the body of these schemes are void of any sense.

An evolution equation for the distribution $f_\alpha$ (i.e. of the averaged microdistribution) can be developed using the continuity equation (28); the logic of the corresponding calculations is well described in Erofeev (1997). This equation has the form

$$\left[\frac{\partial}{\partial t} + v^\beta \frac{\partial}{\partial r^\beta} + \frac{e_\alpha}{c} v_i {}^0F^{i\beta} \frac{\partial}{\partial p^\beta}\right] f_\alpha(\mathbf{r}, \mathbf{p}, t) = -\frac{e_\alpha}{c} v_i \frac{\partial}{\partial p^\beta} \langle \delta F^{i\beta}(\mathbf{r}, t) N_\alpha(\mathbf{r}, \mathbf{p}, t)\rangle_{\mathbf{r}}. \quad (31)$$

It is seen that the distributions $f_{e,i}$ are advanced in time by some moments of the *two-point* correlation function, $\langle \delta F^{i\beta}(\mathbf{r} + \mathbf{R}, t') N_\alpha(\mathbf{r}, \mathbf{p}, t)\rangle_{\mathbf{r}}$. Similarly, the two-point correlation function is advanced in time by the *three-point* correlation function,

$$\langle \delta F^{i\beta}(\mathbf{r} + \mathbf{R}', t'') \delta F^{j\gamma}(\mathbf{r} + \mathbf{R}, t') N_\alpha(\mathbf{r}, \mathbf{p}, t)\rangle_{\mathbf{r}},$$

with the evolution equation

$$\begin{aligned}\left(\frac{\partial}{\partial t} + v^\beta \frac{\partial}{\partial r^\beta} + \frac{e_\alpha}{c} v_i {}^0F^{i\beta} \frac{\partial}{\partial p^\beta}\right) &\langle \delta F^{i\beta}(\mathbf{r} + \mathbf{R}, t') N_\alpha(\mathbf{r}, \mathbf{p}, t)\rangle \\ &= -\frac{e_\alpha}{c} v_j \left\langle \delta F^{i\beta}(\mathbf{r} + \mathbf{R}, t') \delta F^{j\gamma}(\mathbf{r}, t) \frac{\partial}{\partial p^\gamma} N_\alpha(\mathbf{r}, \mathbf{p}, t)\right\rangle. \quad (32)\end{aligned}$$

The three-point correlation function is advanced in time by a four-point correlation function, etc. The corresponding equation hierarchy can be truncated in the case of weak plasma turbulence theory at any desired order, and then one reduces the problem to a study of the coordinated evolution of the distributions $f_{e,i}(\mathbf{p}, t)$ and

https://doi.org/10.1017/S002237780200185X Published online by Cambridge University Press

the *two-time* correlation function

$$\Phi^{i\beta j\gamma}(\mathbf{r}', t', \mathbf{r}, t) = \langle \delta F^{i\beta}(\mathbf{r}', t')\delta F^{j\gamma}(\mathbf{r}, t)\rangle.$$

(For details of our notation consult Erofeev (1997).) We will not develop the corresponding calculations here as they are also well described in Erofeev (1997). The results can be presented as follows. Evolution of the two-time correlation function is governed by simultaneous equations with the structure

$$\frac{1}{c}\frac{\partial}{\partial t}\langle \delta F_{\beta\gamma}(\mathbf{r}, t)\delta F^{kl}(\mathbf{r}', t')\rangle = -\frac{\partial}{\partial r^{\beta}}\langle \delta F_{\gamma 0}(\mathbf{r}, t)\delta F^{kl}(\mathbf{r}', t')\rangle + \frac{\partial}{\partial r^{\gamma}}\langle \delta F_{\beta 0}(\mathbf{r}, t)\delta F^{kl}(\mathbf{r}', t')\rangle, \tag{33}$$

$$\begin{aligned}\frac{1}{c}\frac{\partial}{\partial t}&\langle \delta F^{\beta 0}(\mathbf{r}, t)\delta F^{kl}(\mathbf{r}', t')\rangle \\ &= -\frac{\partial}{\partial r^{\gamma}}\langle \delta F^{\beta\gamma}(\mathbf{r}, t)\delta F^{kl}(\mathbf{r}', t')\rangle \\ &\quad -\frac{4\pi}{c}\int d^3\mathbf{r}_1 dt_1 \sigma^{\beta m\cdot}_{\cdot\cdot\gamma}(\mathbf{r}, t, \mathbf{r}_1, t_1)\langle \delta F^{\cdot\gamma}_{m\cdot}(\mathbf{r}_1, t_1)\delta F^{kl}(\mathbf{r}', t')\rangle \\ &\quad -\frac{4\pi}{c}\sum_{\alpha} e_{\alpha}\int d^3\mathbf{p}\, v^{\beta}\, \mathscr{P}^{kl}_{\alpha}(\mathbf{r}, \mathbf{p}, t, \mathbf{r}', t').\end{aligned} \tag{34}$$

$\sigma^{\beta m\cdot}_{\cdot\cdot\gamma}(\mathbf{r}, t, \mathbf{r}_1, t_1)$ denotes a conductivity tensor. (Following the causality principle, at $t < t_1$ it is identically zero.) It contains both a linear part (that is independent of the plasma turbulence) and nonlinear corrections. The term $\mathscr{P}^{kl}_{\alpha}(\mathbf{r}, \mathbf{p}, t, \mathbf{r}', t')$ is a nonlinear integral of the two-time correlation function. *The evolution of a two-point correlation function satisfies* (33) *and* (34) *irrespective of the order of consideration*: accounting for higher expansion orders only affects the accuracy of the presentation of nonlinear effects in $\hat{\sigma}$ and $\hat{\mathscr{P}}$.

In the graphical notation advanced in Erofeev (1997) the image of the lowest order of $\hat{\mathscr{P}}$ is given by the bottom line in Fig. 17 of that paper. The graphical analogue of $\hat{\sigma}$ is given by the expression in the top two lines of the same figure, which is connected to the entry of a wavy line (of the two-time correlation function).

Let us comment on the origin of (33) and (34). Conceptually, the two-time correlation function satisfies vacuum Maxwell equations (29) and (30) when the external charge and current densities are associated with the respective integrals of the two-point correlation function. Correspondingly, after expressing the two-point correlation function via two-time correlation functions and substituting equations (29) and (30), one arrives at equations (33) and (34).

In the case of homogeneous weak Langmuir turbulence, equations (33) and (34) can be simplified by a transition to the Fourier transform of a two-time correlation function,

$$\Phi^{ijkl}_{\mathbf{k}}(t, t') = \int \frac{1}{(2\pi)^3} d^3\mathbf{R} \exp(-i\mathbf{k}\cdot\mathbf{R})\left\langle \widetilde{\delta F}^{ij}\left(\mathbf{r} + \tfrac{1}{2}\mathbf{R}, t\right)\widetilde{\delta F}^{kl}\left(\mathbf{r} - \tfrac{1}{2}\mathbf{R}, t'\right)\right\rangle.$$

The field of Langmuir waves is a potential one, therefore it can be described by a scalar function

$$\Phi_{\mathbf{k}}(t,t') = \frac{k_\beta k_\gamma}{k^2}\Phi_{\mathbf{k}}{}^{\beta 0\gamma 0}(t,t').$$

With this, the simultaneous equations (33) and (34) reduce to a single evolution equation:

$$\frac{\partial}{\partial t}\Phi_{\mathbf{k}}(t,t') = -4\pi\int dt_1\,\sigma_{\mathbf{k}}(t,t_1)\Phi_{\mathbf{k}}(t_1,t') - \mathscr{B}_{\mathbf{k}}(t,t'). \tag{35}$$

Using this equation, one can express the two-time correlation function $\Phi_{\mathbf{k}}(t,t')$ in the time domain $t > t'$ in terms of spectral characteristics of the turbulence. (In the domain $t < t'$ the function can be reconstructed using the property of self-adjointness $\Phi_{\mathbf{k}}(t,t') = \Phi^*_{\mathbf{k}}(t',t)$.) The logic of the corresponding calculation can be introduced as follows.

Suppose we are dealing with a steady homogeneous plasma and that the turbulence energy is infinitesimally small. That is, one can neglect by $\mathscr{B}_{\mathbf{k}}(t,t')$ and by nonlinear corrections in $\hat{\sigma}$. Then the only solutions to (35) are natural oscillations. If we replace the function $\Phi_{\mathbf{k}}(t,t')$ by the oscillating function $\Phi_{\mathbf{k}}(t')\exp[-i\omega_{\mathbf{k}}(t-t')]$ then (35) reduces to a dispersion equation.

Furthermore, if we return to a time-dependent collisionless plasma and include nonlinear corrections (i.e. restore $\mathscr{B}_{\mathbf{k}}(t,t')$ and corrections in $\hat{\sigma}$), this will modify the situation slightly. Namely, the key part of the two-time correlation function comprises the natural oscillations, and all the remaining terms are the forced oscillations related to the natural oscillations in some way.

If one accepts the given image of the two-time correlation function, one can conclude that the term $\mathscr{B}_{\mathbf{k}}(t,t')$ as a function of $t-t'$ decays at $t > t'$ for a time roughly equal to the inverse frequency width of the spectrum. Note that this inverse width is small compared with the decay time of the oscillations, and therefore at $t - t' > 0$ the term $\mathscr{B}$ slightly modifies the natural oscillations. This is the case provided that the energy density of the turbulent wave field is sufficiently low (more correctly, we are likely to have the well-known applicability condition of weak plasma turbulence theory).

There are two natural frequencies for a given $\mathbf{k}$ that correspond to Langmuir waves. One is a positive frequency corresponding to a wave travelling along $\mathbf{k}$, and another is a negative one corresponding to a wave travelling in the opposite direction. For this reason, within the domain $t > t'$ the expression for the leading order of the two-time correlation function is of the form

$$\Phi_{\mathbf{k}}(t,t') = \sum_{s=\pm} n_{s\mathbf{k}}(t')\exp(-i\int_{t'}^{t}\omega^s_{\mathbf{k}}(\tau)\,d\tau), \tag{36}$$

where $\omega^s_{\mathbf{k}}(t)$ is a solution to the dispersion equation

$$-i\omega^s_{\mathbf{k}}(t) + 4\pi\int_{-\infty}^{t} dt'\,\sigma_{\mathbf{k}}(t,t')\exp\left(i\int_{t'}^{t}\omega^s_{\mathbf{k}}(\tau)\,d\tau\right) = 0. \tag{37}$$

(Note that natural frequencies $\omega^s_{\mathbf{k}}$ are renormalized here: they account for the effect of nonlinear phenomena in the plasma.)

In (36) the function $n_{s\mathbf{k}}(t')$ satisfies the relation

$$\Phi_{\mathbf{k}}(t',t') = \sum_{s=\pm} n_{s\mathbf{k}}(t'). \tag{38}$$

It is real and positive. This feature corresponds conceptually to the traditional spectral density of plasma waves. In view of this, we assign the term *wave spectral density* to the feature $n_{\mathbf{k}}(t)$.

It is worth noting that the solution (36) is not an *arbitrary* approximation to the two-time correlation function. We advanced it as the most natural lowest-order solution satisfying the evolution equation (35). This solution can be improved by successive iterations consisting of direct integration of (35) over time $t$. In weak Langmuir turbulence accounting for higher corrections leads to unimportant modifications in kinetic equations. (To put it more correctly, accounting for the corresponding corrections leads to higher corrections to time derivatives of distributions $f_{e,i}$ and the wave spectral density $n_{\mathbf{k}}$.)

Finally, in the case of stationary plasma and turbulence neither renormalized natural frequencies $\omega_{\mathbf{k}}^{s} = s\omega_{s\mathbf{k}} - i\gamma_{s\mathbf{k}}^{\mathrm{nl}}$ nor the spectral density $n_{\mathbf{k}}$ depend on time. Therefore we have

$$\Phi_{\mathbf{k}}(t,t') = \sum_{s=\pm} n_{s\mathbf{k}} \exp[-is\omega_{s\mathbf{k}}(t-t') - \gamma_{s\mathbf{k}}^{\mathrm{nl}}|t-t'|]. \tag{39}$$

The frequency spectrum of this correlation function splits into two Lorentzian profiles corresponding to Langmuir waves with wavevectors $\mathbf{k}$ and $-\mathbf{k}$, respectively:

$$\Phi_{\mathbf{k}\omega} \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_{\mathbf{k}}(t,t')\exp[i\omega(t-t')] = \frac{1}{\pi}\sum_{s=\pm}\frac{n_{s\mathbf{k}}\gamma_{s\mathbf{k}}^{\mathrm{nl}}}{(\omega - s\omega_{s\mathbf{k}})^2 + \gamma_{s\mathbf{k}}^{\mathrm{nl}\,2}}. \tag{40}$$

Thus, we have shown that the frequency spectrum of the wave correlation function just corresponds to the assumption made by Bass et al. (see (6)).

Note that introduction of natural oscillations in our above procedure appears only as a method of calculation of the two-time correlation function. By no means do these natural oscillations separate the 'wavy' part of the two-time correlation function from the others. Though, undoubtedly, traditional waves are entirely included within our natural oscillations, and in abstraction of traditional weak turbulence of a 'Vlasov plasma' they are the only contributors to the spectral density. For this reason, below we will freely use the term 'wave' to denote imaginary collective motion of our Klimontovich–Dupree plasma that gives the same contribution to the two-point correlation function as the wave of traditional theory.

The lowest-order evolution equation for the electron distribution function is a graphic equation written in Erofeev (1997) in Fig. 4. Translating this equation into analytic form, we obtain

$$\begin{aligned}\frac{\partial}{\partial t} f_e = \frac{e^2}{c^2} v^m \frac{\partial}{\partial p^\beta}\int d^3\mathbf{r}_1\, d^3\mathbf{p}_1 \int_{-\infty}^{t} dt'\,{}^0G_\alpha(\mathbf{r},\mathbf{p},t,\mathbf{r}_1,\mathbf{p}_1,t') \\ \times\langle \widetilde{\delta F}{}_{m\cdot}^{\cdot\beta}(\mathbf{r},t)\,\widetilde{\delta F}{}_{n\cdot}^{\cdot\delta}(\mathbf{r}_1,t')\rangle\, v_1^n \frac{\partial}{\partial p_1^\delta} f_e(\mathbf{r}_1,\mathbf{p}_1,t').\end{aligned} \tag{41}$$

Here ${}^0G_\alpha(\mathbf{r},\mathbf{p},t,\mathbf{r}_1,\mathbf{p}_1,t')$ is a bare Green function for the given species. It is a solution to equation

$$\left(\frac{\partial}{\partial t} + v^\beta\frac{\partial}{\partial r^\beta} + \frac{e_\alpha}{c} v_i\,{}^0F^{i\beta}\frac{\partial}{\partial p^\beta}\right){}^0G_\alpha = \delta^3(\mathbf{p}-\mathbf{p}')\delta^3(\mathbf{r}-\mathbf{r}')\delta(t-t'). \tag{42}$$

The most convenient representation of the bare Green function ${}^0\hat{G}_\alpha$ is its Fourier

https://doi.org/10.1017/S002237780200185X Published online by Cambridge University Press

representation:

$$ {}^0G_{\alpha\mathbf{k}}(\mathbf{p},t,\mathbf{p}',t') \equiv \int d^3\mathbf{r}\, {}^0G_\alpha(\mathbf{r},\mathbf{p},t,\mathbf{r}',\mathbf{p}',t') \exp[-ik_\beta(r^\beta - r'^\beta)], \tag{43} $$

where the calculation is straightforward. In our case of a nonmagnetized plasma it is

$$ {}^0G_{\alpha\mathbf{k}}(\mathbf{p},t,\mathbf{p}',t') \equiv {}^0G_{\alpha\mathbf{k}}(\mathbf{p},t,t')\delta^3(\mathbf{p}-\mathbf{p}') = \exp[-i(\mathbf{k}\cdot\mathbf{v})(t-t')]\delta^3(\mathbf{p}-\mathbf{p}'). \tag{44} $$

This formula is written for $t > t'$: at $t < t'$ the bare Green function is identically zero, because of the causality principle.

As a matter of fact, the reader may arrive at (41) without appeal to formulae from Erofeev (1997). Take (32). The lowest order of its right-hand side is the two-time correlation function multiplied by the momentum derivative of the distribution function $f_\alpha$. Inverting this equation (integrating it over time), one develops a lowest-order expression of the two-point correlation function. After its substitution into the collision integral (31), one obtains (41).

Because of the potentiality of the wave field, only components with $n = m = 0$ of the two-time correlation function are present (have nonzero values). Furthermore, in our order of consideration it is sufficient to substitute the two-time correlation function by its lowest order

$$ \begin{aligned} \langle \widetilde{\delta F}^{0\beta}(\mathbf{r},t)\widetilde{\delta F}^{0\delta}(\mathbf{r}_1,t')\rangle \approx \int d^3\mathbf{k} \exp[i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}_1)]\frac{k^\beta k^\delta}{k^2} \\ \times \sum_{s=\pm} n_{s\mathbf{k}} \exp[-is\omega_{s\mathbf{k}}(t-t') - \gamma^{\mathrm{nl}}_{s\mathbf{k}}|t-t'|]. \end{aligned} \tag{45} $$

It is seen that the effect of different waves can be considered independently. Therefore below we skip contributions corresponding to all but one $\mathbf{k}$. Also, we make use of (44). Then (41) reduces to the next one

$$ \frac{\partial f_e}{\partial t} = e^2 \frac{d}{dp}\left[\int_0^\infty {}^0G_p(r,\tau)\Phi(r,\tau)\, dr\, d\tau\, \frac{d}{dp} f_e(p,t-\tau)\right], \tag{46} $$

where

$$ \Phi(r,\tau) = \exp(i\mathbf{k}\cdot\mathbf{r}) \sum_{s=\pm} n_{s\mathbf{k}} \exp[-is\omega_{s\mathbf{k}}(t-t') - \gamma^{\mathrm{nl}}_{s\mathbf{k}}|t-t'|]. \tag{47} $$

Equation (46) exhibits the one-dimensional character of the problem: it is natural to consider spatial and momentum dependences along the direction of wave propagation only.

The function ${}^0G_p(r,t)$ is an inverted Fourier transform of the function ${}^0G_{\alpha\mathbf{k}}(\mathbf{p},t,t')$, which was introduced in (44).

Time variation of the distribution $f_e(p,t)$ in the right-hand side of (46) can be ignored: this corresponds to the lowest-order approximation in $W/nT$. Owing to this, the evolution equation becomes the usual diffusion equation.

After substitution of ${}^0G_p$ and $\Phi$ by their expressions in Fourier transforms, the diffusion equation (46) takes the form

$$ \left.\begin{aligned} \frac{\partial f_e}{\partial t} &= \frac{d}{dp}\left[D(p)\frac{df_e}{dp}\right], \\ D(p) &= e^2 \int_0^\infty {}^0G_{-kp}(t)\Phi_k(t)\, dk\, dt. \end{aligned}\right\} \tag{48} $$

https://doi.org/10.1017/S002237780200185X Published online by Cambridge University Press

The integration variable $k$ is left here because the mixed contributions of the selected wave and the counterpropagating wave should be divided. (Equation (45) shows that the original wave and its counterpropagating twin enter mixed into the two-time correlation function.) This gives an opportunity to rearrange the integral in right-hand side of (48). The remaining calculations are easy to perform; selecting terms containing $n_{\mathbf{k}}$ and neglecting the dispersion addition to $\omega_{pe}$, one has the result:

$$D(p) = e^2 \frac{2\gamma_{\mathbf{k}}^{\mathrm{nl}} n_{\mathbf{k}}}{(kv - \omega_{pe})^2 + {\gamma_{\mathbf{k}}^{\mathrm{nl}}}^2}. \tag{49}$$

For bulk plasma electrons $v \sim v_{Te} \ll \omega_{pe}/k$, and hence

$$D(p) \approx e^2 \frac{2\gamma_{\mathbf{k}}^{\mathrm{nl}} n_{\mathbf{k}}}{\omega_{pe}^2}. \tag{50}$$

Therefore the rate of bulk electron heating is

$$\frac{dQ}{dt} = \int \frac{p^2 dp}{2m} \frac{d}{dp} \left[ D(p) \frac{df}{dp} \right] = \frac{2\gamma_{\mathbf{k}}^{\mathrm{nl}} n_{\mathbf{k}} n_e e^2}{\omega_{pe}^2 m} = \gamma_{\mathbf{k}}^{\mathrm{nl}} \frac{n_{\mathbf{k}}}{2\pi}. \tag{51}$$

Note that value $n_{\mathbf{k}}/(2\pi)$ is the energy of the wave. Really, in terms of traditional wave amplitudes $\tilde{\mathbf{E}}_{\mathbf{k}}$, the energy of the Langmuir wave is $|\tilde{\mathbf{E}}_{\mathbf{k}}|^2/(2\pi)$. At the same time, in interpretation following the logic of Erofeev (1997, 1998) one has $n_{\mathbf{k}} \equiv |\tilde{\mathbf{E}}_{\mathbf{k}}|^2$. That is, the rate of plasma heating per single wave, i.e. the rate of wave collisionless decay, is $\gamma_{\mathbf{k}}^{\mathrm{nl}}$, in full accordance with the calculation by Bass et al. Note also that $\gamma_{\mathbf{k}}^{\mathrm{nl}}$ cannot take negative values, otherwise the two-time correlation function $\Phi_{\mathbf{k}}(t, t')$ would not have decayed with increasing $|t - t'|$ (see (47)). With this, the bulk plasma electrons can only be *heated* at the expense of wave energy, i.e. 'non-resonant electron cooling' is impossible.

Thus, we have performed a study of nonresonant electron diffusion in a single plasma. This study confirmed both the Lorentzian structure of the wave correlation function and data on nonresonant electron diffusion by Bass et al.

Note one more experimental argument in favour of the above developed kinetics. Vyacheslavov et al. mentioned that hot plasma electrons contain nearly half the energy deposited in the plasma (Vyacheslavov et al. 2000). This feature can be easily explained in our theory. Assume that the wave scattering on density inhomogeneities defines the lower bound for the nonlinear wave damping rate $\gamma^{\mathrm{nl}}$. With this, the rate of wave energy transfer to hot plasma electrons is the same as the rate of direct wave energy transfer to bulk plasma electrons via nonresonant electron diffusion.

Finally, let us discuss one more line of reasoning in favour of our kinetic approach. As was noted at the beginning of the section, the most adequate plasma description is given by the Klimontovich–Dupree–Maxwell equations, but full integration of these equations is technically impossible. This forces theorists to use more simplified approaches for the plasma description. They are either kinetic (of the type of traditional weak plasma turbulence theory and our one) or hydrodynamic (of the type of Zakharov's plasma description). With corresponding simplifications in the plasma description, one inevitably loses its completeness. For this reason, none of the simplified approaches gives a picture of plasma evolution that does not diverge over time from the real plasma macroscopic evolution. Therefore, one can rely upon simplified descriptions of plasma evolution only on restricted time scales. This general position inevitably results in an *asymptotic* character of convergence of *any* perturbation theory that can be advanced for developing plasma kinetics. Regardless of the essence of perturbation theory, consecutive iterations will at best first converge up to some optimal order of consideration, but then inevitably diverge: those familiar with diagrammatic approaches know well about this distinctive feature of perturbation expansions. For this reason, the most consistent goal that one can pursue in developing any kinetics is to describe quantitatively the current evolution of a plasma and turbulence with as high precision as possible. Calculations within our two-time formalism are aimed at this goal by the very intention of successive iterations. In contrast, iteration techniques used in traditional calculations (e.g. by Tsytovich) were developed without understanding the significance of restrictions imposed on the theory by the asymptotic convergence of the perturbation expansions. Let us substantiate this statement.

In traditional kinetic calculations theorists usually develop nonlinear relations for frequency harmonics of the electromagnetic field and the distribution function. The unconditional use of frequency in plasma kinetic calculations implies that the plasma evolves up to $t = \infty$: all the history of this evolution is fixed in frequency spectra of the electromagnetic field and the distribution function. Meanwhile, in real observations the plasma 'lives' usually only for a time comparable with the time of its observation, especially in plasma installations. Therefore, one tries to understand physics that develops in a plasma during a sufficiently restricted period. One has no interest in the influence of delayed periods of plasma evolution on the run of current processes, especially when the plasma has not existed at the corresponding delayed periods. Furthermore, theorists average this history of the plasmas infinite evolution over a plasma probabilistic ensemble. It cannot be the case other than that during this averaging some information about the plasma evolution falls out. One has a right to think that one loses only inessential information about the plasma time evolution in delayed periods and retains all the crucial information about the current plasma evolution. But this thought cannot be substantiated by any consistent argument: the difference in Tsytovich's conclusions regarding the physics of nonresonant electron diffusion and ours undoubtedly is evidence against the consistency of such an argument.

The above statement can be emphasized by the following remark. In no situation can sequential approximations in our two-time formalism and in traditional perturbations (of the Tsytovich's type) approach each other, by virtue of the asymptotic character of their convergency and difference in the leading orders of the wave correlation function.†

## 6. Summary

In this paper we have analysed observations of strong Langmuir turbulence in beam–plasma experiments by Vyacheslavov et al. (BudkerINP) and by Wong et al. (University of California, Los Angeles). Following the tradition of plasma the-

† The reader is reminded that the lowest order of the wave correlation function is usually assumed to be in the form $\Phi_{\mathbf{k}\omega} = N_{\mathbf{k}}\delta(\omega - \omega_{\mathbf{k}})$ within the traditional theory. Here $\omega_{\mathbf{k}}$ is the wave natural frequency, which is real (generally renormalized). That is, the wave is associated with a single line, and any line broadening appears only in the next after leading approximation. The corresponding line shape does not have a Lorentzian structure, in full accordance with Tsytovich's observation.

ory (established after Zakharov (1972)), in both series of experiments Langmuir wave collapse should have taken place. We have discovered that the corresponding experimental data are against the occurrence of Langmuir wave collapse.

We assumed that the developed contradiction indicates that plasma theory undervalues substantially collisionless wave dissipation. We considered the main channel of the corresponding dissipation, the nonresonant electron diffusion in momenta. We have found that apart from the commonly accepted idea of the process following Tsytovich (1972, 1977), the theory comprises an alternative picture of the nonresonant electron diffusion with a substantial increase in the rate of wave energetic decay. We have particularly checked that a new diffusion picture results in suppression of the plasma hydrodynamic instability with respect to spontaneous development of short-wavelength spatial modulation in the plasma density and simultaneous correlated spatial modulation in the energy density of electron Langmuir oscillations. The reader is reminded that the corresponding instability was originally regarded as an initial stage of Zakharov's Langmuir wave collapse. The difference in conclusions regarding the occurrence/nonoccurrence of this instability illustrates that traditional theory cannot provide one with a reliable picture of the physical evolution of a plasma. We have stated that the basic false cornerstone of the theory is the method of the probabilistic ensemble, a key method of nonequilibrium statistical physics. The contradiction in the two pictures of nonresonant electron diffusion shows that the method of the plasma probabilistic ensemble is of no use in kinetic studies of real plasmas.

We have abandoned substitution of a real plasma by a plasma ensemble and performed a study of nonresonant electron diffusion in a single collisionless plasma. Our study confirmed the idea of intense collisionless wave dissipation and the impossibility of Langmuir wave collapse.

In such a way, we have pointed out a more natural direction for the development of the plasma kinetic theory.

## References


Bass, F. G., Fainberg, Ya. B. and Shapiro, V. D. 1965 *Zh. Eksp. Teor. Fiz.* **49**, 329 (1966 *Sov. Phys. JETP* **22**, 230).

Berezin, Yu. A. and Karpman, V. I. 1964 *Sov. Phys. JETP* **19**, 1265.

Berezin, Yu. A. and Karpman, V. I. 1967 *Sov. Phys. JETP* **24**, 1049.

Burmasov, V. S. et al. 1997 *Plasma Phys. Rep.* **23**, 142.

Davidson, R. C. 1972 *Methods in Nonlinear Plasma Theory*. New York: Academic Press.

Dupree, T. H. 1963 *Phys. Fluids* **6**, 1714.

Erofeev, V. I. 1997 *J. Plasma Phys.* **57**, 273.

Erofeev, V. I. 1998 *Austr. J. Phys.* **51**, 843.

Gibbs, J. W. 1902 *Elementary Principles in Statistical Mechanics, Developed with Especial Reference to the Rational Foundation of Thermodynamics*. New York: Yale Univ. Press.

Kadomtsev, B. B. 1965 *Plasma Turbulence*. New York: Academic.

Klimontovich, Yu. L. 1967 *The Statistical Theory of Nonequilibrium Processes in a Plasma*. Cambridge, MA: MIT Press.

Krall, N. A. 1969 *Phys. Fluids* **12**, 1661.

Kovrizhnykh, L. M. 1965a *Zh. Exp. Teor. Fiz.* **49**, 237.

Kovrizhnykh, L. M. 1965b *Zh. Exp. Teor. Fiz.* **49**, 1376.

Malkin, V. M. 1982a *Zh. Eksp. Teor. Fiz.* **83**, 88 (1982 *Sov. Phys. JETP* **56**, 48).

Malkin, V. M. 1982b *Fiz. Plazmy* **8**, 357 (1982 *Sov. J. Plasma Phys.* **8**, 202).

Malkin, V. M. 1982c *Zh. Eksp. Teor. Fiz.* **83**, 1725 (1982 *Sov. Phys. JETP* **56**, 997).

Malkin, V. M. 1984 *Zh. Eksp. Teor. Fiz.* **86**, 1263 (1984 *Sov. Phys. JETP* **59**, 737).

McFarland, M. D. and Wong, A. Y. 2000 *Phys. Rev. Lett.* **84**, 666.

Rogister, A. and Oberman, C. 1969 *J. Plasma Phys.* **3**, 119.

Pozzoli, R. and Ryutov, D. 1979 *Phys. Fluids* **22**, 1782.

Robinson, P. A. 1997 *Rev. Mod. Phys.* **69**, 507.

Tsytovich, V. N. 1970 *Nonlinear Effects in Plasma*. New York: Plenum.

Tsytovich, V. N. 1972 *Usp. Fiz. Nauk* **108**, 143 (1973 *Sov. Phis. Usp.* **15**, 632).

Tsytovich, V. N. 1977 *Theory of Turbulent Plasma*. New York: Consultants Bureau.

Vedenov, A. A. and Rudakov, L. I. 1964 *Dokl. Akad. Nauk SSSR* **159**, 767 (1965 *Sov. Phys. Dokl.* **9**, 1073).

Vyacheslavov, L. N. et al. 1995 *Phys. Plasmas* **2**, 2224.

Vyacheslavov, L. N., Kandaurov, I. V., Kruglyakov, E. P., Meshkov, O. I. and Sanin, A. L. 1996 *Proc. Int. Conf. on Plasma Physics*, Contributed papers, Nagoya (ed. H. Sugai and T. Hayashi), Vol. 1, p. 30.

Vyacheslavov, L. N. et al. 1998a *Plasma Phys. Rep.* **24**, 211.

Vyacheslavov, L. N. et al. 1998b *Plasma Phys. Rep.* **24**, 223.

Vyacheslavov, L. N. et al. 2000 *Proc. IVth Int. Workshop 'Strong Microwaves in Plasmas', Nizhny Novgorod*, Inst. of Applied Physics, 1999 (ed. A.G. Litvak), Vol. 2, p. 405. Moscow: Nauka.

Wong, A. Y. and Cheung, P. Y. 1984 *Phys. Rev. Lett.* **52**, 1222.

Wyld, H. W. 1961 *Ann. Phys.* **14**, 143.

Wong, J. G., Payne, G. L., DuBois, D. F. and Rose, H. A. 1995 *Phys. Plasmas* **2**, 1129.

Zakharov, V. E. 1972 *Zh. Eksp. Teor. Fiz.* **62**, 1745 (1972 *Sov. Phys. JETP* **35**, 908).

Zakharov, V. E. and L'vov, V. S. 1975 *Izv. Vyssh. Uchebn. Zaved., Radiofiz.* **18**, 1470.