Laser and Particle Beams (2012), *30*, 267–273.

doi:10.1017/S026303461200002X

Electric, magnetic Wakefields, and electron acceleration in quantum plasma

P. KUMAR AND C. TEWARI

Department of Physics, University of Lucknow, Lucknow, India

(RECEIVED 6 September 2011; ACCEPTED 2 January 2012)

Abstract

A detailed study of Wakefield excitation in very dense quantum plasma is presented. Electric and magnetic Wakefields have been obtained for a particular profile of the laser pulse, using perturbative technique involving orders of the incident laser beam. The Wakefields can trap electrons and accelerate them to extremely high energies. It is observed that the quantum effects significantly change the classical nature of the Wakefield. The axial and radial forces acting on a test electron due to the Wakefields have been evaluated.

Keywords: Accelerators; Electron acceleration; Quantum plasma; Wakefields

1. INTRODUCTION

Acceleration of electrons by laser Wakefield (Tajima & Dawson, 1979) has been one of the most developed areas of interest during the past 20 years, leading to a number of theoretical and experimental efforts. In a laser Wakefield accelerator (LWFA), a high intensity laser pulse with duration approximately equal to plasma oscillation period, generates a large amplitude plasma wave with phase velocity that is close to the speed of light. A beam of electrons interacting with this wave can be effectively accelerated. The pioneering idea of Tajima and Dawson, has been theoretically explored and experimentally verified (Andreev *et al.*, 1994; Balakirev *et al.*, 2001; Blumenfeld *et al.*, 2007; Faure *et al.*, 2004; Geddes *et al.*, 2004; Gorbunov *et al.*, 2003; Jha *et al.*, 2005; Joshi, 2007; Kawata *et al.*, 2005; Koyama *et al.*, 2006; Leemans *et al.*, 2006; Lifshitz *et al.*, 2006; Lourenco *et al.*, 2010; Lotov, 2001; Luttikhof *et al.*, 2009; Malka & Fritzler, 2004; Malka *et al.*, 2002; Mangles *et al.*, 2004; Marques *et al.*, 1996; Masuda & Miura, 2009; Maltis *et al.*, 2006; Phuoc *et al.*, 2008; Pukhov & Meyer-ter-Vehn, 2002; Schlenvoigt, 2008; Siders *et al.*, 1996; Takahashi *et al.*, 2004; Wang *et al.*, 2009; Xie *et al.*, 2009; Zhou *et al.*, 2007).

Recently, studies concerning high-density plasmas (quantum plasma) that are created by high intensity laser pulses have received attention and pertinent research activities have been observed. Quantum affects appear in ultra-small electronic devices (Markowich *et al.*, 1990), dense astrophysical plasmas (Jung, 2001), and laser plasmas (Kremp *et al.*, 1999). The high-density, low-temperature quantum Fermi plasma is significantly different from the low-density, high temperature "classical plasma" obeying the Maxwell-Boltzmann distribution. In the dense Fermi plasma, the electron degeneracy leads to a consideration of the Fermi-Dirac electron distribution and electron tunneling through the quantum Bohm potential (Gardner & Ringhofer, 1996; Manfredi, 2005). The quantum statistical pressure and the quantum Bohm force affect the electron dynamics that results in collective interactions in dense quantum plasmas. The studies regarding Wakefield generation in quantum plasmas have been recently reported (Shukla *et al.*, 2009).

In this work, we have carried out a detailed analytical study of plasma Wakefield generation in quantum plasma. We have used perturbative technique involving orders of the incident laser beam to obtain explicit electric and magnetic Wakefields. Further, the accelerating force acting on a test electron has been evaluated. The results show that the accelerating force is increased due to the collective contributions of statistical pressure and the quantum Bohm force. Such a study has not been reported in literature before.

In Section 2, the lowest order fast oscillating plasma electron velocities and density perturbations have been derived. Coupling the time averaged current densities with Maxwell's equations, the generated electric and magnetic Wakefields have been obtained in Section 3. In Section 4, the axial and radial forces acting on a moving test electron have

Address correspondence and reprint requests to: Punit Kumar, Department of Physics, University of Lucknow, Indiranagar, Lucknow-226016, India. E-mails: punitlko@yahoo.co.in; punitkumar@hotmail.com

been derived and analyzed graphically. Section 5 is devoted to summary and discussion.

2. LASER-PLASMA INTERACTION

We consider the propagation of linearly polarized laser beam represented by the electric vector $\vec{E} = \hat{e}_x E_o \cos(kz - \omega t)$ ($\hat{e}_x$ is the unit vector of polarization) propagating in an uniform quantum plasma of density n_o. In quantum plasma, the electrons obey the equations

$$\frac{\partial \vec{v}}{\partial t} = -\frac{e}{m}\left[\vec{E} + \frac{1}{c}(\vec{v}\times\vec{B})\right] - \frac{1}{2}\vec{\nabla}(\vec{v}.\vec{v}) + \vec{v}\times(\vec{\nabla}\times\vec{v}) - \frac{v_F^2}{3n_o^2}\frac{\vec{\nabla}n^3}{n} + \frac{\hbar^2}{2m^2}\vec{\nabla}\left(\frac{1}{\sqrt{n}}\vec{\nabla}^2\sqrt{n}\right), \tag{1}$$

$$\frac{\partial n}{\partial t} + \vec{\nabla}.(n\vec{v}) = 0, \tag{2}$$

and

$$\vec{\nabla}.\vec{E} = -4\pi e(n - n_o). \tag{3}$$

$n(= n_o + n^{(1)})$ is the electron density, m is the electron rest mass, $\hbar$ is the Planck's constant divided by 2π, and v_F is the Fermi velocity. The fourth term on the right-hand side of Eq. (1) denotes the Fermi electron pressure. The fifth term is the quantum Bohm potential and is due to the quantum corrections in the density fluctuation. The classical equations may be recovered in the limit of $\hbar = 0$. The ponderomotive force of the high-frequency laser pulse drives longitudinal waves with a frequency much smaller than ω, but fast enough for the dynamics to take place on the electron timescale. The ions form a neutralizing background in dense plasma. A correct relativistic treatment of quantum effects should rely on the moments of a relativistic Wigner function (Bialynicki *et al.*, 1991; Shin, 1996). Perturbatively, expanding Eqs. (1) and (2) for the first order of the electromagnetic field, we get

$$\frac{\partial \vec{v}^{(1)}}{\partial t} = -\frac{e}{m}E - \frac{v_F^2}{n_o}\vec{\nabla}n^{(1)} + \frac{\hbar^2}{4m^2}\left(\frac{1}{n_o}\vec{\nabla}(\vec{\nabla}^2 n^{(1)})\right), \tag{4}$$

and

$$\frac{\partial n^{(1)}}{\partial t} + (n_o.\vec{\nabla}v^{(1)} + v^{(1)}.\vec{\nabla}n_o) = 0. \tag{5}$$

The last term in Eq. (4) has been obtained by using the perturbative expansion (Cao *et al.*, 2008),

$$\vec{\nabla}\left(\frac{1}{\sqrt{n}}\vec{\nabla}^2\sqrt{n}\right) = \frac{1}{n}\left[\frac{1}{2}\vec{\nabla}\vec{\nabla}^2 n^{(1)} - \frac{1}{2n_o}\vec{\nabla}n^{(1)}\vec{\nabla}^2 n_o - \frac{1}{2n_o}\vec{\nabla}n_o\vec{\nabla}^2 n^{(1)} + \frac{n^{(1)}}{2n_o^2}\vec{\nabla}n_o\vec{\nabla}^2 n_o - \frac{1}{4n_o}\vec{\nabla}(2\vec{\nabla}n_o.\vec{\nabla}n^{(1)}) + \frac{1}{4n_o^2}n^{(1)}\vec{\nabla}(\vec{\nabla}n_o)^2 + \frac{1}{2n_o^2}(\vec{\nabla}n_o)^2\vec{\nabla}n^{(1)} + \frac{1}{n_o^2}(\vec{\nabla}n_o.\vec{\nabla}n^{(1)})\vec{\nabla}n_o - \frac{1}{n_o^3}(\vec{\nabla}n_o)^2 n^{(1)}\vec{\nabla}n_o\right].$$

Assuming the perturbed density to vary according to (Bret, 2007) $n^{(1)} = \eta \exp i(kr - \omega t)$, we get from Eqs.(3), (4), and (5),

$$n^{(1)} = -\frac{ekn_oE_o\Omega}{m}\sin(kz - \omega t),$$

where $\Omega = \left\{\omega^2 + k^2V_F^2 + (\hbar^2 k^4/4m^2)\right\}^{-1}$. High frequency laser pulse propagating in collisionless quantum plasma provides a periodic force due to which the plasma electrons oscillate with the frequency of the laser. This high frequency fluctuation in charge density becomes source of a longitudinal electric field and velocity in quantum plasma, which are given by

$$E_z^{(1)} = -\Omega\omega_{po}^2 E_o\cos(kz - \omega t),$$

and

$$v_Z^{(1)} = -ca\Omega\Omega_p\sin(kz - \omega t),$$

respectively, where $\omega_{po}^2 = 4\pi n_o e^2/m$ (the electron plasma frequency), $\Omega_p = \{\omega_{po}^2 + k^2V_F^2 + (\hbar^2 k^4/4m^2)\}$ and $a = eE_o/mc\omega$.

3. WAKEFIELDS

To proceed with the study of excitation of plasma waves, we use the quasistatic approximation where the plasma fluid equations are written in terms of independent variables $\xi = z - ct$ and $\tau = t$. It is assumed that the laser does not evolve significantly as it transits a plasma electron (Esarey *et al.*, 1997). Thus, plasma electrons experience a laser field that is a function of ξ and r variables only. Electric and magnetic Wakefields can be obtained using the time dependent Maxwell's equations

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t},$$

$$\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}.$$

The radial profile of the laser field is Gaussian and the generated fields are assumed to be axisymmetric. The field components may be written as,

$$\frac{\partial E_r}{\partial \xi} - \frac{\partial E_z}{\partial r} = \frac{\partial B_\theta}{\partial \xi}, \tag{6a}$$

$$\frac{\partial E_r}{\partial \xi} - \frac{\partial B_\theta}{\partial \xi} = \frac{4\pi}{c} J_r, \tag{6b}$$

$$\frac{\partial E_z}{\partial \xi} = \frac{4\pi}{c} J_z - \frac{1}{r}\frac{\partial (r B_\theta)}{\partial r}, \tag{6c}$$

where J_r and J_z are transverse and axial current densities, respectively. In order to obtain the second order slow components of the plasma electron velocity, we substitute the first order quantities into the force equation, which gives

$$\frac{\partial \vec{v}_z}{\partial \xi} = \frac{e}{mc} E_z + \frac{1}{4c}\frac{\partial (a^2 c^2 \Omega^2 \Omega_p^2)}{\partial \xi} + \frac{v_F^2}{n_o c} \nabla n^{(2)}, \tag{7}$$

and

$$\frac{\partial \vec{v}_r}{\partial \xi} = \frac{e}{mc} E_r + \frac{1}{4c}\frac{\partial (a^2 c^2 \Omega_s^2)}{\partial r} + \frac{v_F^2}{n_o c} \nabla n^{(2)}, \tag{8}$$

where, $\Omega_s = \left\{-1 + k^2 V_F^2 \Omega + (\hbar^2 k^4 / 4m^2 \Omega)\right\}$; E_r and E_z represents the radial and axial electric Wakefields. The above equations have been obtained by substituting the value of $v_z^{(1)}$ averaged over the polar angle θ. It is observable that the ponderomotive nonlinear effects contribute to transverse as well as longitudinal Wakefield generation. The ponderomotive force is modified due to the contribution of first order longitudinal velocity of plasma electrons. Further, the quantum effects appear significantly in both transverse and longitudinal velocities. From Eqs. (6) and (7), we get

$$\left[\frac{\partial^2}{\partial \xi^2} + k_p^2\right] E_z = -\frac{k_p^2 mc^2 \Omega^2 \Omega_p^2}{4e}\frac{\partial a^2}{\partial \xi} - \frac{\partial}{\partial \xi}\left(\frac{1}{r}\frac{\partial}{\partial r}(r B_\theta)\right), \tag{9}$$

where

$$k_p^2 = k_{po}^2\left[1 + \frac{n^{(1)}}{n_o}\right].$$

Combining Eqs. (6a) and (6b) and substituting in Eq. (8) gives

$$E_r = -\frac{1}{k_p^2}\frac{\partial^2 E_z}{\partial \xi \partial r} - \frac{m}{4e}\frac{\partial a^2 c^2 \Omega_s^2}{\partial r}. \tag{10}$$

Eqs. (9) and (10) are the principal equations for generation of axial and transverse electric Wakefields in a dense quantum plasma. The magnetic Wakefields can be obtained from Eq. (6a). The radial dependence of transverse electric and magnetic Wakefields can be considered to be a higher order effect. The lowest order axial Wakefield is given as

$$\left[\frac{\partial^2}{\partial \xi^2} + k_{po}^2\right] E_z^{(0)} = -\frac{k_{po}^2 mc^2 \Omega^2 \Omega_p^2}{4e}\frac{\partial a^2}{\partial \xi}.$$

Considering a laser pulse $a^2 = a_r^2 \sin^2(\pi\xi/L)$, where $a_r^2 = a_o^2 \exp(-2r^2/r_o^2)$, the solution of the above equation, within the pulse $(0 \le \xi \le L)$ and behind the pulse $(\xi < 0)$ are,

$$E_z^{(0)} = \frac{\varepsilon \Omega^2 \Omega_p^2 k_{po} f}{8}\left[\sin k_{po}(L - \xi) + \frac{k_{po} L}{2\pi} . \sin\frac{2\pi\xi}{L}\right], \tag{11a}$$

and

$$E_z^{(0)} = \frac{\varepsilon \Omega^2 \Omega_p^2 k_{po} f}{8}\left[\sin k_{po}\xi + \sin k_{po}(L - \xi)\right], \tag{11b}$$

where

$$\varepsilon = mc^2 a_r^2 / e \text{ and } f = \left(1 - \frac{k_{po}^2 L^2}{4\pi^2}\right)^{-1}.$$

Similarly, from Eq. (10) the lowest order transverse Wakefields within $(0 \le \xi \le L)$ and behind $(\xi < 0)$ the pulse are

$$\begin{aligned} E_r^{(0)} = -\frac{\varepsilon r}{2 r_o^2}\Big[&\Omega^2 \Omega_p^2 f \cos k_{po}(L - \xi) \\ &+ (\Omega_s^2 - \Omega^2 \Omega_p^2 f)\cos\frac{2\pi\xi}{L} - \Omega_s^2\Big], \end{aligned} \tag{12a}$$

$$\begin{aligned} E_r^{(0)} = -\frac{\varepsilon r}{2 r_o^2}\Big[&\Omega^2 \Omega_p^2\Big\{f \cos k_{po}(L - \xi) \\ &- f \cos k_{po}\xi\Big\} + \Omega_s^2\left(\cos\frac{2\pi\xi}{L} - 1\right)\Big]. \end{aligned} \tag{12b}$$

The lowest order magnetic field, within $(0 \le \xi \le L)$ and behind $(\xi < 0)$ the pulse is obtained by substituting the value axial and transverse electric Wakefield from Eqs. (11) and Eq. (12) in Eq. (6a). The magnetic field within the pulse is $B_\theta^{(0)} = 0$, while the magnetic field behind the pulse is

$$\begin{aligned} B_\theta^{(0)} = -\frac{\varepsilon r}{2 r_o^2}\Big[&\Omega_s^2 \cos\frac{2\pi\xi}{L} \\ &- f \cos k_{po}\xi + \Omega^2 \Omega_p^2 f \cos k_{po}\xi - \Omega_s^2 + f(1 - \Omega^2 \Omega_p^2)\Big]. \end{aligned} \tag{12c}$$

It is evident that the above field is zero for on-axis $(r = 0)$ propagation.

https://doi.org/10.1017/S026303461200002X Published online by Cambridge University Press

On further perturbative expansion of Eq. (9), the equation for first order axial Wakefield comes out to be,

$$\left[\frac{\partial^2}{\partial\xi^2}+k_{po}^2\right]E_z^{(1)}=\Omega_k k_{po}E_z^{(0)}+\frac{k_{po}mc^2\Omega^2\Omega_p^2\Omega_k}{4e}\, a_r^2\frac{\pi}{L}\sin\frac{2\pi\xi}{L}-\frac{\partial}{\partial\xi}\left(\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta^{(0)})\right)$$

where, $\Omega_k=(ekk_{po}\Omega/m)E_o\sin(kz-\omega t)$. In solving the above equation, the first order axial fields within and behind the pulse are found to be

$$E_z^{(1)}=-\frac{\Omega_k\Omega^2\Omega_p^2\varepsilon f}{8}\left[\frac{k_{po}}{2}(L-\xi)\cos k_{po}(L-\xi)+\left(f-\frac{1}{2}\right)\sin k_{po}(L-\xi)+\frac{fk_{po}L}{2\pi}\sin\frac{2\pi\xi}{L}\right] \tag{13a}$$

and

$$\begin{aligned}E_z^{(1)}=\varepsilon f\Bigg[&\frac{\Omega_k\Omega^2\Omega_p^2}{16}\sin k_{po}L\cos k_{po}\xi\\&-\frac{\Omega_k\Omega^2\Omega_p^2Lk_{po}}{16}\cos k_{po}(L-\xi)\\&+\left\{\frac{\Omega_k\Omega^2\Omega_p^2Lk_{po}}{16}-\frac{1}{2r_o^2}\left(1-\frac{2r^2}{r_o^2}\right)(\Omega^2\Omega_p^2-1)L\right\}\\&\cos k_{po}\xi+\left\{-\frac{5\Omega_k\Omega^2\Omega_p^2}{32}+\frac{1}{4k_{po}r_o^2}\left(1-\frac{2r^2}{r_o^2}\right)\right.\\&\left.(\Omega^2\Omega_p^2-1)+\frac{\Omega_s^2}{k_{po}r_o^2}\left(1-\frac{2r^2}{r_o^2}\right)\right\}\sin k_{po}\xi\\&+\left\{-\frac{\Omega_k\Omega^2\Omega_p^2}{32}+\frac{1}{4k_{po}r_o^2}\left(1-\frac{2r^2}{r_o^2}\right)(\Omega^2\Omega_p^2-1)\right\}\\&\sin k_{po}(2L-\xi)+\left\{-\frac{\Omega_k\Omega^2\Omega_p^2}{8}+\frac{\Omega_s^2}{k_{po}r_o^2}\left(1-\frac{2r^2}{r_o^2}\right)\right\}\\&\sin k_{po}(L-\xi)\Bigg],\end{aligned} \tag{13b}$$

respectively. The first order transverse electric Wakefields within and at the back of the pulse are obtained by substituting Eq. (13) into Eq. (10) as

$$E_r^{(1)}=-\frac{\varepsilon r\Omega^2\Omega_p^2\Omega_k f}{2r_o^2k_{po}^2}\left[\frac{k_{po}^2}{2}(L-\xi)\sin k_{po}(L-\xi)-fk_{po}\cos k_{po}(L-\xi)+fk_{po}\cos\frac{2\pi\xi}{L}\right], \tag{14a}$$

and

$$\begin{aligned}E_r^{(1)}=\frac{\varepsilon fr}{r_o^2}\Bigg[&-\frac{\Omega_k\Omega^2\Omega_p^2}{4k_{po}}\sin k_{po}L\sin k_{po}\xi-\frac{\Omega_k\Omega^2\Omega_p^2L}{4}\\&\sin k_{po}(L-\xi)+\left\{-\frac{\Omega_k\Omega^2\Omega_p^2L}{4}\right.\\&\left.+\frac{1}{k_{po}r_o^2}\left(1-\frac{r^2}{r_o^2}\right)(\Omega^2\Omega_p^2-1)L\right\}\\&\sin k_{po}\xi\}+\frac{1}{k_{po}}\left\{-\frac{5\Omega_k\Omega^2\Omega_p^2}{8}+\frac{8\Omega_s^2}{k_{po}r_o^2}\left(1-\frac{r^2}{r_o^2}\right)\right.\\&\left.+\frac{(\Omega^2\Omega_p^2-1)}{k_{po}}\frac{2}{r_o^2}\left(1-\frac{r^2}{r_o^2}\right)\right\}\cos k_{po}\xi\\&+\left\{\frac{\Omega_k\Omega^2\Omega_p^2}{8k_{po}}-\frac{1}{k_{po}^2}\frac{2}{r_o^2}\left(1-\frac{r^2}{r_o^2}\right)(\Omega^2\Omega_p^2-1)\right\}\\&\cos k_{po}(2L-\xi)-\frac{1}{k_{po}}\left\{-\frac{\Omega_k\Omega^2\Omega_p^2}{2}+\frac{8\Omega_s^2}{k_{po}r_o^2}\left(1-\frac{r^2}{r_o^2}\right)\right\}\\&\cos k_{po}(L-\xi)\Bigg],\end{aligned} \tag{14b}$$

respectively. Substitution of Eqs. (13) and (14) into Eq. (6a) give the first order magnetic Wakefields within and behind the pulse.

$$B_\theta^{(1)}=\frac{\Omega_k\varepsilon r\Omega^2\Omega_p^2 f}{2r_o^2k_{po}}[\cos k_{po}(L-\xi)-\cos k_{po}L], \tag{15a}$$

and

$$B_\theta^{(1)}=-\frac{3rL\varepsilon f}{k_{po}r_o^4}(\Omega^2\Omega_p^2-1)\left(1-\frac{r^2}{r_o^2}\right)\sin k_{po}\xi. \tag{15b}$$

The higher order (second and higher) Wakefields can be obtained similarly. It is evident that all the Wakefields are proportional to the intensity of the laser pulse.

4. ELECTRON ACCELERATION

The maximum axial Wakefields within and behind the pulse are obtained using Eqs. (11) and (13) evaluated in the limit $L\rightarrow\lambda_p$ as

$$\begin{aligned}E_{zm}=E_{zm}^{(0)}+E_{zm}^{(1)}=-\frac{\varepsilon\Omega^2\Omega_p^2\pi^2}{4L}&\left[\left(1-\frac{\xi}{L}\right)\cos\frac{2\pi\xi}{L}+\frac{1}{2\pi}\sin\frac{2\pi\xi}{L}\right]+\frac{\varepsilon\Omega_k\Omega^2\Omega_p^2}{64}\left[\left\{4\pi^2\left(1-\frac{\xi}{L}\right)^2+1\right\}\right.\\&\left.\sin\frac{2\pi\xi}{L}+2\pi\left(1-\frac{\xi}{L}\right)\cos\frac{2\pi\xi}{L}\right]\end{aligned}$$

$(0\le\xi\le L)$

$$E_{zm} = \varepsilon\left[\frac{\Omega^2\Omega_p^2\pi^2}{4}\left\{\frac{\Omega_k}{2} - \frac{\pi}{L}\right\} - \frac{\pi^2 L}{r_o^2}\left(1 - \frac{2r^2}{r_o^2}\right)\left\{\frac{3}{8}(\Omega^2\Omega_p^2 - 1) + \frac{\Omega_s^2}{2}\right\}\right] \cos\frac{2\pi\xi}{L} + \frac{\varepsilon\Omega^2\Omega_p^2\Omega_k\pi^2}{8}\sin\frac{2\pi\xi}{L} \qquad (\xi < 0)$$

The maximum transverse electric Wakefield for $L \longrightarrow \lambda_p$ is

$$E_{rm} = E_{rm}^{(0)} + E_{rm}^{(1)} = \frac{\varepsilon r}{2r_o^2}\left[\Omega^2\Omega_p^2\pi\left(1 - \frac{\xi}{L}\right)\sin\frac{2\pi\xi}{L} - \Omega_s^2\cos\frac{2\pi\xi}{L} - \frac{\Omega^2\Omega_p^2\Omega_k L}{8}\left\{3\left(1 - \frac{\xi}{L}\right)\sin\frac{2\pi\xi}{L} - 2\pi\left(1 - \frac{\xi}{L}\right)^2\cos\frac{2\pi\xi}{L}\right\}\right] \qquad (0 \le \xi \le L)$$

$$E_{rm} = \frac{\varepsilon\pi r}{2r_o^2}\left[\Omega^2\Omega_p^2\left(1 - \frac{L\Omega_k}{2\pi}\right) + L^2\left(1 - \frac{r^2}{r_o^2}\right)\left\{\frac{4\Omega_s^2}{\pi^2 r_o^2} + (\Omega^2\Omega_p^2 - 1)\left(2 - \frac{1}{2r_o^2}\right)\right\}\right] \sin\frac{2\pi\xi}{L} + \frac{\varepsilon r}{2r_o^2}\left[\frac{\pi\Omega^2\Omega_p^2\Omega_k L}{2} - \Omega_s^2\right] \cos\frac{2\pi\xi}{L} + \frac{r\varepsilon\Omega_s^2}{2r_o^2} \qquad (\xi < 0)$$

and the maximum magnetic Wakefield is found to be

$$B_{\theta m} = B_{\theta m}^{(0)} + B_{\theta m}^{(1)} = -\frac{r\varepsilon\Omega^2\Omega_p^2\Omega_k L}{4r_o^2}\sin\frac{2\pi\xi}{L} \qquad (0 \le \xi \le L)$$

$$B_{\theta m} = \frac{\varepsilon r}{r_o^2}\left[\frac{3L^2}{4\pi r_o^2}(\Omega^2\Omega_p^2 - 1)\left(1 - \frac{r^2}{r_o^2}\right)\sin\frac{2\pi\xi}{L} + \frac{\Omega_s^2}{2}\left(\cos\frac{2\pi\xi}{L} - 1\right)\right]. \qquad (\xi < 0)$$

In a LWFA, we consider a test electron moving along the z-direction of the laser plasma interaction region. The maximum longitudinal force experienced by test electron is given by,

$$F_z = -eE_{zm}.$$

The electron will be accelerated by this force if $F_z > 0$. The maximum transverse force is,

$$F_r = -eE_{rm} + eB_{\theta m.}$$

The evolution of the normalized force within the pulse $F_{z,r}/\mathrm{mc}\omega_{po}$ with ξ/L is shown in Figure 1 for $\mathrm{n_o}=10^{34}$ m^{-3}, $a_o^2 = 0.05$, $r_o = 15.0\,\mu\mathrm{m}$ and $\mathrm{r} = 4.0\,\mu\mathrm{m}$. The solid curve represents the axial while the dotted line represents the transverse force. The force is greater in the axial direction in comparison to the transverse direction.

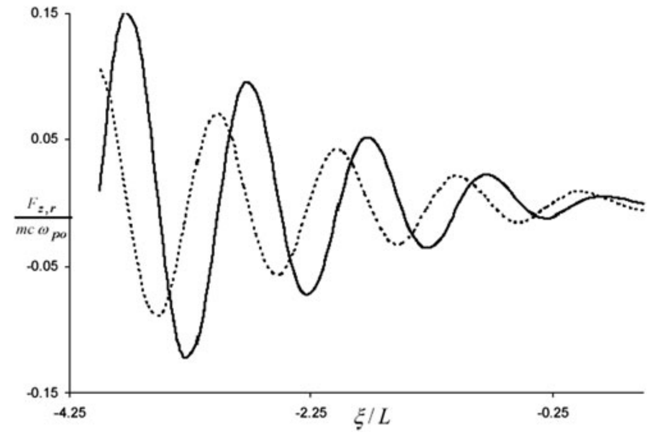

Fig. 1. Variation of the normalized force within the pulse $F_{z,r}/\mathrm{mc}\omega_{po}$ with ξ/L for $n_o = 10^{34}\ \mathrm{m}^{-3}$, $a_o^2 = 0.05$, $r_o = 15.0\ \mu\mathrm{m}$ and $r = 4.0\ \mu\mathrm{m}$.

The variation of the normalized force behind the pulse $F_{\mathrm{z,r}}/\mathrm{mc}\omega_{po}$ with ξ/L is shown in Figure 2 for the same parameters as in Figure 1. The solid line represents the axial white the dotted represents the transverse force. The force is greater in the axial direction then in the transverse direction. The accelerating force produced due to Wakefields in quantum plasma is greater than that produced in classical plasma for similar values of parameters.

5. SUMMARY AND DISCUSSION

We have studied in detail, the Wakefield generation by an intense laser pulse traveling through dense quantum plasma. We have used the electron continuity and electron momentum equation including the quantum statistical pressure and the quantum Bohm force, together with the Poisson equation to obtain the perturbed density and velocity. Equations for

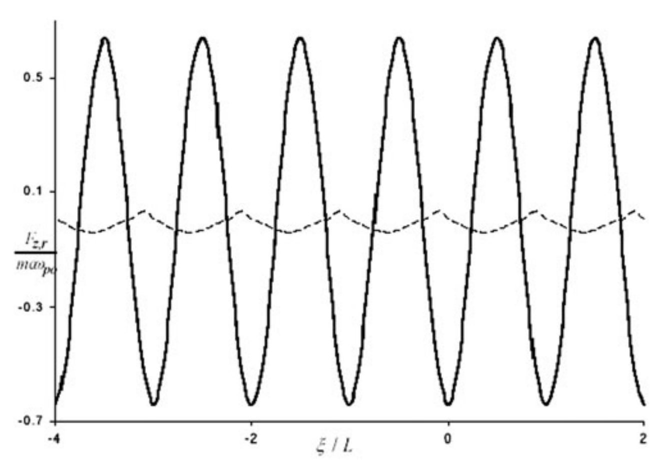

Fig. 2. Variation of the normalized force behind the pulse $F_{z,r}/\mathrm{mc}\omega_{po}$ with ξ/L for $n_o = 10^{34}\ \mathrm{m}^{-3}$, $a_o^2 = 0.05$,$r_o = 15.0\ \mu\mathrm{m}$ and $r = 4.0\ \mu\mathrm{m}$.

https://doi.org/10.1017/S026303461200002X Published online by Cambridge University Press

Wakefields are setup for radial Gaussian field amplitude. Electric and magnetic Wakefields have been derived with the help of time dependent Maxwell's equations using quasi-static approximation. It is found that ponderomotive non-linear effects, quantum force and quantum statistical pressure contribute to transverse as well as longitudinal Wakefield generation. However, nonlinear terms representing vortex motion contribute only to transverse velocity components. The zeroth and first order electric and magnetic Wakefields (both transverse and longitudinal) within and behind the pulse have been obtained for a sinusoidal pulse profile.

The maximum axial and transverse forces acting on a test electron due to the Wakefields have been evaluated. The variation of these forces with ξ/L have been studied graphically. It is found that even weak short pulses are capable of generating considerable Wakefields. The force behind the pulse is greater than within the pulse. Wakefields generated within the pulse are of the same order, whereas behind the pulse, axial field is greater than transverse. Simultaneous acceleration focusing is observed in the regime where $F_Z > 0$ and $F_r < 0$. The laser induced Wakefields can trap electrons and accelerate them to high energies at nanoscales in dense plasmas, such as those in the next generation intense laser-solid density plasma experiments, free electron lasers, plasmonic devices and in compact astrophysical objects (Harding & Lai, 2006).

REFERENCES

Andreev, N.E., Gorbunov, L.M., Kirsonov, V. I. & Pegosova, A.A. (1994). Laser Wakefield accelerator in a plasma pipe with self-modulation of the laser pulse. *JETP Lett*. **60**, 713–717.

Balakirev, V.A., Karas, V.I. & Levchenko, V.D. (2001). Plasma Wakefield excitation by relativistic electron bunches and charged particle acceleration in the presence of external magnetic field. *Laser Part. Beams* **19**, 597–604.

Bialynicki-Birula, L., Gornicki, P. & Rafelski, J. (1991). Phase-space structures of the Dirac vacuum. *Phys. Rev. D* **44**, 1825–1835.

Blumenfeld, I., Clayton, C.E., Decker, F.J., Hogan, M.J., Huang, C., Ischebeck, R., Iverson, R., Joshi, C., Katsouleas, T., Kirby, N., Lu, W., Marsh, K.A., Mori, W.B., Muggli, P., Oz, E., Siemann, R.H., Walz, D. & Zhou, M. (2007). Energy doubling of 42 GeV electrons in a metre-scale plasma Wakefield accelerator. *Nat*. **445**, 741–744.

Bret, A. (2007). Filamentation instability in a quantum plasma. *Phys. Plasmas* **14**, 084503.

Cao, J., Ren, H., Wu, Z. & Chu, P.K. (2008). Quantum effects on Rayleigh-Taylor instability in magnetized plasma. *Phys. Plasmas* **15**, 012110.

Esarey, E., Sprangle, P., Krall, J. & Ting, A. (1997). Self-focusing and guiding of short laser pulses in ionizing gases and plasmas. *IEEE J. Quantum Elec*. **33**, 1879–1914.

Faure, J., Glinec, Y., Pukhov, A., Kiselev, S., Gordienko, S., Lefebvre, E., Rousseau, J.P., Burgy, F. & Malka, V. (2004). A laser-plasma accelerator producing monoenergetic electron beams. *Nat*. **431**, 541–544.

Gardner, C.L. & Ringhofer, C. (1996). Smooth quantum potential for the hydrodynamic model. *Phys. Rev. E* **53**, 157–167.

Geddes, C.G.R., Toth, C., Tilborg, J.V., Esarey, E., Schroeder, C.B., Bruhwiler, D., Nieter, C., Cary, J. & Leemans, W.P. (2004). High-quality electron beams from a laser Wakefield accelerator using plasma-channel guiding. *Nat.* **431**, 538–540.

Gorbunov, L.M., Mora, P. & Solodov, A.A. (2003). Dynamics of a plasma channel created by the Wakefield of a short laser pulse. *Phys. Plasmas* **10**, 1124–1134.

Harding, A.K. & Lai, D. (2006). Physics of strongly magnetized neutron stars. *Rep. Prog. Phys*. **69**, 2631–2706.

Jha, P., Kumar, P., Upadhyaya, A.K. & Raj, G. (2005). Electric and magnetic Wakefields in a plasma channel. *Phys. Rev. Sp. Topics-Accel. Beams* **8**, 071301.

Joshi, C. (2007). The development of laser-and beam-driven plasma accelerators as an experimental field. *Phys. Plasmas* **14**, 055501.

Jung, Y.D. (2001). Quantum mechanical effects on electron-electron scattering in dense high-temperature plasmas. *Phys. Plasmas* **8**, 3842–3844.

Kawata, S., Kong, Q., Miyazaki, S., Miyauchi, K., Sonobe, R., Sakai, K., Nakashima, K., Masuda, S., Ho, Y.K., Miyanakga, N., Limpouch, J. & Andreev, A.A. (2005). Electron bunch acceleration and trapping by ponderomotive force of intense short-pulse laser. *Laser Part. Beams* **23**, 61–68.

Koyama, K., Adachi, M., Miura, E., Kato, S., Masuda, S., Watanabe, T., Ogata, A. & Tanimoto, M. (2006). Monoenergetic electron beam generation from a laser-plasma accelerator. *Laser Part. Beams* **24**, 61–68.

Kremp, D., Bornath, T.H., Bonitz, M. & Schlanges, M. (1999). Quantum kinetic theory of plasmas in strong laser fields. *Phys. Rev. E* **60**, 4725–4732.

Leemans, W.P., Nager, B., Gonsalves, A.J., Toth, C., Nakamura, K., Geddes, C.G.R., Esarey, E., Schroeder, C.B. & Hooker, S.M. (2006). GeV electron beams from a centimeter-scale accelerator. *Nat. Phys*. **2**, 696–699.

Lifshitz, A.F., Faure, J., Glinec, Y., Malka, V. & Mora, P. (2006). Proposed scheme for compact GeV laser plasma accelerator, *Laser Part. Beams* **24**, 255–260.

Lotov, K.V. (2001). Laser Wakefield acceleration in narrow plasma-filled channels. *Laser Part. Beams* **26**, 225–234.

Lourenco, S., Nicolas, K., Werner, S. & Wang, P.X. (2010). Acceleration of electrons and electromagnetic fields of highly intense laser pulses. *Laser Part. Beams* **28**, 195–201.

Luttikhof, M.J.H., Khachatryan, A.G., Van Goor, F.A., Boller, K.-J. & Mora, P. (2009). Electron bunch injection at an angle into a laser Wakefield. *Laser Part. Beams* **27**, 69–77.

Malka, V. & Fritzler, S. (2004). Electron and proton beams produced by ultra short laser pulses in the relativistic regime. *Laser Part. Beams* **22**, 399–405.

Malka, V., Fritzler, S., Lefebvre, E., Aleonard, M.-M., Burgy, F., Chambaret, J.-P., Chemin, J.-F., Krushelnick, K., Malka, G., Mangles, S.P.D., Najmudin, Z., Pittman, M., Rousseau, J.-P., Scheurer, J.-N., Walton, B. & Dangor, A.E. (2002). Electron acceleration by a wake field forced by an intense ultra-short laser pulse. *Sci*. **298**, 1596–1600.

Maltis, N.H., Reed, S., Bulanov, S.S., Chvykov, V., Kalintchenko, G., Matsuoka, T., Rousseau, P., Yanovsky, V., Maksimchuk, A., Kalmykov, S., Shvets, G. & Downer, M.C. (2006). Snapshots of laser Wakefields. *Nat. Phys*. **2**, 749–753.

Manfredi, G. (2005). How to model quantum plasmas. *Fields Inst. Comm*. **46**, 263–287.

Mangels, S.P.D., Murphy, C.D., Najmudin, Z., Thomas, A.G.R., Collier, J.L., Dangor, A.E., Divall, E.J., Foster, P.S., Gallacher, J.G., Hoker, C.J., Jaroszynski, D.A., Langley, A.J., Mori, W.B., Norreys, P.A., Tsung, F.S., Viskup, R., Walton, B.R. & Krushelnick, K. (2004). Monoenergetic beams of relativistic electrons from intense laser-plasma interactions. *Nat*. **431**, 535–537.

Markowich, P.A., Ringhofer, C. & Schmeiser, C. (1990). *Semiconductor Equations*. Vienna: Springer.

Marques, J.R., Geindre, J.P., Amiranoff, F., Audebert, P., Gauthier, J.C., Antonetti, A. & Grillon, G. (1996). Temporal and spatial measurements of the electron density perturbation produced in the wake of an ultrashort laser pulse. *Phys. Rev. Lett*. **76**, 3566–3569.

Masuda, S. & Miura, E. (2009). Generation and analysis of quasi-monoenergetic electron beams by laser-plasma interaction in traditional region from the self-modulated laser Wakefield to bubble accelerator regime. *Phys. Plasmas* **16**, 093105.

Phouc, K.T., Corde, S., Fitour, R., Shah, R., Albert, F., Rousseau, J.P., Burgy, F., Rousse, A., Seredov, V. & Pukhov, A. (2008). Analysis of Wakefield electron orbits in plasma wiggler. *Phys. Plasmas*, **15**, 073106.

Pukhov, A. & Meyer-Ter-Vehn, J. (2002). Laser Wakefield acceleration: The highly non-linear broken wave regime. *Appl. Phys. B: Lasers Opt*. **74**, 355–361.

Schlenovoigt, H.P., Haupt, K., Debus, A., Budde, F, Jackel, O., Pfotenhauer, S., Schwoerer, H., Rohwer, E., Gallacher, J.G., Brunetti, E., Shanks, R.P., Wiggins, S.M. & Jaroszynski, D.A. (2008). Compact synclotron radiation source driven by a laser-plasma Wakefield accelerator. *Nat. Phys*. **4**,130–133.

Shin, G. (1996). The derivation of hydrodynamic equation from quantum field theory. *J. Korean Phys. Soc.* **29**, 571–574.

Shukla, P.K., Brodin, G., Marklund, M. & Stenflo, L. (2009). Excitation of multiple Wakefields by short laser pulses in quantum plasmas. *Phys. Lett. A* **373**, 3165–3168.

Siders, C.W., Le Blane, S.P., Fisher, D., Tajima, T., Downer, M.C., Babine, A., Stepanov & Sergreev, A. (1996). Laser Wakefield excitation and measurement by Femtosecond longitudinal interferometry. *Phys. Rev. Lett*. **76**, 3570–3573.

Tajima, T. & Dawson, J.M. (1979). Laser electron accelerator. *Phys. Rev. Lett*. **43**, 267–270.

Takahashi, E., Honda, H., Miura, E., Yugami, N., Nishida, Y., Katsura, K. & Kondo, K. (2004). Observation of spatial asymmetry of THz oscillating electron plasma wave in a laser Wakefield. *Phys. Rev. E* **62**, 7247–7250.

Wang, W.-M., Sheng, Z.-M. & Zhang, J. (2009). Electron injection into laser Wakefields by colliding circularly-polarized laser pulses. *Laser Part. Beams* **27**, 3–7.

Xie, B.-S., Aimidula, A., Niu, J.-S, Liu, J. & Yu, M.Y. (2009). Electron acceleration in the Wakefield of asymmetric laser pulses. *Laser Part. Beams* **27**, 27–32.

Zhou, C.T., Yu, M.Y. & He, X.T. (2007). Electron acceleration by high current-density relativistic electron bunch in plasmas. *Laser Part. Beams* **25**, 313–319.