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THE PENDULAR–FUNICULAR LIQUID TRANSITION IN SNOW

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ABSTRACT. The way free water is distributed around contact points of ice grains is recorded by measuring the dielectric constant of the porous system which constitutes snow. Within the saturation range of 11% to 15% of the pore volume, a transitional range from the "pendular" to the "funicular" mode of distribution of free water occurs. By measuring the drainage-flow of free water through snow, additional information as to the upper limit of the pendular distribution is obtained. This upper limit is reached at a saturation of approximately 14% of the pore volume.

RÉSUMÉ. *La transition de la répartition pendulaire à la repartition funiculaire de l'eau libre de la neige.* La nature de la répartition de l'eau libre autour de contacts de grains de glace est établie en mesurant la constante diélectrique du système poreux de la neige. Dans la zone de saturation entre 11% et 15% du volume des pores se forme une zone de transition de la répartition "pendulaire" à la répartition "funiculaire" de l'eau libre. Par mesure du passage de l'eau libre au travers du milieu poreux neige, on obtient des informations supplémentaires sur la limite supérieure de la répartition pendulaire. Pour cette limite supérieure on obtient un degré de saturation d'environ 14% du volume des pores.

ZUSAMMENFASSUNG. *Der Übergang von der "pendular" in die "funicular" Verteilung des freien Wassers im Schnee.* Die Art der Anordnung des freien Wassers um Kontaktstellen von Eiskörnern wird durch die Messung der Dielektrizitätskonstante des porösen Systems Schnee erfasst. Im Sättigungsbereich von 11% bis 15% des Porenvolumens bildet sich eine Übergangszone von der "pendular" in die "funikular" Verteilung des freien Wassers aus. Durch Messung des Sickerflusses des freien Wassers durch das poröse Medium Schnee erhält man eine zusätzliche Information über die Obergrenze der "pendular" Verteilung. Für diese Obergrenze ergibt sich ein Sättigungswert von ungefähr 14% des Porenvolumens.

LIST OF SYMBOLS

g_1	depolarization factor
k	intrinsic permeability, m²
L	length of the snow column, m
n	exponent
S	saturation (water volume/pore volume)
S_0	initial saturation
S_i	irreducible saturation
S_0^*	effective saturation, $S_0^* = (S_0 - S_i)/(1 - S_i)$
t	time, s
u	water volume flux, m s^{-1}
u^*	relative volume flux
W	water content (water volume/total volume)
α	constant, m^{-1} s^{-1}
ϵ_∞	dielectric constant extrapolated for $f \rightarrow \infty$
ϵ_i	dielectric constant of ice
ϵ_w	dielectric constant of water at 0°C
ρ	density of wet snow, kg m^{-3}
ρ_i	density of ice, kg m^{-3}
ρ_w	density of water, kg m^{-3}
ϕ	porosity (pore volume/total volume)

INTRODUCTION

The water-saturation of a natural snow cover varies, in general, from zero to approximately 20% of the pore volume. In this case two essentially different types of distribution of free water can be observed: the pendular mode and the funicular mode of distribution. Pendular distribution covers the range of very low saturations corresponding to a water film on an ice grain adsorbed to a degree of saturation where the various isolated water menisci surrounding the ice grains flow together into complicated structures. Funicular distribution covers the range from the upper limit of the pendular mode to the total saturation of the pore volume. The change from the pendular to the funicular mode takes place at a saturation of between 10% and 20% of the pore volume. As there is a considerable change in the distribution of water during this transition from the pendular to the funicular mode, this also leads to a change in the physical characteristics of the snow-cover.

It is the purpose of this paper to determine the saturation regime in which the transition from the pendular to the funicular mode of water distribution in snow appears. For this purpose both the dielectric constant of wet snow and the drainage of water from snow columns were carefully analysed.

Dielectric measurements

A method particularly suitable for field measurements to find the distribution of free water in snow consists in measuring the dielectric constant. Theoretical studies have shown that the dielectric constant of heterogeneous mixtures is essentially influenced by the way the various components are distributed. Here the mixing formula of Polder and van Santen (1946) is particularly suitable for calculating the dielectric constant ϵ_∞ of the porous system which constitutes snow from the dielectric constant of the components ice, air, and water, from their parts per volume, and a depolarization factor (Denoth and Schittelkopf, 1978). The mixing formula of Polder and van Santen in an adapted form for the system of snow reads as follows:

$$\epsilon_\infty \left[1-\frac{W}{3}(\epsilon_w-\epsilon_d)\left(\frac{2}{\epsilon_\infty+(\epsilon_w-\epsilon_\infty)\,g_1}+\frac{1}{\epsilon_\infty+(\epsilon_w-\epsilon_\infty)(1-2g_1)}\right)\right]-\epsilon_d=0, \qquad (1)$$

where

$$\epsilon_d=\frac{3\epsilon_i(1-W)+2(1-\phi)\,\epsilon_i(\epsilon_i-1)}{3\epsilon_i(1-W)-(1-\phi)(\epsilon_i-1)}\,.$$

The geometrical configuration of water inclusions in the dry-snow matrix changes with increasing water content and so consequently does the depolarization factor g_1. In Figure 1 the depolarization factor g_1 calculated from the Polder and van Santen model (Equation (1)) is plotted against saturation S. The dielectric constant of snow samples with different liquid contents was measured by a capacitance bridge within a frequency range from 100 kHz

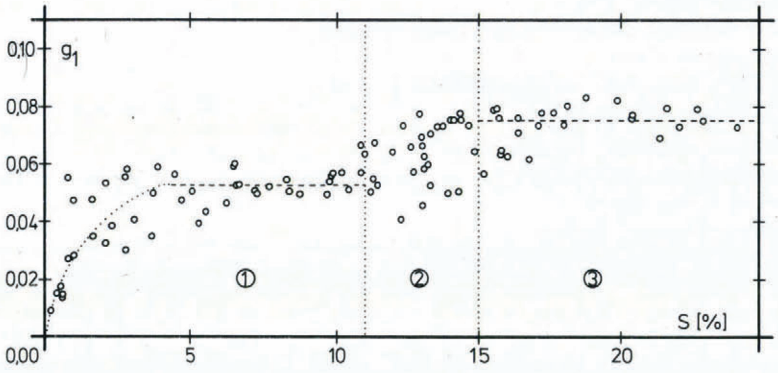

Fig. 1. The depolarization factor g_1 as a function of saturation S. 1: pendular regime; 2: transitional zone; 3: funicular regime.

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up to 20 MHz, and the dielectric constant ϵ_∞ required for Equation (1) was obtained by extrapolation for $f \to \infty$ according to the Cole–Cole model (Ambach and Denoth, 1972). The free-water content W was obtained from a freezing calorimeter with an accuracy of $\pm 0.5\%$ by volume. The density ρ of wet snow was found by weighing; porosity ϕ and saturation S were calculated from:

$$\phi = 1-(\rho-\rho_w W)/\rho_i,$$
$$S = W/\phi.$$

Figure 1 shows two significantly different arrangements of free water: within the saturation range $0 \leqslant S \leqslant 11\%$ with a depolarization factor $g_1 \approx 0.053$ an arrangement corresponding to the pendular mode of distribution, and within the saturation range of $15\% \leqslant S \leqslant 25\%$ with a depolarization-factor $g_1 \approx 0.076$ an arrangement corresponding to the funicular mode of distribution with complicated aggregates formed by the various water menisci flowing together. In between, i.e. within the saturation range from $11\% \leqslant S \leqslant 15\%$, a transitional range from the pendular into the funicular distribution is formed. In the case of very low saturations ($S < 4\%$) the depolarization factor decreases towards zero. This means, that in the case of such low saturations the shape of the water inclusions tends more and more to flat discs, forming a borderline case of oblate spheroids when $g_1 = 0$.

It is obvious that a change in water distribution, as mentioned above, affects the flow characteristics of water in snow. Consequently, additional information as to the upper limit of the pendular distribution can be obtained by measuring the dependence on saturation of flux of the water volume from a snow column. Therefore, drainage experiments were carried out using old, natural snow, repacked in tubes, with saturations of up to 18% of the pore volume. The outflowing water volume was continuously recorded by a capacitative level pick-up.

Drainage measurements

According to the gravity-flow theory (Colbeck, 1971) one obtains for the water volume flux u of a draining snow column with an initially constant saturation S_0 throughout the snow-column length (Denoth and others, 1979):

$$u = \begin{cases} u_0 = \alpha k S_0^{\star n}, & t \leqslant t_0, \\ [\phi L(1-S_i)/n(\alpha k)^{1/n}]^{n/(n-1)} t^{n/(1-n)}, & t_0 \leqslant t, \end{cases} \tag{2a}$$

where t_0 is that time in which a constant flux u_0 appears and is given by:

$$t_0 = L\phi(1-S_i)\, S_0^{\star 1-n}(n\alpha k)^{-1}.$$

For comparison purposes, however, it is practical to rewrite Equation (2a) in dimensionless form:

$$u^\star = u/u_0 = \begin{cases} 1, & t \leqslant t_0, \\ (t/t_0)^{n/(1-n)}, & t \geqslant t_0. \end{cases} \tag{2b}$$

In Figure 2 the relative water-volume flux $u^\star$ (represented by data points) from snow columns* with saturations of $S_0 = 11.6\%$ (symbol: ●), $S_0 = 12.7\%$ (symbol: △), and $S_0 = 14.5\%$ (symbol: ○) respectively is plotted against time (in units of t_0). As a change in flow characteristics is expected when water distribution changes, only drainage experiments with initial saturations corresponding to the transitional zone (cf. Fig. 1) are shown. In addition, the theoretical profile according to Equation (2b) is given (solid line). Satisfactory conformity

* Homogeneous old snow, repacked in tubes, with an average grain-size of 1 mm.

with the experiments is obtained only for saturations less than approximately 13%. For saturations higher than approximately 14%, a substantially higher flux becomes apparent at the beginning than one would have expected according to the gravity-flow theory. This can be explained by the formation of an additional flux in an unstable channel system with total saturation, formed by coalescing water menisci during the transition from the pendular to the funicular regime (Denoth and others, 1979). Consequently, a saturation of about 14% of the pore volume can be assumed as the upper limit of pendular distribution.

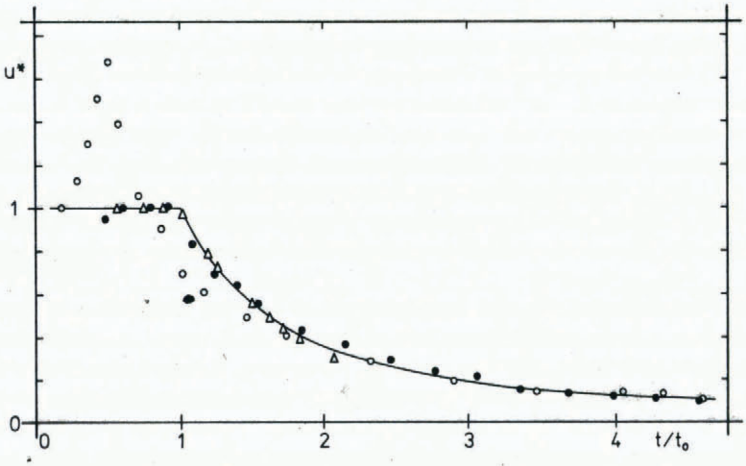

Fig. 2. Plot of relative water volume flux u against time (in units of t_0) according to the gravity-flow theory (Equation (2b)) compared with experimental data of relative volume flux from snow-columns with different initial saturations: $S_0 = 11.6\%$ (●), $S_0 = 12.7\%$ (△), and $S =_0 14.5\%$ (○).*

Discussion

By measuring the dielectric constant, a relationship between the distribution of water inclusions and the water saturation is obtained in the form of the depolarization factor g_1. In Equation (1) the depolarization factor accounts for the shape of inclusions:

$$g_1 = \tfrac{1}{2}a_1a_2a_3 \int_0^\infty [(a_1^2+s)(a_2^2+s)(a_3^2+s)]^{-\frac{1}{2}}(a_1^2+s)^{-1}\,ds,$$

a_1, a_2, and a_3 being the semi-axes of elliptic inclusions. Although the arrangement of free water around the contact points of ice grains (both in the pendular and in the funicular regime) does not appear in the form of ellipsoids or spheroids, it can be assumed that in the case of the low water saturations of a natural snow cover the arrangement of free water is somewhat disc-shaped. Thus the water menisci can approximately be described as oblate spheroids, so that the depolarization-factor g_1 does not reflect quantitatively the geometrical shape of the inclusions, but it offers information as to the relationship between the geometry of distribution and the water saturation.

In the case of greater saturations, several water menisci flow together into complicated water structures (funicular regime), as has already been mentioned. This can be seen from a change in the depolarizing factor from $g_1 \approx 0.053$ to $g_1 \approx 0.076$ when the saturation increases from 11% to 15%. Here it must be noted, however, that with a growing percentage by volume of water, electric interactions between the various water aggregates can no longer be neglected. This limits the application of the basic mixing formula (Equation (1)) to saturations smaller than 20%, which is the case for a natural snow cover.

The drainage experiments show that the flow characteristics of water change substantially if the saturation changes from 11% to 15%: for saturations higher than approximately 14% of the pore volume an additional flow in saturated channels is formed. A saturation of 11% is at the lower end, and the saturation of 15% at the upper end, of the transitional range from the pendular to the funicular regime.

From both the dielectric measurements and the drainage experiments it results that a saturation of about 14% can be assumed as the upper limit of the pendular mode of distribution of water in snow of an advanced degree of metamorphism. This broadly corresponds to an estimation of the upper limit of the pendular regime in snow as 14% of the pore volume given by Colbeck (1973) based on the theoretical work of Smith (1933). However it is likely that the value of the liquid content at the transition will vary with the degree of metamorphism of the snow.

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