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The influence of social networks and homophily on correct voting

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Abstract

There is empirical evidence suggesting that a person's family, friends, or social ties influence who a person votes for. Sokhey & McClurg (2012) find that as political disagreement in a person's social network increases, then a person is less likely to vote correctly. We develop a model where voters have different favorite policies and wish to vote correctly for the candidate whose favorite policy is closest to their own. Voters have beliefs about each candidate's favorite policy which may or may not be correct. Voters update their beliefs about political candidates based on who their conservative and liberal social ties are supporting. We find that if everyone's social network consists only of those most like themselves, then the conditions needed for correct voting to be stable are fairly weak; thus political agreement in one's social network facilitates correct voting. We also give conditions under which correct voting is stable for networks exhibiting homophily and for networks exhibiting random social interactions.

Keywords: *social networks, correct voting, homophily*

1 Introduction

The influence of social networks on voting was first studied by Lazarsfeld et al. (1944) who examined voting decisions in the 1940 presidential election in Erie County, Ohio. They found undecided voters were influenced by family and friends or by their social network. More recently, Sokhey & McClurg (2012) find that voters who cast ballots which are different from family and friends are less likely to vote correctly.[1] In fact, they find that as political disagreement increases in a person's social network, then that person is more likely to vote incorrectly. We develop a model where people update their beliefs about candidate's positions by observing which candidates their liberal and conservative neighbors are supporting and updating their beliefs accordingly. We find that if everyone's network consists only of those most like themselves then the conditions needed for correct voting to be stable are fairly weak.[2] Thus political agreement in one's social network facilitates correct voting as in Sokhey & McClurg (2012).

[1] A person is said to vote correctly if he votes the same way he would if he had full information regarding the candidates; (see Lau & Redlawsk, 1997; Lau et al., 2008).

[2] Specifically, stability requires conditions on a voter's initial beliefs as well as conditions on the favorite policies of the most conservative liberal voters and the most liberal conservative voters (given the favorite policies of the second most conservative liberal voter and the second most liberal conservative voter are fixed). However, conditions on the favorite policies of all other voters are not needed; in other networks stability of the correct vote requires restrictions on more voters' favorite policies.

Additionally, we find that voting correctly is more likely to be stable in a network exhibiting homophily (where people are more likely to interact with those similar to themselves and less likely to interact with those who are different) than in a random interactions network (where people have an equal probability of interacting with all others) if voters can be divided into two correct voting groups (say conservatives and liberals) where one group contains voters others consider to be radical. Then with homophily moderate voters will ignore radicals in their own group and vote correctly while with random interactions these radical voters are not ignored and thus moderates may vote incorrectly with the other voting group. Voting correctly is less likely to be stable in a homophily network than in a random interactions network if both voting groups contain agents others consider to be radical. Then with homophily moderates may ignore radicals in the other voting group but pay attention to radicals in their own group and thus may vote incorrectly with the other voting group. Note that in Western Europe researchers have found an increase in voter support of the radical right and a decrease in voter support of the radical left; (see Rydgren, 2007; Oesch, 2008; Mudde & March, 2005). Thus, we might expect here for voting correctly to have a greater likelihood of being stable assuming people have homophilic networks.

We are interested in how homophily in social networks affects who people vote for and specifically affects whether or not people vote correctly. Thus, we are interested in how homophily affects voters' decision making where homophily is defined as the tendency for people who are similar to associate with each other more frequently than they associate with others; this term was first coined by Lazarsfeld & Merten (1954). There is a previous literature which shows the existence of homophily based on race, gender, age, religion, education, social class, behavior, political ideology, and other characteristics. An extensive review of this literature is provided by McPherson et al. (2001); see also Jackson (2011). More recent studies have examined what causes homophily or how homophilic relationships form. (See Currarini et al., 2009; Currarini & Vega Redondo, 2011; Joyner & Kao, 2000; Kossinets & Watts, 2009). It is also important to understand how homophily affects decision making. Jackson & López-Pintado (2012) and Golub & Jackson (2012) examine how homophily affects the diffusion of a new idea and the speed of diffusion (respectively), while we show that the presence and absence of homophily can even affect correct voting decisions.

Empirical studies show that decisions on whether or not to vote and on who to vote for often are influenced by close social ties such as one's spouse, immediate family, or close personal friends (see Lazarsfeld et al., 1944; Nickerson, 2008). Our voting network model provides a mathematical framework representing these findings. Our approach is slightly different in that we ask how these ties influence voting and examine the affects of different network configurations on correct voting. There is a literature focusing on the flow of information from influencers or opinion leaders to those connected to them. Katz & Lazarsfeld (1955) and Lazarsfeld et al. (1944) were pioneers of this literature. More recent developments on the theory of influencers and their emergence can be found in Galeotti & Goyal (2010), see also Jackson (2008) for a literature review. We do not focus on the role of influencers, but instead focus on how the shape of the network affects correct voting.

Our model is related to those of information aggregation as in Golub & Jackson (2012), Acemoglu et al. (2010), and DeGroot (1974).[3] Our model is different in that it is a voting model not a consensus model and our diffusion process is different from the average based updating processes discussed in these information aggregation models. In average based updating a person's opinion of a new product is the average of his friend's opinions about the product. In our model, a person's beliefs about candidate X is an average of the favorite policies of her friends who support X not an average of her friends beliefs about X. For example, in a city council election a person decides that candidate X is conservative because her conservative neighbors are supporting him not because she learns her neighbor's specific beliefs about the candidate. Beliefs are modeled in this way to capture the notion that it may be easier to learn who a person supports than to learn why a person supports a certain candidate. Note that modeling beliefs this way allows agents to obtain information that may be mistaken. It is possible for an agent's friend to mistakenly be supporting the wrong candidate thus using the friend's favorite policy as a proxy for the candidate's favorite policy will be inaccurate. We are interested in when do agents vote correctly even amid the possibility of mistaken information.

There exists a large voting literature which answers questions such as why do people vote (Feddersen, 2004; Feddersen & Sandroni, 2006; Evren, 2012) as well as normative questions such as which voting policies or rules are socially beneficial (Krasa & Polborn, 2009; Börgers, 2004), does the correct candidate win (Young, 1988; Krishna & Morgan, 2012; McMurray, 2012), and is an efficient outcome achieved (Besley & Coate, 1997; Krishna & Morgan, 2012). We add to this debate in that we also examine a normative question of do people vote correctly; however, our paper is different from this voting literature in that voters are modeled in a network where voters update their beliefs about candidates based on information received from their social networks.

Lastly, correct voting is related to sincere voting; see Austen-Smith & Banks (1996), Groseclose & Milyo (2010; 2013), and Wolitzky (2009) for an analysis of sincere voting. A person votes sincerely if he votes for the candidate he thinks is best, while a person votes correctly if he votes for the candidate that he thinks is best when he has complete information regarding the candidates. Thus, a sincere vote may not be a correct vote. In our model, agents are myopic (as in Golub & Jackson (2012) and Acemoglu et al. (2010)) in that when they update their candidate support they do not consider how this update will affect other voters' candidate support. Thus, in our model agents always vote sincerely, but they do not always vote correctly.

The paper proceeds as follows. We present the model in Section 2 and the results in Section 3. Conclusions are provided in Section 4.

[3] Krishna & Morgan (2012) and McMurray (2012) consider information aggregation in a voting context however in their models information is aggregated when players vote instead of in a diffusion of information process.

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2 Model

There is a set $C = \{c_1, c_2\}$ of two candidates who are running for election. There is a set $N = \{1, 2, \ldots, n\}$ of voters who interact over a social network, g to exchange information regarding the candidates. A link in g between $i \in N$ and $j \in N$, $i \neq j$, is represented by $ij \in g$. We assume for each $i \in N$ there exists at least one $j \neq i$, $j \in N$ such that $ij \in g$. Thus, each voter has at least one neighbor.

Each candidate $c_k \in C$ has a favorite policy $y_k \in [0,1]$. At time 0, each voter $i \in N$ has an initial belief about c_k's favorite policy y_k represented by $y_k^i(0) \in [0,1]$. Each voter i has favorite policy $x_i \in [0,1]$, which is known to himself and to all of his friends and neighbors. We order voters so that $x_1 < x_2 < \ldots < x_n$ and let $x \equiv \{x_1, x_2, \ldots, x_n\}$. At time 0, voter i supports candidate $c^i(0) = c_k$ if $k = \arg\min_\ell\{|\ y_\ell^i(0) - x_i\ |\}$. If multiple k solve the minimization problem, then i selects one of these candidates at random to support.

Time is divided into periods $T \equiv \{0, 1, 2, \ldots, t, \ldots\}$ and at each period every agent i interacts with each of his neighbors. The strength of i's interaction with his neighbor j is measured by p_{ij} where $p_{ij} > 0$ if and only if $ij \in g$. If $ij \notin g$, then $p_{ij} = 0$. If $p_{ij} > p_{i\ell}$ we say that i spends more time interacting with j than with ℓ or that j has a bigger influence on i than does ℓ. Additionally, we assume $\sum_{j=1}^n p_{ij} = 1$ and $p_{ii} = 0$. Let $p_i \equiv \{p_{i1}, p_{i2}, \ldots, p_{in}\}$ and $p \equiv \{p_1, p_2, \ldots, p_n\}$.

A network g with corresponding interaction strengths p exhibits *homophily*[4] if $p_{ij} > p_{ik}$ for all $|x_i - x_j| < |x_i - x_k|$ where $i \neq j$ and $i \neq k$. Thus, players in these networks spend more time interacting with those similar to themselves. A network g exhibits *random interactions* if $p_{ij} = \frac{1}{n-1}$ for all $i \neq j$. Thus, here each player interacts equally with all other players. A network is called a *line network* if $p_{ij} = \frac{1}{2}$ for all $j = i+1$, $j = i-1$ and $i \in \{2, 3, \ldots, n-1\}$; $p_{ij} = 1$ for $i = 1$, $j = 2$ or $i = n$, $j = n-1$; and $p_{ij} = 0$ for all other $i, j \in N$. In a line network, each $i \in \{2, 3, \ldots, n-1\}$ is connected only to $i+1$ and $i-1$ and interacts with each neighbor equally. If $\max\{|x_i - x_{i-1}|, |x_i - x_{i+1}|\} < |x_i - x_j|$ for all $i \in N$ and $j \notin \{i-1, i, i+1\}$, then the line network exhibits homophily.

Define $S_k^i(t) = \{j | c^j(t) = c_k,\ j \neq i \text{ and } ij \in g\}$ as the set of i's neighbors who support k at time t. Define $|S_k^i(t)|$ as the number of i's neighbors who support k at time t.

Players observe who their neighbors support and they observe their neighbors' favorite policies, but they do not observe their neighbors' beliefs. Players use these observations to update their beliefs regarding candidate favorite policies as follows. At time $t \geqslant 1$, i updates his beliefs such that for all $k \in \{1, 2, \ldots, m\}$

$$y_k^i(t) = \frac{y_k^i(t-1) + \frac{\sum_{j \in S_k^i(t-1)} p_{ij} x_j}{\sum_{j \in S_k^i(t-1)} p_{ij}}}{2} \qquad \text{if } S_k^i(t-1) \neq \varnothing \text{ and}$$

$$y_k^i(t) = y_k^i(t-1) \qquad \text{if } S_k^i(t-1) = \varnothing$$

Thus if i has neighbors who support k in the previous period, then i updates his beliefs about k by taking an average of his previous period beliefs with a weighted

[4] Our notion of homophily is more simplistic than that of Golub & Jackson (2012) who define a measure of the degree of homophily based on the second largest eigenvalue of a matrix of relative densities of links between various groups.

average of the favorite policies of his neighbors who support k.[5] If none of i's neighbors support k in the previous period then i keeps his $(t-1)$ beliefs. Let $y^i(t) \equiv \{y^i_1(t), y^i_2(t), \ldots, y^i_m(t)\}$ and $y(t) \equiv \{y^1(t), y^2(t), \ldots, y^n(t)\}$.

At the end of time t, voter i updates his candidate preferences and will support candidate $c^i(t) = c_k$ if $k = \arg\min_\ell\{| y^i_\ell(t) - x_i |\}$. If multiple candidates solve this minimization and if $c^i(t-1) = c_{\widetilde{k}}$ where $\widetilde{k} \in \arg\min_\ell\{| y^i_\ell(t) - x_i |\}$ then $c^i(t) = c_{\widetilde{k}}$ otherwise i will pick a candidate that solves the minimization at random to support. Let $c(t) \equiv \{c^1(t), c^2(t), \ldots, c^n(t)\}$.

Voter beliefs y are *stable* if $y^i_k(t) = y^i_k$ implies $y^i_k(t+1) = y^i_k$ for all $i \in N$, $k \in \{1, 2, \ldots, m\}$ and all t. Thus no individual voter will update his beliefs.

Candidate support c is *stable* if the corresponding voter beliefs y are stable. Note that if candidate support is stable then no individual voter will change his candidate choice or support.

Agent i votes *correctly* at time t if $c^i(t) = c_k$ and $| x_i - y_k | \leqslant | x_i - y_j |$ for all $j \neq k$.

We say that voting correctly is stable as $t \to \infty$ if voter beliefs $y(t)$ converge to stable voter beliefs y as $t \to \infty$, in which every voter votes correctly.

Agent i is an *expert* if $y^i_k(t) = y_k$ for all k and t. Note that an expert will always vote correctly and an expert's beliefs are not influenced by his friends.

In the appendix, we examine a variant of the model in which agents always pay some attention to initial beliefs:

$$y^i_k(t) = \alpha(y^i_k(0)) + (1-\alpha)\left(\frac{y^i_k(t-1) + \frac{\sum_{j\in S^i_k(t-1)} p_{ij}x_j}{\sum_{j\in S^i_k(t-1)} p_{ij}}}{2}\right) \qquad \text{if } S^i_k(t-1) \neq \varnothing \text{ and}$$

$$y^i_k(t) = \alpha(y^i_k(0)) + (1-\alpha)(y^i_k(t-1)) \qquad \text{if } S^i_k(t-1) = \varnothing$$

for $0 \leqslant \alpha \leqslant 1$. Such a model changes the conditions presented in the propositions as would be expected in that the results become dependent on initial beliefs; however, the results are not substantially changed. Therefore, for simplicity we focus on the $\alpha = 0$ case in the main text and refer the interested reader to the appendix.

3 Results

For the first proposition, we fix the candidate choices of all agents other than i at $c^j(t) = c^j(0)$ for all $t \geqslant 0$ and all $j \in N$, $j \neq i$. Then we ask whether or not $y^i_k(t)$ converges as $t \to \infty$ for all $k \in \{1, 2, \ldots, m\}$. Note that although we focus on the two candidate case, Proposition 1 also holds true for the m candidate case.

Proposition 1

Let $c^j(t) = c^j(0)$ for all $t \geqslant 0$ and all $j \in N$, $j \neq i$. If $S^i_k(0) \neq \varnothing$, then $y^i_k(t) \to \frac{\sum_{j\in S^i_k(0)} p_{ij}x_j}{\sum_{j\in S^i_k(0)} p_{ij}}$ as $t \to \infty$. If $S^i_k(0) = \varnothing$, then $y^i_k(t) = y^i_k(0)$ for all $t \geqslant 0$.

[5] Alternatively, we could assume that voter i learns something about k if i's neighbor j does not support k. In the two candidate case, i could assume that his neighbor j dislikes k if j does not support k. Perhaps i might believe that y_k is closer to $(1-x_j)$ than to x_j since j does not support k and i could update his beliefs about k accordingly. Such an updating rule should not qualitatively affect the results but would make them more complicated; for simplicity, we focus on the case where i updates his beliefs about k based on the favorite policies of his neighbors who support k.

Proof. First consider the case where $S_k^i(0) \neq \emptyset$. Note that since $c^j(t) = c^j(0)$ for all t and $j \neq i$, it must be that $S_k^i(0) = S_k^i(t)$. Let

$$Y_k^i(t) \equiv \begin{bmatrix} y_k^i(t) \\ x_1 \\ x_2 \\ \vdots \\ x_{i-1} \\ x_{i+1} \\ \vdots \\ x_n \end{bmatrix}$$

and

$$T_k^i \equiv \begin{bmatrix} \frac{1}{2} & \frac{p_{i1}}{2\sum_{j\in S_k^i(0)} p_{ij}} & \cdots & \frac{p_{i(i-1)}}{2\sum_{j\in S_k^i(0)} p_{ij}} & \frac{p_{i(i+1)}}{2\sum_{j\in S_k^i(0)} p_{ij}} & \cdots & \frac{p_{in}}{2\sum_{j\in S_k^i(0)} p_{ij}} \\ 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \dots & 0 \\ \vdots & & & \ddots & & & \vdots \\ 0 & 0 & \dots & & \dots & 0 & 1 \end{bmatrix}.$$

Then

$$Y_k^i(t) = T_k^i \cdot [Y_k^i(t-1)]$$

and

$$Y_k^i(t) = (T_k^i)^t \cdot Y_k^i(0).$$

Thus, if $(T_k^i)^t$ converges as $t \to \infty$, then $y_k^i(t)$ converges. It is easy to show that

$$(T_k^i)^t = \begin{bmatrix} (\frac{1}{2})^t & \frac{\alpha p_{i1}}{2\sum_{j\in S_k^i(0)} p_{ij}} & \cdots & \frac{\alpha p_{i(i-1)}}{2\sum_{j\in S_k^i(0)} p_{ij}} & \frac{\alpha p_{i(i+1)}}{2\sum_{j\in S_k^i(0)} p_{ij}} & \cdots & \frac{\alpha p_{in}}{2\sum_{j\in S_k^i(0)} p_{ij}} \\ 0 & 1 & 0 & 0 & \dots & & 0 \\ 0 & 0 & 1 & 0 & \dots & & 0 \\ \vdots & & & \ddots & & & \vdots \\ 0 & 0 & \dots & & \dots & 0 & 1 \end{bmatrix}.$$

where $\alpha = \sum_{\ell=0}^{t-1}(\frac{1}{2})^\ell$.

Here $(\frac{1}{2})^t \to 0$ as $t \to \infty$ and $\sum_{\ell=0}^{t-1}(\frac{1}{2})^\ell \to 2$. Thus, $y_k^i(t) \to \frac{\sum_{j\in S_k^i(0)} p_{ij}x_j}{\sum_{j\in S_k^i(0)} p_{ij}}$ as $t \to \infty$.

Next consider the case where $S_k^i(0) = \emptyset$. By definition, i never updates his beliefs about k and so $y_k^i(t) = y_k^i(0)$ for all $t \geqslant 0$.

Proposition 1 shows that i's beliefs about candidate k converge to a weighted average of his neighbor's favorite policies and do not depend at all on i's initial beliefs about k. Additionally, if none of i's neighbors ever support candidate k then i will keep his initial beliefs about k and will not update them.

Corollary 1
Let $c^j(t) = c^j(0)$ for all $t \geqslant 0$ and all $j \in N$, $j \neq i$ and let $p_{ij} = \frac{1}{n-1}$ for all i, j. If $S_k^i(0) \neq \emptyset$, then as $t \to \infty$

$$y_k^i(t) \to \frac{\sum_{j\in S_k^i(0)} x_j}{|S_k^i(0)|}.$$

If candidate choice for all agents except for i is fixed and if all agents interact with equal probability, then i's expected beliefs regarding candidate k converge to a simple average of the favorite policies of i's neighbors who support k.

Note from Proposition 1 that if i is only linked to j and if j always supports k then $y_k^i(t) \to x_j$ as $t \to \infty$. Thus, by changing the j that i is linked to we change what i's beliefs converge to. So the configuration of the network and/or the interaction strengths are quite important in determining how beliefs converge[6] which is why we focus most of our results on random networks and homophilic networks as many empirical networks have evidence of these types of interactions (see McPherson et al., 2001).

For the remainder of the paper, we make the following assumption. Assume that if all agents vote correctly then $\{1, 2, \ldots, i\}$ vote for c_1 and $\{i+1, i+2, \ldots, n\}$ vote for c_2 or that $x_i < \frac{y_1+y_2}{2}$ and $x_{i+1} > \frac{y_1+y_2}{2}$ for some $i \in \{2, .., n-2\}$.[7]

Proposition 2
Let the network be a line network and let $(x_i - x_{i-1}) < (x_{i+1} - x_i)$ and $(x_{i+2} - x_{i+1}) < (x_{i+1} - x_i)$. Let $y_2^j(0) \notin [x_{j-1}, x_{j+1}]$ for all $1 < j < i$ and $y_1^j(0) \notin [x_{j-1}, x_{j+1}]$ for all $i+1 < j < n$ and let $y_k^1(0) \notin [x_1, x_2]$ and $y_k^n(0) \notin [x_{n-1}, x_n]$. Then voting correctly is stable as $t \to \infty$. Furthermore, voting correctly is uniquely stable if i and $(i+1)$ are experts and if $(x_{j+1} - x_j) < (x_j - x_{j-1})$ for all $j \in \{1, 2, \ldots, i-1\}$ and $(x_j - x_{j-1}) < (x_{j+1} - x_j)$ for all $j \in \{i+2, \ldots, n\}$ as $t \to \infty$.

Proof. First, we show that voting correctly is stable. Assume t is sufficiently large so that the convergence results of Proposition 1 hold true.[8] We show that if all agents $k \neq j$ vote correctly, then so will agent $j \in N$, $j \neq i, i+1$. First let $1 < j < i$. As all other agents vote correctly, agents $(j+1)$ and $(j-1)$ must both vote for c_1. Therefore, from Proposition 1 $y_1^j(t) \to \frac{x_{j-1}+x_{j+1}}{2}$ as $t \to \infty$ and $y_2^j(t) = y_2^j(0)$. By assumption, $y_2^j(0) \notin [x_{j-1}, x_{j+1}]$, then it must be that $\mid y_2^j(0) - x_j \mid > \mid \frac{x_{j-1}+x_{j+1}}{2} - x_j \mid$. Thus, agent j prefers to vote for c_1. If $j = 1$, then $y_1^j(t) \to x_2$ and by assumption $\mid y_2^j(0) - x_1 \mid > \mid x_2 - x_1 \mid$ and j will vote for c_1. Similarly, $j > (i+1)$ will choose to vote for c_2. Next we show that i will vote for c_1 given that $i-1$ and $i+1$ both vote correctly for c_1 and c_2 respectively. By Proposition 1, $y_1^i(t) \to x_{i-1}$ and $y_2^i(t) \to x_{i+1}$. Thus i votes correctly for c_1 if $\mid y_1^i(t) - x_i \mid < \mid y_2^i(t) - x_i \mid$ or if $|x_{i-1} - x_i| < |x_{i+1} - x_i|$ which is true by assumption. Similarly, $i+1$ will vote correctly for c_2.

Next, we show that voting correctly is the unique stable outcome given the conditions stated in the proposition. Assume t is sufficiently large. As i and $i+1$ are experts we know i votes for c_1 and $i+1$ votes for c_2. Next consider agent $(i-1)$. If $(i-2)$ votes for c_1, then $i-1$ will vote for c_1 since $y_1^{i-1}(t) \to \frac{x_{i-2}+x_i}{2}$ as $t \to \infty$ and

[6] In fact if $c^j(t) = c^j(0)$ for all t and $j \neq i$, then there exists a network and interaction strengths p_i such that $y_k^i(t)$ converges to any specific number in the convex hull of $\{x_j\}$ for j who initially support k.

[7] We assume $i \notin \{1, n-1\}$ or that when agents vote correctly at least two agents vote for each candidate. This assumption is made for simplicity as including the case of $i \in \{1, n-1\}$ does not change the ensuing propositions except to add extra conditions on the initial beliefs of agents 1 and n.

[8] We assume that t is sufficiently large, say $t > T$. Players interact in the first T periods, but may or may not vote correctly. After period T we show that if all other players vote correctly then so will j. Additional conditions on initial beliefs could be added to the proposition which would guarantee that voting correctly is stable for all t. However, these conditions would add to the length of the proposition without changing its essence. (Such conditions would require for instance that agent $j < i$ prefer to vote for 1 even in period 0 which would require $|y_1^j(0) - x_j| < |y_2^j(0) - x_j|$.) Instead of adding such conditions we choose to focus on the $t > T$ case so that the results of Proposition 1 can be applied.

$y_2^{i-1}(t) = y_2^{i-1}(0) \notin [x_{i-2}, x_i]$. If $i-2$ votes for c_2, then $y_1^{i-1}(t) \to x_i$ and $y_2^{i-1}(t) \to x_{i-2}$ as $t \to \infty$. Since $(x_i - x_{i-1}) < (x_{i-1} - x_{i-2})$, then $i-1$ votes for c_1. Similarly, $i-2$ will choose to vote for c_1 as will all $j \in \{2, \ldots, i-1\}$. Next consider $j-1$. Since agent 2 votes for c_1 we know $y_1^1(t) \to x_2$ as $t \to \infty$. Since $y_2^1(t) = y_2^1(0) \notin [x_1, x_2]$ then 1 votes for c_1. Similar reasoning shows all $j \geqslant i+2$ will vote for c_2. Thus, correct voting is the unique stable outcome.

In Proposition 2, voting correctly is stable since each person $j \notin \{i, i+1\}$ prefers to vote with his two neighbors rather than against them as they have similar views to j's views. And agent i (respectively, $i+1$) prefers to vote with $i-1$ (resp., $i+2$) over $i+1$ (resp. i), thus i (resp. $i+1$) will vote correctly if $i-1$ (resp. $i+2$) does. Voting correctly is the unique stable outcome if every agent has more in common with or prefers to vote with his neighbor who is closest to the expert rather than with his other neighbor. Note that the assumptions needed for correct voting to be stable are fairly weak as they simply require that each agent j initially believes that his incorrect candidate's favorite policy is a sufficient distance from j's own as well as conditions on the location of the favorite policies of voters i and $i+1$ who are the voters where the correct vote changes (as i should correctly vote for 1 and $i+1$ for 2).

Note that experts are needed to guarantee that the correct vote is the unique stable outcome. Experts always vote correctly and their presence allows other voters to learn how to vote correctly. If there are no experts then having everyone vote opposite of the correct vote where $\{1, 2, \ldots, i\}$ vote for c_2 and $\{i+1, \ldots, n\}$ vote for c_1 would also be stable.

Proposition 3

Let the network exhibit random interactions. Voting correctly is stable if $|\frac{\sum_{k=1}^{i-1} x_k}{i-1} - x_i| < |\frac{\sum_{k=i+1}^{n} x_k}{n-i} - x_i|$ and $|\frac{\sum_{k=i+2}^{n} x_k}{n-i-1} - x_{i+1}| < |\frac{\sum_{k=1}^{i} x_k}{i} - x_{i+1}|$ as $t \to \infty$.

Proof. For $j \leqslant i$ and t is sufficiently large, if all other players vote correctly, then j will also vote correctly if $|y_1^j(t) - x_j| < |y_2^j(t) - x_j|$. By Proposition 1 as $t \to \infty$, $y_1^j(t) \to \frac{\sum_{k=1,k\neq j}^{i} x_k}{i-1}$ and $y_2^j(t) \to \frac{\sum_{k=i+1}^{n} x_k}{n-i}$. Thus, j votes correctly if $|\frac{\sum_{k=1,k\neq j}^{i} x_k}{i-1} - x_j| < |\frac{\sum_{k=i+1}^{n} x_k}{n-i} - x_j|$ which is true by assumption for $j = i$. Next we show that if $|\frac{\sum_{k=1}^{i-1} x_k}{i-1} - x_i| < |\frac{\sum_{k=i+1}^{n} x_k}{n-i} - x_i|$ then $|\frac{\sum_{k=1,k\neq j}^{i} x_k}{i-1} - x_j| < |\frac{\sum_{k=i+1}^{n} x_k}{n-i} - x_j|$ for $j < i$. First assume that $x_j \geqslant \frac{\sum_{k=1,k\neq j}^{i} x_k}{i-1}$. Since $x_1 < x_2 < \ldots < x_j < \ldots < x_i$ it must be that $\frac{\sum_{k=1}^{i-1} x_k}{i-1} < \frac{\sum_{k=1,k\neq j}^{i} x_k}{i-1}$ and that $x_i - \frac{\sum_{k=1}^{i-1} x_k}{i-1} > x_j - \frac{\sum_{k=1,k\neq j}^{i} x_k}{i-1} \geqslant 0$. And so $|\frac{\sum_{k=1,k\neq j}^{i} x_k}{i-1} - x_j| < |\frac{\sum_{k=1}^{i-1} x_k}{i-1} - x_i| < |\frac{\sum_{k=i+1}^{n} x_k}{n-i} - x_i| < |\frac{\sum_{k=i+1}^{n} x_k}{n-i} - x_j|$ and j will vote correctly. Second assume to the contrary that $x_j < \frac{\sum_{k=1,k\neq j}^{i} x_k}{i-1}$. Then $x_j < \frac{\sum_{k=1,k\neq j}^{i} x_k}{i-1} < \frac{\sum_{k=i+1}^{n} x_k}{n-i}$ and so $|\frac{\sum_{k=1,k\neq j}^{i} x_k}{i-1} - x_j| < |\frac{\sum_{k=i+1}^{n} x_k}{n-i} - x_j|$ and again j votes correctly. Similarly analysis shows that if $j > i$ and if everyone else votes correctly, then j prefers to vote correctly as well.

With random interactions each person votes correctly given that others do if each person has more in common with the average favorite policies of those in his own

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group than with the average favorite policies of those in the other group.[9] Comparing Proposition 2 to Proposition 3, we see that Proposition 2 involves conditions on favorite policies guaranteeing that i and $i+1$ vote correctly and that these conditions only restrict x_{i-1}, x_i, x_{i+1}, and x_{i+2}. However, Proposition 3 involves conditions on favorite policies guaranteeing that i and $i+1$ vote correctly and each of these conditions restricts all of the x_j's. Thus when interactions are uniformly at random and not within the line network, correct voting requires restrictions involving all of the x_j's instead of restrictions involving only x_{i-1}, x_i, x_{i+1}, and x_{i+2}.

Proposition 4

There exists g, p that exhibits homophily and such that the correct vote with homophily is stable as $t \to \infty$ if for all $j \leqslant i$ there exists $1 \leqslant \ell \leqslant j$ such that $|\frac{\sum_{k=\ell,k\neq j}^{i} x_k}{i-\ell} - x_j| < |\frac{\sum_{k=i+1}^{n} x_k}{n-i} - x_j|$ and if $\ell > 1$, then $|x_{\ell-1} - x_j| > |x_n - x_j|$. And for all $j \geqslant i+1$ there exists $j \leqslant \ell \leqslant n$ such that $|\frac{\sum_{k=i+1,k\neq j}^{\ell} x_k}{\ell-i} - x_j| < |\frac{\sum_{k=1}^{i} x_k}{i} - x_j|$ and if $\ell < n$, then $|x_{\ell+1} - x_j| > |x_1 - x_j|$.

Proof. We show that there exists p such that g, p exhibits homophily and such that the correct vote with homophily is stable. Assume t is sufficiently large. Consider any agent $j \leqslant i$. If $\ell \geqslant 2$, then let $p_{jk} = \epsilon_k > 0$ for all $k \in \{1, 2, \ldots, \ell-1\}$ and $k \neq j$ and if $\ell = 1$, then let $p_{j1} = \frac{1}{n} + \epsilon_1$ where $\epsilon_1 > 0$. For all other p_{jk}, $k \in \{1, 2, \ldots, i\}$ and $k \neq j$, let $p_{jk} = \frac{1}{n-\ell+1} + \epsilon_k$ where $\epsilon_k < 0$. Size the ϵ_k's so that $\sum_{k=1,k\neq j}^{i} \epsilon_k = 0$ and so that $p_{jk} > p_{jm}$ for all $|x_k - x_j| < |x_m - x_j|$ and $k, m \in N$ and $k \neq j, m$. We assume $|x_{\ell-1} - x_j| > |x_n - x_j|$ so that only $x_{\ell-1}$ not close to x_j is given a weight of $\epsilon_{\ell-1}$ which is needed for homophily. We will show that if all other agents vote correctly, then so will agent j. Agent j votes correctly if $|y_1^j(t) - x_j| < |y_2^j(t) - x_j|$. By Proposition 1, $y_1^j(t) \to \frac{\sum_{k=1,k\neq j}^{i} p_{jk} x_k}{\sum_{k=1,k\neq j}^{i} p_{jk}}$ and $y_2^j(t) \to \frac{\sum_{k=i+1}^{n} p_{jk} x_k}{\sum_{k=i+1}^{n} p_{jk}}$ as $t \to \infty$. From the way we defined the p_{jk}'s, we know that $\frac{\sum_{k=1,k\neq j}^{i} p_{jk} x_k}{\sum_{k=1,k\neq j}^{i} p_{jk}} = \frac{\sum_{k=1}^{\ell-1} \epsilon_{jk} x_k + \sum_{k=\ell,k\neq j}^{i} (\frac{1}{n-\ell+1} + \epsilon_k) x_k}{\sum_{k=1}^{\ell-1} \epsilon_{jk} + \sum_{k=\ell,k\neq j}^{i} (\frac{1}{n-\ell+1} + \epsilon_k)} \approx \frac{\sum_{k=\ell}^{i} x_k}{i-\ell}$ for ϵ_k's small enough. Similarly, $\frac{\sum_{k=i+1}^{n} p_{jk} x_k}{\sum_{k=i+1}^{n} p_{jk}} = \frac{\sum_{k=i+\ell}^{n} (\frac{1}{n-\ell+1} + \epsilon_k) x_k}{\sum_{k=i+\ell}^{n} (\frac{1}{n-\ell+1} + \epsilon_k)} \approx \frac{\sum_{k=i+\ell}^{n} x_k}{n-i}$ for ϵ_k's small enough. Thus, j votes correctly if $|\frac{\sum_{k=\ell}^{i} x_k}{i-\ell} - x_j| < |\frac{\sum_{k=i+\ell}^{n} x_k}{n-i} - x_j|$ which is true by assumption. Similarly, one can find p_j for $j > i$ such that the assumptions of homophily are met and such that j votes correctly given that everyone else votes correctly.

Proposition 4 states that there exists a homophily network such that each player votes correctly given others do if each agent j has more in common with the average favorite policies of those in his own group (excluding any agents in j's group that j considers radical[10]) than with the average favorite policies of those in the other

[9] Note that with random interactions if i and $i+1$ are both experts it is still difficult to guarantee that voting correctly is the unique stable outcome. To ensure uniqueness, any voting configuration where a subset of non-expert agents vote incorrectly must be unstable or have at least one agent who does not want to vote the perscribed way. Thus with n voters the number of such subsets is quite large and so the number of conditions needed to guarantee uniqueness is quite large. In the line network, we can rule out such a scenario as each voter interacts with at most two other voters and so the expert's influence can spread to his neighbors and then to his neighbor's neighbors, etc. With random interactions each voter interacts with every other voter equally thus the influence of the two experts on any other voter or on the rest of the network is much smaller and so the correct vote may not always spread and thus the correct vote may not be the only stable vote.

[10] Let $j \leqslant i$. If in Proposition 4 there exists $\ell > 1$, then we say that j considers $\{1, \ldots, \ell-1\}$ to be "radical" agents. From j's point of view these agents have ideal points that are far away from j's ideal

group. Comparing Propositions 3 and 4, if correct voting with random interactions is stable, then there exists a network with homophily where correct voting is stable. However, this does not guarantee that correct voting in all homophily networks is stable. Similarly, stability of correct voting in a network with homophily does not guarantee stability of correct voting in a network with random interactions. The next two propositions illustrate these points.

Proposition 5
Let correct voting be stable with random interactions as $t \to \infty$ and assume there exists either a $j \leqslant i$ and a $1 \leqslant \ell < n-i$ such that $|\frac{\sum_{k=i+1}^{i+\ell} x_k}{\ell} - x_j| < |\frac{\sum_{k=1,k\neq j}^{i} x_k}{i-1} - x_j|$ and $|x_{i+\ell+1} - x_j| > \max\{|x_1 - x_j|, |x_i - x_j|\}$ or that there exists a $j > i$ and $1 < \ell \leqslant i$ such that $|\frac{\sum_{k=\ell}^{i} x_k}{i-\ell+1} - x_j| < |\frac{\sum_{k=i+1,k\neq j}^{n} x_k}{n-i} - x_j|$ and $|x_{\ell-j} - x_j| > \max\{|x_n - x_j|, |x_{i+1} - x_j|\}$. Then there exists g, p that exhibits homophily and such that the correct vote with homophily is not stable as $t \to \infty$.

Proof. Let t be sufficiently large. First assume there exists such a $j \leqslant i$ and $1 \leqslant \ell \leqslant n-i-1$ that meet the assumptions of the proposition. Let $p_{j(i+\ell+1)} = \epsilon_{i+\ell+1}$, $p_{j(i+\ell+2)} = \epsilon_{i+\ell+2}, \ldots, p_{jn} = \epsilon_n$ and let $p_{jk} = \frac{1}{i+\ell} - \epsilon_k$ for $k \in \{1, 2, \ldots, i+\ell\}, k \neq j$. Assume $\epsilon_k > \epsilon_{k+1} > 0$ for all $k \in \{1, 2, \ldots, j-2\} \cup \{j+1, j+2, \ldots, n-1\}$ and that $\epsilon_{i+\ell+1} + \ldots + \epsilon_n - (\epsilon_1 + \ldots + \epsilon_{i+\ell}) = 0$. Thus our assumption of homophily is satisfied. Note that our assumption $|x_{i+\ell+1} - x_j| > \max\{|x_1 - x_j|, |x_i - x_j|\}$ ensures that no x_k close to x_j is given a weight of only ϵ_k which is needed for homophily to be satisfied. Choose ϵ_i's small enough so that $|\frac{\sum_{k=1,k\neq j}^{i}(\frac{1}{i+\ell} - \epsilon_k)x_k}{\sum_{k=1,k\neq j}^{i}(\frac{1}{i+\ell} - \epsilon_k)} - x_j| \approx |\frac{\sum_{k=1,k\neq j}^{i} x_k}{i-1} - x_j|$ and $|\frac{\sum_{k=i+1}^{i+\ell}(\frac{1}{i+\ell} - \epsilon_k)x_k + \sum_{k=i+\ell+1}^{n} \epsilon_k x_k}{\sum_{k=i+1}^{i+\ell}(\frac{1}{i+\ell} - \epsilon_k) + \sum_{k=i+\ell+1}^{n} \epsilon_k} - x_j| \approx |\frac{\sum_{k=i+1}^{i+\ell} x_k}{\ell} - x_j|$. Given everyone else votes correctly, agent j does too if $|y_1^j(t) - x_j| < |y_2^j(t) - x_j|$. By Proposition 1, $y_1^j(t) \to \frac{\sum_{k=1,k\neq j}^{i} p_{jk} x_k}{\sum_{k=1,k\neq j}^{i} p_{jk}}$ and $y_2^j(t) \to \frac{\sum_{k=i+1}^{n} p_{jk} x_k}{\sum_{k=i+1}^{n} p_{jk}}$ as $t \to \infty$. Thus as $t \to \infty$, j votes correctly if $|\frac{\sum_{k=1,k\neq j}^{i} p_{jk} x_k}{\sum_{k=1,k\neq j}^{i} p_{jk}} - x_j| < |\frac{\sum_{k=i+1}^{n} p_{jk} x_k}{\sum_{k=i+1}^{n} p_{jk}} - x_j|$. However, $|\frac{\sum_{k=1,k\neq j}^{i} p_{jk} x_k}{\sum_{k=1,k\neq j}^{i} p_{jk}} - x_j| \approx |\frac{\sum_{k=1,k\neq j}^{i} x_k}{i-1} - x_j|$ and $|\frac{\sum_{k=i+1}^{n} p_{jk} x_k}{\sum_{k=i+1}^{n} p_{jk}} - x_j| \approx |\frac{\sum_{k=i+1}^{i+\ell} x_k}{\ell} - x_j|$. By assumption $|\frac{\sum_{k=i+1}^{i+\ell} x_k}{\ell} - x_j| < |\frac{\sum_{k=1,k\neq j}^{i} x_k}{i-1} - x_j|$, thus j will not vote correctly in this homophily network. The case for $j \geqslant i+1$ is similar.

Proposition 5 shows that it is possible for correct voting to be stable with random interactions and not stable with homophily. In Proposition 5, there exists an agent j such that if j ignores the agents in the other group that j considers to be "radicals" (or agents $k > i + \ell$ for $j \leqslant i$ who are the agents in the other group most different from j), then j prefers the opinions of the other group to the opinions of those in his own voting group. This preference will be especially true if j's own group has agents j considers radical with favorite policies closer to x_j than the favorite policies of the radicals in the other group. Thus, with homophily j may ignore the agents he considers to be "radical" in the other group but pay attention to the radicals in his own group, which can cause j to vote incorrectly. However, with random interactions j interacts with the agents he considers to be "radical" in the other group and learns their opinions. Since their opinions are quite different from

point. Note that this set may vary with j, but all such non-empty sets of such radicals would have some overlap as for instance all sets would contain agent 1.

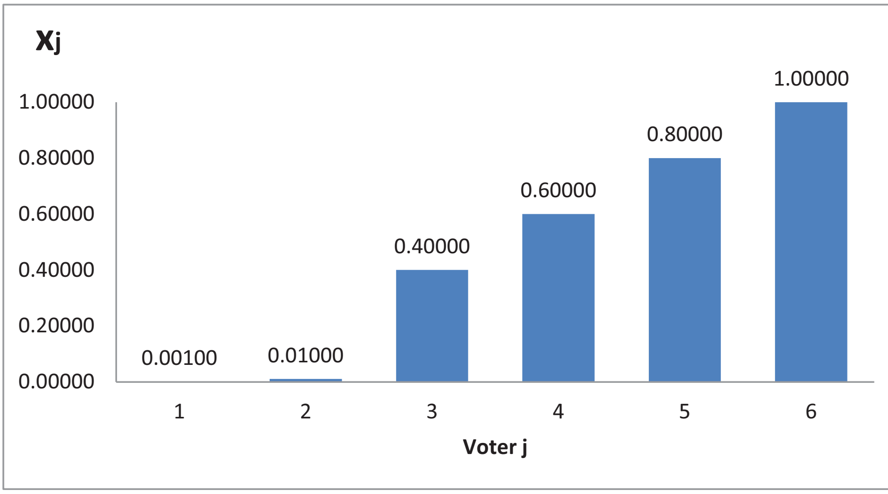

Fig. 1. Each voter's favorite policy. (color online)

j's he chooses not to vote with these agents. The following example illustrates this proposition.

Example 1
In this example we show that correct voting may be stable with random interactions but unstable with homophily. Let $n = 6$ and let $\{x_1, x_2, \ldots, x_6\} = \{0.001, 0.1, 0.4, 0.6, 0.8, 1\}$. This data is presented in Figure 1 and shows that agents 1, 2, 5, and 6 are quite different from agent 3. Assume that if all agents vote correctly then $\{1,2,3\}$ vote for c_1 and $\{4,5,6\}$ vote for c_2. It is easy to check that under random interactions correct voting is stable. To see this first consider voter 3. We will show that if all other voters vote correctly, then so will 3. Agent 3 votes correctly if $|y_1^3(t) - x_3| < |y_2^3(t) - x_3|$. By Proposition 1, $y_1^3(t) \to \frac{x_1+x_2}{2}$ and $y_2^3(t) \to \frac{x_4+x_5+x_6}{3}$ as $t \to \infty$. Plugging in values we get $y_1^3(t) \to 0.05$ and $y_2^3(t) \to 0.8$. Thus, 3 votes correctly if $|0.05 - x_3| < |0.8 - x_3|$ which is always true for $x_3 = .4$. Similar analysis shows that all other agents vote correctly. Next we show that there exists p such that g, p exhibits homophily and such that the correct vote with homophily is not stable. Let p be such that $p_3 = \{0.33233, 0.33323, 0, 0.33333, 0.001, 0.0001\}$. This data is presented in Figure 2 and shows that 3 does not interact much with the agents most different from him in the other group. (Recall that by definition $p_{33} = 0$.) Let all remaining p_i be anything such that the assumptions of homophily are met. Assume that all $i \neq 3$ vote correctly; we will show that 3 does not vote correctly. Agent 3 votes correctly if $|y_1^3(t) - x_3| < |y_2^3(t) - x_3|$. By Proposition 1, $y_1^3(t) \to \frac{p_{31}x_1+p_{32}x_2}{p_{31}+p_{32}}$ and $y_2^3(t) \to \frac{p_{34}x_4+p_{35}x_5+p_{36}x_6}{p_{34}+p_{35}+p_{36}}$ as $t \to \infty$. Plugging in values we get $y_1^3(t) \to 0.05$ and $y_2^3(t) \to 0.6$. Thus, 3 votes correctly if $|0.05 - x_3| < |0.6 - x_3|$ which is not true for $x_3 = .4$. In this network with homophily agent 3 votes incorrectly for c_2 and so correct voting is not stable.

Proposition 6
Let g, p exhibit homophily and let the correct vote with homophily be stable as $t \to \infty$. If there exists a $j \leqslant i$ and a $1 < \ell \leqslant j$ such that $|\frac{\sum_{k=\ell,k\neq j}^{i} x_k}{i-\ell} - x_j| <$

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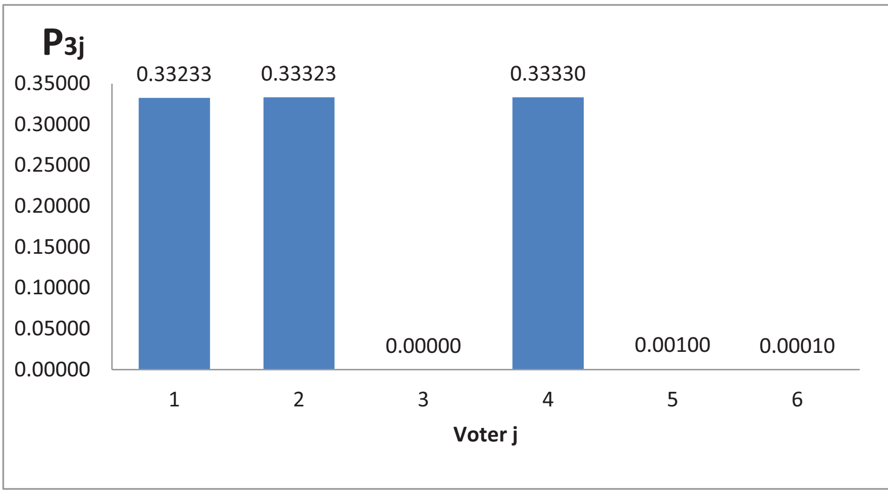

Fig. 2. Voter 3's interaction strengths with voter j. (color online)

$|\frac{\sum_{k=i+1}^{n} x_k}{n-i} - x_j| < |\frac{\sum_{k=1,k\neq j}^{i} x_k}{i-1} - x_j|$ and $|x_{\ell-1} - x_j| > |x_n - x_j|$ or if there exists a $j > i$ and a $j \leqslant \ell < n$ such that $|\frac{\sum_{k=i+1,k\neq j}^{\ell} x_k}{\ell-i} - x_j| < |\frac{\sum_{k=1}^{i} x_k}{i} - x_j| < |\frac{\sum_{k=i+1,k\neq j}^{n} x_k}{n-i-1} - x_j|$ and $|x_{\ell+1} - x_j| > |x_1 - x_j|$, then the correct vote is not stable with random interactions as $t \to \infty$.

Proof. Let t be sufficiently large. We show that if such a $j \leqslant i$ exists then with random interactions j prefers to vote for c_2 which is incorrect. Given everyone else votes correctly, j does as well if $|y_1^j(t) - x_j| < |y_2^j(t) - x_j|$. By Proposition 1, $y_1^j(t) \to \frac{\sum_{k=1,k\neq j}^{i} x_k}{i-1}$ and $y_2^j(t) \to \frac{\sum_{k=i+1}^{n} x_k}{n-i}$ as $t \to \infty$. Thus, j votes correctly if $|\frac{\sum_{k=1,k\neq j}^{i} x_k}{i-1} - x_j| < |\frac{\sum_{k=i+1}^{n} x_k}{n-i} - x_j|$ which is not true by assumption. However, as $|\frac{\sum_{k=\ell,k\neq j}^{i} x_k}{i-\ell} - x_j| < |\frac{\sum_{k=i+1}^{n} x_k}{n-i} - x_j|$ and $|x_{\ell-1} - x_j| > |x_n - x_j|$ our assumptions for Proposition 6 are met for j and thus j votes correctly with homophily for g, p. Similarly, if the conditions for $j > i$ are met then agent j will vote incorrectly for c_1 in a random interactions network, but will vote correctly for c_2 with homophily for g, p.

Proposition 6 shows that it is possible for the correct vote to be stable with homophily and not stable with random interactions. In Proposition 6, there exists an agent j who votes incorrectly with random interactions, because in j's voting group there are agents j considers "radical" (agents $k < \ell$ for $j \leqslant i$) who j interacts with and j prefers not to vote with these agents. However, with homophily j ignores or interacts less with the agents he considers to be "radicals" in his own group and j votes correctly with the moderate agents in his own group.

Proposition 6 can also provide intuition for when correct voting is stable with random interactions. Roughly speaking if only one correct voting group contains agents some consider radical then correct voting may not be stable with random interactions. This is illustrated in the following example where only $j(=4)$'s correct voting group contains agents considered radical by j and correct voting is not stable with random interactions.

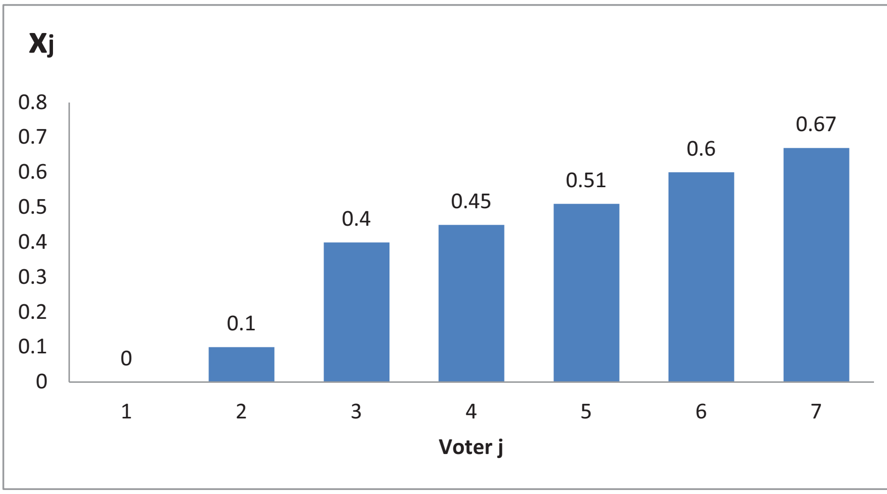

Fig. 3. Each voter's favorite policy. (color online)

Example 2

In this example we show that correct voting may be stable with homophily, but unstable with random interactions. Let $n = 7$ and let $x = \{0, 0.1, 0.4, 0.45, 0.51, 0.6, 0.67\}$. This data is presented in Figure 3 and shows that agents 1 and 2 are quite different from agent 4. Assume that if all agents vote correctly then $\{1,2,3,4\}$ vote for c_1 and $\{5,6,7\}$ vote for c_2. First we will show that there exists p satisfying homophily for which correct voting is stable. Let $p_3 = \{0.001, 0.002, 0, 0.985, 0.005, 0.004, 0.003\}$, $p_4 = \{0.001, 0.002, 0.985, 0, 0.005, 0.004, 0.003\}$, and $p_5 = \{0.1649, 0.1651, 0.167, 0.169, 0, 0.168, 0.166\}$. Given these p's it is straight forward to show that if everyone else votes correctly then so does $i \in \{3,4,5\}$. For instance, consider agent 4. Agent 4 votes correctly if $|y_1^4(t) - x_4| < |y_2^4(t) - x_4|$. By Proposition 1, $y_1^4(t) \to \frac{\sum_{k=1}^{3} p_{4k} x_k}{\sum_{k=1}^{3} p_{4k}}$ and $y_2^4(t) \to \frac{\sum_{k=5}^{7} p_{4k} x_k}{\sum_{k=5}^{7} p_{4k}}$ as $t \to \infty$. Plugging in values we get $y_1^4(t) \to 0.399$ and $y_2^4(t) \to 0.58$. Thus, 4 votes correctly if $|0.399 - 0.45| < |0.58 - 0.45|$ which is always true as $t \to \infty$. Similarly, 3 and 5 always vote correctly. Using the techniques described in the proof of Proposition 5 it is easy to find values for p_1, p_2, p_6 and p_7 that meet the conditions of homophily and for which these agents also vote correctly given everyone else does.

Next we show that voting correctly is not stable with random interactions. Here we show that given everyone else votes correctly, agent 4 votes for c_2 which is incorrect. Agent 4 votes correctly if $|y_1^4(t) - x_4| < |y_2^4(t) - x_4|$. By Proposition 1, $y_1^4(t) \to \frac{x_1+x_2+x_3}{3}$ and $y_2^4(t) \to \frac{x_5+x_6+x_7}{3}$ as $t \to \infty$. Plugging in values $y_1^4(t) \to 0.167$ and $y_2^4(t) \to 0.593$. So, 4 votes correctly as $t \to \infty$ if $|0.167 - 0.45| < |0.593 - 0.45|$ which is not true; and so 4 votes incorrectly for c_2 as 4 wants to avoid voting with 1 and 2 who are the agents 4 considers to be radical in his own group.

With homophily voters may ignore or give less weight to some information. This action can encourage correct voting if voters ignore those they consider to be "radicals" in their own voting group (Proposition 6) but can discourage correct voting if they ignore those they consider to be "radicals" in the other voting group (Proposition 5).

4 Conclusion

There is a body of empirical evidence illustrating that social networks influence voting decisions. We provide conditions for when voting correctly is a stable outcome in a line network, random interactions network, and homophily network. Additionally, conditions are provided under which correct voting is stable in a homophily network, but not in a random interactions network and vice versa; these conditions can be interpreted as conditions on the distribution of favorite policies among voters.

There are a number of possible extensions to the current research. For instance, our model can be applied to other situations besides voting. Consider the decision of whether or not to attend college where some agents have the ability and/or drive to succeed in higher education while others may lack such ability and/or drive. Agents may decide whether or not to apply to college based on recommendations from their social network. One could ask for which types of social networks is it more likely that high ability and/or highly motivated agents choose to apply to college and for which networks do they choose not to apply. Additionally, our model could be applied to a decision of religious affiliation. Each person may have a set of religious beliefs or an absence of religious beliefs and may also be influenced by her spouse, friends, and social network about whether or not to attend a church and about which church to attend. Again one could determine for which social network configurations do people choose a religious institution which most closely matches their beliefs. Such extensions would be an additional step toward understanding how social networks influence other types of correct and incorrect decision making.

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A Appendix

Next we examine a variant of the model in which agents give some weight to initial beliefs. Specifically, the updating rule is changed to the following:

$$y_k^i(t) = \alpha\big(y_k^i(0)\big) + (1-\alpha)\left(\frac{y_k^i(t-1) + \frac{\sum_{j\in S_k^i(t-1)} p_{ij}x_j}{\sum_{j\in S_k^i(t-1)} p_{ij}}}{2}\right) \qquad \text{if } S_k^i(t-1) \neq \varnothing \text{ and}$$

$$y_k^i(t) = \alpha\left(y_k^i(0)\right) + (1-\alpha)\left(y_k^i(t-1)\right) \qquad \text{if } S_k^i(t-1) = \varnothing$$

for $0 \leqslant \alpha \leqslant 1$.

In what follows we give updated versions of the propositions, but not the proofs as these can easily be derived using the proofs given in the main text. Note that most of the propositions extend in the obvious way making all propositions dependent on initial beliefs.

Proposition A1 *Let $c^j(t) = c^j(0)$ for all $t \geqslant 0$ and all $j \in N$, $j \neq i$. If $S_k^i(0) \neq \varnothing$, then $y_k^i(t) \to \alpha(y_k^i(0)) + (1-\alpha)(\frac{\sum_{j\in S_k^i(0)} p_{ij}x_j}{\sum_{j\in S_k^i(0)} p_{ij}})$ as $t \to \infty$. If $S_k^i(0) = \varnothing$, then $y_k^i(t) = y_k^i(0)$ for all $t \geqslant 0$.*

Proposition A2 *Let the network be a line network and let $|\alpha(y_1^i(0)) + (1-\alpha)(x_{i-1}) - x_i| < |\alpha(y_2^i(0)) + (1-\alpha)(x_{i+1}) - x_i|$ and $|\alpha(y_2^{i+1}(0)) + (1-\alpha)(x_{i+2}) - x_{i+1}| < |\alpha(y_1^{i+1}(0)) + (1-\alpha)(x_i) - x_{i+1}|$. Let $y_2^j(0) \notin [x_{j-1}, x_{j+1}]$ for all $1 < j < i$ and $y_1^j(0) \notin [x_{j-1}, x_{j+1}]$ for all $i+1 < j < n$ and let $y_k^1(0) \notin [x_1, x_2]$ and $y_k^n(0) \notin [x_{n-1}, x_n]$. Let $|y_1^j(0) - x_j| < |y_2^j(0) - x_j|$ for $j \leqslant i$ and let $|y_2^j(0) - x_j| < |y_1^j(0) - x_j|$ for $j > i$. Then voting correctly is stable as $t \to \infty$. Furthermore, voting correctly is uniquely stable if i and $(i+1)$ are experts and if $|\alpha(y_1^j(0)) + (1-\alpha)(x_{j+1}) - x_j| < |\alpha(y_2^j(0)) + (1-\alpha)(x_{j-1}) - x_j|$ for all $j \in \{1, 2, \ldots, i-1\}$ and $|\alpha(y_2^j(0)) + (1-\alpha)(x_{j-1}) - x_j|| < |\alpha(y_1^j(0)) + (1-\alpha)(x_{j+1}) - x_j|$ for all $j \in \{i+2, \ldots, n\}$ as $t \to \infty$.*

Proposition A3 *Let the network exhibit random interactions. Voting correctly is stable if for all $j \leqslant i$, $|\alpha(y_1^j(0)) + (1-\alpha)(\frac{\sum_{k=1,k\neq j}^{i} x_k}{i-1}) - x_j| < |\alpha(y_2^j(0)) + (1-\alpha)(\frac{\sum_{k=i+1}^{n} x_k}{n-i}) - x_j|$ and for all $j > i$, $|\alpha(y_2^j(0)) + (1-\alpha)(\frac{\sum_{k=i+1,k\neq j}^{n} x_k}{n-i-1}) - x_j| < |\alpha(y_1^j(0)) + (1-\alpha)(\frac{\sum_{k=1}^{i} x_k}{i}) - x_j|$ as $t \to \infty$.*

Proposition A4 *There exists p such that g, p exhibits homophily and such that the correct vote with homophily is stable as $t \to \infty$ if for all $j \leqslant i$ there exists $1 \leqslant \ell \leqslant j$ such that $|\alpha(y_1^j(0)) + (1-\alpha)(\frac{\sum_{k=\ell,k\neq j}^{i} x_k}{i-\ell}) - x_j| < |\alpha(y_2^j(0)) + (1-\alpha)(\frac{\sum_{k=i+1}^{n} x_k}{n-i}) - x_j|$ and $|x_{\ell-1} - x_j| > |x_n - x_j|$ for $\ell > 1$. And for all $j \geqslant i+1$ there exists $j \leqslant \ell \leqslant n$ such that $|\alpha(y_2^j(0)) + (1-\alpha)(\frac{\sum_{k=i+1,k\neq j}^{\ell} x_k}{\ell-i}) - x_j| < |\alpha(y_1^j(0)) + (1-\alpha)(\frac{\sum_{k=1}^{i} x_k}{i}) - x_j|$ and $|x_{\ell+1} - x_j| > |x_1 - x_j|$ for $\ell < n$.*

Proposition A5 *Let correct voting be stable with random interactions as $t \to \infty$ and assume there exists either a $j \leqslant i$ and a $1 \leqslant \ell < n-i$ such that $|\alpha(y_2^j(0)) + (1-$*

$\alpha)(\frac{\sum_{k=i+1}^{i+\ell} x_k}{\ell}) - x_j| < |\alpha(y_1^j(0)) + (1-\alpha)(\frac{\sum_{k=1,k\neq j}^{i} x_k}{i-1}) - x_j|$ *and* $|x_{i+\ell+1} - x_j| > \max\{|x_1 - x_j|, |x_i - x_j|\}$ *or that there exists a* $j > i$ *and* $1 < \ell \leqslant i$ *such that* $|\alpha(y_1^j(0)) + (1-\alpha)(\frac{\sum_{k=\ell}^{i} x_k}{i-\ell+1}) - x_j| < |\alpha(y_2^j(0)) + (1-\alpha)(\frac{\sum_{k=i+1,k\neq j}^{n} x_k}{n-i}) - x_j|$ *and* $|x_{\ell-j} - x_j| > \max\{|x_n - x_j|, |x_{i+1} - x_j|\}$. *Then there exists* p *such that* g, p *exhibits homophily and such that the correct vote with homophily is not stable as* $t \to \infty$.

Proposition A6 *Let* g, p *exhibit homophily and let the correct vote with homophily be stable as* $t \to \infty$. *If there exists a* $j \leqslant i$ *and a* $1 < \ell \leqslant j$ *such that* $|\alpha(y_1^j(0)) + (1-\alpha)(\frac{\sum_{k=\ell,k\neq j}^{i} x_k}{i-\ell}) - x_j| < |\alpha(y_2^j(0)) + (1-\alpha)(\frac{\sum_{k=i+1}^{n} x_k}{n-i}) - x_j| < |\alpha(y_1^j(0)) + (1-\alpha)(\frac{\sum_{k=1,k\neq j}^{i} x_k}{i-1}) - x_j|$ *and* $|x_{\ell-1} - x_j| > |x_n - x_j|$ *or if there exists a* $j > i$ *and a* $j \leqslant \ell < n$ *such that* $|\alpha(y_2^j(0)) + (1-\alpha)(\frac{\sum_{k=i+1,k\neq j}^{\ell} x_k}{\ell-i}) - x_j| < |\alpha(y_1^j(0)) + (1-\alpha)(\frac{\sum_{k=1}^{i} x_k}{i}) - x_j| < |\alpha(y_2^j(0)) + (1-\alpha)(\frac{\sum_{k=i+1,k\neq j}^{n} x_k}{n-i-1}) - x_j|$ *and* $|x_{\ell+1} - x_j| > |x_1 - x_j|$, *then the correct vote is not stable with random interactions as* $t \to \infty$.