Macroeconomic Dynamics, **15**, 2011, 725–770. Printed in the United States of America.
doi:10.1017/S1365100510000180

VINTAGE ARTICLE

CONSUMPTION AND SAVING OVER THE LIFE CYCLE: HOW IMPORTANT ARE CONSUMER DURABLES?

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In this paper we investigate whether a standard life-cycle model in which households purchase nondurable consumption and consumer durables and face idiosyncratic income and mortality risk as well as endogenous borrowing constraints can account for two key patterns of consumption and asset holdings over the life cycle. First, consumption expenditures on both durable and nondurable goods are hump-shaped. Second, young households keep very few liquid assets and hold most of their wealth in consumer durables. In our model durables play a dual role: they both provide consumption services and act as collateral for loans. A plausibly parameterized version of the model predicts that the interaction of consumer durables and endogenous borrowing constraints induces durables accumulation early in life and higher consumption of nondurables and accumulation of financial assets later in the life cycle, of an order of magnitude consistent with observed data.

Keywords: Consumer Durables, Life Cycle, Consumption and Saving

PREAMBLE

When we wrote this paper in 2000–2001, we wanted to stress the triple role of consumer durables in life-cycle models as an asset, as a long-lived good generating utility flows, and as collateral for borrowing. To do so, we incorporated durable goods serving these three roles into an otherwise standard overlapping-generations life-cycle consumption–savings model with an endogenous borrowing constraint. We calibrated the model to match U.S. macro and micro observations and evaluated the importance of consumer durables for life-cycle consumption and asset choices.

After the completion of this paper, substantial new work emerged in three areas related to our work. We structure our brief discussion of this literature along the

We would like to thank Andrew Atkeson, Michele Boldrin, Hal Cole, MariaCristina De Nardi, Narayana Kocherlakota, Lee Ohanian, Luigi Pistaferri, Edward Prescott, and Tao Zha, as well as an anonymous referee, for helpful comments. All remaining errors are our own. Address correspondence to: Dirk Krueger, Department of Economics, University of Pennsylvania, 3718 Locust Walk, Philadelphia, PA 19104, USA; e-mail: dkrueger@econ.upenn.edu.

1365-1005/11

lines of the three main roles consumer durables play in our own research, although of course recognizing that in most of the literature durables and housing play multiple roles. In order to keep this discussion brief and concise, we will not discuss the large literature modeling house prices endogenously, because our own work maintains the assumption of exogenous and constant prices for durables and thus does not contribute to this literature. To leave the integrity of the paper intact from its original state, we opted to review the subsequent literature here, rather than incorporating the literature review into the original paper.

First, a substantial empirical and theoretical-quantitative literature studies the interaction of household nondurable consumption choices and consumer durables (especially housing), often in a life-cycle context. Our own companion paper, Fernández-Villaverde and Krueger (2007), and Yang (2009) provide empirical evidence on expenditures for consumer durables over the life cycle. Other relevant papers include Martin (2003), Ortalo-Magne and Rady (2006), Li and Yao (2007), Diaz and Luengo-Prado (2008), Kiyotaki et al. (2008), and Iacoviello and Pavan (2009).

The importance of housing for the accumulation of wealth and for portfolio composition is the theme of an important body of work, from Grossman and Laroque (1990) and Berkovec and Fullerton (1992) to the recent contributions by Gervais (2002), Gruber and Martin (2003), Martin and Gruber (2004), Cocco (2005), Nakajima (2005), Yao and Zhang (2005), Silos (2007a, 2007b), Diaz and Luengo-Prado (in press), and Hintermaier and Koeniger (2009).

Third, the role as collateral of consumer durables in general and housing in particular in the main theme of a recent literature that focuses on the joint housing and mortgage choice. Important examples of this work include Hurst and Stafford (2004), Luengo-Prado (2006), and Chambers et al. (2009a, 2009b). The recent increase in defaults on mortgages has motivated a small but growing literature on structural models of foreclosures within this context. See, for instance, Jeske and Krueger (2005) and Garriga and Schlagenhauf (2009). The same issue is analyzed empirically, among others, by Carroll and Li (2008). Finally, a few recent papers have attempted to estimate models of housing choice [Sanchez (2007) is an important example], often with a focus on quantifying key structural parameters, especially the elasticity of substitution between nondurable consumption and services from consumer durables or housing. Examples of this line of work include Bajari et al. (2008) and Li et al. (2009).

This paper was last substantially revised in 2001 and prior to the solicitation from *Macroeconomics Dynamics* had not been submitted for publication to any journal.

1. INTRODUCTION

In this paper we investigate whether a standard life-cycle model in which households purchase nondurable consumption and consumer durables and face idiosyncratic income and mortality risk as well as endogenous borrowing constraints can account for two key patterns of consumption and asset holdings over the life cycle. *First*, consumption expenditures on both durable and nondurable goods are hump-shaped: expenditures are low early in life, then rise considerably until about age 50, and fall again. The average household in the Consumer Expenditure Survey (CEX) spends 63% more when the head of the household is 50 than when he or she is 25 and around 70% more than when he or she is 65. *Second*, young households keep very few liquid assets and hold most of their wealth in consumer durables.[1] Later in life, families accumulate significant amounts of financial assets for retirement. The importance of durables is also mirrored in the aggregate composition of wealth: households hold 35% of their total assets in real estate and other consumer durables and only 28% in equity.[2]

As we will discuss below, the first fact, the hump in consumption expenditures, persists even after controlling for family size and constitutes a puzzle from the perspective of the standard life-cycle model with complete financial markets, according to which individuals smooth consumption over their lifetimes and across states of the world. Understanding this puzzle is not only interesting from a theoretical perspective, but also crucial for applied policy analysis. As individual behavior changes over the life cycle, so will the effects of social security reform, the public provision of saving incentives, or the welfare consequences of progressive taxation vary by age groups. An accurate quantitative assessment of policy reforms is thus essential to establish a coherent explanation for changes in the consumption and savings behavior over the life cycle that gives rise to the hump in consumption.

The second fact, the life-cycle pattern of households' portfolio composition, suggests that it is necessary to model purchase decisions on consumer durables explicitly to understand households' consumption and portfolio allocation decisions. This constitutes a departure from the tradition in the life-cycle consumption literature, which has largely ignored the presence of durables. This omission is problematic if the purchase of durables and of the flows of services generated by them interacts with nondurable consumption in a nonseparable way. For example, recent explanations of the life-cycle hump in nondurable consumption provided by Carroll (1997) or Gourinchas and Parker (2002) rely crucially on households consuming, up to the age of 40, all of their income, apart from some small buffer stock used to insure against future bad income shocks. We will argue that, in the presence of consumer durables in the model, households behave quite differently: they accumulate durables early in life and consume the rest of disposable income, without any saving in financial assets. In this period of their lives durables not only provide consumption service flows, but also are used as collateralizable insurance against uninsurable idiosyncratic income risk.

Our model also distinguishes itself from the standard household portfolio choice model, which, on one hand, explicitly include the presence of durables such as housing, but, on the other hand, usually ignore the life-cycle dimension of a household.[3] Thus these models by construction cannot explain the observation of little accumulation of liquid assets early in life in the data. Instead of interpreting this fact as indicating that young households do not to save, as one is forced to when considering a standard life-cycle model of consumption without durables, we will demonstrate that it is optimal for households to save *in durables* early in life and to shift to the accumulation of financial assets only later in the life cycle.

To substantiate our claims, we construct a dynamic general-equilibrium life-cycle model of consumption and saving with labor-income uncertainty and borrowing constraints to formally evaluate whether such a model can explain the two empirical observations mentioned above in a unified framework. The crucial elements of our model are (a) a highly persistent stochastic idiosyncratic labor-income process with (b) a hump-shaped mean over the life cycle; (c) the presence of durables that yield consumption services *and* can be used as collateral, in addition to a standard one-period bond whose short sales are subject to a borrowing constraint; and (d) endogenous determination of the interest rate in general equilibrium.

We now justify the key elements of our model. The first two elements are fairly standard in the literature on life-cycle consumption and mainly motivated by the empirical observation that, even if households face substantial idiosyncratic labor-income risk [see, e.g., Gottschalk and Moffitt (1994)], on the average labor income follows a hump-shaped pattern over the life cycle with a peak around the age of 50 [see Hansen (1993)]. The third and fourth elements of the model are novel features in the literature we wish to contribute to. The presence of consumer durables is motivated by the empirical observation that they constitute a large fraction of households' asset holdings. Furthermore, as mentioned above, the life-cycle pattern of nondurable consumption may be intimately related to the life-cycle pattern of the accumulation of durables, so that abstracting from them may severely bias any study of the life-cycle profile of nondurable consumption and asset accumulation. Finally, we determine the interest rate of the economy endogenously in general equilibrium because the life-cycle profile of consumption and asset accumulation depends crucially on the ratio between the subjective time-discount factor and the interest rate. The discipline of general equilibrium determines this ratio endogenously in our model and therefore restricts our possibility of predetermining the results by appropriate choice of *both* the interest rate *and* the time discount factor.[4] In addition, because we want to extend our line of research to the study of the interactions between durables and the business cycle and to evaluations of the welfare effects of different fiscal policies, we view endogenous price and interest-rate determination not only as attractive from a theoretical perspective, but also as quantitatively crucial for addressing these questions.

Our main findings are summarized as follows. In the empirical part of the paper we use data from the Consumer Expenditure Survey (CEX) to show that consumption expenditure on *both* durables and nondurables follows a hump-shape pattern over the life cycle. We also use data from the Survey of Consumer Finances (SCF) to document that young households own virtually no liquid financial assets, but hold a major fraction of their wealth portfolios as durables, whereas later in life the composition of household portfolios shifts in favor of financial assets.

Our second contribution is to demonstrate that a plausibly parameterized version of our model can quantitatively explain these empirical findings as arising from rational choices of consumers facing an increasing wage profile and income uncertainty. The interaction between consumer durables that provide both consumption and collateral services and endogenous borrowing constraints gives rise to accumulation of durables (and no accumulation of financial assets) early in life and substitution toward higher consumption of nondurables and the accumulation of financial assets later in life.

This work is related to several strands of the literature. On the empirical side it adds to the discussion about the life-cycle profile of consumption observed in cross-sectional micro data. Important references include Blundell et al. (1994), Attanasio and Browning (1995), Attanasio and Weber (1995), and Gourinchas and Parker (2002). The key question in the context of our paper is whether, once changes in family size are controlled for, consumption still follows a hump-shaped life-cycle profile. The key empirical finding of our paper is the presence of a hump-shaped profile not only for nondurable consumption, but also for expenditures on consumer durables.

On the theoretical side, the basic building block of our model is the classic income-fluctuation problem in which a consumer faces a stochastic income process and decides how much to consume and how much to save. Contributors to this literature include Deaton (1991), Carroll (1992, 1997), and Gourinchas and Parker (2002). Following Bewley (1986), the income fluctuation problem has been embedded by Huggett (1993) and Aiyagari (1994) into general equilibrium, giving rise to the endogenous determination of the interest rate as well as a nontrivial income, wealth, and consumption distribution at equilibrium. Our paper incorporates consumer durables and endogenous borrowing constraints into Aiyagari's framework; the specification of the borrowing constraint is adapted from the recent endogenous incomplete-markets literature [see Kehoe and Levine (1993), Kocherlakota (1996), Krueger and Perri (1999, 2006), and Alvarez and Jermann (2000)]. Lustig (2004) also presents a model with a durable asset and endogenous borrowing constraints to explain the equity premium puzzle; in his model, however, agents have access to a full set of Arrow securities and are infinitely lived. Our model shares some of his elements, but our focus is on life-cycle consumption and asset allocation, whereas Lustig studies the pricing implications of endogenous borrowing constraints.

Although our focus is on the life-cycle pattern of consumption of durables and nondurables, our model also has implications for optimal portfolio allocation between financial assets and durables at each point of the life cycle. Therefore the paper makes contact with the literature on optimal portfolio choice in the presence of consumer durables, such as Grossman and Laroque (1990), Eberly (1994), Chah et al. (1995), and Flavin and Yamashita (2002). Finally, the paper also relates to the literature on real business cycles with a household production sector [see Greenwood et al. (1995) for a review]. We share its focus on an explicit treatment of the household sector in dynamic general equilibrium and its dynamic behavior; we do not, however, model aggregate uncertainty.

The paper is organized as follows. In Section 2 we present empirical results from the CEX documenting a hump shape in both nondurable and durable consumption expenditures even after controlling for household size. We also discuss the evidence on the life-cycle pattern of wealth composition derived from the SCF. In Section 3 we present our model and define equilibrium. Section 4 is devoted to a discussion of the model calibration and Section 5 presents the quantitative results obtained from the benchmark calibration of the model. Section 6 performs a sensitivity analysis and Section 7 concludes. Technical discussions about our empirical methods, the data used, and the computational algorithm are contained in the Appendix.

2. EMPIRICAL FINDINGS

This section presents our empirical findings on consumption and wealth accumulation over the life cycle. We first document the life-cycle profile of consumption using data from the CEX dealing explicitly with the issue of changing household size over the life cycle. A more extensive discussion of the results in this section can be found in our empirical companion paper, Fernández-Villaverde and Krueger (2007). We then turn to the SCF in order to document the life-cycle pattern of wealth and portfolio allocation. We point out that the structure of household portfolios changes with age, and that housing and other consumer durables play a quantitatively crucial role in most households' portfolios.

2.1. Life-Cycle Profiles of Consumption

A basic prediction of the life-cycle model with complete financial markets is that the life-cycle consumption profiles should be smooth. If the period utility function is time-invariant and the time discount factor is constant, then households choose consumption plans to equate marginal utility across time and states of the world, possibly with some growth rate, depending on the relative size of the real interest rate and the discount factor. Under Constant Relative Risk Aversion (CRRA), period utility consumption growth itself should be constant across time. With complete markets, consumption smoothing can be achieved through the transfer of contingent claims across periods and states.[5] The first empirical question, motivated by standard economic theory, that we want to answer is therefore whether smooth consumption profiles are indeed observed over the life cycle in household-level consumption data.

During the eighties some agreement arose that the answer to this question was negative [see Deaton (1992) for an overview]. The main stylized fact emerging from this literature was that consumption seems to track income over the life cycle, changing only when income changes, and not when it becomes known that income will change, as economic theory predicts. This evidence was interpreted as indicating the presence of liquidity constraints or other financial market imperfections.

Because labor income follows a hump over the life cycle, these imperfections then imply the consumption hump over the life cycle in the data.

Recently this view has been disputed by Blundell et al. (1994), Attanasio and Browning (1995), and especially Attanasio et al. (1999). These authors argue that once the consumption data are appropriately adjusted for changes in household size (which is also hump-shaped over the life-cycle) and composition the hump in life cycle consumption disappears. We will challenge this view and argue that consumption follows a hump even after controlling for demographics. For that we will use 1980–2001 data from the CEX.

The Consumer Expenditure Survey data. During the last few years, the CEX has become one of the main sources for empirical work on consumption [see Attanasio (1999) for a survey]. The CEX is a rotating panel of about 5,000 households, where each household is interviewed every three months over five calendar quarters, and every quarter 20% of the sample is replaced by new households.

Two main problems in documenting the life-cycle profile for consumption in the way economic theory envisions it arise from the CEX. First, the CEX only measures consumption expenditures, and not the consumption service flows from these expenditures. Second, the CEX only contains a very limited panel dimension, as households provide at most four subsequent quarters of consumption data. In Fernandez-Villaverde and Krueger (2007) we discuss in detail how we address both issues. Here we provide a short summary of our discussion.

Expenditures versus consumption. Because the CEX does not report a measure of *consumption services*, we are left with analyzing *expenditures* on consumption goods. Although this distinction is not very relevant to nondurable goods, it is crucial in the case of consumer durables. However, the CEX does not allow us to impute service flows reliably from information on the stock of consumer durables. Nevertheless, our theoretical model below has implications for the life-cycle profile of expenditures on nondurables and durables, and thus it is useful to document the empirical profiles from the CEX here. Our empirical results may serve as a benchmark against which the quantitative predictions of our model as well as other models can be evaluated.

Constructing a pseudopanel. Because the CEX does not have a significant panel dimension, we construct a pseudopanel or synthetic cohort panel [see Deaton (1985)] to document life-cycle consumption profiles. We use the age of the reference person to associate a household with a cohort and define ten cohorts with a length of five years. To generate a balanced panel with consumption data over the life cycle we compute the means of cohort consumption, using CEX-provided population weights.

Specification and estimation of life-cycle profiles. We propose to estimate life-cycle consumption expenditure profiles by a simple and flexible seminonparametric regression that controls for cohort and time effects with dummy variables and puts relatively little parametric structure on the dependence of consumption on age. In particular, we specify the partially linear model

$$c_{it} = \pi_i \text{cohort}_i + \pi_t \gamma_t + m(\text{age}_{it}) + \varepsilon_{it}, \tag{1}$$

where c_{it} is the cohort-i average of log-consumption at time t, cohort_i is a dummy for each cohort (except the youngest one), γ_t is a dummy for each quarter, age_{it} is the age of cohort i at time t, measured in years, $m(\text{age}_{it}) = E(c_{it} \mid \text{age}_{it})$ is a smooth function of age_{it}, and ε_{it} is an independent, zero-mean random error. The random term captures multiplicative measurement error in consumption expenditures (because the dependent variable is log consumption) as well as unobserved cross-sectional heterogeneity. We estimate the partially linear model using the two-step estimator proposed by Speckman (1988). This estimator combines ordinary least squares to estimate the parametric component with a standard kernel-smoothing estimator to estimate the nonparametric component.[6] To separately identify time, age, and cohort effects we follow Deaton (1997) and assume that time effects are orthogonal to a time trend and that their sum is normalized to zero.

Controlling for family size: Household equivalence scales. Before carrying out the estimation of consumption profiles, we need to quantify how much of the hump in consumption over the life cycle is explained by changes in household size and composition. For this purpose we employ household equivalence scales, which measure the change in consumption expenditures needed to keep the welfare of a family constant when its size varies.[7] In our companion paper we survey the large literature on household equivalence scales before settling on a particular scale.[8]

We now deflate consumption expenditure measures C_{jt} from the CEX for household j at quarter t by the equivalence scale es_{jt} obtained from the demographic information of the household. This results in adjusted household consumption $\hat{c}_{jt} = \log(C_{jt}/es_{jt})$. Let $\tilde{c}_{it}$ denote the synthetic cohort-i average of $\hat{c}_{jt}$, which we now use to estimate the partially linear model in equation (1).

Results. Figures 1–3 show life-cycle profiles of total expenditure (Figure 1), expenditures on nondurables (Figure 2), and expenditure on durables (Figure 3), controlling for cohort and time effects but *not* for family size. In all figures, consumption expenditures follow a clear hump. Note that the humps for durables and nondurables are of similar magnitude and occur at the same age, roughly in the late forties of the household.

We now control for household size using equivalence scales and repeat our analysis.[9] The results are summarized in Figures 4–6. Adjusting consumption for family size reduces the magnitude of the hump, measured as the ratio between consumption at the peak and at the beginning of the life cycle, by about 50%. However, half of the hump persists even after controlling for family size: adjusted quarterly total consumption increases from around \$2,550 to nearly \$3,300 and then decreases to about \$1,800 (see Figure 4). Relative to the unadjusted data we

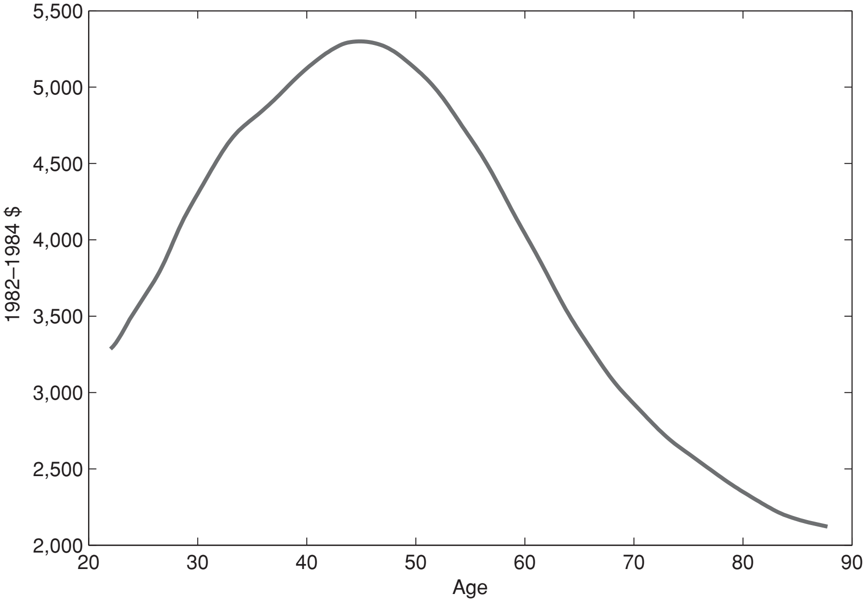

FIGURE 1. Total expenditure.

also observe that the peak in life-cycle consumption is postponed, roughly to the age of 50.

In Figure 5 we show adjusted nondurable consumption. Again the size of the hump is reduced by 50%, as a comparison with Figure 2 reveals. With respect

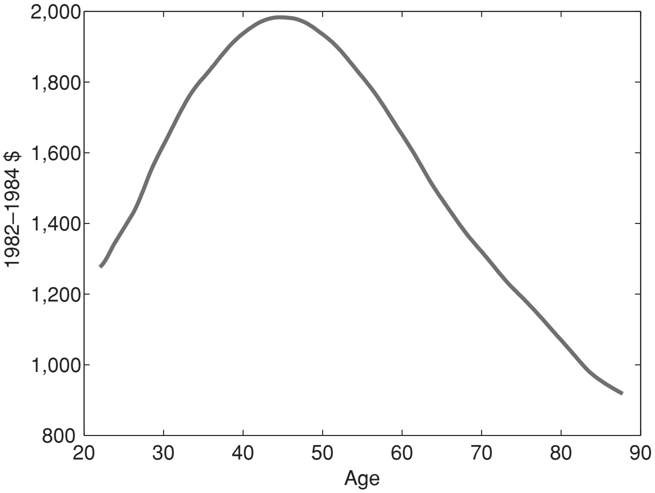

FIGURE 2. Nondurable expenditure.

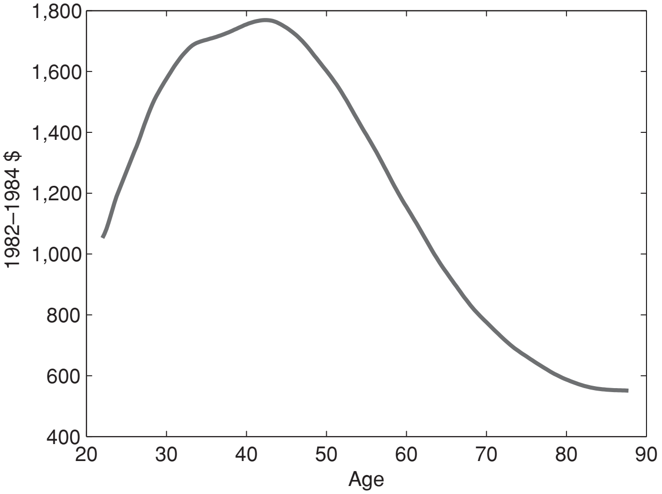

FIGURE 3. Durable expenditure.

to expenditures of consumer durables, Figure 6 yet again displays a clear hump. Now, however, expenditures are already fairly high at the beginning of the life cycle, capturing first purchases of durable goods by young households. Comparing Figures 5 and 6, we observe that the reduction of the hump due to adjustment for

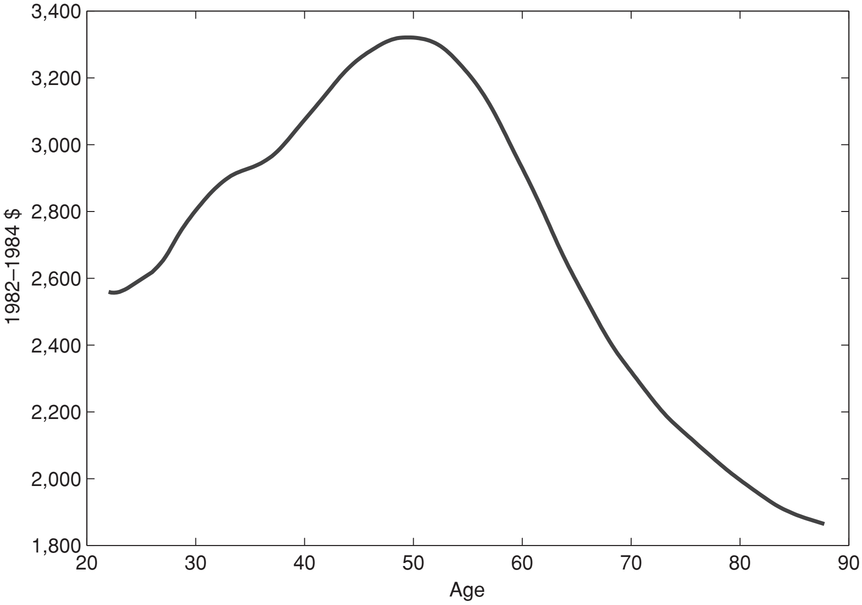

FIGURE 4. Total expenditure, adult equivalent.

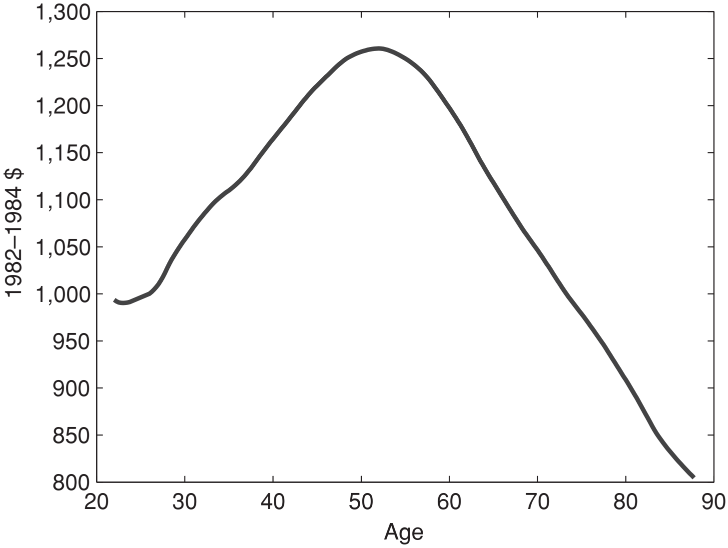

FIGURE 5. Nondurable expenditure, adult equivalent.

household size is quite similar for nondurables and durables. Furthermore, both profiles peak at around the same household age.[10]

In summary, our analysis demonstrates that even though changes in household size can account for around half of the hump in life-cycle consumption and thus are

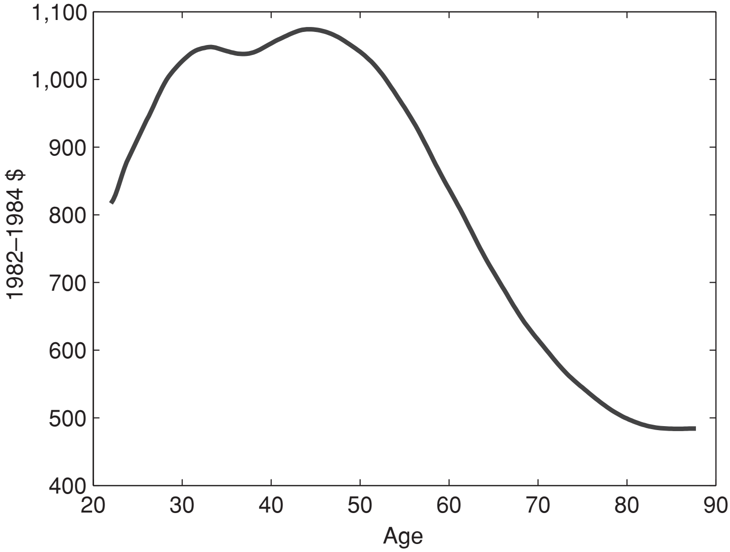

FIGURE 6. Durable expenditure, adult equivalent.

crucial in understanding life-cycle profiles, the other half remains a puzzle from the perspective of the standard life-cycle model with complete financial markets. This is especially true for the profile of expenditures on consumer durables. If the period utility function is separable in nondurable consumption and services from durables, and the real interest rate is equal to the time discount factor and constant over time, then it is optimal simply to purchase the desired stock of durables early in life and only replace depreciation from there on. In contrast, our data display that the process of durables accumulation is incremental over the life cycle, consistent with other work that has documented liquidity constraints in the purchases of consumption durables [Eberly (1994); Alessie et al. (1997); Barrow and McGranahan (2000); Attanasio et al. (2005)], or argued for the importance of nonseparabilities in the utility function [Attanasio and Weber (1995)].

2.2. Life-Cycle Profiles of Wealth

How do the findings in the previous section relate to observed patterns of wealth accumulation and portfolio composition over the life cycle? One of the most authoritative sources on households' asset holdings is the SCF, a triennial survey of U.S. households undertaken by the Federal Reserve System. The SCF interviews a representative cross section of over 4,000 households and collects data about their demographic characteristics, assets, and debts. Because the small number of repeated surveys (six, of which only four are directly comparable) precludes the building of a pseudo-panel, we will focus on the cross-sectional aspect of the data. We will use the 1995 survey information to document several important aspects of the life-cycle profile of households assets.

The pattern of life-cycle wealth is shown in Figure 7. We plot the households' mean and median net worth along the life cycle.[11] Two main points arise from this figure. First, as for consumption expenditures, wealth follows a hump-shaped pattern over the life cycle. Households accumulate wealth from the beginning of their lives until retirement, the moment at which they begin to run down their wealth. It is noticeable, however, that wealth stays relatively high even after the age of 80. Second, wealth is highly unequally distributed, as can be seen from the large ratio between household mean and median net worth; this ratio attains its maximum over the life cycle at around 400% just before retirement.

Additional interesting information is contained in data on the composition of household wealth. Figure 8[12] shows the importance of durables in most households' portfolios. We order households along the dimension of total wealth and plot the percentages of their portfolios held in real estate (which consists of the primary residence for most households) and the percentages invested in corporate equity. For 60% of all families, those between the thirtieth and ninetieth percentiles, real estate represents most of their total assets, whereas vehicles and other durables are an important part of the remaining portfolios. Below the thirtieth percentile, households have no or low wealth, most of it in vehicles and other

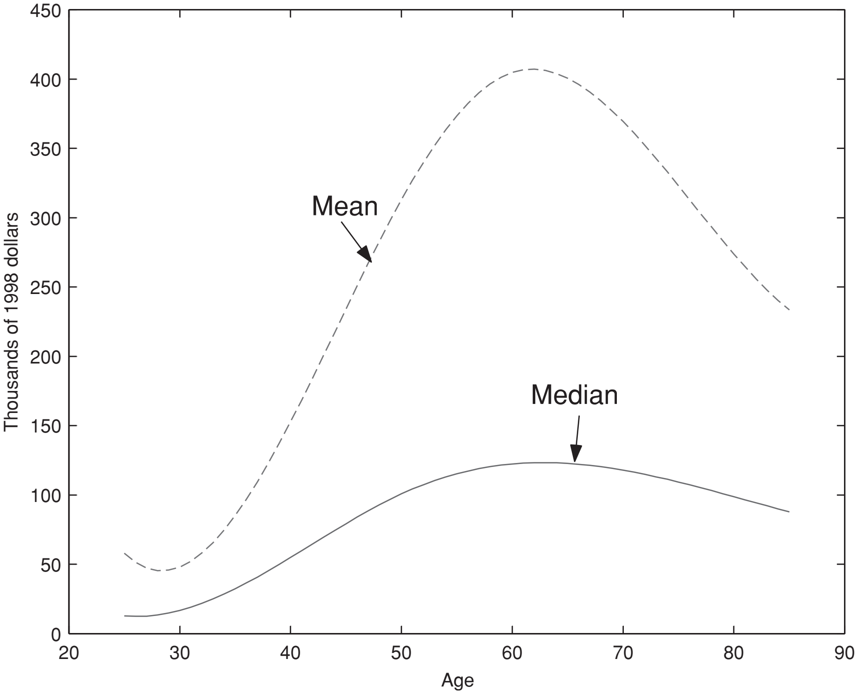

FIGURE 7. Household net worth, 1995 SCF.

durables. For example, note that 65.4% of the households in the lowest quartile of the net worth distribution own vehicles with median value of $4,800, whereas only 7.9% of this group have financial assets beyond a transaction account. If we include the transaction accounts as financial assets, 78% of these last-quartile households have some financial assets, but with a median value of only $1,100. These facts leave the highest 10% of the wealth distribution as the only households for which financial assets are a fundamentally important part of their portfolios. A theory of life-cycle consumption and saving should account for the low levels of financial wealth of most households.

Even more important, given our focus on life-cycle consumption, is the portfolio composition of assets along the life cycle. Figure 9 shows that the primary residence is a basic component of the median assets of households over their lives. Before households reach the age of 40, the median value of a homeowner's primary residence exceeds the median value of total assets for households holding some assets. After the age of 40, the median primary residence value always stays above 50% of the median of total assets. The same picture arises in a percentile decomposition of homeowners' portfolios. Up to the age of 45, the portion of homeowners' assets in real estate is around two-thirds. After that time and until retirement it decreases, but never falls below 57%.[13]

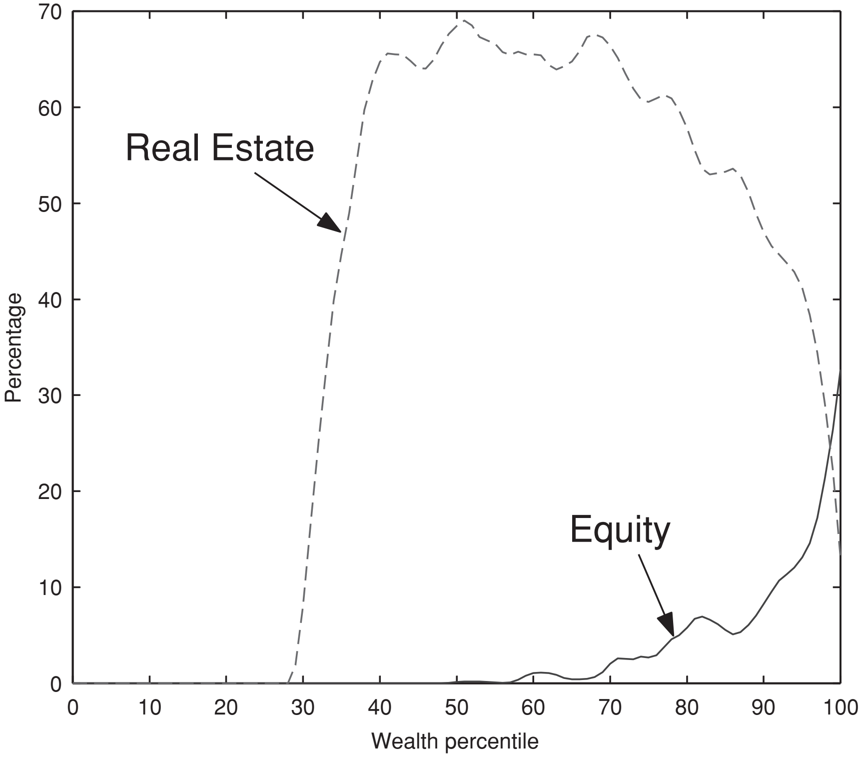

FIGURE 8. Household assets distribution by wealth, 1995 SCF.

Again, theory needs to explain why housing and other durable goods have such a primary role in the life-cycle accumulation of wealth for the median-wealth household.

To summarize the empirical part of this paper: expenditures on nondurable and durable consumption goods follow a hump over the life cycle, and the stock of durables seem to accumulate only progressively. At the same time, young households hold most of their wealth in consumer durables, with financial assets gaining importance in later periods of a household's life. We now present a model to jointly reproduce these stylized facts of life-cycle consumption and portfolio decisions.

3. THE ENVIRONMENT

In this section we present a dynamic general equilibrium model of life-cycle consumption. We use a standard dynamic–general equilibrium, life-cycle model with income uncertainty, with two novel features: first, we will introduce service flows of consumer durables into the utility function and second, we will restrict

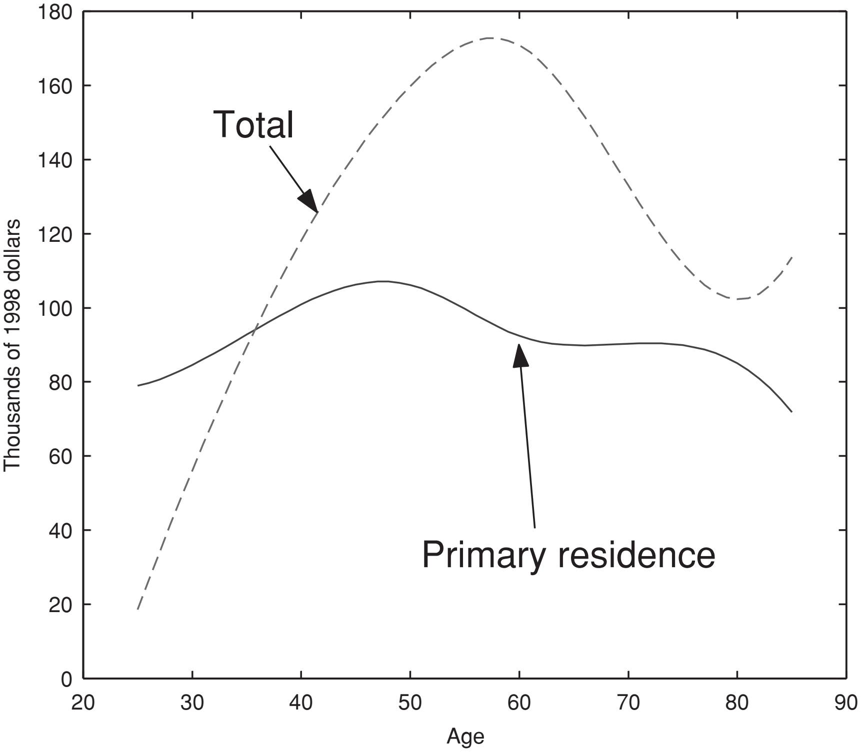

FIGURE 9. Median household assets, 1995 SCF.

intertemporal trade by endogenous borrowing constraints, as explained below in detail.

3.1. Demographics

There is a continuum of individuals of measure 1 at each point in time in our economy. Each individual lives at most J periods. In each period $j \leq J$ of his or her life the conditional probability of surviving and living in period $j+1$ is denoted by $\alpha_j \in (0, 1)$. Define $\alpha_0 = 1$ and $\alpha_J = 0$. The probability of survival, assumed to be equal across individuals of the same cohort, is beyond the control of the individual and independent of other characteristics of the individual (such as income or wealth). We assume that α_j is not only the probability of survival for a particular individual, but also the (deterministic) fraction of agents[14] that, having survived until age j, will survive to age $j+1$. Annuity markets are assumed to be absent and accidental bequests are assumed to be uniformly distributed among all agents currently alive. In each period a number $\mu_1 = (1 + \sum_{j=1}^{J-1} \prod_{i=1}^{j} \alpha_i)^{-1}$ of newborns enter the economy, and the fraction of people in the economy of age j is defined recursively as $\mu_{j+1} = \alpha_j \mu_j$, with $\mu_{J+1} = \alpha_J = 0$. Let $\mathcal{J} = \{0, 1, \ldots, J\}$ denote the set of possible ages of an individual.

3.2. Technology

There is one good produced according to the aggregate production function $F(K_t, L_t)$, where K_t is the aggregate capital stock and L_t is the aggregate labor input. We assume that F is strictly increasing in both inputs, is strictly concave, has decreasing marginal products that obey the Inada conditions, and is homogeneous at degree one. As usual with constant–returns to scale production technologies, at equilibrium the number of firms is indeterminate and without loss of generality we assume that there is a single representative firm.

The final good can be either consumed or invested into physical capital or consumer durables. Let K_t^d denote the aggregate stock of consumer durables in period t. The aggregate resource constraint then reads as

$$C_t + K_{t+1} - (1-\delta)K_t + K_{t+1}^d - (1-\delta^d)K_t^d = F(K_t, L_t), \tag{2}$$

where C_t are aggregate consumption expenditures and δ and δ^d are the depreciation rates on physical capital and consumer durables, respectively.

3.3. Preferences and Endowments

Individuals are endowed with one unit of time in each period that they supply inelastically in the labor market. Individuals differ in their labor productivity due to differences in age and realizations of idiosyncratic uncertainty. The labor productivity of an individual of age j is given by $\varepsilon_j \eta$, where $\{\varepsilon_j\}_{j=1}^J$ denotes the age profile of average labor productivity. The stochastic component of labor productivity, η, follows a finite-state Markov chain with state space $\eta \in E = \{\eta_1, \ldots \eta_N\}$ and transition probabilities given by the matrix $\pi(\eta' \mid \eta)$. Let Π denote the unique invariant measure associated with π. The initial realization of the stochastic part of labor productivity is assumed to be drawn from Π for all agents.

We assume that all agents, independent of age and other characteristics, face the same Markov transition probabilities and that the fraction of the population experiencing a transition from η to η' is also given by π. This law of large numbers and the model demographic structure ensure that the aggregate labor input is constant. As with lifetime uncertainty, we assume that individuals cannot insure against idiosyncratic labor productivity by trading contingent claims. Moral hazard problems justify the absence of these markets.

In addition to his or her time endowment, an individual also possesses an initial endowment of the durable consumption good, $k_1^d \geq 0$, and an initial position of capital, $k_1 \geq 0$. In most of our applications we will assume that $k_1 = k_1^d = 0$.

Individuals derive utility from consumption of the nondurable good, c, and from the services of the stock k^d of the durable good. Individuals value streams of consumption and durables $\{c_j, k_j^d\}_{j=1}^J$ according to

$$E_0 \left\{ \sum_{j=1}^{J} \beta^{j-1} u\left(c_j, k_j^d\right) \right\}, \tag{3}$$

where β is the time-discount factor and E_0 is the expectation operator, conditional on information available at time 0. The period utility function u is assumed to be strictly increasing in both arguments, strictly concave, with diminishing marginal utility from both arguments, and obeying the Inada conditions with respect to nondurable consumption. The instantaneous utility from being dead is normalized to zero and expectations are taken with respect to the stochastic processes governing survival and labor productivity.

3.4. Timing and Information

The timing of events in a given period is as follows. Households observe their idiosyncratic shock η and receive transfers from accidental bequests. Then labor and capital are supplied to firms and production takes place. Next households receive factor payments and make their consumption and asset allocation decisions. Finally uncertainty about early death is revealed. Durables are not transferred until the end of the period. In that way, even if the household sells its stock of durables and uses the payment to finance present consumption of nondurables, it will hold the durables (and receive utility from the service flow) until the end of the period. Analogously, the addition or subtraction to the stock will not influence the present period service flow. All information is publicly held and the idiosyncratic labor productivity status (as well as the survival status) becomes common knowledge upon realization.

3.5. Equilibrium

We will limit our attention to stationary equilibria in which prices, wages, and interest rates are constant across time. Individuals are assumed to be price takers in the goods and factor markets they participate in. In each moment of time individuals are characterized by their position of capital and holdings of consumer durables, as well as their age and labor productivity status (k, k^d, η, j). Let $\Phi(k, k^d, \eta, j)$ denote the measure of agents of type (k, k^d, η, j), constant in a stationary equilibrium.

We normalize the price of the final good to 1 and let r and w denote the interest rate and wage rate for one efficiency unit of labor, respectively. Also let Tr denote transfers from accidental bequests. The consumer problem can now be formulated recursively as

$$V(k, k^d, \eta, j) = \max_{c, k' k^{d'}} u(c, k^d) + \beta\alpha_j \sum_{\eta'} \pi(\eta' \mid \eta) V(k', k^{d\prime}, \eta', j+1) \quad \textbf{(4)}$$

$$\text{s.t.}$$

$$c + k' + k^{d\prime} = w\eta\varepsilon_j + (1+r)k + (1-\delta^d)k^d + \text{Tr}$$

$$k' \geq \bar{b}(k^{d'}, \eta, j)$$

$$c \geq 0, k^{d'} \geq 0.$$

Several specifications of the constraints $\bar{b}$ that limit short sales of capital will be discussed below. Note that these constraints are allowed to vary by age and current labor productivity status[15] to reflect differences in future earning potentials among agents, and are allowed to vary by durable holdings for the next period to allow for collateralized borrowing.

We are now ready to define a stationary equilibrium. Let $\mathcal{J}$ and $\mathcal{E}$ be the power sets of J and E, respectively, and $\mathcal{B}$ be the Borel sets of **R** Let $S = \mathbf{R} \times \mathbf{R} \times \mathbf{E} \times \mathbf{J}$ and $\mathcal{S} = \mathcal{B} \times \mathcal{B} \times \mathcal{E} \times \mathcal{J}$ and M be the set of finite measures over the measurable space $(S, \mathcal{S})$.

DEFINITION 1. *A stationary equilibrium is a value function* V, *policy functions for the household,* $(c, k', k^{d\prime})$, *labor and capital demand for the representative firm,* (K, L), *prices* (w, r), *transfers* Tr, *and a finite measure* $\Phi \in M$ *such that*

1. *Given* (w, r) *and* Tr, V *solves the functional equation* (4) *and* $(c, k', k^{d\prime})$ *are the associated policy functions.*
2. *Input prices satisfy*

$$r = F_K(K, L) - \delta$$

$$w = F_L(K, L).$$

3. *Markets clear:*

$$\int c(k, k^d, \eta, j)\, d\Phi + \delta \int k'(k, k^d, \eta, j)\, d\Phi + \delta^d \int k^{d\prime}(k, k^d, \eta, j)\, d\Phi$$

$$= F(K, L)\,\textit{(Goods Market)}$$

$$\int k'(k, k^d, \eta, j)\, d\Phi = K\ \textit{(Capital Market)}$$

$$\int \eta\varepsilon_j\, d\Phi = L\ \textit{(Labor Market).}$$

4. *Transfers are given by*

$$\text{Tr} = \int [k'(k, k^d, \eta, j) - k] d\Phi + \int [k^{d\prime}(k, k^d, \eta, j) - k^d] d\Phi.$$

5. *The measure follows*

$$\Phi = T(\Phi)$$

where T is the law of motion generated by π and the policies k' and $k^{d,\prime}$ as described below.

This definition is standard, possibly apart form the definition of transfers. The distribution of agents at the beginning of the period, Φ, does not include the individuals that died at the end of last period. Hence total accidental bequests of capital from deceased households at the end of last period equal

$$\int [k'(k, k^d, \eta, j) - k] d\Phi, \tag{5}$$

where we also use the fact that the total number of agents in the economy is normalized to 1. A similar argument holds for bequests of consumer durables.

We now describe what we mean by the law of motion T being generated by π and the policies k' and $k^{d\prime}$. The operator T maps M into M in the following way. Define the *transition function* $Q : (S, \mathcal{S}) \rightarrow [0, 1]$ by

DEFINITION 2. *For all $S' = R' \times Z' \times E' \times J' \in \mathcal{B} \times \mathcal{B} \times \mathcal{E} \times \mathcal{J}$ and all $s = (k, k^d, \eta, j) \in S$,*

$$Q(s, S') = \sum_{\eta' \in E'} \begin{cases} \alpha_j \pi(\eta' \mid \eta) & \text{if } j + 1 \in J', k'(k, k^d, \eta, j) \in R', k^{d\prime}(k, k^d, \eta, j) \in Z' \\ 0 & \text{else.} \end{cases}$$

Thus for all $J' \in \mathcal{J}$ such that $0 \notin J'$ we have

$$T(\Phi)(S') = \int Q(s, S')\, d\Phi.$$

For $J' = 0$ we have

$$T(\Phi)(R' \times Z' \times E' \times \{0\}) = \sum_{\eta' \in E'} \begin{cases} \Pi(\eta')\mu_1 & \text{if } 0 \in R', 0 \in Z' \\ 0 & \text{else.} \end{cases}$$

Note that this definition implicitly assumes that individuals are born with zero assets (capital and consumer durables).

To complete the description of the model, our specifications of the borrowing limits are as follows. In our benchmark economy we specify the borrowing limits $\bar{b}(k^{d'}, \eta, j)$ to be the smallest number to satisfy

$$V(\bar{b}(k^{d'}, \eta, j), k^{d'}, \eta', j + 1) \geq V(0, 0, \eta', j + 1) \quad \text{for all } \eta' \in E;$$

that is, households can borrow up to the point at which, for all possible realizations of the stochastic labor productivity shock tomorrow, they have an incentive to repay their debt rather than to default, with the default consequence being specified as losing their debt, but also their consumer durables. Thus consumer durables play an important role not only in generating consumption services but also as collateralizable assets against which agents can borrow.

We will also report results for economies in which borrowing limits are specified as

$$\bar{b}(k^{d'}, \eta, j) = 0$$

and as

$$\bar{b}(k^{d'}, \eta, j) = -\kappa k^{d'}.$$

The first specification prevents borrowing altogether. Although we do not view this specification as reasonable in an economy with collateralizable assets, because a large fraction of previous work on life-cycle consumption (in the absence of durables) has explicitly or implicitly (via judicious choice of the income process) used this specification, we want to present similar results for comparison. The second specification allows households to borrow up to a percentage κ against their stock of consumer durables.

We finish this section by discussing an important element of our model: the absence of a durable goods rental market. Suppose that households, in addition to buying durable goods, can also rent them from competitive providers of durable services for a rental rate of p_r. Consistent with the timing of the model, suppose a unit of durables rented today yields consumption services tomorrow. Then the rental price has to satisfy $p_r = r + \delta^d$, and the net cost of renting one unit of services for tomorrow is $r + \delta^d$, whereas the net cost of obtaining one unit of durables services via buying is $1 - (1 - \delta^d)/(1 + r) = (r + \delta^d)/(1 + r) < r + \delta^d$ as long as the interest rate is positive.[16] In addition, purchased consumer durables relax the borrowing constraint and thus make buying instead of renting even more attractive. Therefore our modeling choices (households can buy and sell durables without adjustment cost) would imply that the option to rent the durable is strictly dominated by purchasing it, using it, and selling it afterward.

Obviously the introduction of transactions and agency costs associated with purchases of consumer durables (but also with repeatedly renting them) changes the argument; in our model with cross-sectional heterogeneity both positive rentals and purchases of durables would potentially occur in equilibrium.[17] Are these effects important? Our answer is that, even if they may be of some importance, the insights of including an explicit rental market may not compensate for the additional computational burden involved. Note also that the existence of collateralized loans reduces the theoretical role of rental markets. Households, by judicious choice of when and how much to borrow, will be able to reproduce nearly the same intertemporal allocation of consumption in our model as in a model with an explicit rental market.

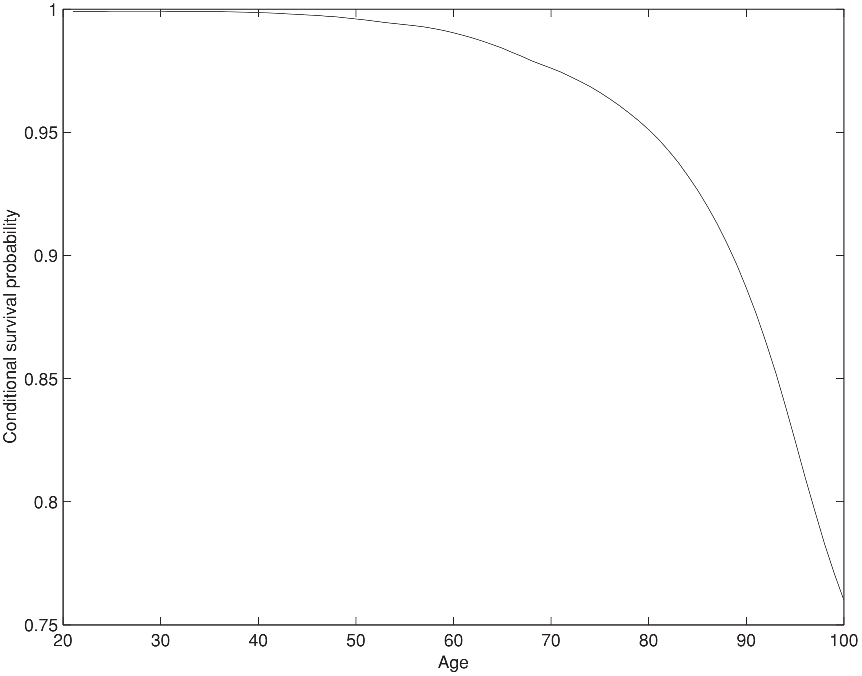

FIGURE 10. Conditional survival probabilities.

4. CALIBRATION

We choose the benchmark parameterization of our economy partly on the basis of microeconomic evidence and partly so that the stationary equilibrium for our economy matches selected long-run averages of U.S. data.

4.1. Demographics

We define a year as our unit of time. Then, with respect to demographics, we will have $J = 81$ generations. Therefore we can interpret our model as one in which households become economically active at age 20, and live up to age 100. The conditional survival probabilities $\{\alpha_j\}_{j=1}^{J}$ are taken from Faber (1982).[18] We plot these survival probabilities in Figure 10.

4.2. Technology

We select a Cobb–Douglas production function $F(K_t, L_t) = AK_t^{\alpha}L_t^{1-\alpha}$ as a representation of the technology that produces the final good. We normalize $A = 1$ and set $\alpha = 0.3$ so that the equilibrium of our economy matches the long-run labor share of national income for the United States of approximately $1 - \alpha = 0.7$.

We choose the depreciation rates δ and δ^d of physical capital and consumer durables to match investment shares of output and capital–output ratios for the U.S. economy. In the steady state of our model $I = \delta K$ and $I^d = \delta^d K^d$ and hence $\delta = (I/Y)/(K/Y)$ and $\delta^d = (I^d/Y)/(K^d/Y)$.

We use data from the 2000 comprehensive revision of NIPA and *Fixed Assets and Consumer Durable Goods* of the Bureau of Economic Analysis (see http://www.bea.doc.gov for detailed information and downloadable tables) to compute K, defined as Private Nonresidential Fixed Assets (equipment, software, and nonresidential structures), and K^d, defined as Private Residential Structures and Consumer Durable Goods. Because the NIPA are somewhat inconsistent in the treatment of the household sector (the accounts do include the imputed flow of services from owner-occupied housing as part of GDP, but not the services from other durables), we adjust NIPA data when needed to reflect the measurement definitions in our economy: final, physical goods produced in the period. We use as our benchmark calibration $\delta = (I/Y)/(K/Y) = 0.135/1.2 = 0.1125$ and $\delta^d = (I^d/Y)/(K^d/Y) = 0.12/1.45 = 0.0857$.

4.3. Preferences and Endowments

In each period agents supply one unit of time, the productivity of which is given by $\varepsilon_j \eta$. The deterministic age profile of the unconditional mean of labor productivity $\{\varepsilon_j\}_{j=1}^J$ is taken from Hansen (1993). We take $\varepsilon_j = 0$ for $j \geq 46$, in effect imposing mandatory retirement at the age of 65.

In the parameterization of the stochastic idiosyncratic labor-productivity process we follow Storesletten et al. (2007). They build a rotating panel from the Panel Study of Income Dynamics (PSID) to estimate the stochastic part $u_{it} = \ln(\eta_{it})$ of the labor income process for household i at time t,

$$\begin{aligned} u_{it} &= z_{it} + \varepsilon_{it} \\ z_{it} &= \rho z_{it-1} + \nu_{it}, \end{aligned} \tag{6}$$

where $\varepsilon_{it} \sim N(0, \sigma_\varepsilon^2)$ and $\nu_{it} \sim N(0, \sigma_\nu^2)$ are innovation processes. Their point estimates are $\rho = 0.935$, $\sigma_\varepsilon^2 = 0.017$, and $\sigma_\nu^2 = 0.061$.

This process differs from other specifications in the literature [see Abowd and Card (1989), Carroll (1992), or Gourinchas and Parker (2002), among others] in two aspects. First, we do not allow labor income to go to zero. Even if this event has a very low probability [Carroll (1992) estimates this probability as 0.003 for a year], its effects on intertemporal allocations are substantial: households will not borrow any positive amount because they may face a lifelong sequence of zero labor income and may be unable to consume a positive amount and repay their debt in some period. This implication seems debatable, in particular in light of the existence of a collection of public income-support programs in the United States, given that the notion of labor income in the model should be interpreted as after-tax, after–government transfer labor income. Second, we do not impose a unit root in the autoregressive process for z_{it} because Storesletten et al. (2001) are able to reject the null of a unit root.[19] This choice remains, however, an open and debated issue. In small samples it is very difficult to separate a unit root from our value $\rho = 0.935$, especially because with finitely lived families, the stochastic process cannot drift away too much from its initial condition.[20] Fortunately, our results are not very sensitive to this choice, as shown below when we perform sensitivity analysis by increasing ρ toward unity.

Using the method proposed by Tauchen and Hussey (1991), we approximate this continuous state $AR(1)$ process with a three-state Markov chain,[21] which results in

$$E = \{0.57, 0.93, 1.51\}, \tag{7}$$

$$\pi = \begin{bmatrix} 0.75 & 0.24 & 0.01 \\ 0.19 & 0.62 & 0.19 \\ 0.01 & 0.24 & 0.75 \end{bmatrix}, \tag{8}$$

$$\Pi = [0.31, 0.38, 0.31]. \tag{9}$$

As initial endowments of physical capital and durables, we assume $k_1 = k_1^d = 0$.

With respect to preferences, we assume that the period utility function is of CRRA type:

$$u(c, k^d) = \frac{[g(c, k^d)]^{1-\sigma} - 1}{1-\sigma}, \tag{10}$$

where $g(\cdot, \cdot)$ is an aggregator function of the services flows from durables and nondurables. A simple but quite general choice for the aggregator is a CES aggregator of the form

$$g(c, k^d) = [\theta c^\tau + (1-\theta)\,(k^d + \varepsilon)^\tau]^{1/\tau}, \tag{11}$$

where ε is a number small enough to be irrelevant to our quantitative exercises, but makes the utility function finite for $k^d = 0$ (the intuition being that one can survive without a house and other consumer durables, but one cannot survive without food).

Unfortunately, we do not have conclusive empirical evidence about the value of τ. Eichenbaum and Hansen (1990) find that the substitutability between durables and nondurables is highly sensitive to the overall specification of preferences. McGrattan et al. (1997) use aggregate data to estimate, in a model where labor input is needed to complement durables to produce consumption services, a value of $\tau = 0.429$ with standard error of 0.116. Rupert et al. (1995) explore a number of different specifications of a model similar to that of McGrattan et al. (1997) using PSID data. They find that the estimated values of τ differ greatly with changes in the sample composition and that overall, their results are not particularly informative. For example, they estimate $\tau = -0.065$ with standard error 0.471 for single males, whereas the equivalent estimates for single females are 0.445 and 0.121 and for couples 0.083 and 0.292. It is interesting that two of the three results are not significantly different from zero. Ogaki and Reinhart (1998), using aggregate data and a specification similar to ours, estimate $\tau = 0.143$, not significantly different from zero at the 5% level. Given this range of estimates, we find it reasonable to adopt as a benchmark the case $\tau = 0$ (the aggregator function takes a Cobb–Douglas form) and test later for sensitivity of the results to our choice. The resulting period utility function is then given by

$$u(c, k^d) = \frac{(c^\theta (k^d + \varepsilon)^{1-\theta})^{1-\sigma} - 1}{1-\sigma}. \quad \mathbf{(12)}$$

Also, for our benchmark calibration, we choose a coefficient of relative risk aversion $\sigma = 2$, a value in the middle of the range commonly used in the literature.

We jointly pick the parameters θ and the time discount factor β so that the steady-state equilibrium for our benchmark calibration has an interest rate of $r = 4\%$ [see McGrattan and Prescott (2001) for a justification of this number based on their measure of the return on capital and on the risk-free rate of inflation-protected U.S. treasury bonds] and a ratio of expenditures on nondurables and durables of $C/I^d = (C/Y)/(I^d/Y) = 6.2$, the long-run average for U.S. data. This results in the choices $\beta = 0.9375$ and $\theta = 0.81$.

5. RESULTS

5.1. Aggregate Variables

In Table 1 we report values for aggregate variables for our benchmark economy.

Because we calibrated β and θ to match an interest rate of 4% and a ratio between expenditures on nondurables and durables of 6.2, the first and last entries of Table 1 are obtained by construction. GDP in our economy is used for consumption (78%) and investment in physical capital (22%). These figures are in line with long-run averages for the U.S. economy (remember that our definition of GDP does not include housing services). The capital–output ratio $(K + K^d)/Y$ equals

TABLE 1. Steady state

Variable	Steady state value
r	4%
C/Y	0.67
I/Y	0.22
I^d/Y	0.11
w/Y	0.54
L	1.29
K/Y	1.97
K^d/Y	1.26
C/I^d	6.2

approximately 3.23, and the aggregate capital stock is composed 60% of physical capital and 40% of consumer durables. The average annual wage wl amounts to about 100% of GDP per capita Y, where l is the average labor productivity of the working population, given by

$$l = \frac{L}{\text{fraction of population that works}} = \frac{1.29}{0.7}.$$

Note that the capital–output ratio for physical capital is quite a bit higher than the one used in the calibration section. There are two possible explanations for this finding. First, it may indicate that the interest rate we try to match is too low. However, if we insist on our choices $\delta = 11.25\%$ and $\alpha = 0.3$, then to obtain $K/Y = 1.2$ would require $r = 13.75\%$ because in a stationary equilibrium

$$\frac{K}{Y} = \frac{\alpha}{r + \delta}.$$

This interest rate is well beyond the plausible range for risk-free rates.

A second explanation for the high ratio of physical capital is the absence of social security in our model. It is known that, in a standard dynamic general equilibrium model, a pay-as-you-go social security system that is not perfectly linked to contributions but redistributive, such as the current system in the United States, tends to reduce the level of asset accumulation in equilibrium.[22] As a consequence, our model should overpredict the amount of physical capital in the economy. The absence of social security biases the results against our main argument: the importance of durables in explaining the life-cycle profiles of consumption and assets accumulation. If we show that durables are key to explaining these profiles even when no social security exists and the incentive for financial accumulation is higher, the result will hold even more tightly with a redistributive social security system. The size of this bias is, however, uncertain and the effects of social security in an economy with durables deserve further research.

5.2. Life-Cycle Profiles

Figures 11 and 12 show the average life-cycle patterns of labor income, nondurable consumption expenditures, and the stocks of consumer durables, financial assets, and total net worth. We plot age on the x-axis, following our interpretation that agents start their economic life at the age of 20 and live up to the age of 100.

These averages are obtained by integrating the policy functions with respect to the equilibrium measure of agents, holding age fixed. For example, average nondurable consumption expenditures by cohort j are given by

$$C_j = \int c(k, k^d, \eta, j)\Phi(dk \times dk^d \times d\eta \times \{j\}).$$

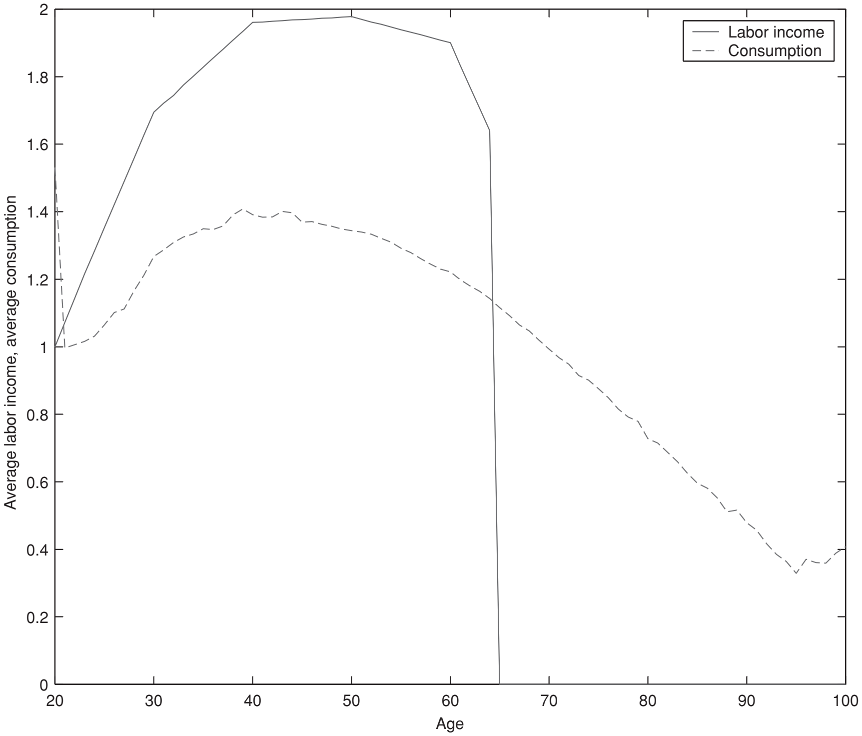

FIGURE 11. Life-cycle pattern of labor income and nondurable consumption.

Note that due to stochastic death, cohorts are not of the same size, so that population averages are *weighted* averages of cohort averages.

From Figure 11 we see the hump shape of average labor income. This hump shape arises by construction because the life-cycle profile for labor income parallels the life-cycle profile of average labor productivity $\{\varepsilon_j\}_{j=1}^{J}$, which obeys a hump shape over the life cycle, with peak around the age of 50. Also, note that at age 65 agents retire in our model, which is induced by assuming $\varepsilon_j = 0$ for $j \geq 46$.

Also, in Figure 11 we can see that expenditures on nondurable consumption obey a hump-shaped life-cycle pattern, with peak around the age of 45[23] and a pattern and, more importantly, a *size (*about 40% bigger than age 20) quite similar to the ones reported in Figure 5. The increase in nondurable consumption in the early part of life is due to two factors in our model, both of which are crucially dependent on the presence of consumer durables. First, because durables generate service flows, early in life it is optimal to build up the stock of consumer durables and compromise on the consumption of nondurables. Second, once the stock of nondurables is built up, due to the nonseparabilities in the utility function the marginal utility from nondurable consumption is higher, because of a higher stock

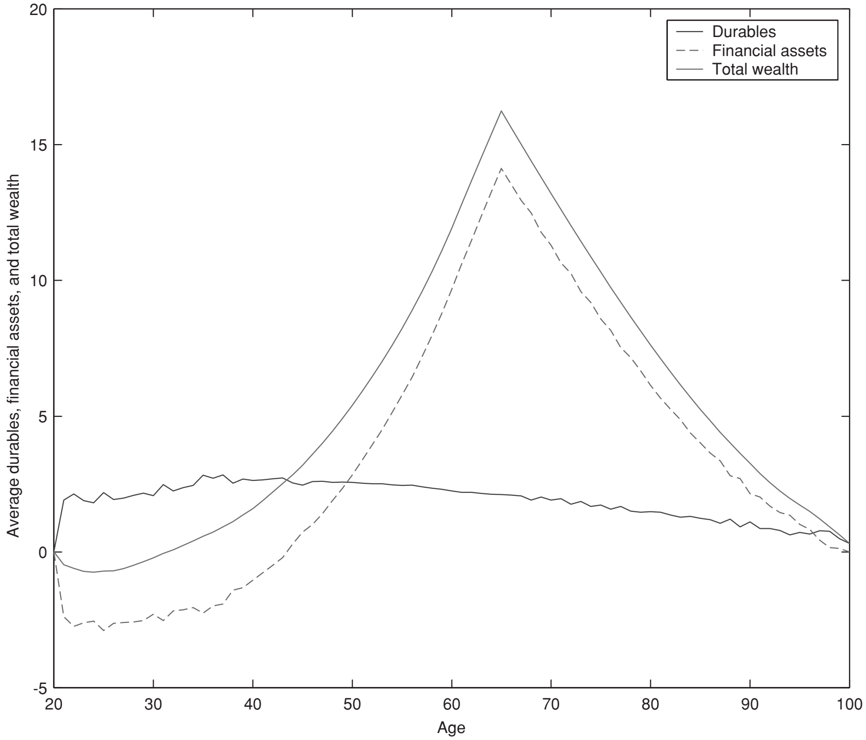

FIGURE 12. Life-cycle pattern of consumer durables, financial wealth and total wealth.

of durables. The hump shape in nondurable consumption is *not* due to buffer stock behavior per se as in Carroll (1992) or Gourinchas and Parker (2002): households in our model, once they have accumulated consumer durables, can use these as collateralizable insurance against unfavorable labor productivity shocks as their borrowing capacity increases with their holding of durables.

It is important to note the increase of consumption late in life. This small increase is due to lifetime uncertainty. Households want to buffer until almost the end of their lives, then they consume in the last periods, because survival probabilities are low or zero. Also, from Figure 11, we see how average consumption tracks deterministic average labor income. Because this income increase is perfectly forecastable, our model displays excess sensitivity of consumption to income, as suggested by empirical data, and contrary to the predictions of the basic life cycle–permanent income model [see Deaton (1992) for a review].

In Figure 12 we show how the average wealth portfolio evolves over the life cycle. Early in life, households borrow as much as possible to buy houses and other consumer durables. As time goes by, the stock of durables is built up and holdings of financial assets, as well as nondurable consumption, increase. Because households can borrow against their durable assets, the accumulation of financial

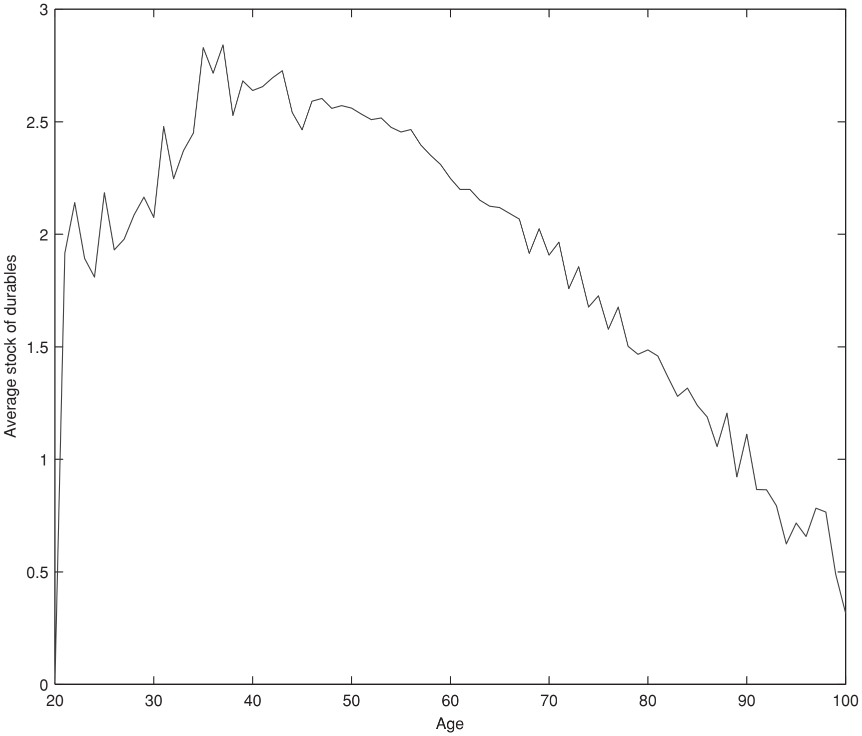

FIGURE 13. Average stock of durables.

assets occurs for life-cycle and not for insurance purposes. Our model reproduces important facts about the life-cycle composition of wealth: young households do save (net worth becomes significantly positive by the age of 35), but they do not save in financial assets, but rather in consumer durables. As households become older, financial assets become a more important part of the household's wealth portfolio, with these assets being accumulated primarily to finance consumption in retirement. Note that total net worth peaks at age 64, the year prior to retirement. Also, note that households hold substantial net worth until high ages, mainly for insurance against living too long. This corresponds to the observation that elderly households seem to overaccumulate assets (or more precisely they do not run down their wealth fast enough). This peak in net worth at age 64 would be far less pronounced in the presence of a pay-as-you go social security system, because part of the life-cycle motive for savings and the precautionary savings motive due to stochastic mortality disappears.

In Figure 13 we plot the total stock of consumer durables. The stock follows a hump shape, differing from a complete-markets model where the desired stock is built up in the first period and only an amount equal to depreciation is spent each period thereafter. We plot the average expenditure on durables in Figure 14. From

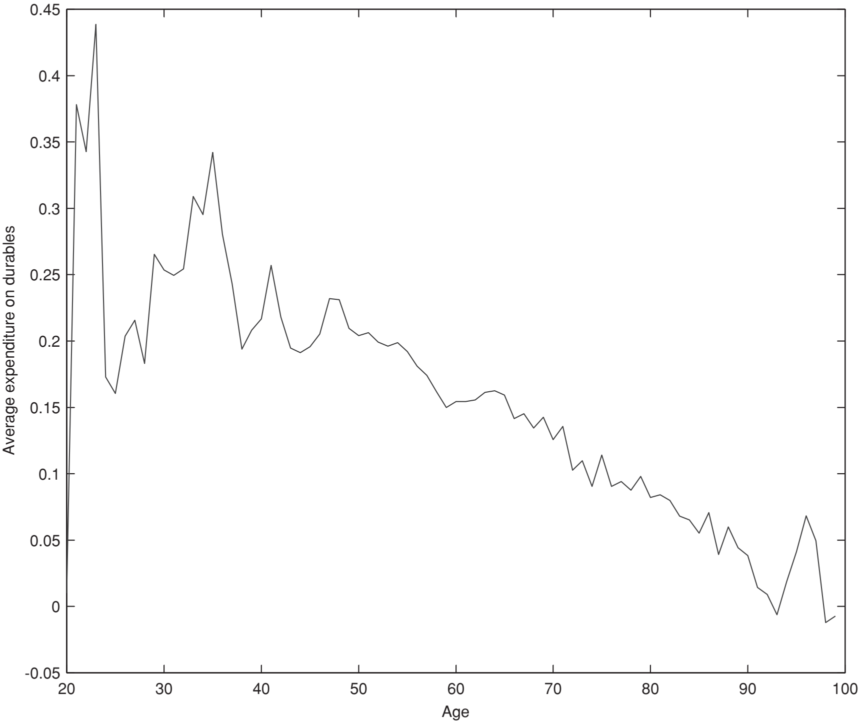

FIGURE 14. Average expenditure on durables.

this graph we notice that the model generates a pattern of consumer durables that somehow diverges from the observed pattern: there is a big peak in the first years and then it falls, even though it is possible to see something of a hump after the first spike. One possible explanation is that in the data young families obtain bequests, which in large part come as consumer durables. A second possible explanation is the endogenous formation of households in the data. In our model, all households enter their active economic life at age 20, a time period where they want to build up the desired stock of capital. In the data, however, economically active (in the sense of our model) households are created endogenously due to differences in marriage timing and education. This endogeneity smooths out the first big spike of durable expenditures in the data and leads to the pattern of life-cycle durables expenditure reported in Section 2.[24]

In Figure 15 we plot several simulated life-cycle patterns, from which we observe how households adjust their consumption decisions to labor income shocks. These shocks, however, although quantitatively important, are not able to overcome, the general pattern of a life-cycle hump in consumption on average.

The stochastic patterns in Figure 15 raise the question of the role of idiosyncratic income uncertainty. To address this issue, we solve the model, setting the variances

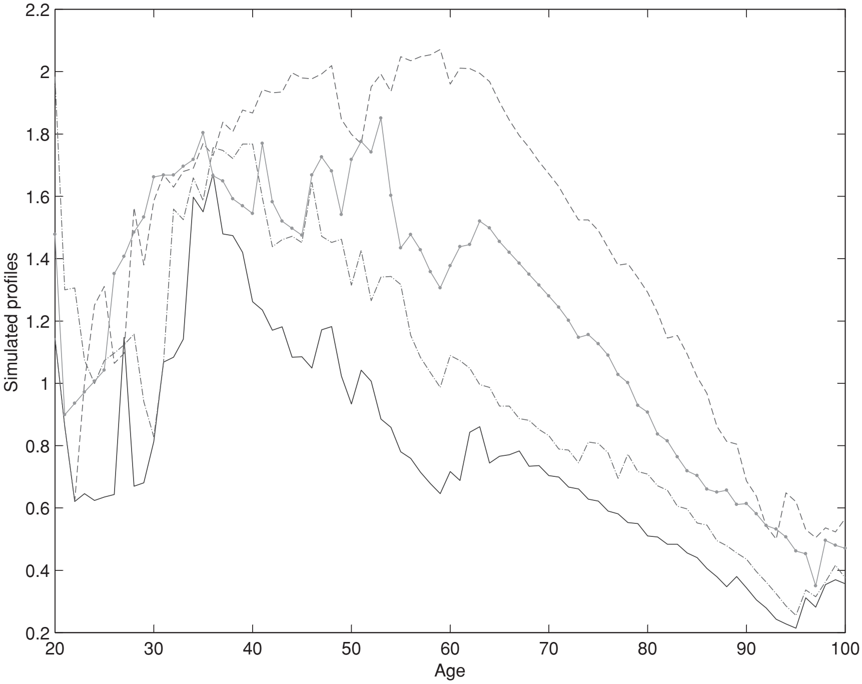

FIGURE 15. Some simulated life-cycle consumption profiles.

of the labor income innovations to zero, i.e., computing the model without labor income uncertainty. The results are plotted in Figures 16 and 17. Two results are worth mentioning. First, from the life-cycle pattern of nondurable consumption in Figure 16, we can see that, once uncertainty is eliminated, half of the hump in nondurable consumption disappears: the new profile is substantially smoother than before (see Figure 11). With uncertainty, borrowing constraints are tighter, because default has to be prevented in all income states tomorrow, in particular in the high-income states. A tighter borrowing constraint makes consumption comove more with income. In addition, risk-averse households postpone a larger fraction of consumption until an important degree of uncertainty is revealed. In contrast, without labor income uncertainty, this effect is absent and higher nondurable consumption sets in earlier in life.

Second, the average holding of durables is smaller. As explained before, in an environment with uninsurable stochastic labor income, durables are also accumulated because of the collateral services they provide in allowing borrowing to smooth nondurable consumption. Comparing the stock of durables from Figure 17 with the stock of durables from Figure 12, we can see that, for prime age households, the average holding of durables is reduced by around 15%.

Finally, our model is also able to cast some light on two other important issues. First, the model predicts that only 58% of the households hold any financial wealth

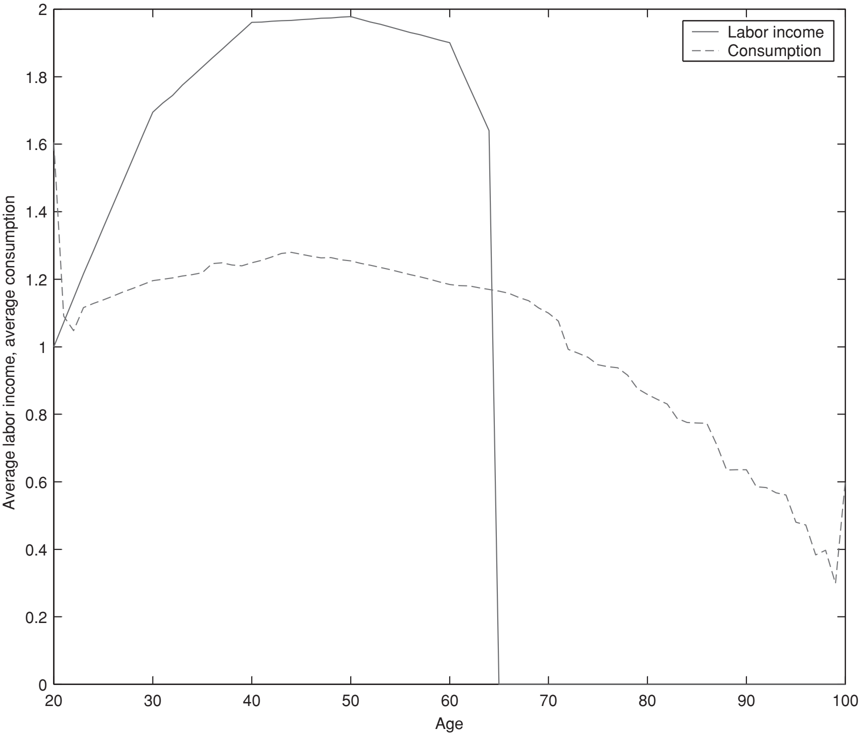

FIGURE 16. Life-cycle pattern of labor income and nondurable consumption, no uncertainty.

in equilibrium, and these are households in later periods of their life. This low participation corresponds to the evidence on financial assets from the SCF and is obtained without the need for model elements such as a very high elasticity of intertemporal substitution or transaction costs, commonly used in the literature to generate similar results. The presence of an alternative asset that also generates consumption and collateral services is enough to discourage 42% of households from participating in financial markets.

Second, consumer durables can help to explain why households with higher life-cycle income save proportionally more than poor households [see the empirical evidence presented in Dynan et al. (2004)]. Figure 18 plots the life-cycle profile of financial assets for the average household, for a household that always enjoys the high labor income shock, *ex post*, and for a household that always suffers the low income shock, also *ex post*. We can see from this plot how the high-income household, despite having a realized lifetime income that is only 2.6 times higher than that of the low-income households, has accumulated over six times more financial assets at age 65. This result comes from the strong nonhomogeneity introduced by the dual role of durables as a saving instrument and a consumption good: as the household becomes richer, the marginal utility of durables decreases

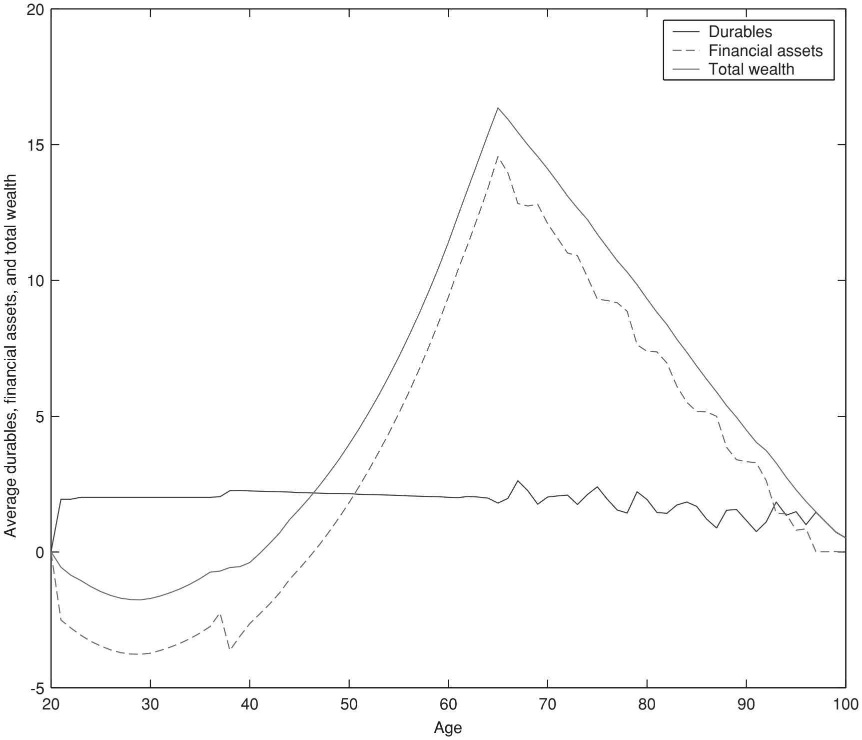

FIGURE 17. Life-cycle pattern of consumer durables, financial wealth, and total wealth, no uncertainty.

and financial assets become relatively more attractive. Quantitatively, around 50, the high-income household only has around twice as many durables as the low-income household, but has already accumulated an important stock of financial assets when the poor household still has negative financial wealth.

6. SENSITIVITY ANALYSIS

In this section we will consider two different issues. First, Sections 6.1 and 6.2 will study the behavior of the model under the two alternative borrowing constraints outlined in Section 3. Second, in Section 6.3 we will check the robustness of our calibration to different changes in parameter values.

6.1. Ad Hoc Borrowing Constraints

In this section we describe how our main results change as we adopt a different form of borrowing constraint. The case on which most of the literature on life-cycle consumption without durable goods has focused is an ad hoc specification

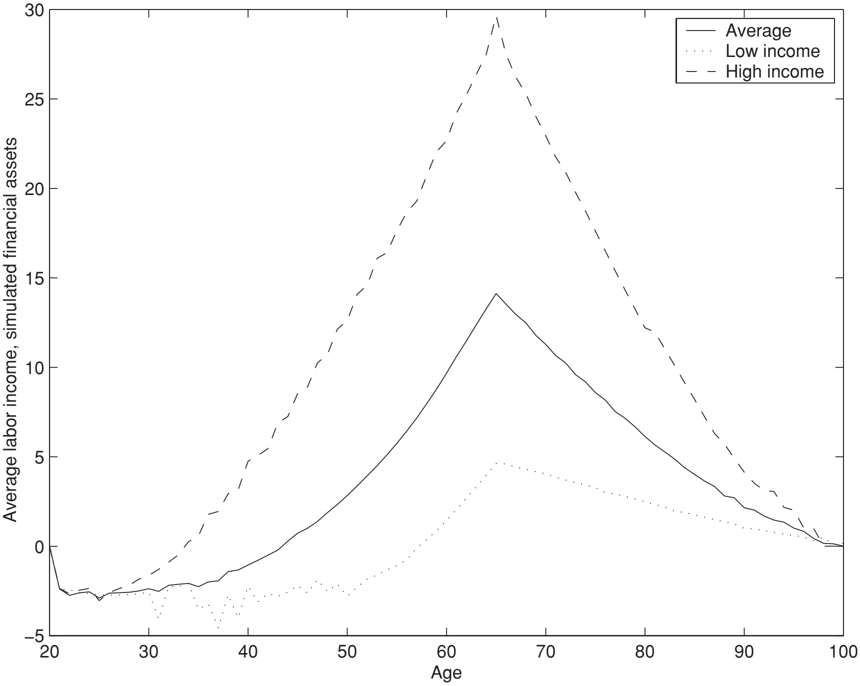

FIGURE 18. Simulated and average life-cycle pattern of financial assets.

limiting short sales of bonds to a fixed number $\bar{b}$. Often this number is set to $\bar{b} = 0$ [see Aiyagari (1994) and Krusell and Smith (1998), among others].[25] Although such a specification of the borrowing constraint in the presence of a collateralizable asset seems somewhat unreasonable, we want to relate our results to the existing literature and hence adopt the constraint $\bar{b} = 0$. Thus, durables, while still providing services and hence utility to households, lose their role as collateralizable assets. We leave all other parameters unchanged from the benchmark calibration.

In Table 2 we report values for aggregate variables for the economy in which agents are prevented from borrowing. The main difference between the economy with the endogenous borrowing constraint and the economy with no borrowing is that the interest rate is significantly lower in the latter case. This is a direct consequence of less demand for capital (loans), due to the fact that households are prevented from borrowing. It can also be argued that households now are prevented from smoothing bad income realizations and hence wish to hold a higher buffer stock of financial assets to self-insure against this income risk, particularly early in life, when mean income is low. We will argue below when discussing life-cycle patterns of consumption and asset accumulation that, in the presence of consumer durables, this is not the case.

TABLE 2. Steady state

Variable	Steady state value
r	2.25%
C/Y	0.65
I/Y	0.25
I^d/Y	0.1
w/Y	0.54
L	1.29
K/Y	2.21
K^d/Y	1.17
C/I^d	6.5

The total stock of physical capital is higher in the no-borrowing economy, and because by construction total labor input is fixed exogenously, total output in the economy increases. This results mirrors the theoretical findings of Aiyagari (1994) with infinitely lived agents. The higher physical capital stock requires higher investment to replace the depreciated capital; hence the investment share of output increases from 22% to 25% of GDP. Consumption as share of GDP of both nondurables and durables declines to 65% and 10%, respectively.

In Figure 19 we plot the life-cycle patterns of average labor income and consumption. By construction, the income profile is identical across the two economies. Comparing the consumption profiles, we see that in the economy with no borrowing the hump shape in consumption is more pronounced, the peak of consumption occurs later in life, and consumption declines more rapidly toward the end of life. Also, the size of the hump is much bigger than that reported in Figure 5, indicating that this specification of borrowing constraints is too extreme in imposing frictions on intertemporal trade. Because the interest rate is lower in the no-borrowing constraint economy, agents, ceteris paribus, prefer a consumption profile that declines more rapidly toward the end of life, compared to the economy with endogenous borrowing constraints. This governs behavior after the age of around 45. Prior to that the constraint on borrowing determines the consumption choice. Young households expect increasing labor income, and hence would like to borrow, which they are prevented from doing. Consequently they spend all their income and accumulate no financial assets (see Figure 20). Conditional on this fact, a decision that remains is the allocation of income between expenditures on nondurables and investment into consumer durables. Figure 19 shows that between the ages of 20 and 30 an increasing fraction of income is devoted to nondurables: at the beginning of life agents build up the stock of consumer durables, as with endogenous borrowing constraints. Because this accumulation cannot be credit-financed and hence comes at the expense of nondurable consumption, however, this process is slower in the economy with borrowing constraints that prevent all borrowing.

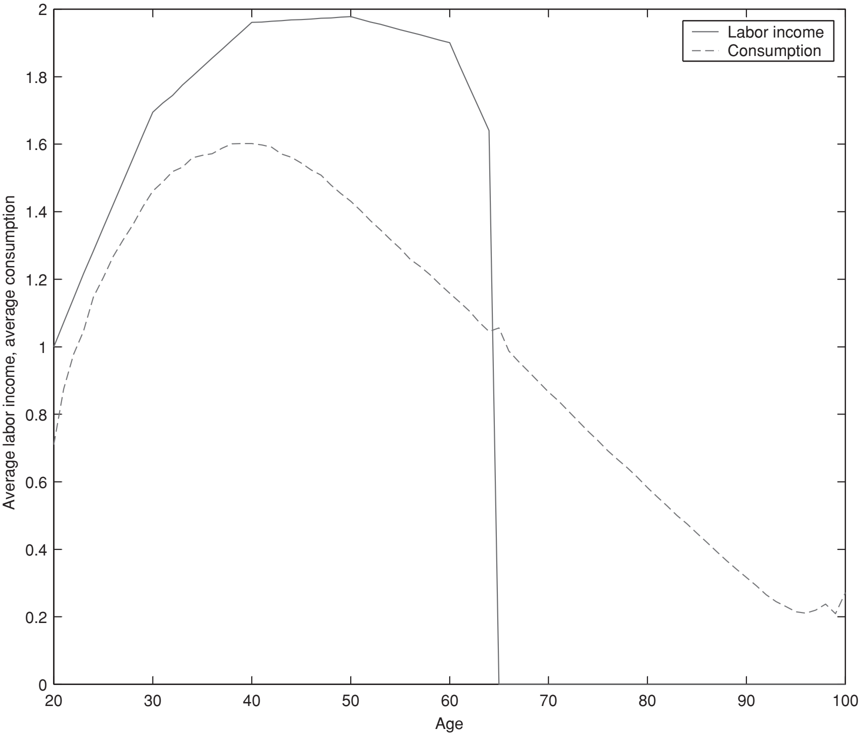

FIGURE 19. Life-cycle pattern of labor income and nondurable consumption.

From Figure 20 we also see that financial assets are not used to buffer bad income shocks early in life; this is accomplished by holding durables, which both yield services and can be sold if necessary. Financial assets are used for retirement saving, as they become the dominant asset in the average household's portfolio after the age of 40.

One important difference between the two economies is that with endogenous borrowing constraints the net worth of the average young generation is (slightly) negative: even when taking account of consumer durables, households up to the age of 30 borrow, on net, against their higher expected future labor income; mostly to finance the accumulation of durables, but also to smooth consumption over time and states (in equilibrium the low interest rate relative to the time discount factor implies that a declining consumption profile is optimal).

6.2. Consumer Durables and Fixed Down Payment

As argued before, the presence of durables suggests that, using them as collateral, households should be able to borrow up to some amount. This consideration motivates a borrowing constraint of the form $\bar{b}(k^{d'}, \eta, j) = -\kappa k^{d'}$. We pick

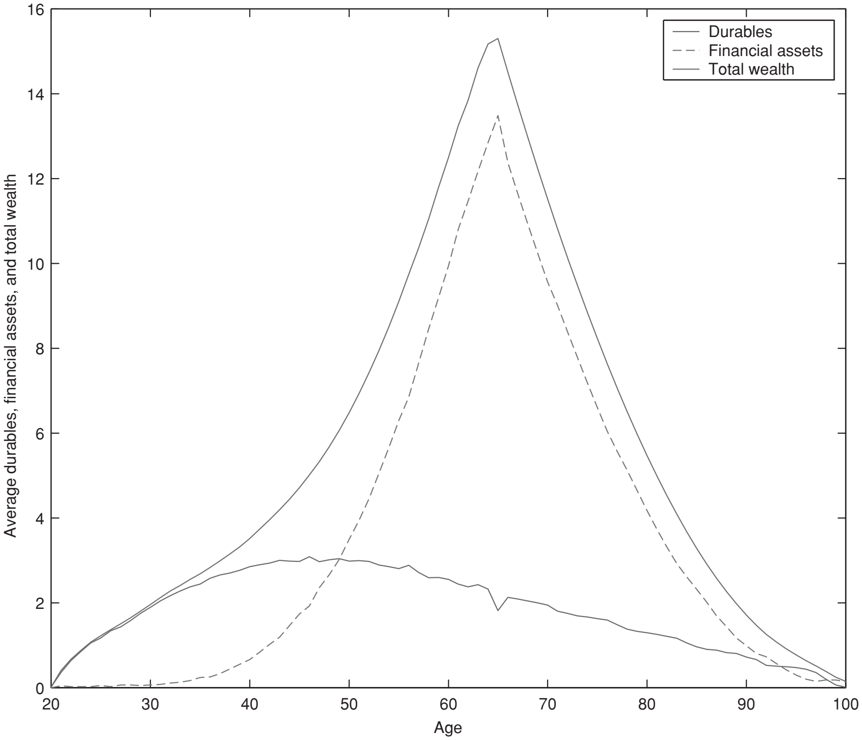

FIGURE 20. Life-cycle pattern of consumer durables, financial wealth, and total wealth.

$\kappa = 0.8$, which corresponds to a down payment requirement for purchases of consumer durables of 20%, following real estate market practices.[26] We leave all the remaining parameters of the benchmark calibration unchanged.

We present the results for aggregate variables for this economy in Table 3. As in the previous case, the main difference, compared to the endogenous–borrowing

TABLE 3. Steady state

Variable	Steady state value
r	2.24%
C/Y	0.63
I/Y	0.25
I^d/Y	0.12
w/Y	0.54
L	1.29
K/Y	2.15
K^d/Y	1.37
C/I^d	5.25

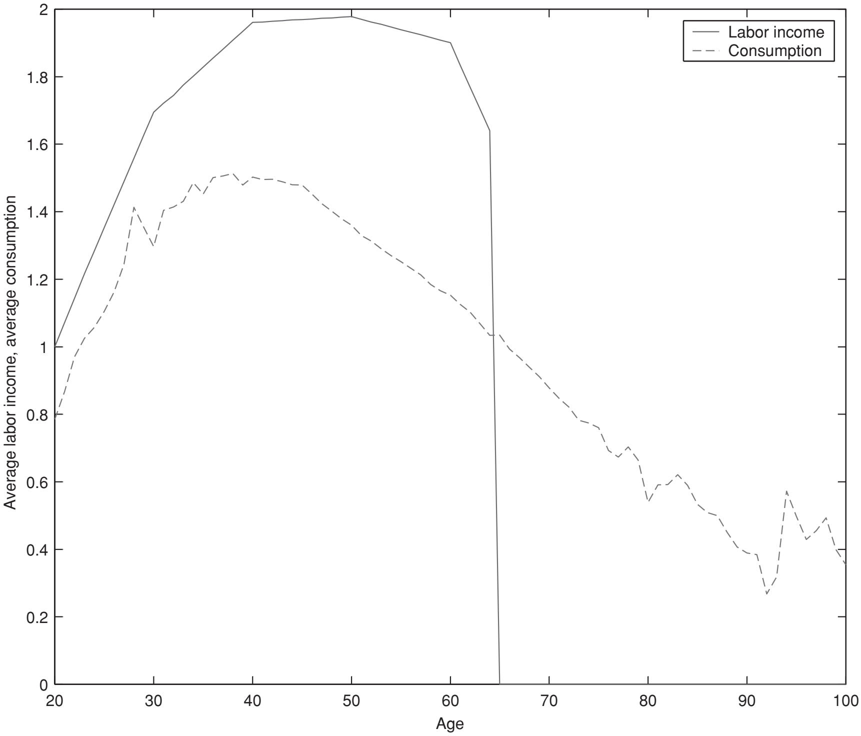

FIGURE 21. Life-cycle pattern of labor income and nondurable consumption.

constraint economy, is that the interest rate is substantially lower. Also, the stocks of physical capital and durables are higher. In fact, the stock of durables is even higher than in our benchmark economy. The result is closely related to the dual role of durables as collateral and as a generator of utility. The use of durables as collateral also reduces the importance of physical capital as a buffer to smooth income fluctuations and, as a consequence, the stock of physical capital is lower than in the case in borrowing is not permitted altogether.

In Figure 21 we plot the life-cycle pattern of average labor income and consumption. The possibility of partially financing the acquisition of durables reduces the hump with respect to the case of no borrowing, but it is still bigger than in our benchmark economy. In Figure 22 we plot the life-cycle patterns of consumer durables, financial wealth, and total wealth. In this picture we can see how households take advantage of durables as a collateral and how they have negative financial wealth until their midforties, a point at which they begin to save for retirement. However, in comparison to the benchmark case with endogenous borrowing constraint, total wealth is always strictly positive, because all borrowing has to be fully collateralized.

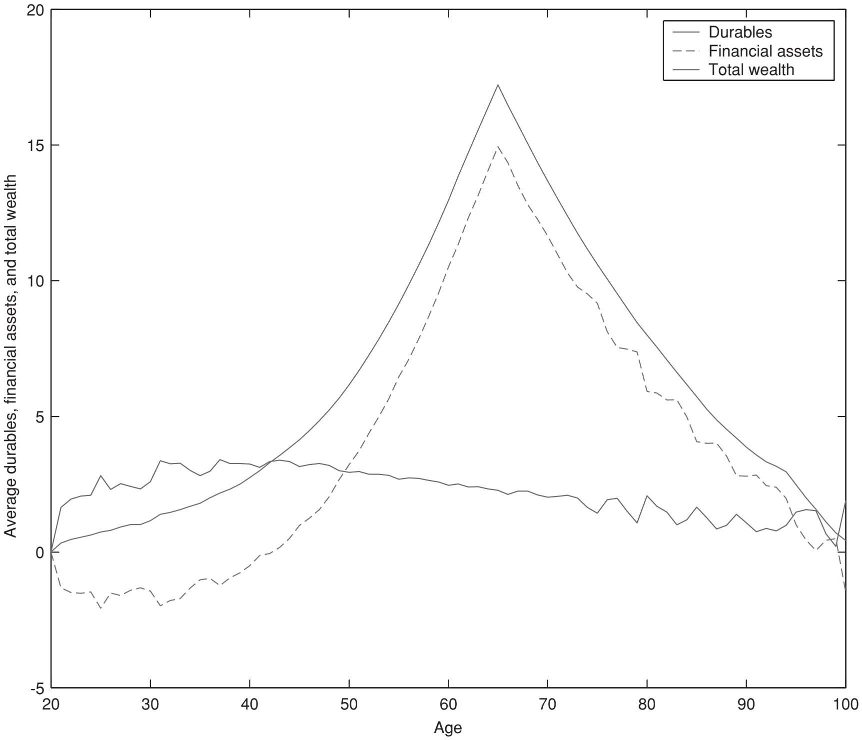

FIGURE 22. Life-cycle pattern of consumer durables, financial wealth, and total wealth.

6.3. Changes in Parameters

In this section we will check the robustness of the results to changes in parameters. First we study the effects of increasing the persistence parameter ρ. The main consequence of this increment is a bigger hump in nondurable consumption. Here the reverse arguments we used to discuss the effects of uncertainty apply. A higher persistence of labor income makes borrowing constraints tighter. High-shock households have a higher incentive to default on debts when the shock is more persistent: they can leave behind their debts and rebuild their durables stock under the better labor-income perspectives. Also, as a higher degree of uncertainty needs to be revealed through life when shocks are more persistent (the reversion to the mean of a particular realization of the shock is lower or nonexistent if a unit root is present), risk-averse households will wait until more of this uncertainty is revealed before increasing their consumption. Quantitatively we found that the effects of moving ρ from our benchmark calibration to 0.99 are small.

Also small are the effects of changing the elasticity of substitution between durables and nondurables. Even if this elasticity is crucial in the case of real–business cycles models with household production [see Greenwood et al. (1995)]

because it governs the margin of substitution between the two sectors in the economy, in our model without endogenous labor choice it is of minor quantitative importance. Higher elasticities of substitution help to delay the building of a stock of durables slightly, improving the performance of the model to match the hump in durables, but they decrease the hump in nondurables consumption.

Finally, to assess the importance of durables, we drive the weight of durables in the utility function to zero. In this case our models nests a dynamic general equilibrium version of Carroll (1997) and Gourinchas and Parker (2002).[27] We find two main differences from our benchmark calibration. First, the hump in consumption is too big. Second, households accumulate a buffer stock of financial assets at the beginnings of their lives. With our benchmark choice of endogenous debt constraints, in the absence of durables, the constraints collapses to a standard positive-borrowing condition, because without the punishment associated with the loss of durables, households do not have any incentive to pay back any negative amount of debt.[28] This highlights the importance of durables: they interact with nondurables to get the profile of life cycle right, they serve as a self-insurance device and they are key to properly accounting for the size and composition of household wealth.

7. CONCLUSION

In this paper we demonstrate that consumer durables are crucial to explain the life-cycle profiles of consumption and savings. Households begin their economic lives without a stock of durables and they are precluded from building this stock immediately by the presence of limited intertemporal markets. As a consequence, during the first part of their life cycles, households are forced to progressively accumulate durables and compromise on their consumption of nondurables and accumulation of financial assets. This phenomenon can explain why we observe that empirical life-cycle consumption profiles, both of durables and of nondurables, are hump-shaped, even after controlling for demographic characteristics, and why most households do not hold any substantial financial wealth until they enter into their forties.

To quantitatively explore this mechanism, we build a dynamic general-equilibrium life-cycle model with exogenous and uninsurable labor income risk and borrowing constraints. We parameterize the model using long-run considerations and microeconomic evidence and we use it to generate life-cycle profiles of durables and nondurables consumption.

The model is able to match two basic aspects of the data: the hump in nondurables consumption and the life-cycle component of wealth level and composition. The model also accounts for most of the hump in durables expenditures, although some improvements still remain along this dimension. In addition, the model casts some light on several other important issues such as (a) the tracking of income by consumption, (b) the amount of savings undertaken by young households, (c) the importance of insurable labor risk, (d) the low participation of households in financial markets, and (e) the higher savings rates of households with higher life-cycle income. An interesting final point is that, checking the behavior of the model under different borrowing constraints, we find that our choice of endogenous borrowing constraint outperforms the more commonly used exogenous specification of the constraint.

NOTES

1. From now on we will use the term consumer durables or, more simply, durables to include houses and other consumer durable goods. See Section 2 for detailed data on these two empirical observations.

2. See Flow of Funds Accounts, second quarter 1998.

3. A notable exception is Cocco et al. (2005).

4. A complementary approach, taken by Gourinchas and Parker (2002), fixes the interest rate and estimates the time-discount factor from cross-section micro data using the simulated methods of moments. In contrast to Gourinchas and Parker's partial-equilibrium model, in our general-equilibrium model all markets clear at each point of time.

5. This statement relies on the further assumption that leisure and consumption are separable in the period utility function. For instance, Ghez and Becker (1973) propose a model where consumption services are produced with time and consumption goods as inputs. When time becomes more expensive (i.e., labor income is higher), agents substitute goods for time in the production function of consumer services, generating a correlation between labor income and consumption.

6. Our estimator is described in detail in Fernandez-Villaverde and Krueger (2007), where we also provide a detailed discussion of the advantages of our seminonparamtric procedure.

7. Early papers that deflate household consumption expenditure by a function of family size include Zeldes (1989), who adds adjusted food requirements as a regressor in some of his Euler equation estimates, and Blundell et al. (1994), who plot the life-cycle path of consumption, deflated by the number of adults plus 0.4 times the number of children in the household, for U.K. data.

8. This scale implies that a household of two needs 1.34 the consumption expenditure of a single household, with further additions to household size requiring an increment of 65%, 97%, and so on. See Fernandez-Villaverde and Krueger (2007) for the details. There we also provide sensitivity analysis with respect to our particular choice of the equivalence scale.

9. The time effects are small, with the exception of significantly negative values in 1992 and significantly positive values for the quarters in 1984 and in 1997 and 1998. This pattern is consistent with standard business cycle dating. The cohort effects are fairly small as well.

10. In Fernandez-Villaverde and Krueger (2007) we perform an extended bootstrap analysis to document that the confidence bands around our point estimates for life-cycle consumption profiles are tight. Thus our findings are not due to sampling uncertainty in the CEX data.

11. The age used is the age of the "head" of the family as defined by the SCF: the male in a mixed-sex couple, the older person in a same-sex couple, or the main individual earner otherwise.

12. The data on asset composition by wealth percentiles was kindly supplied to us by Joseph Tracy. See Tracy et al. (1999) for the study in which these data were first used.

13. It can be argued that concentrating on homeowners' portfolios does introduce a selection bias in favor of primary residences. However, because most nonhomeowner households hold very little wealth, including these nonhomeowners in Figure 8 will reduce the level of the curves, but not necessarily the relation between them. We do not include nonhomeowners to avoid the jump in primary residence value associated with the median household acquiring its first home.

14. In other words, we assume a law of large numbers to hold in our economy. See Feldman and Gilles (1985) for a justification; note that we do not require realizations of the underlying stochastic process to be independent across agents.

15. As markets for contingent claims are assumed to be inoperative, the borrowing constraints cannot depend on the realization of the productivity shock *next* period.

16. Note that if the providers of the durable rental could rent the durable in the same period in which it is acquired by them, then the rental price would satisfy $p_r = (r + \delta^d)/(1 + r)$, and both the rent and buy options have the same cost associated with them. Still, because consumer durables have collateral value (they relax the borrowing constraint) for agents that face binding borrowing constraints, at least for these agents buying strictly dominates the renting option.

17. See Platania and Schlagenhauf (2000) for an explicit life-cycle analysis of the purchase vs. renting decision for housing.

18. Because we care about the life-cycle consumption of households after demographic adjustments, we do not need to worry about the different mortality rates in the household. Faber's numbers refer to women's survival probabilities.

19. Part of the appeal of the unit root assumption derives from the fact that, as pointed out by Deaton (1991), it simplifies the computation of the household problem, because one state variable can be eliminated.

20. We performed Monte Carlo simulations to check that, when individual wages generated with our chosen process are estimated with a unit root process, we in general cannot reject the null of nonstationarity.

21. An approximation with more than three states would be desirable; computational constraints prevents this at the moment.

22. See Conesa and Krueger (1999) for a quantitative exploration.

23. The spike in the first period is due to the fact that agents start with $k^d = 0$. To avoid very low utility, households choose a high consumption of nondurables. A possible remedy would be to endow agents with a small positive stock of consumer durables at birth.

24. This divergence between data and model may also indicate that our borrowing constraint is specified too loosely, allowing households to invest in consumer durables at too rapid a pace when young.

25. Other important contributors to the life-cycle consumption literature specify income processes with positive probability of zero lifetime income. The Inada condition on the utility function then leads to a self-imposed borrowing constraint at 0: no agent facing a chance of zero lifetime income would ever borrow and risk zero or negative consumption for certain realizations of the stochastic income process. See, e.g., Carroll (1992, 1997) and Gourinchas and Parker (2002).

26. Grossman and Laroque (1990) justify this practice based on liquidity costs associated with durables.

27. Even if some details in the labor income process or the treatment of retirement are different, the households' problems are esentially equivalent.

28. A specification such as Alvarez and Jermann's (2000) borrowing constraints would allow some borrowing because there is an effective punishment for default: agents are forced to autarchy from the moment of default on. The quantitative effect of this specification in a life-cycle model is an open question and is deferred to future research. See Azariadis and Lambertini (2003) for a treatment in a three-period OLG economy.

29. The presence of durables makes the computation of this "natural debt" limit dependent on the stock of durables. To avoid this problem, we take the limit implied by the first point in the grid of durables. This limit is strictly above the real "natural debt" limit for households with larger quantities of durables and was chosen to avoid optimizing in highly negative points of assets holdings. We check that in the optimization routine the borrowing constraint is always equal to or higher than the lower bound of the grid.

30. Our most important deviation from the standard Newton–Raphson method is that we impose relatively conservative bounds on the size of the update to avoid overshooting. These bounds proved more reliable than cooling down the algorithm because of the discontinuity in the computable objective function caused by the penalty correction.

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APPENDIX: COMPUTING THE MODEL

To compute the steady state of our model, first we discretize the state space for durable goods and asset holdings, $\mathcal{K} \times \mathcal{D} = \{k_1, \dots, k_n\} \times \{k_1^d, \dots, k_m^d\}$. We do not restrict the choices, though, to lie in the grid, but use interpolation to cover any intermediate choices. The upper bounds on the grids are chosen large enough so that they do not constitute a constraint on the optimization problem. Using this grid, we can store the value function V and the distribution of households Φ as finite-dimensional arrays.

We solve for the steady state equilibrium as follows:

(1) Guess r and use the equilibrium conditions in the factor markets to obtain w.

(2) Given $V(\cdot, \cdot, \cdot, J+1) = 0$, solve the value function for the last period of life for each of the points of the grid, setting consumption to the total level of labor income plus the value of assets and the undepreciated stock of durables.

(3) Given $V(\cdot, \cdot, \cdot, J)$, find the value of the borrowing constraints $\bar{b}(k^{d'}, \eta, J-1)$ that makes the participation constraint hold with equality. If the borrowing constraint is specified as exogenous, this value is trivially set equal to zero.

(4) With the value of the borrowing constraint and $V(\cdot, \cdot, \cdot, J)$, solve for $V(\cdot, \cdot, \cdot, J-1)$, following the optimization routine described below in detail.

(5) By backward induction, repeat the steps (3) and (4) until the first period in life. This yields policy functions $k', k^{d'}, c$.

(6) Compute the associated stationary distribution of households Φ. Note that, because lives are finite, only forward induction using the policy functions is needed, starting from the known distribution over types of age 1.

(7) Given the stationary distribution Φ and prices, compute factor input demands and supplies and check market clearing.

(8) If all markets clear, we have found an equilibrium. If not, go to step 1 and update r.

We now comment in more detail on several aspects of this computation. The presence of state-dependent borrowing constraints presents a challenge for the grid generation. In the simplest case, when these constraints are exogenously set to zero, the grid along the asset dimension can be generated using a standard procedure, distributing the different points along the positive real line. The case for endogenous borrowing constraints is more involved. We generate a dynamic multigrid for assets with the following procedure. Given a price vector, we compute the "natural debt" limit implied by the discounted value of the remaining life-cycle income at the worst possible realization of the shock. With these limits, we generate a different grid for each period of life, ranging from this lower limit to some arbitrary, age-dependent, positive number.[29] Because in general these "natural debt" limits will be lower than the borrowing constraint limit, the household may face the situation that, for a particular point of the grid, no positive consumption is feasible. Because, for negative consumption, the utility function is not defined, we address this issue numerically using a penalty correction in the utility function: negative consumption implies such a huge negative utility that the household will never visit the areas of the multigrid that imply this negative consumption even after an arbitrary sequence of the worst possible realizations of the stochastic shock. The highest points in the grid for all type of borrowing constraints were chosen in such a way that the stationary distribution Φ does not put mass at these points. The distribution of point in the grid is uniform.

Finding the level of the borrowing constraint $\bar{b}$ at time j amounts to inverting the value function $V(\cdot, \cdot, \eta', j+1)$ along its first dimension. To do so, we solve the root of the linear equation $V(\cdot, k^{d'}, \eta', j+1) - V(0, 0, \eta', j+1)$ at each point of the grid of durables and each possible point η'. Then, because we are looking for the tightest borrowing constraint values in the stochastic income process, we find the vector of maximum values in η' for each possible choice of durables stock. This vector contains the value of the borrowing constraint $\bar{b}(k^{d'}, \eta, j+1)$ evaluated at each point in the durables grid, for each η and each age.

With this vector as an input, the optimization routine searches on the grid of durables and performs a quasi-Newton update[30] on asset holdings conditional on each point of the durables grid except for values of the asset holding close to the borrowing constraints. For these values we substitute the quasi-Newton with a variant of the bisection method. This change allows a correct treatment of the borrowing constraint in which the derivative of the objective function is not zero. We computed the needed numerical derivatives using a forward scheme. To improve efficiency, we tried a two-dimensional Newton search instead of a combination of grid search and quasi-Newton. This alternative, however, was not adopted, because of numerical instability problems.

Where needed, we use a simple linear or bilinear interpolation scheme. As a robustness check, we also tried a cubic spline interpolation with a "not-a-knot" condition. Cubic splines exactly match the function values at the grid points with continuous first and second derivatives. The "not-a-knot" condition requires that the third derivative of the spline be continuous in the last $n-1$ and $n-2$ points of the grid. We found that this more sophisticated interpolation scheme implied outcomes that were virtually indistinguishable from the ones reported in the text and resulted in a large degradation of time performance.

To simulate the stationary distribution Φ and life-cycle profiles and because we store only finite-dimensional arrays, an individual choice $k'(k, k^d, \eta, j)$ is interpreted as choosing asset holdings k_1 and k_2 with probabilities ν and $1-\nu$. These probabilities solve the linear equation $k'(k, k^d, \eta, j) = \nu k_1 + (1-\nu)k_2$. A similar procedure is followed for the durable stock choice. We build average life cycles with a simulation of a cross section of individuals and check the law of large numbers with the first two moments of the distribution.

All the programs needed for the computation of the model were programed in Fortran 95 and compiled in Compaq Visual Fortran 6.1 to run on a Windows PC. The computational materials are available upon request from the authors.