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PENSION FUND ASSET VALUATION AND INVESTMENT

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ABSTRACT

The theoretical basis for, and practical application of, the discounted income method for valuing U.K. pension fund assets is discussed with particular reference to the widely adopted application to variable income (equity type) assets, as proposed by Day & McKelvey (1964), in the context of both the management and compliance objectives of pension fund valuation. An alternative methodology is proposed in which consistency with assets, liabilities, and market values is demanded, with smoothing of the valuation result achieved on an explicit rather than implicit basis. It is then demonstrated that the explicit smoothing parameter can be set so as to achieve the historic smoothness of the discounted income asset valuation method and that the overall approach leads to a rational framework for establishing pension fund investment policy.

In conclusion the paper suggests greater emphasis on market-related methodologies for compliance valuations and leaves open the choice of methodology for management valuations and monitoring purposes, on the grounds that there is a large subjective element in any realistic basis. However, it is demonstrated that while smoother than unadjusted market-related methods, other aspects of the dynamics of the funding level under the method of Day & McKelvey can be perverse and it is suggested that this method should not be allowed to dictate investment decisions.

KEYWORDS

Assets; Pension Schemes; Disclosure; Solvency; Funding; Investment; Management

1. INTRODUCTION

Human nature is such that by sheer repetition one becomes increasingly 'comfortable' with using a particular set of assumptions which it is then all too easy to reassure others not in a position to judge is 'typical' or 'normal'. Thornton & Wilson (1992)

1.1 These words by Thornton & Wilson in their paper on pension funding were directed at actuarial bases generally, but in our opinion apply forcefully to the method which United Kingdom pension fund actuaries have come to regard as 'typical' or 'normal' in the valuation of equity assets. The approach is by no means universally accepted, but those who doubt the supremacy of this approach in the U.K. should look to the Social Security Pensions Act 1975 Market Level Indicators, to the Finance Act 1986 and to the acceptance of the method both as 'true and fair' under SSAP24 and as a basis for disclosing to scheme members the likelihood of their pension expectations being met under the Disclosure Regulations issued under the Social Security Act 1985. More recently, it has also been mooted in connection with an equity-linked basis for establishing a

minimum funding standard for U.K. pension schemes under the Government's 1994 White Paper. Furthermore, those who read the U.K. financial press will often see references to U.K. pension fund actuaries in connection with investment issues associated with pension funds' preference for current income and this is, in many cases, derived from the asset valuation principles enshrined in the technique. The method has therefore become increasingly associated with the public perception of the profession in the U.K.

1.2 The approach was first proposed by Day & McKelvey (1964) following the earlier work of Heywood & Lander (1961) (henceforth, for convenience, the method is referred to as the equity valuation method of Day & McKelvey, although we must acknowledge here the major contribution made by Heywood and Lander), in the context of a rather approximate statement of the relative size of pension fund assets and liabilities, usually on an aggregate basis, with the future service contribution rate the balancing item (see quotations heading Sections 2 and 3).

1.3 Those who still see Day & McKelvey's method quite firmly in the context of the broad targeting and monitoring of the ongoing position perhaps need read no further. However, this view is somewhat naive in the modern world, given that actuarial valuations are no longer carried out solely to establish and control the pace of funding of pension schemes. In the U.K. in particular there are now many statutory and other uses with different objectives into which Day & McKelvey's technique has been transposed. Furthermore, even if the method is still confined to setting a contribution rate, it is at least appropriate to question whether the approximations underlying the method are still acceptable and valid, given the major economic, financial and other developments over the last thirty years.

1.4 Section 2 therefore commences with a description of the method and its history and, in particular, looks at the various changes since its proposal in 1964. Having set the method in this historical context, Section 3 considers the various objectives of U.K. pension fund valuations (in particular establishing from the outset a distinction between a management and a compliance valuation) and sets out two broad characteristics against which different asset valuation methods can be judged, aspects of which deserve different emphasis according to the application. Sections 4 and 5 then take a first look at each of these characteristics with specific reference to the Day & McKelvey method. This effectively concludes the first part of our paper.

1.5 In the second part (Section 6) we put forward for discussion an alternative market-related methodology which has (in theory) been available in the U.K. since index-linked gilts became widely available in 1983. We believe this alternative to be broadly in line with the one exception to the conventional approach described by Thornton & Wilson (1992), and it is by no means new in principle. However, despite the importance of the asset valuation method in determining the financial position of U.K. pension funds (with assets of around £500 billion as at 31 December 1993), we would again remark (as have many

other observers in the past) that there have been surprisingly few papers presented to the Institute or Faculty on the subject over the last 30 years, notable exceptions being Wise (1984), Arthur & Randall (1990) (who also set out a market-related methodology) and Clark (1992) (the latter in the context of asset and liability modelling).

1.6 The merits of these two methods are compared quantitatively in Section 7, which then proceeds to develop a simple *explicit* smoothing process for the market-related methodology. Section 8 then expands on the quantitative analysis of the market-related methodology with an analysis of some of the asset and liability management implications which flow from it. Section 9 considers some of the potential drawbacks of this alternative methodology and our conclusions are then set out in Section 10.

1.7 Finally, as an historical footnote, we conclude this introduction by noting with some interest that the alternative method proposed in the second part of this paper in essence pre-dates the work of Day & McKelvey since it is very much as proposed by Gilley & Funnell (1958), but with the benefit of index-linked gilt yields replacing 2½% Consol yields (a luxury with which neither Gilley & Funnell nor Day & McKelvey were blessed). The long quotation at the beginning of Section 6 is therefore an acknowledgement of their work.

2. The Method of Day & McKelvey in Historical Context

Inflation and the secular economic trend are both so indeterminate that any approach is open to criticism; a valuation is, at best, very approximate and with a series of valuations, one is merely aiming to regulate the pace of funding of the liabilities. Day & McKelvey (1964)

2.1 *Description of Method*

Although not necessarily a criticism in itself, the asset valuation methodology proposed by Day & McKelvey was very elementary indeed. They expressed their method algebraically as follows:

Actuarial value of equities =

$$\frac{R \times T}{j - f} \times \text{(Market value)} \times \text{(Dividend yield on ordinary share index)} \quad (1)$$

where j was the rate of interest used to discount the dividend income, f was the assumed annual rate of inflation (so $(j - f)$ was an implied real rate of interest), R was allowing for last year's dividends being misleading and T effectively allowed for the grossing up of the net dividends to allow for the reclaim of tax (in modern applications, following the introduction of ACT in 1973 the dividend yield on the Ordinary (All) Share Index is expressed as a gross amount, removing the necessity for the taxation adjustment).

2.2 Thirty years on, the method has changed little in common application except that R (the arbitrary multiplier, as it was well described by Day &

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McKelvey in expressing the obvious defects of the method) is now dispensed with. Without this factor, the method assumes that dividends will grow exactly in line with price inflation (the dividend yield therefore representing the real rate of return). It is more common today instead to use an explicit assumption of real dividend growth m (on this point, Day & McKelvey mused that something between the earnings yield and the dividend yield on the index might be appropriate in the numerator of equation (1).

2.3 Thus, using r for Day & McKelvey's real rate of return $(j - f)$, the modern form would be:

$$\text{Actuarial value of equities} = \text{Market value} \times \frac{\text{Gross dividend yield}}{r - m} \qquad (2)$$

the denominator value $(r - m)$ now representing a par yield. For example, if the valuation rate of interest was 9% p.a. and the assumed rate of inflation was 4½% p.a. (a real return r of 4½% p.a.) and real dividend growth m was ½% p.a., this would give a 'par yield' of 4% (4½% less ½%). If the gross dividend yield on the All Share Index is above this par yield, then the market value at the valuation date will be written up by the actuary, and if it is below this par yield it will be written down. On this basis, reducing the dividend growth assumption by ½% p.a. (no real growth) would reduce the actuarial value by about 11%, equivalent to an (arbitrary) 'R' factor of 0.89. One of the main thrusts of our argument will be that m is so subjective and the valuation result so sensitive to small changes in this parameter that one cannot place an objective value on the assets of the pension scheme using this method, and in practical effect the choice of m is little different from the choice of an arbitrary multiplier, despite the impression of sophistication which it engenders.

2.4 The valuation rate of interest proposed by Day & McKelvey was "the actuary's estimate of the rate of interest at which it will be possible, on average over the long period, ..., to invest the future net investments in the fund or carry out disinvestment after the fund has reached its maximum". It was not necessarily envisaged that this would reflect current market rates of interest. Therefore, under this approach, if M is the market value of assets, y is the yield on the All Share Index and L are the liabilities (based on *fixed* 'long-term' real rates of interest r and inflation expectations f), then for a scheme with a proportion ζ invested in equities and $(1 - \zeta)$ invested in gilts, the funding level can be expressed as:

$$\text{Funding level (Day \& McKelvey)} = \left[\zeta \cdot \frac{y}{r - m} + (1 - \zeta) \cdot g\,(r, f)\right] \frac{M}{L\,(r, f)}$$

$$= \frac{K\,(m, r, f)}{L\,(r, f)} \times M \qquad (3)$$

defining $K(m, r, f)$ as the adjustment factor applied to the market value,

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dependent on the current market conditions (through ζ, g and y omitted as explicit arguments, with $g(r, f)$ being the ratio of the discounted income value to market value of the gilt portfolio). The explicit arguments (included for subsequent discussion) are the actuarial assumptions described above, and by construction K is unity when these assumptions equal the respective market rates.

2.5 Not only does the above formula have elegance and appealing simplicity, but the emergence of the reverse yield gap in equities in 1958, at a time when both Heywood & Lander and Day & McKelvey were developing their equity valuation techniques, powerfully illustrates the extent to which their thinking at the time was also both contemporary and, to a certain extent, bold. There was, at that time, insufficient past data to support the view that the reverse yield gap would be a permanent feature. Furthermore, it would have been difficult to predict at that time the effect of the Trustee Investment Act 1961 on equity valuations (in the same way as one could not have predicted the effect of the abolition of exchange controls, abolition of credit controls and financial deregulation in the 1980s).

2.6 Perhaps rightly at the time, Day & McKelvey were also concerned that market rates of interest were inconsistent with the long-term nature of their valuation process (''market rates can vary quite sharply over a short interval when there has been no change in one's estimate of the average long-term rate of interest and no change in the future receipts that one expects to receive from a given asset''). With some justification, given the market environment at that time, they took the view that the actuary was better able to decide the long-term rates of interest by professional judgement than was possible by use of market rates of interest.

2.7 One of the main catalysts for our paper is, however, the wide range of changes in a variety of fields which have taken place since this method was originally proposed. To convince readers of the need for a fundamental re-examination of the method, in the rest of this section we therefore itemise the principal changes which readers may wish to keep in mind throughout subsequent sections. It will be seen that, almost without exception, these changes have served to weaken rather than strengthen the original justifications for the approach.

2.8 *Changes to Pension Funds*

Pension funds are considerably more mature than in 1964, increasing the relative size of past-service liabilities to future-service liabilities. At that time the prevalent actuarial valuation method was the aggregate method. Today it is the projected unit method coupled with, in many cases, much shorter spreading of surpluses (contribution holidays), which again places much greater emphasis on an accurate assessment of the past-service funding position. Thus it is considerably more important to build up a reliable picture of the past-service funding ratio than was the case in 1964.

2.9 *Changes to Investment Strategy*

Pension funds now invest predominantly in equities, partly reflecting a view that equities will deliver the highest return and, in our opinion, partly encouraged by the Day & McKelvey method itself (as we shall discuss in Section 5). It is not unreasonable to suggest that this large-scale investment in U.K. equities has created an element of bidding up of the price of equities (to which the method has contributed). Any such bidding up would inflate historic returns on equities above their (future) long-run levels, further emphasising the degree of caution that should be applied in using historic equity returns to infer future returns and, to the extent that other assets must be priced relative to equity returns, suggesting that measurable prospective returns on alternative assets under more objective methodologies may provide a more reliable basis for expectations.

2.10 In 1964 the average pension fund had 45% in equities, virtually all domestic. Today it has around 85%, of which roughly one third is invested overseas. Thus the method is being applied to a much higher proportion of the assets (indeed model funds of 100% in equities are not uncommon). Moreover a significant part of that exposure is in overseas equity assets, whose income is very different and likely to have behaviour very different from that assumed by the method (the theoretical basis for the assumptions behind the method is considered in Section 4).

2.11 *Changes to Investment Markets*

Perhaps the most significant of all the changes we discuss is the existence of index-linked gilts which offer a guaranteed real return (above price inflation). These were first issued in the early 1980s. One of Day & McKelvey's main justifications for their method was that it might "lead to a consistent approach as to the degree of inflation and secular improvement ... on both sides of the valuation balance sheet". Index-linked gilts provide a structure of market real interest rates that was not available in 1964, and thus open the possibility for consistent treatment of assets and the price-related element of liabilities on an entirely market-related basis.

2.12 This emphasis on inflation in 1964 showed significant foresight, since inflation was then generally much lower (average inflation was of the order of 2% p.a. over the previous five years with expected growth, by contrast, around 4% p.a.). However, Day & McKelvey, in particular, seemed to regard negative inflation as a real possibility, and part of their rationale was to give equal weight also to the importance of the consistent treatment of secular economic growth in the increase in salaries in adopting equity-based valuation techniques. (In the words of Day & McKelvey, "looking ahead, severe inflation is unlikely to be a problem but technical change, overcapacity and capital shortage seem likely to be major factors").

2.13 The lack of precision in the link between domestic economic growth and U.K. equity dividends is discussed quantitatively in Section 4. However, it should also be noted that, when Day & McKelvey proposed their technique, the

overwhelming majority of U.K. company earnings were domestic and directly linked to the domestic economy. It was therefore perhaps more meaningful then to consider a linkage between the salary growth of U.K. employees and the growth of U.K. equity earnings due to domestic growth which they proposed. In 1994 this linkage is much weaker, with around 45% of listed U.K. company earnings derived from overseas operations. Accordingly, with the benefit of hindsight, we feel that a less ambitious focus on consistency in the treatment of price inflation only is now a more realistic objective (salary inflation exposure having generally declined in favour of price inflation due to increased maturity, in any event).

2.14 World stock and bond markets are also increasingly globalised. Likewise capital is now free to move between different countries and different markets. This changes the nature of the fundamental economics of the U.K. which is no longer a closed system and whose decision-makers must cater for these international capital flows. The actuary's implicit view, referred to in ¶2.6, that interest rates are high or low becomes the view on world markets (rather than the U.K. alone) in a free-flowing capital system. This reinforces the caution needed before adopting such a position.

2.15 We would also venture to suggest that in the 1960s the notion of distinguishing between income and capital was perceived to be more important than is the case today, possibly encouraged by the widespread book value accounting methods. More recently such a distinction was also emphasised by Thornton & Wilson (1992) when they cited one of the advantages of the method of Day & McKelvey as "it places proper emphasis on the significance of future growth in the income stream as the major factor influencing the emergence of surplus in schemes". From an investment perspective there is far less justification for this distinction now than there was thirty years ago. Liquidity is much higher, giving greater confidence that capital values and income prospects are treated by investors in an actuarially consistent manner. Indeed, the existence of index futures contracts, whose price reflects both income and capital in the underlying asset, has, in our opinion, blurred any remaining distinction between equity income and capital. Whereas in 1964 a pension fund could only invest physically in equities, in theory a fund can now hold cash as its only physical asset, while gaining synthetic equity exposure. Such a fund could therefore be effectively invested in equities indefinitely, receiving dividend income implicitly as capital in the pricing of the contract. (It is very likely that income will be less volatile than capital, but that merely necessitates the use of smoothing techniques, rather than representing an investment distinction between the two).

2.16 *Changes in Technology*

The development of information technology since 1964 has brought with it, not only a greater wealth of global economic and market data available in real time, but also the computer power to perform economic analysis which was not

previously possible. Indeed, it has brought the emergence of econometrics as a profession in its own right and, whereas in 1964 there was no real alternative to the actuarial profession's extrapolation of past averages as a basis for establishing long-term future expectations, a wealth of economic forecasts are now available, including published market consensus surveys with up to ten-year average forecasts of inflation and other key economic variables. Although, with the benefit of hindsight these (and all) forecasts will invariably be wrong, for valuation consistency purposes, they have the advantage that the market uses the best forecasts available at the time and they are implicit in the market pricing of assets.

2.17 Furthermore, the method of Day & McKelvey would today be recognised as fundamentally a dividend discount model. Again aided by computer technology, much more elaborate versions of the same model are now used by some investment managers. Given that these are neither universal nor universally successful, we should take greater care over placing too much confidence in a very rudimentary version thereof.

2.18 *Changes in Technical Factors*

The increase in institutional dominance has also reduced the impact of personal taxation effects in the pricing of assets. Furthermore, the introduction of personal equity plans in 1987 has reduced the scope for taxation anomalies in the personal sector in any event. Added to all these effects, marginal rates of tax on income have also reduced considerably to a maximum of 40% in 1994. This is not to say that taxation anomalies do not exist. However, they are of much less significance, and it is clear that Day & McKelvey's concerns over the effects of high taxation on *raising* overall yields above normal (prudent) levels are not now a significant issue (to quote Day & McKelvey in 1964: ''in times of high taxation, gross yields are likely to be higher than those regarded as 'normal' in the past'').

2.19 It is also possible, in considering the scope for the mispricing of the 'fundamental' value of equities, to overlook the role of companies themselves in controlling the *supply* of assets, both bonds and equities. This role was extended by the Companies Act 1985 which permitted companies to buy back their own equity capital. Essentially, companies will issue more capital when they perceive the market to be 'expensive' and will buy other companies whose share value they perceive to be 'cheap'. Overall this is clearly another factor which acts to dilute the extent to which markets can become cheap or dear.

2.20 Financial deregulation in the mid 1980s also had a number of important effects:

(1) As shown by Figure 2.1, equity market volume (defined by average number of bargains per day) increased significantly.
(2) Transaction costs fell, with the introduction of negotiable commissions and reductions in stamp duty.
(3) Futures markets added additional liquidity.

(4) The Financial Services Act 1986 reduced further the scope for prices to be set on the basis of 'inside information'.

Overall there are therefore many more technical factors operating to remove distortions from markets which serve to discourage any universally accepted view that one market is manifestly expensive or cheap *relative* to another at a given point in time.

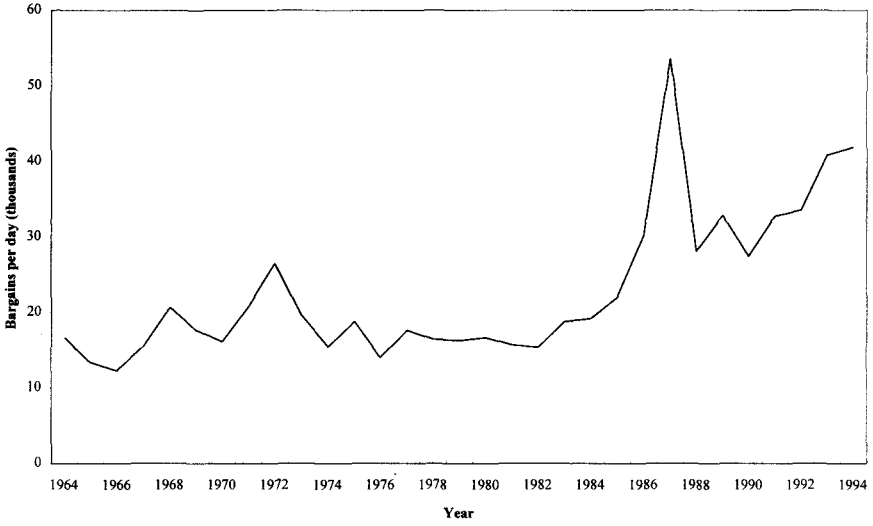

Figure 2.1. Activity in U.K. equity market (Source: London Stock Exchange)

2.21 *Changes in Attitudes to Markets (and Volatility)*

We conclude with perhaps the most quoted legacy from the paper of 1964, namely concern over the reliability of prices 'which balance marginal buyers and marginal sellers', the inference being (since this was and is quoted as an *objection* to the use of market values) that they were volatile and do not reflect normal value. Such concern is misplaced if the existence of marginal activity simply reflects that those who regard the price as too high are balanced by those who regard it as too low.

2.22 Furthermore, these concerns are slightly at odds with modern analysis which tends to assume that *increased* activity is associated with *increased* volatility, as increased activity suggests the arrival of new information (see for example Board & Sutcliffe (1993)). Dealing with volatility (arrival of new information) is in our view an issue of *smoothing* which should not be confused with the establishment of rational expectations at any point in time.

2.23 *Historical Overview*

We have developed a number of themes in this summary of the historical

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background of the method, to which we will return in more detail in subsequent sections. However, the general perception is of an elegant and simple method which in the early 1960s was at the forefront, of not only actuarial thinking on pension funds, but also leading the thinking on the valuation of equities. Although not anticipated at the time, this was followed in the U.K. by a period of financial chaos and high inflation, particularly around 1974 which, in our opinion, represented the heyday of the method, with much of the justification for the approximate approach disappearing with the issue of index-linked gilts in the early 1980s.

2.24 Furthermore, as we have suggested above, with the visionary concept becoming embedded in the conventional wisdom of pricing equities, the need for the actuarial profession to take a position at odds with the market has also declined and, whereas in 1964 the simplicity of the technique conveyed powerful ideas, it now, in our opinion, largely reflects the simplicity of the assumptions (uniform real dividend growth). By contrast with the early 1960s, it is therefore difficult to believe now that the actuarial profession's elegant but elementary technique (which, however presented, requires an arbitrary multiplier) could consistently give great insight into the value of the equity market. Ironically, however, as we outline in our introduction and in the next section, rather than declining, the importance of the method has increased dramatically in recent years.

3. OBJECTIVES

..the net liability disclosed by the valuation of a pension fund is not a figure which can be regarded as uniquely determined, but rather something which can be regarded as lying in a range of possible values from x *to* y.

– Heywood & Lander (1961)

3.1 In this section we first consider the various objectives of pension fund valuations at present and in the foreseeable future within the U.K. and establish the different emphases which these objectives place on two broad characteristics of asset valuation methodologies, which we then discuss throughout the rest of our paper. In discussing these objectives we immediately recognise a distinction between two types of valuation processes, namely:

(1) *management valuations*, mainly concerned with setting and monitoring of the pace of funding by the trustees and company management; and
(2) *compliance valuations,* concerned with the statutory supervision of pension funds by various authorities and public reporting on their financial position.

Such distinctions have commonly been acknowledged more widely in life office and general insurance applications than has traditionally been the case in the U.K. pension fund environment.

3.2 For the avoidance of doubt, it will be noted that in some applications of

both management and compliance valuations the market valuation adjustment is applied to the liabilities (the adjustment being the multiplicative inverse of the equivalent asset value adjustment). Such applications raise no new principles for our purposes except that, where the adjustment is made to the assets, the currency of actuarial rather than actual money amounts is introduced and may require comment. Subject to this point of clarification, we now consider the various applications in turn.

3.3 *Management Valuations.*

3.3.1 *The ongoing valuation: setting contribution rates*

3.3.1.1 The regular ongoing valuation is the quintessential management valuation for a U.K. pension fund. This is because, subject to the constraints (if any) of minimum funding and Inland Revenue approval (both discussed below), the pace at which a pension scheme is funded is largely at the discretion of the trustees and the sponsoring company. In this context, if the company and trustees, on the advice of the actuary, choose to monitor this pace of funding on the assumption that U.K. equity dividends will grow at a uniform real rate based on past average experience (as in the Day & McKelvey method for example), or by reference to some other yardstick, then so be it : valuation surplus or deficit will emerge when dividends exceed or fail to meet these expectations and the sponsor will adjust contributions accordingly. Provided the company continues as a going concern, the company may be primarily concerned with ensuring that the assumptions are *realistic* and that excessive amounts of capital are not tied up in the pension fund unnecessarily. In addition, possibly even of equal importance in practice, the trustees and company will seek *smoothness* and predictability of the funding level and contribution rate. However, we would draw attention to a number of important issues which flow from this focus on realism and smoothness.

3.3.1.2 Firstly, realism tends to bring with it a greater need for subjectivity in the assumptions, when compared with *prudence*. For example, a typical, prudent actuarial approach would simply be based on the principle of establishing a gilt portfolio matched to the assumed liabilities. On the other hand, a realistic valuation of a U.K. pension fund is likely to take some account of expected outperformance of the actual assets held relative to gilts. Setting this excess return must involve a higher degree of subjectivity. This subjectivity may not be of concern for the purposes of a management valuation, since the actuary can be comfortable in the knowledge that the errors in this subjective assumption can be corrected by a subsequent adjustment to future contributions, while the company remains a going concern. This is, of course, only a restatement of the flexibility in the U.K. in the pace of funding an ongoing scheme, although it does beg the question as to why a detailed valuation is necessary for such a broad brush process and emphasises the need for caution in declaring the existence of large surpluses.

3.3.1.3 Secondly, issues arising from the *dynamics* of the funding level over

time need to be considered. In particular, it must be acknowledged that any smoothing process is almost certainly not giving full weight to the information available at a particular time (in general terms it is probably giving a certain weight to prior expectations based on information available previously). Since the valuation result will determine any adjustment to the future service contribution rate, this has important *investment implications* in terms of the *timing* of cash flows into the fund.

3.3.1.4 Furthermore, if the investment model underlying this process is too simplistic and does not incorporate new information in a rational manner, the dynamics of this funding level over time, which can have a critical impact on *asset allocation* policies, may be perverse (regardless of the smoothness). More generally, we would highlight the need for any valuation methodology to provide a clear and rational view of underlying investment issues. After all, it must be stressed that whilst the choice of basis dictates the pace of funding, the actual investment returns achieved in practice dictate the true cost of a given pension scheme. One of our main concerns in this paper is that the dynamics of the funding level under the Day & McKelvey method can obscure some of these issues.

3.3.1.5 Finally, as well as considering the dynamics over time, attention may also be drawn to the dynamics at a given point in time. For example, unless the valuation result is independent of the asset allocation at the valuation date, it may be possible to change the valuation result by switching the asset mix, or by realising assets to meet immediate liabilities on a particular day (these effects can be attributed to the subjectivity in asset valuations which depart from market valuations, however for the purposes of discussion they are grouped under our general heading of 'dynamics').

3.3.2 *Sales and purchases*

Actuarial valuations for the purposes of company sales and purchases became an increasingly important application of valuation methods during the last decade. We regard such valuations (subject again to the minimum protection of transferees' rights specified by legislation or scheme rules) as essentially management valuations, the main difference with the process described above being that actual money moves between two companies on the basis of the valuation calculations, and any experience shortfall (or excess) arising from subjective assumptions is likewise transferred rather than being an issue of timing of funding by a single company.

3.4 *Compliance Valuations*

3.4.1 *Discontinuance solvency valuations*

The trust deed and rules of many U.K. pension schemes and indeed the current professional guidance note GN9 have always required some regard for the security of the accrued benefits on winding-up. For such valuations, the principles of *objectivity* and *prudence* would appear to be paramount. Accepting

a valuation methodology which is "at best very approximate and ... aiming to regulate the pace of funding" as their approach was described by Day & McKelvey, should be unacceptable in situations where the company is no longer available to fine tune subjective assumptions as the beneficiaries approach retirement.

3.4.2 *Minimum funding standards*

3.4.2.1 A more recent potential application of compliance valuations in the U.K. is the Government proposal to introduce some form of minimum funding requirement for pension schemes. A weak form of minimum funding already exists for schemes contracted out of the State Earnings Related Pension Scheme (under the Social Security Pensions Act 1975), in the form of actuarial certification of the cover of certain priority liabilities on a prescribed basis in the normal course of events. This basis already incorporates elements of the methodology of Day & McKelvey in respect of non-pensioner liabilities. The certification requirement has, however, had little impact on U.K. pension funds generally due to the limited nature of the liabilities covered. The current proposal is to extend some form of minimum funding to cover a greater proportion of the accrued liabilities.

3.4.2.2 If primary emphasis is placed on comparative measurement of funding among similar schemes on a prescribed basis then, provided that these schemes also follow similar investment policies, it may be reasonable to place some emphasis or even primary emphasis on realism (the degree of subjectivity being prescribed) and smoothing (likewise prescribed). However, in doing so our comments in ¶¶3.3.1.2 to 3.3.1.5 are of major significance, since the choice of method may have a fundamental impact on the *dynamics* of the funding level which can, in particular, have crucial (and possibly perverse) implications for investment behaviour (as discussed later) unless a scheme is funded well above the minimum standard.

3.4.3 *Over-funding legislation*

The Finance Act 1986 has already introduced a maximum level of funding for tax exempt U.K. pension schemes on a prescribed basis. The regulations issued under this Act again embraced the principles of the Day & McKelvey method, and many of the same comments apply as above (except that the potential investment impact falls on comparatively well-funded schemes).

3.4.4 *Company accounts: SSAP24*

The incorporation of ongoing valuation surplus in U.K. company accounts under SSAP24 has tended to follow the emphasis on realism and smoothness adopted for management valuations. However, we would suggest that greater account of objectivity and the dynamics of the methodology are appropriate in this application, for the following reasons:

(1) The method should be consistent between different companies with similar pension schemes. The method should therefore minimise the number of

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subjective assumptions to which the valuation result is sensitive if it is to provide the most reliable 'true and fair' basis for comparison by investors. In our opinion, this requirement demands that the profession adopts methods which provide a transparent basis for comparison by a non-actuary.

(2) It is, in our view, important that the asset valuation method is consistent between those companies with a large pension fund and those with no pension obligations. In other words, an investor should be able to take £1 billion of pension assets or liabilities at face value. As an example, a company could issue £1 of equities, receive £1 from investors and invest this in the pension fund at a 'value' of £1.25, say. This is a trivial example of the need for consistency in the dynamics. Demanding that movement of funds from the company to the fund on a particular day has a neutral effect implies taking assets at market value (and adjusting the liabilities) whatever the methodology of adjustment.

3.4.5 *Disclosure to members (actuarial certificates and valuation reports)*

The ongoing valuation cannot continue to be regarded only as a broad targeting of funding when the results are formally disclosed to third parties who should reasonably be able to take the results at face value. On a subjective and smoothed approach, it may be acceptable to show a significant surplus in respect of the funding of final salary liabilities, but actuaries need to be aware that this will lead to an expectation that, in the event of the fund being discontinued, the assets are then sufficient to secure a *lower level* of discontinuance benefit. In many ways, the requirement under the Social Security Act 1985 to disclose the results of the formal ongoing valuation therefore brought this within the realm of a compliance valuation, and not only ties together the various issues above, but also reinforces the need to consider the public perception of the profession in the widespread dissemination of valuation results.

3.5 *Key Characteristics of Valuation Methodologies*

We have identified above two broad characteristics of valuation methodologies, with the appropriate emphasis on various aspects under these headings depending on the particular application (management and compliance roles requiring particular distinction):

(1) the degree of *objectivity, prudence* and *transparency*; although at first sight these may be considered as separate characteristics, we believe that, in general, realism (which we juxtapose with prudence) leads to subjectivity and lack of transparency (our maxim being that the greater the number of subjective parameters the lower the transparency of the result) and vice versa; and

(2) the *dynamics* of the valuation result, including obviously the *smoothness* of the funding level over time (or, alternatively, the weight given to prior expectations in striking a valuation result at a particular point in time), but also including consideration of the perversity of the changes both over time

and at a given point in time (arising from switching the asset allocation or cash flows), with particular reference to the impact on *investment policy.*

In the next two sections we take a first look at each of these broad characteristics with reference to the method of Day & McKelvey.

4. Analysis of the Assumptions Underlying the Day & McKelvey Method

There are long fluctuations in the position and a judgement has to be made as to how cyclical increases in dividends are and where in the cycle one is. Thornton & Wilson (1992)

4.1 In this section we set out to analyse the soundness of the theoretical principles upon which the major assumptions of the Day & McKelvey method are based. In doing so we draw attention to the following two issues (where K is as defined in equation (3) in ¶2.4):

(1) The valuation result is *highly sensitive* to the choice of m ('long-term average' real dividend growth), the sensitivity being $\dot{K}_m$ (where we adopt the notation $\dot{K}_m = \partial/\partial m \log K$), or $\zeta/(r - m)$ when $y = r - m$ and K is unity. This sensitivity factor would be between 20 and 25 for typical values of $(r - m)$, so a change of 1% p.a. in m changes K by between 20% and 25%. The method therefore hinges critically on the reliability of the subjective parameter m. How confident can the actuary be in setting the parameter?

(2) If g and ζ are not (or cannot) be chosen (with ζ in the range $0 \leq \zeta \leq 1$) so that $\dot{K}_r = \dot{L}_r$ and $\dot{K}_f = \dot{L}_f$ (using same notation as in (1)) then the result can also be highly sensitive to the choice of r and f (real interest rates and inflation expectations). How justified is the actuary in departing from the implicit values of these parameters suggested by investment markets?

These are now considered below in turn.

4.2 *Real Dividend Growth Assumptions: Economic Arguments*

4.2.1 Despite common use of the comparison in explaining publicly the principles of the method of Day & McKelvey, it is of course (as explained by Wilkie (1984)) in no way sufficient in establishing the dividend growth assumption simply to compare the two generally increasing series in level form to establish linkage between dividends and the retail prices index. The two series must be detrended into rates of growth (most conveniently by differencing the logarithms) before any sensible economic analysis can be performed. Various authors, including Wilkie, have fitted a lagged link between past inflation *rates* and the growth *rate* of dividends. For example, Wilkie found that a combination of the previous year's inflation plus an exponentially weighted average of past

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inflation gave the best fit over the period 1920 to 1982 (an average lag of 3.2 years) while Thornton & Wilson found that (using a simpler formula) a 3-year lag between price inflation and dividends gave the best fit over the period 1940 to 1990.

4.2.2 However, the fit of these models would not appear to be sufficient to give a firm indication of how dividends will grow from a given point in time. The results tabulated by Wilkie (1984), for example, give an unexplained residual error in dividend growth of about 7.3% p.a. over the data period 1920 to 1982, compared with the annual standard deviation of around 10.4% p.a. in the underlying series, implying a within-sample *R*-squared value of around 50% with six parameters having been fitted successively to the data. Also, although Thornton & Wilson give a 'first principles' justification for a lag of around 2 years, the lags found by multiple least-squares regression appear to be highly sensitive to the period chosen. For example, using the mid-year inflation and equity income indices published by Wilkie (1984), extended to 1990 using the U.K. Retail Prices Index and the FT-A All Share Dividend Index gives the following coefficient estimates, regressing the force of dividend growth on lagged forces of inflation:

	Inflation lag						Constant	Standard error of regression	$\bar{R}^2$
	0	1	2	3	4	5	(%)	(%)	(%)
1940-1965	-0.35	-0.82	-0.11	0.34	0.04	0.05	8.80	5.6	24
1965-1990	0.46	-0.22	-0.41	0.45	-0.08	0.54	3.41	4.9	37

The instability of the results suggests that any simple formula assuming constant real growth in excess of some lagged function of inflation only is highly mis-specified.

4.2.3 We analyse below some of the reasons for this mis-specification of the model underlying the Day & McKelvey method and consider the degree of confidence which can be attached to subjective estimates of long-term real dividend growth *m*. To establish this degree of confidence, we look at a range of different factors which contribute to the observed historic dividend growth and the variation generated by each.

4.3.1 *Linkage to economic growth*

4.3.1.1 We would not dispute the broad macroeconomic justification for some form of linkage between dividends and nominal economic growth, the theory upon which the (approximate) methodology of Day & McKelvey was based being that dividends and salaries will essentially remain fairly stable proportions of Gross Domestic Product (GDP) over time. Figure 4.1 shows the historic variation in 'Income from employment' and 'Gross trading profits' as a proportion of U.K. GDP since 1946. Both proportions in fact show similar

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variations as a percentage of GDP. However, a variation over a given period of 3½% (one standard deviation being 1.6% in each case) in the average of around 70% of GDP represented by income from employment amounts to a 5% difference between total income growth and GDP growth. In the case of company trading profits, the same variation would amount to a growth differential of nearer 25%. Therefore, if this change from peak to trough (or vice versa) occurred over 20 to 25 years (the approximate duration of the adjustment made under the Day & McKelvey method), it would account for a discrepancy of about 1% p.a. between company profits and GDP growth.

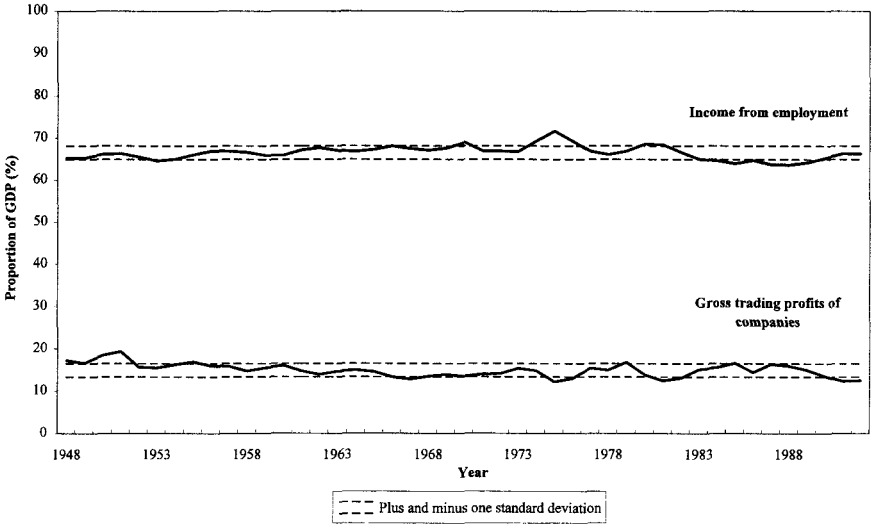

Figure 4.1. Variations in components of Gross Domestic Product (Source: Central Statistics Office)

4.3.1.2 This fluctuation alone would be sufficient to throw doubt on the validity of any stable long-term assumption for the method of Day & McKelvey, and would suggest that separate consideration needs to be given by the actuary to prospective changes in profits as a proportion of GDP, clearly a subjective view. However, even if the growth of gross trading profits of companies was a stable component of GDP, the following further factors would need to be taken into account.

4.3.2 *Changes in number of shares*

The growth in company earnings per share will be affected by the rate of issue of new capital. This feature is shared with the growth of average salaries which is also affected by the growth in the numbers employed. However, whereas the

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latter is constrained by demographics, the growth in the number of shares is largely at the discretion of companies themselves and we see no particular reason why a long-term average rate of growth should be a reliable estimator of future prospective behaviour. In effect, the growth in number of shares should be regarded as a policy variable rather than a fundamentally stable macroeconomic factor. Measures of the variation in this factor over time are not readily available on an objective basis (the relative growth of market capitalisation versus index value requiring some adjustment to reflect simple transfers of ownership, such as privatisations, as suggested by Thornton & Wilson). However, Thornton & Wilson suggested a range of 1% to 2% p.a. for this factor in isolation, indicating a considerable degree of uncertainty.

4.3.3 *Changes in payout ratio*

Another policy variable is clearly the pace at which earnings are distributed by companies. As well as being influenced by taxation, even if it is assumed that companies are run solely for the benefit of shareholders, practice in this respect may be influenced by the perceived preferences of those shareholders for income. Furthermore, if a broader view is taken of shareholders as a provider of capital only, with other stakeholders (such as employees, suppliers, customers) having influence on management, then, in theory, this policy variable will be set at a level sufficient to attract the required capital and may be lower when interest rates are low. Thornton & Wilson also suggested that the statistic was distorted by the overstatement of profits during high inflation. All of these factors are relevant, and again they give little confidence in the reliability of past data. Statistics provided by Thornton & Wilson showed the dividend cover of the FT-A All Share Index to fluctuate widely over the period 1973 (introduction of ACT) to 1992, with a decline of around one third over the final ten years in particular. Such fluctuations could easily account for a variation in dividend growth of 1% p.a. or more over 20 to 25 years.

4.3.4 *Changes in taxation*

Taxation changes affect, not only the net earnings of companies (with corporation tax falling from 52% in 1973 to 33% in 1991), but also, as noted by Day & McKelvey, net investors' preference for income or capital growth. The changes in advance corporation tax also distort the growth in gross dividends. Thus taxation is another policy variable which has the ability to alter permanently the proportion of earnings distributed as dividends, again making more difficult any attempt to forecast future real dividend increases objectively based on past averages alone.

4.3.5 *Overseas earnings of U.K. companies*

As noted in ¶2.13, an increasing proportion of total earnings of U.K. companies are derived from overseas operations, not directly linked to domestic GDP in any event. This weakens further the implicit link in the methodology between company earnings, dividends and GDP.

4.3.6 *Conclusion*

There may be elements in some of the above variations which partially offset variations in other factors. However, overall, the uncertainty in each suggests strongly to us that when taken together, the level of confidence in any estimate of m is likely to be very low indeed.

4.4 *Real Dividend Growth Assumptions: Past Experience*

4.4.1 Given the above distortions, it is perhaps not surprising that actual past data give no stable, objective guide as to the appropriate rate of real dividend growth to be assumed as a basis for the method of Day & McKelvey. For example, the statistics below show the rate of real dividend growth from U.K. equities over seven individual decades (source: mid-year data, as for ¶4.2.2).

	Real dividend growth	Nominal dividend growth
1920-1929	9.0% p.a.	6.1% p.a.
1930-1939	-4.2% p.a.	-4.6% p.a.
1940-1949	-0.9% p.a.	2.7% p.a.
1950-1959	2.2% p.a.	6.5% p.a.
1960-1969	3.2% p.a.	6.9% p.a.
1970-1979	-3.2% p.a.	8.7% p.a.
1980-1989	5.7% p.a.	13.7% p.a.

4.4.2 The use of rolling averages instead of discrete periods will add the appearance of stability over long periods, for example, one could take rolling twenty-year periods from 1950 to 1990 (most of which would overlap), but there remain only two *independent* post-war twenty-year periods giving real growth of 2.7% p.a. and 1.2% p.a. respectively. It is certainly feasible to adopt 1.9% p.a. as the forty-year average and refer to an assumption of such growth as a long-term assumption. However, although it is objective in a very limited sense of the word (that is, it can be measured over a prescribed past period), it is somewhat disingenuous to present it as a reliable or robust out-of-sample (prospective) estimator. Long-term past averages always exist for any time-series, but this existence alone does not imply reliability as an *ex ante* predictor.

4.4.3 Given the conclusions of this very simple analysis, we see no merit in a more detailed investigation into the 'long-run average' available from these data. Our view is that it is not a stable entity and our degree of confidence in a single long-term average is very low indeed for out-of-sample forecasts. We therefore suggest that the method of Day & McKelvey takes as its most critical assumption a parameter whose future value is highly subjective and, in particular, cannot easily be estimated from theory or from past averaging techniques.

4.4.4 Finally, we stress that, if a relationship does exist between the growth

in prices, output and dividends (albeit more complicated than that underlying the method of Day & McKelvey), then, to the extent that changes in the market value of equities anticipate such linkage (which markets should do if they are rational and incorporate all known information), market-related valuation methods should be equally valid at capturing this. In fact (as shown in Appendix B) an alternative method of analysis does give some support to a view that market movements anticipate real growth of dividends after an initial implied lag, broadly consistent with the work of Wilkie (1984).

4.5 *Interest Rate and Inflation Assumptions: Theory*

4.5.1 As stated in ¶4.1, as well as the assumption that dividends will grow at a stable 'long-term' rate relative to inflation, to the extent that the duration of the asset adjustment is different to that of the liabilities, the choice of long-term rates of inflation and real return will also have an impact on the valuation result. It was implicit in the arguments of Day & McKelvey (and Thornton & Wilson) that by careful analysis of long-term data the actuary is better able to assess the total nominal rate of return than is implied by the asymptotic limit of the yield curve. Following the issue of index-linked gilts, this argument must be seen to apply to real as well as nominal interest rates (and hence to long-term inflation expectations), although this was not the case when the method was first proposed by Day & McKelvey.

4.5.2 As always, a careful distinction must be made here between the different objectives of the method. In the case of a management valuation, the actuary may wish to take credit in advance for investment returns in excess of those implied by the current yield curves (equivalently he may deliberately seek to understate the cost of future fixed or capped pensions by assuming inflation in excess of current implied long-term market expectations). However, it must be recognised that this introduces a strong element of subjectivity into this process which may be at odds with the consensus defined by the market. If so, it can also be interpreted as an investment view. For example, if the longest implied spot real returns from index-linked gilts is 4% p.a. and the longest implied nominal interest rate is 7% p.a. then the most objective estimate of inflation expectations is 3% p.a. We see no reason to adopt an assumption of 5.5% p.a. (unless it is prudent or 'neutral' to do so) simply because it has averaged 5.5% in the past (the implicit investment view of this assumption being that index-linked gilts would be cheap relative to conventional gilts). We must be careful when presenting a long-term view that we are looking a long way forward and not just a long way backward.

4.6 *Interest Rate and Inflation Assumptions: Past Experience*

As suggested above, a traditional actuarial approach to establishing these assumptions involves calculating 'long-run averages'. We would hesitate to suggest that this is based partly on the statistical myth (explained admirably by Lee (1991)) that over the long term annualised rates converge on a long-run

average. In fact, as shown by Lee, such convergence of annualised returns is largely a mathematical play on the effect of geometric averaging.

4.6.1 As at the date of writing this paper the longest available rate on index-linked gilts implied a long-term real return of around 4% p.a. We would therefore expect equities to offer a real return somewhat greater than this. Historic data show the following real returns on equities over four independent decades (source: BZW (1994)):

	Annualised real return (% p.a.)
1950-59	13.7
1960-69	4.2
1970-79	-1.8
1980-89	15.3

4.6.2 Similar considerations apply as in ¶4.4.2 and, as we have stressed earlier, it is always important when looking at such past data to bear in mind that underlying these statistics can be a secular increase in the valuation rating of equities. It would certainly appear reasonable to us to use a real return from equities in excess of 4% p.a. for a management valuation, although the choice of margin above this rate is inevitably subjective. It is less clear cut that such a margin is appropriate for compliance valuations and, if it is, it should perhaps be prescribed.

5. Investment Implications Associated with the Dynamics of the Method of Day & McKelvey

To continue to enjoy and deserve this (acceptance as a source of expert advice on pension fund investment) we suggest that the actuary must resist all temptation to let the basis and technique of valuation determine the investment policy. Day & McKelvey (1964) (Our paraphrasing)

5.1 Having considered the two major subjective elements of the method of Day & McKelvey, we now turn to our second broad characteristic, namely the dynamics. We consider the essentially quantitative (and comparative) issue of *smoothness* in Section 7. This leaves us to focus here only on the *investment implications* of these dynamics from a qualitative perspective (these relate mainly to asset allocation issues, the impact of the method on the *timing* of cash flows is considered briefly in Section 7 also). In doing so we would give qualified support to the above maxim, although we would suggest that it should apply only to *subjective* bases and techniques (since one could not dispute that investment policy for fixed pension liabilities, for example, should have regard to the existence of a matching gilt portfolio and the risk taken relative to this).

5.2 There are a number of different ways in which the method of Day & McKelvey is applied in practice. In particular, it may be applied to the actual asset allocation of the fund or to a 'model portfolio' (or to a combination of the two). As part of the overall approach, gilt holdings are usually treated in a consistent manner, that is, by discounting the income at the nominal (or for index-linked gilts, real) rate of interest used to value the liabilities. Nevertheless, the general impact of the method tends to be similar, however it is applied. Where there are differences, these are brought out in the comments below, where we focus on four main issues.

5.3 *The Pension Fund Actuary's Perceived Income Preference*

5.3.1 One obvious concern to which any casual reader of the financial press will be alert is the way in which the method appears to have dragged the pension fund actuary unwittingly into the debate on income distribution versus capital growth, with the profession perceived to have a strong preference for the former. This issue has acquired particular sensitivity recently, with some political comments on possible over-distribution of income. There is, however, no particular reason why the profession should prefer income over capital growth. A pension fund should prefer the course of action which leads to the highest expected gross rate of return from the equities it holds. If this is achieved by companies retaining more income, then so be it.

5.3.2 This focus on income has potential to create quite obvious contradictions. For example, if a change in the market, economic, taxation or management environment were to discourage rights issues and encourage the retention of earnings, then companies might well reduce dividends in the short term (or not increase them as expected). In these circumstances it would be sensible for the price of equities to remain constant and the yield to fall (on expectation of higher future earnings per share growth). To avoid writing down assets unnecessarily, the actuary would need, subjectively, to adjust the long-term dividend growth assumptions. Yet in practice (particularly in compliance applications) the method may be applied mechanistically without adjustment. The ramifications flowing from this are wholly unnecessary.

5.4 *The Distortion of Perceived Asset Allocation Risk*

5.4.1 Our second concern is the manner is which the method distorts perceived risk associated with different asset classes. We have already noted Day & McKelvey's view that the valuation method should not drive asset allocation, and yet it is quite clear that the method has enormous potential to do so. To illustrate how this operates it is necessary to establish a conceptual model of the asset allocation decision process. Fundamentally, this is a reflection of various different objectives, but typically trustees may, quite sensibly, seek to maximise the total return (on market values) subject to the desired degree of likely stability in the funding level (or contribution rate or pension expense). Clearly, the valuation methodology will legitimately have an effect on the second item, but it should not disguise the importance of total return (whether by way of income

or capital). We would advocate extreme caution in allowing *subjective* elements of the valuation basis to distort the perception of risk attached to different investment strategies and hence the choice of strategy itself (as a corollary, great care should therefore be taken when prescribing such subjectivity within a compliance valuation basis). In drawing attention to this issue, the two methods of application described above can provide different (but equally paradoxical) distortions.

5.4.2 *Method applied to actual asset allocation*

5.4.2.1 Under this approach, the discounted income method is applied in turn to each asset type held in the fund at the valuation date.

5.4.2.2 If this approach is adopted without changing long-term assumptions in the basis, then it can be demonstrated that, provided the growth of cash deposits responds to short-term inflation, these are the least volatile asset for a pension fund in terms of resultant variability of funding level. This is because, on the asset side, the capital value of the deposits will be stable in real terms. Likewise, changes in the liabilities depend mainly on inter-valuation (historic) inflation and are not directly sensitive to changes in views reflected by securities markets. Hence, if the method is applied mechanistically in this way, cash deposits emerge as an acceptable low-risk asset for a pension fund. Investment in short-dated gilts may be appropriate for some mature funds; however the conclusion applies equally to immature funds, in which case the asset allocation implications of the method may be regarded as quite perverse and failing to appreciate the very long-term nature of the liabilities (the analysis in Section 8 gives, in our view, a more rational framework for establishing the risk associated with different equity, cash and long gilt combinations according to maturity and the variability in inflation expectations). Similar distortions can also apply to other assets which offer apparent market value stability (such as property), although in these cases the contradictions in the conclusions may be less clear-cut.

5.4.2.3 As a separate issue, considering the dynamics of the funding level on a particular day (rather than changes over time), it should also be stressed that the discounted income value of an asset may differ from that placed on another asset with the same market value at that date. As a consequence, the funding level can change if the asset allocation is switched on a particular day. For example, if interest rates are low relative to the actuary's long-term assumptions, surplus can be created on mechanistic application of the method simply by moving from long-term securities into shorter-dated issues. Furthermore, of greater practical concern in the discussion which follows, it may be possible to have a situation where the yield on equities (par dividend yield 4%) was high (say 6%) *because the market correctly expected dividends to fall* by 33%, but the yield on an equal duration index-linked gilt was only 4½% (equal to the real rate of interest underlying the valuation). A complete switch from equities (written up by 50%)

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to index-linked gilts of the same duration (taken at market value) would then reduce the asset value by 33% at the valuation date for no justifiable reason. These contradictions were brought out quite clearly by Arthur & Randall (1990).

5.4.2.4 Again, even if such contradictions were tolerable in the context of a (subjective) management valuation, it is, in our view, undesirable for a method with such dynamics to be adopted as a prescribed basis for compliance valuation purposes. Effectively, the unadjusted method is saying not only that one asset is cheap relative to another, from the perspective of a pension fund, but that, if an asset allocation switch is implemented, immediate credit will be given for this relative difference. Given the subjectivity of the assumptions discussed in the previous sections (dividend growth in particular for equity market switches and inflation expectations for switches within the gilt market), this would appear to be a clear contradiction of Day & McKelvey's maxim at the beginning of this section.

5.4.3 *Method applied to 'model' portfolio*

5.4.3.1 Under this approach, the assets are assumed to be notionally re-invested (at market value) into some model portfolio either as judged appropriate by the actuary for management applications, or on some prescribed basis.

5.4.3.2 If this portfolio consists entirely of gilts (index-linked and fixed-interest) which have the same duration as the liabilities, then this is essentially the same as the method we propose in the following section, but the 'switch factor' is applied to the assets rather than the liabilities (if the duration of the liabilities cannot be matched by existing gilts, then the only issue is whether the 'ultimate' rates of interest and inflation should move with the market's expectations or remain on the actuary's 'long-term' rates). We consider some criticisms of this approach in Section 9. However, our main concern here is the issues raised by the application of the method to a 'model' portfolio which includes equities. The concern arises once again from the way in which the dynamics of equities (or, more particularly, U.K. equities if the yield on the FTSE-A All Share Index is used for y in equation (3)) are distorted.

5.4.3.3 This is not simply an issue of smoothness. There are at least three other issues which need to be highlighted:

(1) As will be shown in Section 7, U.K. equities are not 'matched' to the liabilities in the sense that the value of U.K. equities does not move precisely in tandem with the value placed on U.K. pension fund liabilities. Accordingly large surpluses and deficits can be generated (which would not occur if assets and liabilities were matched in any traditional sense of the word). This does not render equities inappropriate as investments, but the valuation method can disguise their risks and discourage switches out of equities which are justified on other grounds. For example, it would not seem unreasonable to suggest that funds could lock-in to the surpluses generated from equities by switching into alternative assets if they so wished.

(2) The method of Day & McKelvey gives no consistent basis for valuing alternative assets such as overseas investments, and as noted earlier, there is therefore a tendency to include only U.K. equities in the model fund. From the perspective of controlling the funding level, this means that these overseas investments become more 'risky' than U.K. equities, simply due to the fact that overseas equities might not move in tandem with U.K. equities (the subjectively defined 'matched' asset). Our concern here is that application of the Day & McKelvey method for compliance valuations may therefore lead to an unnecessarily high proportion of U.K. pension fund equity exposure being domestic rather than overseas. Whilst the U.K. market has performed well relative to the major overseas markets overall over the last decade, it is not impossible for the situation to reverse over the next ten years. Similar problems are associated with other assets which do not easily fit into the framework of the Day & McKelvey methodology.

(3) Allied to (2), if a gilt portfolio existed which matched closely the incidence of the fund's cash flows, but the model fund was chosen instead to include a significant equity element, trustees could be in a position where a move into these same gilts was deemed to be 'risky' (because U.K. equities could rise dramatically, reducing the yield in the 'model fund' and writing down all assets in the funding level calculation). This highlights the need to ensure that prudent and sensible investment behaviour is not distorted by the dynamics of a subjective asset valuation method.

5.4.3.4 The potential effects of these issues should not be underestimated. For example, consider the position of trustees with a view that the market value of equities was unsustainably high in September 1987. Backing this view by switching into index-linked gilts would be risky in funding level terms because, if the equity content of the model fund was not likewise reduced and equity markets rose further (equity yields fall), then they could suffer a write down of their assets and a reduction in funding level if these gilts did not rise in parallel with equities. By continuing to hold (U.K.) equities, on the other hand, they are exposed to less short-term variation in funding level (that is, U.K. equities are *defined* by the methodology as the least risk position). This clearly has potential to act as a strong deterrent to backing quite prudent investment judgements. We do not therefore necessarily see the use of the method of Day & McKelvey as a triumph in 1987. It is at least plausible that, if more trustees had been encouraged to consider both the actual gains in equity market values in the months leading to October 1987 and the yields available on alternative assets (index-linked gilts in particular), then there would have been a lower exposure to equities by the end of the period.

5.4.3.5 The application of the method of Day & McKelvey in the proposed minimum funding standard for U.K. pension funds (multiplying the liabilities by the inverse of the market adjustment factor) using an implicit (U.K. equity) model fund has the potential to increase the distortions described above. It should be

noted that the period for comparing the performance of alternative assets may shorten to a single year with an annual funding test and the effect may be to tilt the balance of risk perceptions further in favour of U.K. equities and away from overseas exposure in particular, possibly unnecessarily and to the detriment of overall investment performance.

5.4.3.6 For a quantitative demonstration of the way in which asset and liability studies produce different 'optimal' strategies according to the asset allocation of the model fund, readers are referred to the paper by Lockyer (1990).

5.5 *The Distortion of Sensitivity to Changes in U.K. Interest Rates and Inflation Expectations*

5.5.1 The funding level may be little changed by changes in these expectations if the valuation method adjusts the assets or the liabilities (or both) in a consistent manner. However, in the case of the application of the Day & McKelvey method in particular, as shown in Section 4, the assumptions become important when the duration of assets and liabilities are not matched. This is particularly relevant in the context of recent sharp fluctuations in both long-term real interest rates and inflation expectations in the U.K., and we would question the extent to which the previous use of the method has encouraged trustees and companies (and affected the perception of Governments in imposing legislation) to incur benefit improvement liabilities costed on long-term assumptions without full understanding of these duration matching implications. It is, in our view, not necessarily desirable to infer that the cost of a long-term liability can be estimated on the basis of a stable set of long-term assumptions. It would be far better, surely, for trustees, companies, governments and scheme members to understand that our best estimate of cost can vary as perceptions of the future environment change.

5.5.2 Similar comments apply, of course, to the estimation of the cost of each year's accrual of benefit. In our view, it is prudent and objective to reflect the fact that the cost of accrual is higher when long-term real interest rates (or inflation expectations for fixed liabilities) are low, and that, if they remain at low levels, the pension scheme will be more expensive to fund. The management decision as to whether to increase actual contributions in this environment (with the danger that markets may be high) should be an investment decision.

5.5.3 Furthermore, we would also suggest that, although the use of comparative performance measurement has had a powerful (and well publicised) impact on the asset allocation of U.K. pension funds, the use of fixed liability bases has also, in our view, encouraged a disregard for matching issues. A trustee told that the pension liability will rise by 9% for each 1% p.a. fall in fixed-interest rates is perhaps more likely to reduce his equity allocation than one who believes that his liabilities are insensitive to changes in market interest rates and the only risk is that annual dividend growth fails to attain some hurdle rate.

Nor will the use of modern asset and liability studies necessarily address this matching issue, unless the study recognises the effect of falling interest rates on liabilities. Indeed, many such studies (as described by Lockyer (1990) for example) are based on assumptions very similar to the method of Day & McKelvey, namely that it is sensible to define a long-term average for variables such as inflation and dividend growth and to use a fixed long-term basis to value the liabilities. Whilst such long-term studies certainly give insight, unless they allow for changing the valuation basis in line with changes in interest rates and inflation expectations, the study may understate duration risks and reinforce perceptions of the long term as a stable entity.

5.6 *Management of Investment Risk*

5.6.1 Finally, we would draw attention to the general lack of interest shown by pension funds in the control of investment risk, despite the size of pension assets relative to the average U.K. company (for FTSE-100 companies, for example, the mean pension fund size is around one third of market capitalisation). This lack of concern for the control of investment risk contrasts sharply, in our experience, with the risk aversion observed in the general asset management activities of most companies (for example in the hedging of currency or commodity price exposure through derivative markets).

5.6.2 Although attitudes are changing, the effect of the above can be seen in the general lack of interest shown by finance directors towards pension funds (when compared with corporate treasury activities) in relation to the sophisticated risk management techniques which developed during the 1980s. This is partly a tribute to the strength of the method of Day & McKelvey in damping the volatility of funding levels in management valuations (and hence contribution rates) in which lies a genuine corporate interest. However, the weakness of the method is that the dynamics of the residual volatility are driven by linkage to U.K. equity dividend growth which cannot be hedged by exchange-traded derivative instruments. By contrast, a market-related methodology would be significantly more straightforward to hedge.

5.7 *Historical Context*

As a final historical comment, we would stress that in 1964 none of the above were real concerns when compared with the enormous benefit of the method of Day & McKelvey in giving trustees and company management insight into the essential characteristics of equity investment, characteristics which were then unproven, but which are now taken for granted. It is only when we weigh the value of this insight now in the contemporary environment, that we feel the above drawbacks outweigh the benefits. Accordingly we now revisit a modern version of the alternative market value approach suggested by Gilley & Funnell.

6. Modern Revisit of Market-Related Methodology

It may be asked whether the market's assessment of the value of an equity share relative to 2½% Consols is a reliable one to adopt; in pursuit of this point it can be shown that the ratio of the yield on Consols to the overall yield on industrial ordinary shares varies over time. However, if this ratio rises it is presumably in expectation of dividend increases and if the increases materialise the ratio will automatically fall again until the next wave of optimism; if the anticipated increases do not materialise the prices will fall and equilibrium will be restored in this way. Conversely, the ratio will fall if a fall in dividend is anticipated. If therefore we have no reason to anticipate a long-term change in the mean value of the ratio, and deviations from the mean are the result of short-term expectations, it is correct to base the value of equity shares on their unadjusted market value and to follow the method of valuing them which we have suggested, since it is reasonable to take note of expectations of increases or falls in dividend.....In applying the above method of valuation of equities, however, it should be remembered that the yield on equities is to some extent depressed where, as at the time of writing, they are being bought as a hedge against future inflation....We see no way of assessing the amount by which the market value of equities is increased as a result of such buying and we can only suggest that an arbitrary deduction from the value calculated by the above method should be made.

Gilley & Funnell (1958: Twenty five years before the widespread issue of index-linked gilts in the U.K.)

6.1 In this section we outline briefly our alternative market-related methodology (see also Wise (1984) and Arthur & Randall (1990)) and derive formally the two major discrepancies between this method and that of Day & McKelvey. The section then concludes with an outline of a simple dynamic model of the assets and liabilities under this alternative method, a model which we will invoke in Sections 7 and 8.

6.2 *Unsmoothed Market-Related Method*

Our alternative method simply bases the long-term rate of interest for the valuation of liabilities on the series of spot interest rates implied by the U.K. nominal yield curve at the valuation date (j^*) and real yield curve (r^*) implied by U.K. index-linked gilts (at the short-dated end this yield curve must be calculated on current rates of inflation to avoid distortions due to the 8-month lag; at the long end the ultimate rate must be assumed to be the asymptotic value). Our implicit assumption is then that these rates of interest are consistent with the market valuation of equities at the valuation date (incorporating the markets expectation of dividend growth). Subtracting these series then gives, in our view, an acceptable estimate of the equivalent inflation expectations series f^* (the distortions in this estimate ultimately need to be balanced against the uncertainty in Day & McKelvey's factor R). Under this approach, using Λ to denote the liability valuation on r^* and f^* (asterisks denote market estimates), the funding level at a given valuation date becomes:

$$\text{Unsmoothed funding level (market related basis)} = \frac{M}{\Lambda\,(r^*, f^*)} \fallingdotseq \frac{M}{L\,(\bar{r}^*, \bar{f}^*)} \qquad (4)$$

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where $\bar{r}^*$ and $\bar{f}^*$ are (scalar) averages of the individual components of $\boldsymbol{r}^*$ and $\boldsymbol{f}^*$ respectively over the duration of the liabilities, it being noted in the case of $\boldsymbol{r}^*$ in particular that, with the issue of very long-dated index-linked stocks in the U.K. (2½% 2024 and 4⅛% 2030, for example), the liabilities of many (relatively mature) U.K. pension schemes now fall, in aggregate duration terms, within the space spanned by market real rates of interest.

6.3 *Applications of General Methodology*

6.3.1 The funding level in equation (4) can be applied in two specific ways:

(1) *On an objective and prudent basis, with no advance allowance for the expected excess return from the actual investment policy over the prospective rates of return implied by Government securities.* This form of application is appropriate for solvency valuations (where it is also generally possible to value the liabilities on a relatively objective basis). However, it can also be adopted as a compliance valuation methodology with more subjective ongoing liabilities if certain items of the basis remain implicit. For example, one could allow for real salary escalation of accrued liabilities implicitly by franking this against the expected excess return from the actual investment policy. Not only is this approach similar in many ways to the statutory net premium valuation of a life office (the quintessential compliance valuation), it is also very similar in principle to the annual statutory valuation methodology adopted for funded pension schemes in the Netherlands and in Switzerland (although with the significant benefit of market real rates of interest).

(2) *On a subjective, realistic basis with explicit allowance for both real salary increases and for a rate of expected investment return in excess of that implied by Government securities.* We consider this approach to be more transparent than the method of Day & McKelvey because it recognises from the outset, explicitly, that the key issue is the quantum of excess return and no artificial distinction needs to be made between achieving this through superior active management, through investment in corporate (rather than Government) bonds or from the *total return* on equity-type assets relative to bonds. We will also demonstrate, that although less smooth, the dynamics of this methodology are not as perverse as those of the Day & McKelvey method and the investment policy implications are more rational.

6.3.2 In the analysis which follows in Sections 7 and 8 we make no distinction between these two applications, it being recognised that the addition of a constant margin $\boldsymbol{u}$, (denoting a vector with identical components u), for example, to the expected rate of real return in respect of liabilities of duration α in practice merely multiplies the liabilities by a constant. That is;

$$\Lambda\ (\boldsymbol{r} + \boldsymbol{u}, \boldsymbol{f}\ ) \doteq e^{-\alpha u}.\Lambda\ (\boldsymbol{r}, \boldsymbol{f}\ ).$$

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The practical effect of this is to add a fixed drift of magnitude u to the development of the expected funding level over time, when compared with use of the unadjusted prospective return r. Similar considerations apply to explicit real salary growth assumptions.

6.3.3 However, the important issue to note is that, although they impact upon the disclosed funding *level*, the additions of constant margins to market rates of return do not affect its *dynamics* other than through the constant drift over time. We consider the dynamics over time in the following sections, but it will be noted immediately that, since we are in a valuation frame of reference where all assets are taken at market value, many of the criticisms and inconsistencies described in the previous section in connection with the method of Day & McKelvey certainly do not apply. In particular the valuation result is not altered by altering the asset allocation on a particular day. Neither can the actuary be seen to be implying that one asset is cheap or expensive relative to another.

6.4 *Formal Derivation of Differences in Methodology*

Comparing equation (4) with equation (3) in ¶2.4, the difference between the unsmoothed market-related funding level (FL) and the smoothed funding level under the method of Day & McKelvey $\hat{FL}^{D\&M}$ (using circumflex to denote an implicitly smoothed estimate) is to first order given by:

$$\log \hat{FL}^{D\&M} - \log FL = \dot{K}_m.(m - \bar{r}^* + y) + (\dot{K}_r - \dot{L}_r).(r - \bar{r}^*) + (\dot{K}_f - \dot{L}_f).(f - \bar{f}^*).$$

Hence, to first order, the difference between the two methods is accounted for by:

(a) the difference between the market estimate of long-term average real dividend growth (first term); and

(b) differences between the valuation real rates of interest and inflation expectations (second two terms respectively) if the model portfolio is not matched (that is, if $\dot{K}_r \neq \dot{L}_r$ or $\dot{K}_f \neq \dot{L}_f$).

6.5 *Simple (First Order) Model Of Asset And Liability Dynamics*

6.5.1 *Basic outline*

6.5.1.1 We conclude this section with an outline of a simple, first order, model of the asset and liability dynamics under the market-related methodology, which we will invoke in the following two sections. This is the simplest version of the model which we feel is of value for our purposes, incorporating the main principles while ignoring the detail which would be required in full application.

6.5.1.2 We assume that the market gives rational prospective expectations at any point in time and hence that market movements will be primarily driven by changes in expectations due to the arrival of new information. In particular we

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define the following changes in expectations which drive the dynamics of the assets and liabilities:

(1) changes in real return expectations ($\Delta\bar{r}^*$);
(2) changes in inflation expectations ($\Delta\bar{f}^*$);
(3) changes in real dividend growth expectations ($\Delta\bar{m}^*$); and
(4) changes in demographic expectations ($\Delta\bar{q}^*$).

6.5.1.3 For simplicity our model assumes that the real return, inflation rate, dividend growth rate and demographic experience over the first time period can be perfectly predicted and are known exactly at the start of the time period (for convenience we define an information set I_n to represent all known information at time n), but information which is received during the time period essentially changes the expectations for these factors for each subsequent time period by a uniform amount (that is, it considers only parallel shifts in the yield curve, for example). The model is set out in more detail in Appendix A, more complicated changes can be considered by modelling pivot points along the expectation curves but this is beyond the scope of the paper.

6.5.2 *General form of dynamic equations*

6.5.2.1 *Assets*

Using this approach, the general form of first order dynamic equation for an asset allocation A among various asset types derived in Appendix A is as follows (where $\dot{\Delta}A = \log A_n - \log A_{n-1}$):

$$\dot{\Delta}A = \delta_A - \alpha_A\Delta\bar{r}^* - \beta_A\Delta\bar{f}^* + \gamma_A\Delta\bar{m}^* \qquad (\delta_A \in I_0).$$

6.5.2.2 *Liabilities*

Using the above notation, the growth of the liabilities is, to first order, given by:

$$\dot{\Delta}L \fallingdotseq \delta_L - \alpha_L\Delta\bar{r}^* - \beta_L\Delta\bar{f}^* + \gamma_L\Delta\bar{q}^* \qquad (\delta_L \in I_0).$$

6.5.2.3 *Funding levels*

Finally, combining the above, the general first order dynamic form for the change in funding level is as follows:

$$\dot{\Delta}A - \dot{\Delta}L \fallingdotseq (\delta_A - \delta_L) - (\alpha_A - \alpha_L)\Delta\bar{r}^* - (\beta_A - \beta_L)\Delta\bar{f}^* + \gamma_A\Delta\bar{m}^* - \gamma_L\Delta\bar{q}^*.$$

We now turn to analysis of the alternative methods on both an empirical basis and under the above model. (Henceforth, for convenience, we will assume $\Delta\bar{q}^* = 0$).

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7. Comparison of Alternative Methodologies

Data snooping is a term used...to describe the bias which occurs when the properties of a data set influence the choice of estimator or test statistic. This leads to overstatement of the significance of estimated relationships. In turn out-of-sample performance of a fitted model is likely to be inferior...Even apparently innocuous data snooping (for instance ascertaining the long-term mean of the series which is being predicted) significantly enhances reported forecast quality.

Dimson & Marsh (1990)

7.1 In this section we attempt comparative analysis between the method of Day & McKelvey and our market-related methodology under the headings of the two broad characteristics described earlier:

(1) the *objectivity* of the valuation result; and
(2) the *dynamics* of the funding level (where we introduce simple explicit smoothing techniques for the market-related result).

Our analysis of the dynamics of the market-related methodology is then pursued further in more detail in the following section, which focuses in particular on the asset and liability management implications.

7.2 *Outline of Empirical Analysis*

7.2.1 *Back-testing issues*

Although our empirical analysis of the market-related methodology is relatively straightforward and objective, the analysis of the Day & McKelvey method is bedevilled by problems with which we are very familiar in the context of the 'back testing' of all quantitative investment techniques; namely, the removal of post-event knowledge from the test, the most obvious knowledge in this case being the known *ex-post* average dividend yield. A proper test of the method would require use of the 'par dividend yield', based only on information available at the time. Unfortunately such tests are rarely attainable (this study being no exception), with the result that methods invariably appear more successful retrospectively than is the case prospectively (see, for example, Dimson & Marsh (1990)).

7.2.2 *Choice of past periods*

Subject to the caution expressed above, we have analysed the methods using actual past data to draw out our main inferences. These data periods are:

(a) quarterly from 1984 Q1 to 1993 Q4 (10 years); and
(b) annually from 1964 to 1993 (30 years).

Despite the specificity of these past data to particular environments, the analysis does have the advantage of showing the actual impact of the method for a typical pension fund on a model neutral basis. However, we have also included where possible a sketch of the theoretical derivation of the same result using the simple model of asset and liability dynamics described in the preceding section.

7.2.3 *Specification of the analysis*

7.2.3.1 For simplicity we have modelled a pension scheme providing fully-index-linked pensions. This model fund is described in more detail in Appendix C.

7.2.3.2 Clearly, prior to the availability of index-linked gilts to all investors in 1983, the full application of our proposed alternative method was not possible. In order to establish the longer annual series of results, we therefore kept all the real liabilities on a fixed basis prior to 1984. The approach is therefore likely to overstate the variance of the market-related methodology over the first twenty years of period (b). For the quarterly analysis and the annual data after 1983, however, we have applied an approximation to the full method (discounting all real liabilities at the real interest rates implied by Treasury 2½% 2006).

7.2.3.3 As described above, specification of the method of Day & McKelvey is less straightforward because our impression is that application of the method in practice has been far from consistent, despite the supposed long-term nature of the assumptions. This is particularly the case for fixed liabilities or those capped at a nominal limit (that is sensitive to changes in inflation expectations) since our survey of colleagues practising at the time confirms an impression of nominal annualised interest rate assumptions in valuation bases rising from around 7% in the late 1960s to around 10% in the 1980s then falling to 9% in the early 1990s and subsequently falling to around 8% at present. Whilst we welcome such changes, they only reinforce the impression of a method which has elements of a (subjective and implicit) market-smoothing technique. It is difficult to obtain reliable statistics for these basis changes prior to the introduction of SSAP24 and, accordingly, a constant real basis has been adopted throughout.

7.3 *Notation*

We adopt the following notation for the analysis in this section:

FL_n;	funding level (asset value divided by liability value) at end of time period n.
$\hat{FL}_n$;	as in Section 6, we adopt a circumflex to denote a smoothed result, which includes the method of Day & McKelvey (by smoothed we mean that the result places some weight on prior expectations based on information available at the end of the previous period).
$\hat{FL}_n^{D\&M}$, $\hat{FL}_n^{SM(\lambda)}$;	these superscripts denote the Day & McKelvey and smoothed market-related methods respectively, (λ being a smoothing parameter).
FL_n^*;	an asterisk is used to denote the expected funding level based on information available at the end of the previous time period ($n-1$).
ε_n;	this represents the percentage error between the (logarithm of) the expected funding level and the

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actual funding level under a particular method. That is:

$$\varepsilon_n = 100.\log\,(FL_n \,/\, FL_n^*).$$

7.4 *Analysis of Market-Related Methodology assuming 'Matched' Investment Policies*

7.4.1 *Fully-matched liabilities*

We take it as trivial that application of the market-related methodology to a situation where a matched gilt portfolio exists (as in the case of a typical cohort of fixed pension liabilities, for example) will lead to an objective valuation result with no dynamics, provided that the fund remains invested in the matched portfolio. That is:

$$E(\varepsilon_n) = 0, \text{ and } \mathrm{Var}(\varepsilon_n) = 0. \tag{5}$$

7.4.2 *Duration (first order)-matched liabilities*

For longer, deferred liabilities (for example a cohort of index-linked pension liabilities payable in ten years time) only a weaker form of duration-based matching may be possible (as described in the following section). However, it is still trivial to show that, provided the assets remain invested in the matched gilt portfolio defined by this method, equation (5) will hold as a first order approximation. In terms of the aggregate position, this is not dissimilar from that of a typical (mature) U.K. pension scheme where the majority of accrued liabilities will relate to older members and pensioners. This result should therefore be contrasted with the analysis in the following paragraphs where the Day & McKelvey method defines U.K. equities as a matched asset for such a fund, and yet investing wholly in this asset does not satisfy either of the conditions in equation (5).

7.4.3 *Immature liabilities*

7.4.3.1 For very long deferred liabilities (for example in the case of an index-linked pension payable in 30 years time), it must be acknowledged that even a duration-based match is not possible in the current U.K. index-linked gilt market. Our assertion here is only that the use of the longest market-related real interest rate is a more objective and prudent measure of prospective return than the assumptions underlying the Day & McKelvey method. However, we must acknowledge that in practice a degree of subjectivity is required in any such valuation, and it may be unrealistic to adopt the most prudent basis. More importantly, linking very-long-term liabilities to an (extrapolated) asymptotic limit of a much shorter market real yield curve may overstate the coupling with index-linked gilts in the overall dynamics, possibly to the detriment of investment policy.

7.4.3.2 This is unlikely to be a major issue for typical U.K. pension fund valuations, where the bulk of the liabilities relates to much older members

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(falling within ¶¶7.4.1 and 7.4.2). However, it may be of particular relevance in the matching of individual transfer value liabilities. Although closely allied to valuation methodologies, the subject of individual transfer value calculations is beyond the scope of our paper. We would only suggest here that the Day & McKelvey methodology should not be regarded as the only technique which can partly link liability movements to equity markets in such applications (if linkage is deemed appropriate). For example, traditional *retrospective* calculation techniques (although not envisaged within the current guidance note GN11) can provide similar linkage on a more objective and equitable basis.

7.5 *Comparison of Methodologies assuming 100% U.K. Equity Investment*

7.5.1 We now consider the application of either methodology to a fund invested totally in U.K. equities. Figure 7.1 compares the quarterly development of the disclosed funding level of our model fund under both the (unsmoothed) market-related approach and the method of Day & McKelvey. In our experience this is the type of analysis commonly presented publicly as a demonstration of the workings of the latter method. However, whatever the merits of the theoretical justification of the method (as discussed in Section 4) this particular back testing exercise raises a number of important issues which we consider below under our two broad characteristics of valuation methodologies.

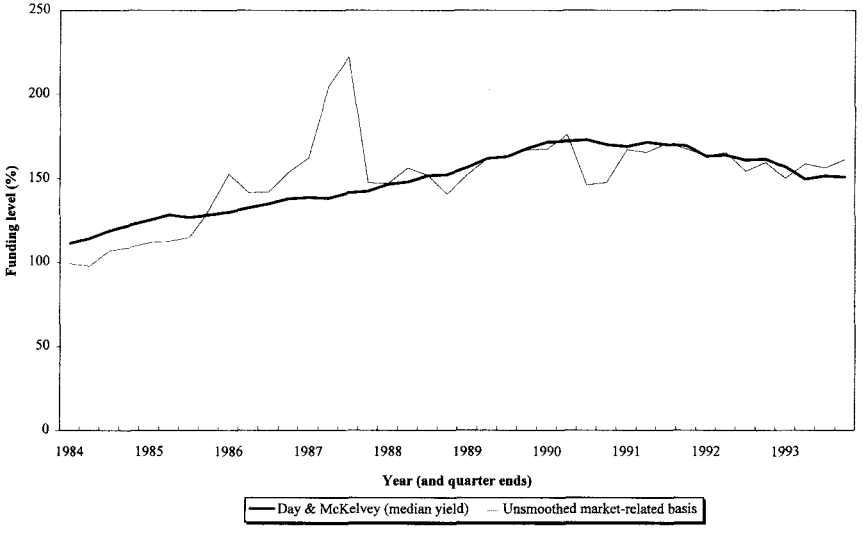

Figure 7.1. Comparison of funding level development under alternative methods

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7.5.2 *Objectivity of valuation result*

7.5.2.1 Firstly, as we have stressed repeatedly in this paper, the parameter m in the formula is highly subjective and is only a formalisation of the arbitrary multiplier R defined by Day & McKelvey. This means that for a given real rate of return used to value the liabilities, there is no unique valuation (par yield) for the assets. The development shown for the Day & McKelvey method in Figure 7.1 assumes a 'par yield' equal to the *actual* median yield over the period. This could not have been predicted in advance (the use of the actual average of a series which is supposedly being predicted is a classic example of apparently innocuous data-snooping discussed in detail by Dimson & Marsh (1990)). Indeed, as Thornton & Wilson commented "there is slight evidence of a random step function, with discrete changes in the mid 1960s, the mid 1970s and the early 1980s", as demonstrated by the following averages (source: mid-year data Wilkie (1984) extended using FT-A All Share Index gross dividend yield):

	Mean yield
1964 - 1973	4.29
1974 - 1983	5.89
1984 - 1993	4.36

7.5.2.2 The difficulty of predicting whether it will average 4% or 5% (or even 3.5% or 5.5%) over the next decade is well illustrated by Thornton & Wilson's musing that "it is perhaps too early to say whether there has recently been a step upwards to 5%" (we recall that a fashionable theory at the time was that the increased weighting of utilities in the U.K. market after privatisation issues had increased the level of dividend payout). However, at the date of writing this paper the yield has remained stubbornly around 4% for well over a year (and we would not wish to predict where it will be next year nor in ten years' time). Figure 7.2 shows the effect of changing the 'par yield' by ½% in each direction at any point in time.

7.5.2.3 Although this is only a small change in the par yield, it is interesting to note that the funding level disclosed on the unsmoothed market-related basis rarely falls significantly outside this envelope and, with the exception of the 'spike' in 1987 (to which we return below) and, to a lesser extent, the Gulf War during 1990, one must question why this market assessment is not in the normal course of events a reasonable compromise.

7.5.3 *Dynamics of the funding level: qualitative analysis*

7.5.3.1 *Capturing long-term trends*

The second issue which needs to be explained is why the path of the funding level on the market-related basis visually appears to follow the trend of the

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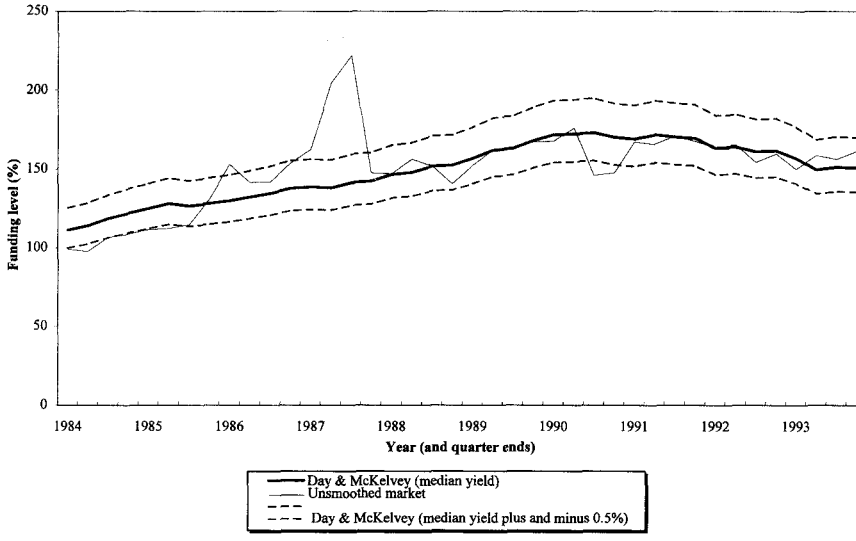

Figure 7.2. Demonstration of the impact of changing the par yield by ½% under the Day & McKelvey method

envelope of Day & McKelvey curves in Figure 7.2. The driving factor behind this common trend is, of course, the dividend growth over the period, the key point to note being that the method of Day & McKelvey is applied to the *actual* series of dividends (which have not conformed in any way with the *expected* growth over the period). This error between expected and actual dividend growth clearly results in an emergence of surplus of no less than 60% of the liabilities over six years, accrued through the mechanism of the Day & McKelvey asset valuation continually rebasing the equity valuation on the *actual* level of dividends each quarter. These long-term dynamics are no different in principle to those of the market's valuation of equities. At any time, the market will similarly start with this current level of dividends and then project (and discount) future expected growth leading to a market valuation multiple (the reciprocal of this valuation multiple being the yield). The fact that this multiple (or yield) has remained within a particular range over any period is only a reflection of the fact that the market's long-term expectations are not that dissimilar from typical actuarial assumptions.

7.5.3.2 *Capturing 'mismatch profit' (tactical asset allocation)*

We now focus more closely on the development of the funding level using the unsmoothed market value, where attention is immediately drawn to the 'spike' in 1987. This is almost entirely an intra-year event, not particularly noticeable when using annual calendar year data as below. The very emergence of such a

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significant and exceptional surplus on our objective market-related basis suggests that, whether or not they are suitable investments, equities do not match our model pension schemes's liabilities in any conventional sense. As discussed in Section 5, if equities are an appropriate investment, then the Day & McKelvey method provides one approach to smoothing the resulting funding level, but this should not, in our view, be confused with the issue of matching. By contrast, as explained above, with 100% investment in index-linked gilts, we would obtain a funding level which was broadly constant and close to 100%, more consistent with traditional matching principles.

7.5.3.3 Under the market-related method, the spike therefore represents no more than an extraordinary mismatch profit to the extent that a fund could have switched into our matched asset and the same feature would emerge from the asset and liability dynamics on either of the two applications described in Section 6 (that is, regardless of any subjective allowance for excess return above the real return available from index-linked gilts). Accordingly, the sequence of events in 1987 should not necessarily be seen as a triumph for the Day & McKelvey method in terms of its impact on investment behaviour (as suggested also by the discussion in Section 5.4).

7.5.4 *Dynamics of the funding level: quantitative analysis*

7.5.4.1 *Smoothness*

For management valuation purposes in particular, as we have discussed previously, the volatility of the funding level by the above market-related basis may, however, be too great for practical application. Again we stress that the way in which this volatility is dealt with is, in our view, an entirely separate issue, and in this respect the smoothed market-related methodology described below in our opinion differs from that of Day & McKelvey primarily in the explicit (rather than implicit) nature of the smoothing function.

7.5.4.2 This leads us informally to the following general class of smoothed market-related valuation results:

$$\hat{FL}_n = \lambda \hat{FL}_n^* + (1 - \lambda)\, FL_n$$

where FL_n is the unsmoothed (market-related) funding level. Essentially this function applies a weight of λ to the estimate at time $n - 1$ (which ignores all information during the next time period) and a weight of $(1 - \lambda)$ to the 'best estimate' incorporating all available market information. For an alternative theoretical derivation of smoothing techniques, applying weight to the retrospective and prospective result in turn, readers are referred to the paper by Smith (1993).

7.5.4.3 In fact, as we show below, the Day & McKelvey method displays properties very similar to an implicit market value smoothing belonging to the above class; the difference being that the smoothing process under the Day & McKelvey method is not divorced from the valuation methodology and is instead

hidden within a series of assumptions which define a value independently of the market, based on long-term assumptions.

7.5.4.4 Application of the above formula to the development of the funding level of our model fund gives the series of characteristics shown in Figures 7.3a and 7.3b using quarterly data over the period 1984 Q1 to 1993 Q4 and annual data over the period 1964 to 1993 respectively. (The initial smoothed funding level on the market-related basis is taken as 100% in each case).

7.5.4.5 When placed within these series of characteristics, it becomes apparent again that the method of Day & McKelvey displays, in practice, many of the attributes of an explicit market smoothing technique. In fact, using $\mathrm{Var}(\varepsilon_n)$ as our measure of smoothness we have:

$$\mathrm{Var}(\varepsilon_n^{D\&M}) = \mathrm{Var}(\varepsilon_n^{SM(\lambda)})$$

for the following parameter settings over these periods:

	λ
Quarterly (1984-1993)	0.921
Annual (1964-1993)	0.788

In fact, these are broadly consistent, since the quarterly method gives a weight (λ) of:

$$1 - 0.079\ (1 + 0.921 \times \tfrac{3}{4} + 0.921^2 \times \tfrac{1}{2} + 0.921^3 \times \tfrac{1}{4}) = 0.817 \text{ (c.f. } 0.788)$$

to the previous funding level, if the level over the past four quarters is estimated by interpolation between the two annual results. This suggests that the market-related methodology can be combined with a specific choice of smoothing function to produce a funding level development which is as smooth or smoother than under the method of Day & McKelvey.

7.5.4.6 *Efficiency (contribution timing impact)*

As noted in Section 3, the dynamics of any asset valuation process will inevitably have an impact on the timing of cash flows into a pension fund, if disclosed surplus or deficit is amortised by an adjustment to the future-service contribution rate. In quantitative terms it also is clear that, if the methodology is *efficient* at calling market peaks and troughs, then this impact, in terms of total return, will tend to be adverse (this is if the assets are *correctly* written up, this will obviate the need to increase contributions into a depressed market and vice versa).

7.5.4.7 Figure 7.4 shows the difference in contribution rate (as a percentage of pensionable salaries) between the method of Day & McKelvey and the unsmoothed market-related basis annually over the period 1964 to 1993 (as described in Appendix C, contributions are *normalised* in our model so that the

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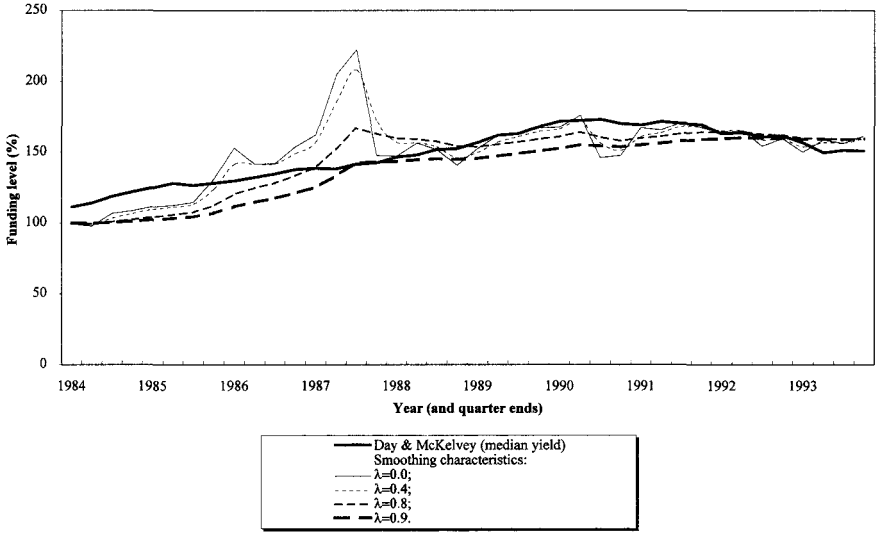

Figure 7.3a. Smoothed market-related funding characteristics compared with Day & McKelvey method (quarterly steps, 1984 Q1 to 1993 Q4)

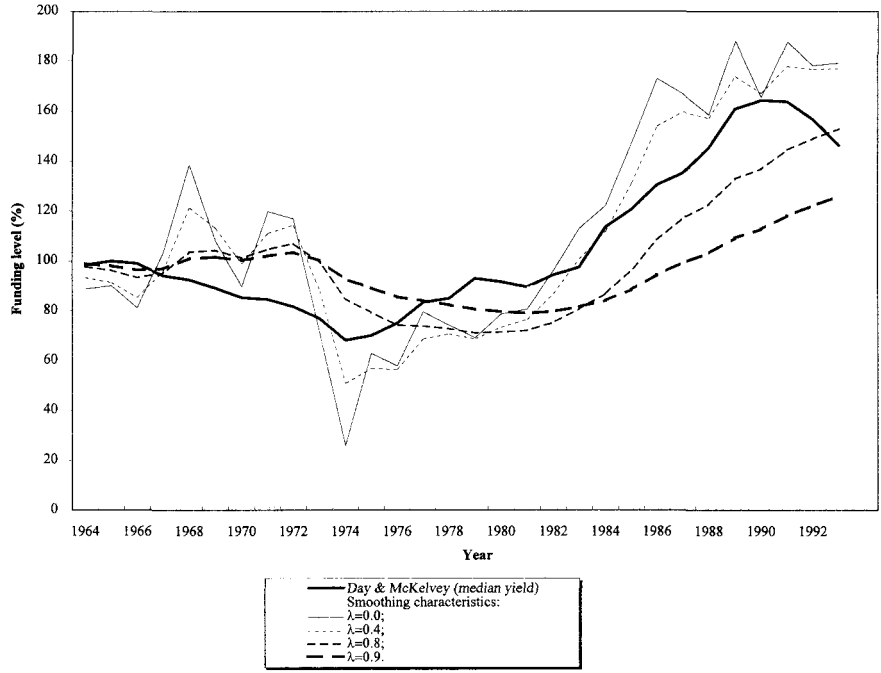

Figure 7.3b. Smoothed market-related funding characteristics compared with Day & McKelvey method (annual steps, 1964 to 1993)

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average contribution rate over the period is the same under both methods). A positive excess indicates that the Day & McKelvey method would have produced a higher contribution in the year concerned.

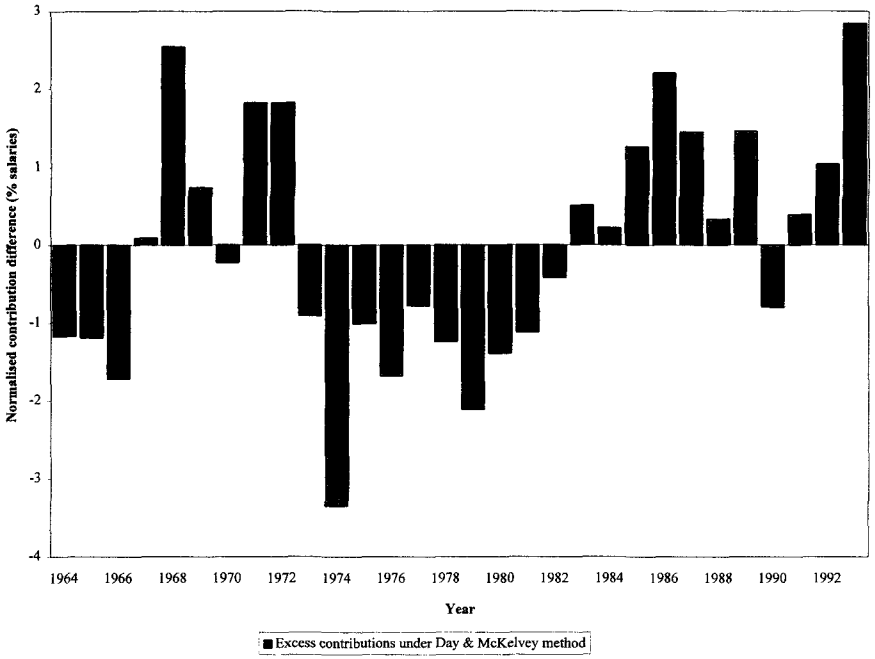

Figure 7.4. Difference in normalised contribution rates under alternative methods

7.5.4.8 The important issue here is that it is post-event knowledge that markets recover from the depths of the 1970s (so that higher contributions at this time can increase the overall real internal rate of return) and that they achieve unsustainably high levels in 1987 (so higher contributions here have the reverse effect). However, it still seems appropriate to analyse whether *empirically* the Day & McKelvey method has been efficient at calling these peaks and troughs when compared with smoothed market-related methodologies; the conveniently weighted measure of (*ex-post*) 'efficiency' for our analysis being the *adversity* of impact on total internal real rates of return over a given period.

7.5.4.9 The impact of timing on the real (in excess of salary inflation) internal rate of return achieved by our model fund over the period, (assuming amortisation of surplus or deficit over 15 years) is shown in Figure 7.5, plotted against the variability of the funding level defined by the formula above (for computational convenience the funding variation for the market-related method

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is based on the unnormalised contribution rate, that is excluding a small constant percentage of salary adjustment the effect of which is not material). The line shows the attributes of smoothed market-related methods while the attributes of the Day & McKelvey method are shown as a single point close to the line at a λ value of 0.788 (see ¶7.5.4.5).

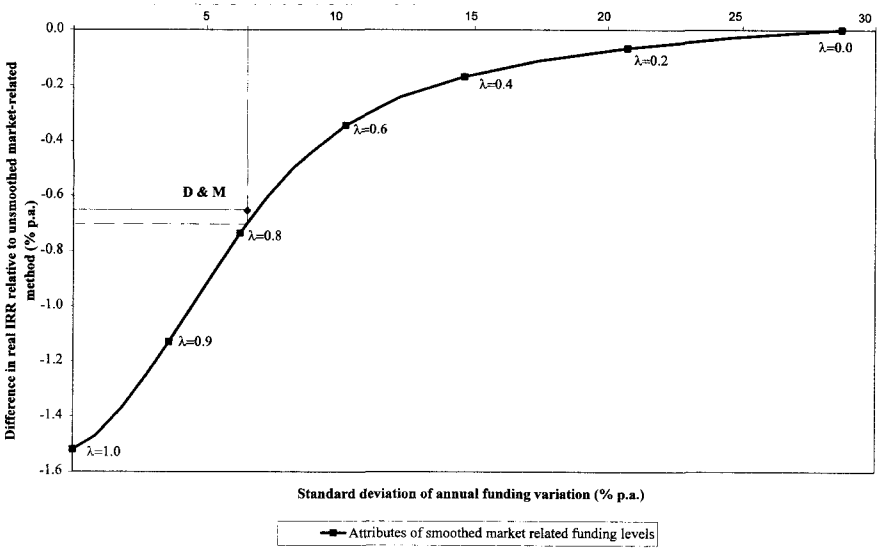

Figure 7.5. Comparison of efficiency of alternative methods using real internal rates of return (annual steps, 1963 to 1994)

7.5.4.10 It can be seen that in these *empirical* terms the behaviour of the method of Day & McKelvey is hardly distinguishable from that of the equivalent smoothed market basis (λ = 0.788) over the period 1964 to 1993, with both reducing overall returns in the order of 0.7% p.a. relative to the unsmoothed market basis due to the timing effect (in other words, the trade-off between volatility and the historic return achieved is the same under both methodologies).

7.5.4.11 Owing to our normalisation process, this result is also not particularly sensitive to the choice of par yield, and reducing the spreading periods increases the magnitude of the differences in the real internal rates of return without changing the relative positioning of the two methods. It will, however, be noted that the assumption of an immediate alteration to the contribution rate is unrealistic in practical applications of either method. As stated above, the contribution adjustment is being used here primarily as a convenient measure of (*ex-post*) efficiency at calling markets rather than to achieve full realism. In fact, allowing for one, two and three-year lags in the contribution adjustment *appears*

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to increase the historic rate of return using the Day & McKelvey method relative to the smoothed market method. Nevertheless, we are concerned that this 'lag' effect is specific to the particular past data period and the conclusion is likely to depend on the *period* of the cycles in funding levels and markets under the two methods which are observable post-event (but not, in our opinion, predictable *ex-ante*).

7.5.4.12 *Perversity; weak 'rational expectations' test*

Finally we analyse the perversity or otherwise of the dynamics of the funding levels under the two methodologies using the theory of 'rational expectations' (Muth (1961)). In its simplest form the theory states that if y_n^* is an estimate at time $(n - 1)$ for y_n (the actual outcome known at time n) and:

$$y_n = y_n^* + \varepsilon_n$$

then for y_n^* to be a 'rational expectation' the error term ϵ_n must have zero expectation (unbiased) and must be uncorrelated with all elements of the information set I_{n-1} available at time $n - 1$. In particular (the weak version of the rational expectation hypothesis), since the error (ε_{n-1}) in the previous estimate is known (therefore, $\varepsilon_{n-1} \in I_{n-1}$) we must have:

$$\text{Cor}\ (\varepsilon_n, \varepsilon_{n-1}) = 0.$$

7.5.4.13 This theory can be applied to our valuation methodology if the 'best estimate' of the funding level at time n is taken as:

$$FL_n^* = \frac{1}{v}\left((v - 1)FL_{n-1} + 1\right)$$

where v is the spreading period.

7.5.4.14 Now, the test for unbiasedness $(E(\varepsilon_n) = 0)$ only leads us back to the data snooping since this expectation is a function of the strength or prudence of the basis in each case. In particular, the expectation will be nil for the method of Day & McKelvey only if we can choose the *ex-post* average real dividend growth assumption as our value of m over each period, and will be nil for the market-related method only if we use the *ex-post* outperformance of the actual investment policy as our value of u in ¶6.3.2. Of more interest here is therefore the test for weak rationality based on the hypothesis that $\alpha_1 = 0$ in the following regression:

$$\varepsilon_n = \alpha_0 + \alpha_1\varepsilon_{n-1} + u_n.$$

In this formulation unbiasedness would imply $\alpha_0 = 0$, so α_0 can be regarded as a *prudence* coefficient and α_1 as a *perversity* coefficient. Using both the method of Day & McKelvey and our (unsmoothed) market-related method in turn, we obtain the following results for α_1 regressing ε_n on ε_{n-1} (and a constant) over the

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periods shown:

	Day & McKelvey		Unsmoothed market basis	
Period	Coefficient	(Standard error)	Coefficient	(Standard error)
1964-1993 (Annual)	0.55	(0.15)	-0.15	(0.19)
1984-1993 (Quarterly)	0.34	(0.15)	-0.03	(0.16)

7.5.4.15 During both periods the hypothesis that $\alpha_1 = 0$ is rejected (at the 5% confidence level) on the method of Day & McKelvey, whereas the results of the market-related methodology are consistent with the hypothesis (within one standard error). This result is perhaps not surprising. It is, of course, implicit in our simple model introduced in the previous section, since the error in the method of Day & McKelvey is broadly (to first order):

$$\varepsilon_n^{D\&M} \fallingdotseq m_0 - m$$

where m is the actuary's 'long-term' real dividend growth assumption and m_0 is the dividend growth in the next time period. In our simple model, the value of m_0 is known (to first order) by the market (that is $\varepsilon_n^{D\&M} \in I_{n-1}$) so the inconsistency with rational expectations is trivial. However, by way of empirical justification Figure 7.6a shows the observed autocorrelation of m_0 (quarterly dividend growth in excess of national average earnings). As expected, the correlations here are significant both at the one standard error test and, more convincingly, on a runs test.

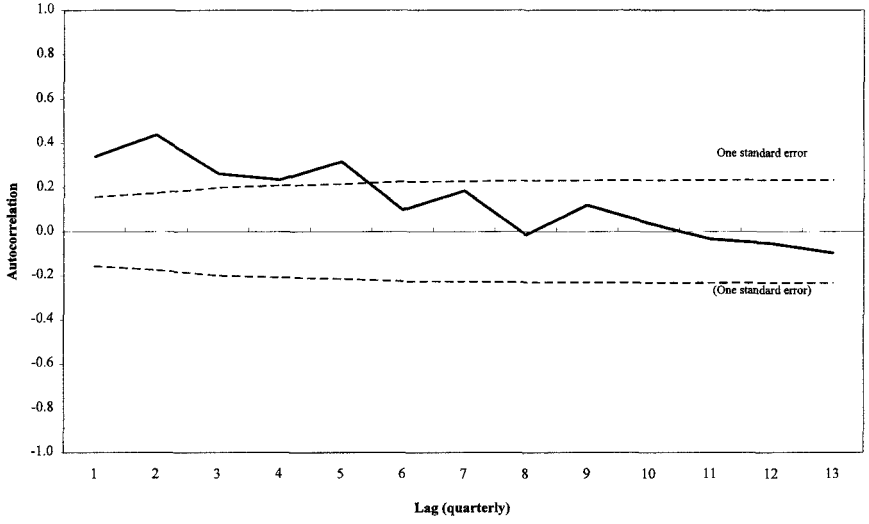

Figure 7.6a. Autocorrelation of residual error under Day & McKelvey's method (quarterly steps; 1984 Q1 to 1993 Q4)

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7.5.4.16 By contrast, assuming that the duration of the assets and liabilities are matched (with respect to real interest rates and inflation expectations) the error in the market-related method is :

$$\varepsilon_n^{\text{market}} \fallingdotseq \gamma_e \Delta \overline{m}^*$$

and to first order, if the market is arbitrage free, this change in expectations cannot be known at time 0 (so $\varepsilon_n^{\text{market}} \notin I_{n-1}$). Again, more empirically, equating $\Delta \overline{m}^*$ with the residual error in our regression of U.K. equity price movements over 1984 Q1 to 1993 Q4 (see Appendix B) we obtain the autocorrelation function shown in Figure 7.6b which, in contrast to Figure 7.6a, shows only weak autocorrelation.

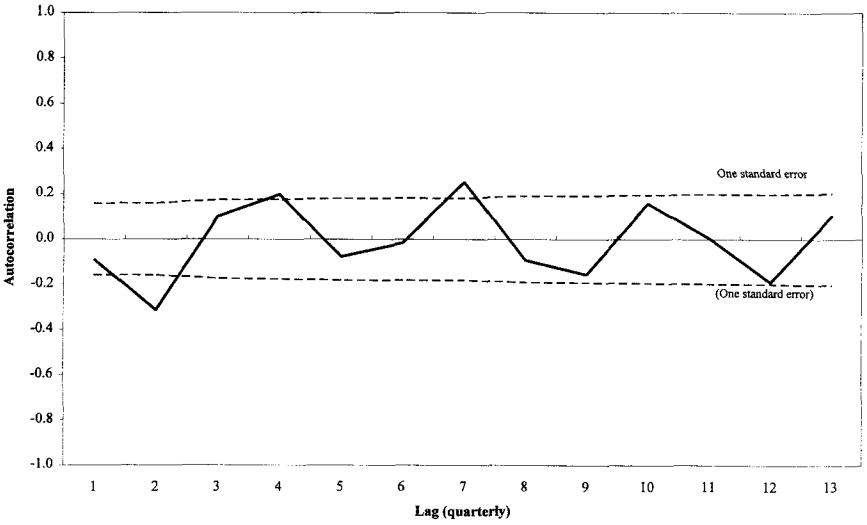

Figure 7.6b. Autocorrelation of residual error under unsmoothed market-related method (quarterly steps 1984 Q1 to 1993 Q4)

7.5.4.17 In our minds these two results are intuitive. There is no reason why all rational investors cannot expect dividend growth during the next time period to be above some subjective long-run average and for them all to be right. On the other hand, the marginal investor cannot (by definition) believe the rate of future dividend growth expectation implied by the market price of equities to be wrong. In other words, we consider that the dynamics of the market-related methodology are rational whereas those of the Day & McKelvey method are not.

7.5.4.18 Given these dynamics, it seems to us unlikely that adoption of the

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method of Day & McKelvey on a *mechanistic basis* will form a reliable platform for establishing investment policy. However, in the next section we demonstrate how even an elementary model of the asset and liability dynamics under the market-related basis leads to a consistent investment framework.

8. Asset and Liability Management Implications of Market-Related Methodologies

... complexity arises not only from the randomness but also the information structure. At each time t *more information becomes available on which future decisions may be based. It is in general not possible to specify optimal asset allocations explicitly more than one step ahead, since the decision will depend on information not yet known.* Smith (1993)

8.1 In this section we assess the potential asset and liability management implications of adopting a market-related valuation basis. One of our main concerns here is to allay fears that such a move will necessarily lead to a reduction in equity exposure of U.K. pension funds. In many ways we hope that the preceding discussion of the smoothing techniques available have already partly addressed this issue. However, it is clear (as also discussed previously) that use of the Day & McKelvey method based on the FT-SE-A All Share yield immediately ascribes matching properties to U.K. equities which may not persist under alternative valuation methods (the fact that they are manifestly not matched is shown in Section 7). It is fully recognised that over the thirty-year period since 1964 this equity preference has been wholly vindicated by returns (*a.k.a.* mismatch profit) achieved on U.K. equities relative to other assets. However, it should be clear from the foregoing that we find the circular argument, which essentially *assumes* U.K. equities to be matched for valuation purposes and then deduces that the matched position is therefore this same asset, to be wholly unsatisfactory and, indeed, undesirable to the extent that it leads to distortion between preferences for other risky assets such as overseas bonds and equities or property; in essence the dynamics of the method do not, in our view, form a rational basis for setting investment policy.

8.2 We therefore use our first-order model of the asset and liability dynamics on our market-related basis to demonstrate, firstly, that based on the more recent behaviour of U.K. equities relative to gilts, the new approach endorses, in a much more objective manner, much of the conventional wisdom regarding equity preference (including many of the assertions of Day & McKelvey concerning preference for equities over bonds in the U.K. at that time). Furthermore, the approach potentially leads to a more objective method of determining and monitoring the appropriate level of exposure to alternative risky assets without the distortions of the Day & McKelvey method. We believe that the methodology follows many of the general elements of the approach suggested by Wise (1984). However, at that time the lack of market information on implied

real rates of return prevented any empirical calculation of the key characteristics of equity-type assets in our model.

8.3 *Simple Matching Results*

8.3.1 For the purposes of our analysis we define an asset allocation A as a first order match for liabilities L if:

$$\text{Var}\ (\dot{\Delta}A\ -\ \dot{\Delta}L)\ =\ 0.$$

Now, using our simple model and assuming $\Delta\bar{r}^*$ and $\Delta\bar{f}^*$ to be independent (a simplifying assumption, but see the tables in ¶B.2 for orthogonality over 1984 Q1 to 1993 Q4):

$$\text{Var}\,(\dot{\Delta}A-\dot{\Delta}L)=(\alpha_A-\alpha_L)^2\,\text{Var}\,(\Delta\bar{r}^*)+(\beta_A-\beta_L)^2\,\text{Var}\,(\Delta\bar{f}^*)+\gamma_A{}^2\,\text{Var}\,(\Delta\bar{m}^*).$$

As shown in Appendix B, the following approximate component variances are obtained from the period 1984 Q1 to 1993 Q4 (we investigate below the effect of changing these assumptions):

Factor change	Quarterly standard deviation (% p.q.)
$\Delta\,\bar{r}^*$	0.25%
$\Delta\,\bar{f}^*$	0.5%
$\Delta\,\bar{m}^*$	0.25%

We now illustrate a number of elementary matching results obtainable from this simple model.

8.3.2 *The conventional gilt/equity balance as schemes mature*

8.3.2.1 We first consider a scheme with index-linked pension increases and hypothesise an (artificial) environment where the real rate of interest can be measured, but investment is restricted to conventional gilts and equities, and, for simplicity, assume that only two gilts are available with durations of 0 ('cash') and 10 ('long gilt') respectively (*duration*, in this context, being defined as $-\partial/\partial i$ log $p(i)$, where $p(i)$ is the price of the asset at interest rate i).

8.3.2.2 Now, since $\beta_L = 0$ (real liabilities only) and using the above statistics, we need to minimise:

$$\text{Var}\ (\dot{\Delta}A\ -\ \dot{\Delta}L)\ =\ (\alpha_A\ -\ \alpha_L)^2\ \sigma^2_{\Delta r}\ +\ \beta^2_A\ \sigma^2_{\Delta f}\ +\ \gamma^2_A\sigma^2_{\Delta m}.$$

8.3.2.3 However, if the asset distribution is limited as described, then taking x_0 and x_1 as the cash and long gilt proportions respectively and x_2 as the equity

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proportion, gives (using the equity coefficients estimated in Appendix B):

$$\text{Minimise } \left[(10x_1 + 16x_2 - \alpha_L)^2 \sigma^2_{\Delta r} + (10x_1 + 4x_2)^2 \sigma^2_{\Delta y} + 16^2 x_2^2 \sigma^2_{\Delta m}\right]$$

subject to $x_0 + x_1 + x_2 = 1$ and $x_i \geq 0$ for all i (assuming no short selling).

8.3.2.4 Without the constraint on short selling this can be solved analytically using the method of Lagrange, as shown in Appendix D. However, it is more convenient to use a standard numerical optimiser with the short selling constraint imposed. Figure 8.1 shows the optimal equity allocation for values of α_L from 40 (very immature scheme) down to 0 (we assume throughout, for convenience, that α_L is stationary over the time period considered).

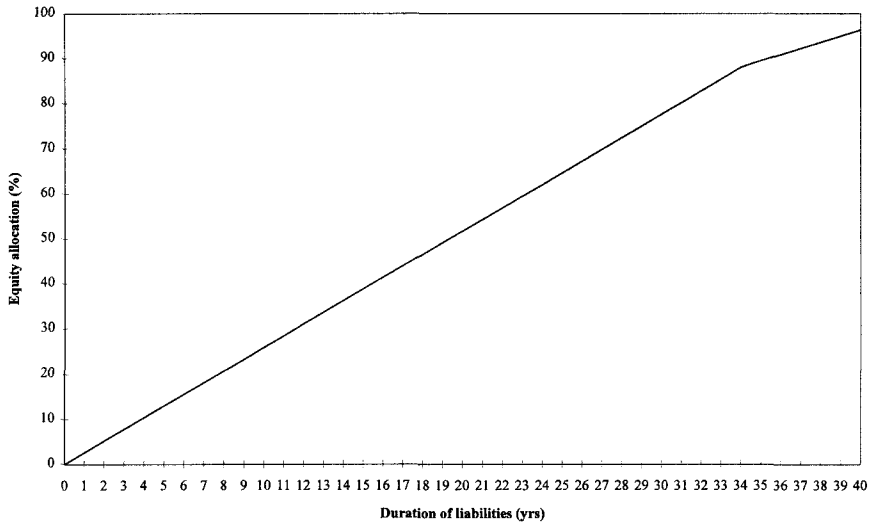

Figure 8.1. Optimal equity allocation according to duration of liabilities in an equity and conventional bond framework (assuming market-related valuation basis)

8.3.2.5 This analysis (based on empirical analysis of equities which has only been possible subsequent to the issue of index-linked gilts) therefore gives a justification on our market-related basis for the assertion of Heywood & Lander and Day & McKelvey that equities were the most consistent match for an immature pension scheme in an equity, cash and conventional gilt environment. However, it should also be stressed that index-linked gilts provide a superior match. Indeed, provided an appropriate duration can be obtained (the longest currently available being 2½%, 2024 and 4⅛%, 2030), they are a first order match under this simple method.

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8.3.2.6 For schemes where pension increases are fixed in payment, similar analysis can be performed, but the conventional gilt yield used to value the future pensions in payment must be based on the forward rates, and realistic results are obtained only if changes in the yield curves are modelled more accurately, rather than assuming only parallel changes.

8.3.3 *Accounting for country differences in preference for equities versus bonds in pension fund asset allocation*

8.3.3.1 A second simple result can be obtained by considering the effect of increasing the volatility of inflation expectations from a low (e.g. Deutschemark block) to high (e.g. U.K. in the 1970s) level. Repeating the analysis in Section 8.3.2 using different standard deviations of Δf^* then gives the surface shown in Figure 8.2, with liability duration and volatility of changes in inflation expectations on separate axes.

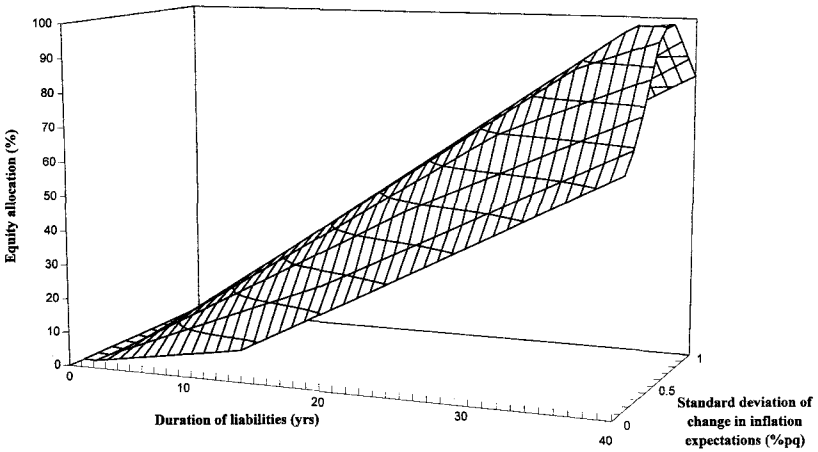

Figure 8.2. Optimal equity allocation according to both duration of liabilities and volatility of changes in inflation expectations in an equity and conventional bond framework (assuming market-related valuation basis)

8.3.3.2 It will be noted that the equity allocation in the best matched portfolio *increases* initially as the variability of inflation expectations rises, but then *falls* back after a maximum is reached. This feature is illustrated by Figure 8.3, which looks at a cross-section of the same graph (at a duration of 20 years). Superimposed on this is also the duration of the residual bond portfolios (the area of sharp decline is the area where the non-numerical solution of Appendix D applies, elsewhere the short selling constraint applies to either cash or bonds).

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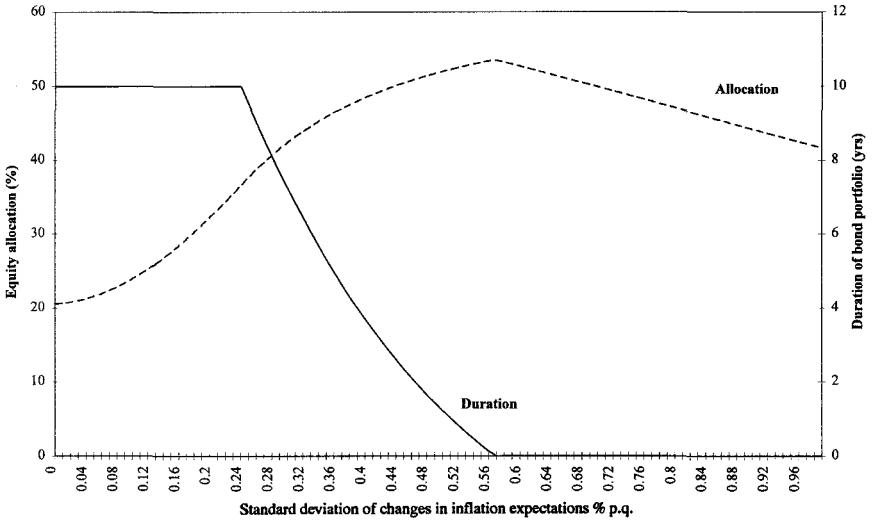

Figure 8.3. Optimal equity allocation and duration of residual bond portfolio according to volatility of changes in inflation expectations in an equity and conventional bond framework (assuming market-related valuation basis)

8.3.3.3 It can therefore be seen further that, as variability of inflation expectations rises initially, the need to retain adequate exposure to real interest variation leads to a straight trade-off between equities and long bonds (the lower the variation the higher the allocation to bonds). However, as the variability of inflation expectation rises further, the variability of real interest rate expectations becomes increasingly insignificant in relative terms, and of more concern is minimising the exposure to changes in inflation expectations (keeping the inflation expectation duration as small as possible). This leads to an increasing preference for cash, firstly as an alternative to long bonds and then, once the standard deviation of inflation expectation variability exceeds around 0.6% per quarter, as an alternative to equities.

8.3.4 *Overseas asset and property exposures*

In theory, the general form of the asset analysis (including for overseas assets an additional term, say Δx_A for changes in exchange rates) can now be extended to other risky assets fitting empirically the equation:

$$\dot{\Delta}A = \delta_A + \hat{\alpha}_A \Delta \bar{r}^* + \hat{\beta}_A \Delta \bar{f}^* + \hat{\gamma}_A \Delta \bar{m}_A^* + \Delta x_A.$$

8.3.4.1 Such analysis provides a natural means of incorporating alternative

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assets into pension fund investment strategies on a rational (if empirical) basis, without the bias towards U.K. equities implicit in the method of Day & McKelvey.

8.3.4.2 Detailed consideration is beyond the scope of this paper. However, it is possible to obtain here an approximate solution to the most efficient domestic and overseas equity proportion which conforms with standard practice of U.K. pension funds, by assuming that the residual non-equity portfolio is arranged to eliminate terms in $\Delta\bar{r}^*$ and Δf^*, leaving the following mismatch equation (where p_{UK} and p_{OS} are the U.K. and overseas allocations respectively):

$$\text{Minimise } \left[p_{UK}^2\,\hat{\gamma}_{UK}^2\,\sigma^2_{\Delta\bar{m}^*_{UK}} + p_{OS}^2\left(\hat{\gamma}_{OS}^2\,\sigma^2_{\Delta\bar{m}^*_{OS}} + \sigma^2_{\Delta x}\right)\right]$$

subject to $p_{UK} + p_{OS} = p$, assuming, for convenience, independence of $\Delta\bar{m}^*_{UK}$, $\Delta\bar{m}^*_{OS}$ and Δx.

This then gives the optimal overseas equity allocation as approximately:

$$\frac{p_{OS}}{p} = \frac{\hat{\gamma}_{UK}^2\,\sigma^2_{\Delta\bar{m}^*_{UK}}}{\hat{\gamma}_{UK}^2\,\sigma^2_{\Delta\bar{m}^*_{UK}} + \hat{\gamma}_{OS}^2\,\sigma^2_{\Delta\bar{m}^*_{OS}} + \sigma^2_{\Delta x}}$$

$$\fallingdotseq \frac{1}{3}\left(\text{since } \hat{\gamma}_{UK}^2\,\sigma^2_{\Delta\bar{m}^*_{UK}} \fallingdotseq \hat{\gamma}_{OS}^2\,\sigma^2_{\Delta\bar{m}^*_{OS}} \fallingdotseq \sigma^2_{\Delta x} \fallingdotseq 16\%^2 pq\right)$$

or the typical ratio of 2:1 (U.K. : overseas) seen in many U.K. pension funds.

8.3.5 *Dynamic matching*

8.3.5.1 Another consequence of the market-related methodology is that the best-matched portfolio becomes a dynamic rather than a static entity. This can be seen, in the very simple example of the matching of a current pension liability with Limited Price Indexation of pension increases in payment (that is retail price inflation linkage capped at 5% p.a.). For this liability the coefficient β_L displays the following step function behaviour with respect to $\bar{f}^*$:

$$\beta_L = 0 \text{ if } \bar{f}^* < 5\% \text{ p.a., or } \beta_L = \alpha_L \text{ if } \bar{f}^* > 5\% \text{ p.a.}$$

8.3.5.2 Accordingly, the best match varies according to whether $\bar{f}^*$ is above or below 5% p.a. Although immediately obvious, this behaviour is not necessarily highlighted by alternative valuation methodologies.

8.3.6 *Derivative markets*

The method is consistent with the pricing of derivatives, and indeed certain market checks can be made on the overall volatilities of the components, (for

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example, the implied volatility of the notional long-gilt future option gives an estimate of Var $(\Delta \bar{r}^* + \Delta f^*)$). This linkage could be strengthened significantly if a market could be made in a notional long index-linked future and option. (This would also facilitate hedging of real interest rate risk exposure under the method).

8.4 *Performance Measurement*

8.4.1 *Monitoring mismatch profit/loss*

8.4.1.1 With the increasing maturity of U.K. pension funds (and with the issue of longer-dated stocks such as index-linked 2½%, 2024 and 4⅛%, 2030) a first-order match exists in many circumstances. For these funds, it is now therefore possible to monitor the performance of the investment *strategy* against the theoretically matched position on the market-related valuation basis.

8.4.1.2 The result of this monitoring can then be equated with the investment profit (or loss) at each valuation, (making no distinction between the income or capital gains) with demographic and real salary growth variations the only remaining sources of fluctuation. This contrasts with monitoring under the method of Day & McKelvey, which tends to focus on the growth of assets relative to the retail prices index, possibly after adjustment to actuarial values, which then ties actuarial returns to U.K. dividend growth. However, this latter approach leads to a comparison of returns from actual investment strategies against those of a hypothetical asset in which a scheme cannot physically invest, so that, although they can be monitored, the trustees cannot really control their exposure to these two factors in any event, and such monitoring is not focusing on controllable risks.

8.4.2 *Monitoring and controlling tactical asset allocation and stock selection risk*

The approach also presents the possibility of other aspects of investment risk (for example, tactical asset allocation and stock selection) being combined into the overall asset and liability management framework. For example, assuming as a first approximation that the active management risk (active variance) is independent of the movement in these benchmark assets relative to the least risk asset allocation (strategic variance) it can be shown that:

$$\text{Total investment variance} = \text{Strategic variance} + \text{Active variance}$$

or, more simply:

$$\text{Total investment risk} = \text{Strategic risk} + \text{Active risk.}$$

The important issue arising from the use of our market-related approach being that strategic risk and active risk can be measured in the same terms over the same time periods and they differ only in relative magnitude.

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9. Criticisms of Market-related Methodology

... index-linked ... now takes the place of Consols in the 19th century, as the basic risk-free long-term security. It does not matter whether all investors hold the stock. All investors are free to hold it, and the quantities in issue, although smaller than shares or conventional fixed-interest stock, are not so trivial that the yield is distorted by scarcity. Wilkie (1993) (*J.I.A.* **120**, 299-300)

9.1 In this section we acknowledge some of the criticisms of our market-related methodology. It will be noted that most of these are practical considerations rather than theoretical.

9.2 The most apposite criticisms of our proposed methodology, in our opinion, are that it relies unduly on the real return implied by asset classes in which pension funds do not generally invest, and that these asset classes are too small to provide a reliable guide to the prospective rates of return embedded in the pricing of other asset classes. As we shall discuss below, such criticism has some merit, although it can be overstated and must be set against our criticisms of the alternative Day & McKelvey methodology discussed earlier in the paper.

9.3 *Market Considerations*

9.3.1 *Market size*

9.3.1.1 It would clearly be preferable to be able to deal with asset classes whose size was such that all pension funds could invest or disinvest without any impact on price. However, when dealing with total assets of nearly £500 billion at the end of 1993, this is not attainable. As discussed earlier, it can be argued that equity valuations have themselves been affected by pension fund demand, but equally in the international market described in Section 2, the demand for U.K. equities from overseas buyers must also be acknowledged. Furthermore, it must be recognised that in the longer term the supply of all assets is influenced by demand, so it can equally be argued that the size of the U.K. equity market relative to the corporate bond market is itself partly influenced by the past preferences of pension funds for equities.

9.3.1.2 Besides, size alone does not immunise markets from (usually with the benefit of hindsight) 'distortions' due to excess demand or supply. For the purposes of our preferred valuation method, of most concern are situations where, for example, an extreme rise in bond prices (fall in bond yields) is not matched by similar rises in other asset prices.

9.3.1.3 The size of the U.K. index-linked gilt market from 1987 to 1993, both in market-value terms and as a percentage of total pension fund assets, together with the size of the total Government funding requirement in each year is as follows (sources: the Central Statistics Office, the London Stock Exchange and

the Bank of England):

	Year ending 31 December						
	1987	1988	1989	1990	1991	1992	1993
Index-linked market value (£bn)	12.7	13.8	15.4	15.1	16.6	21.9	30.7
U.K. pension fund assets (£bn)	227.6	267.4	339.0	302.7	343.7	393.6	491.2
Index-linked market as proportion of U.K. pension assets (%)	5.6	5.2	4.5	5.0	4.8	5.6	6.3
Nominal amount of funding (year ending following April) (£bn)	15.0	2.8	-	2.3	16.7	35.2	51.4

9.3.1.4 It is clear that the index-linked market is dwarfed by the total size of pension assets, but equally, in recent years, particularly, sufficient supply of gilts was available to suggest that a quite significant level of additional demand could have been accommodated. Furthermore, even in lean years there is, in fact, within the net funding requirement shown, at present a roll-over redemption requirement of around £5-10 billion which will presumably be issued in the form most demanded by investors, so the market is also able to adjust gradually even if the public sector is in financial balance.

9.3.1.5 At around £33.6 billion (31 December 1993) the long-dated (over 15 years) conventional gilt market is currently very similar in size to the total index-linked gilt market so similar comments apply.

9.3.2 *Liquidity and volume*

9.3.2.1 The past liquidity of the market is illustrated by a number of statistics provided by the London Stock Exchange, for example the average dealing spreads and normal market size. Volume can also be measured by the average amounts traded each day.

	Liquidity and volume comparison (year ending 31 December 1993)		
	Index-linked gilts	Long-dated conventional gilts	U.K. equities
Annual turnover	220%	640%	70%
Dealing spreads	< 0.1%	< 0.1%	0.2 - 0.6%
Normal market size (average bargain)	£2 m	£2 m	(£54 K)
Stamp duty	-	-	0.5%
Total daily volume (customer)	£0.3bn (0.15)	£0.9bn (0.45)	£2.3bn

Note: Customer volume is estimated *pro rata* from total gilt market statistics

9.3.2.2. By all of these measures, the U.K. gilt market in general and both the index-linked and the long-dated conventional gilt market in particular appear reasonably 'liquid' relative to the equity market, although the volume is low in comparison and the measure of 'normal market size' is difficult to compare directly. However, it is worth stressing that the Bank of England will supply liquidity to market makers, whereas no such facility exists in equity markets. Furthermore, although the current proposals to introduce generalised sale and repurchase ('repo') arrangements may increase volatility, a repo market is also likely to add additional trading and investment activity.

9.3.3 *Ownership*

9.3.3.1 It can also be argued that the pricing of the gilt market relative to the equity market is distorted by the different composition of the participants in the two markets. We are unable to obtain registration details separately for each of the different sectors of the U.K. gilt market. However, at the total market level, the composition of ownership is as follows in the major directly comparable categories (source: The Bank of England and PDFM (1994)):

	Ownership (%)	
	U.K. gilt market 31 March 1993	U.K. equity market 31 December 1993
Pension funds	18.2	34.7
Insurance companies	37.7	16.5
Personal sector	11.9	21.3
Overseas	19.7	12.8
Others (not directly comparable)	12.5	14.7

9.3.3.2 There are clearly differences in the relative weight of both insurance companies and pension funds as well as in the personal sector (we understand (see PDFM (1994)) that a large proportion of the overseas holdings of U.K. gilts is concentrated in the short-dated sector and hence the difference between the overseas holdings of longer-dated gilts and equities may be less pronounced than above). We do not consider that this analysis demonstrates conclusively the likelihood of a material distortion due to ownership differences.

9.3.4 *The inflexibility of capital markets*

9.3.4.1 However, although these past statistics give no real cause for concern, the possibility remains that future behaviour may be changed by a change in pension fund valuation methods. In many ways, there is therefore a parallel here between the uncertainty of outcome when Day & McKelvey first proposed their method. At that time, the move by pension funds into equities occurred quite gradually and there was time for the supply of assets to adjust. Similarly, in our opinion, a gradual move from equities into bonds could be accommodated by a gradual increase in the supply of corporate issues. It should be noted here that, at an aggregate level, a move by pension funds from equities into corporate bonds should be a cost neutral exercise to the extent that any loss of return suffered by pension funds (bonds being too expensive) will be gained by the companies themselves (capital raised cheaply) and vice versa. This can be seen by considering a company which simultaneously issues £1m of corporate debt and buys back £1m of equity, while the pension fund sells a £1m equity portfolio into the market and buys a £1m bond portfolio. Alternatively, the company could implement an overlay by trading index futures (we suggest that companies' reluctance to do so is primarily due to the distortion of pension fund versus company risk perceptions described in Section 5).

9.3.4.2 However, despite their apparent suitability as a form of company debt (assuming company earnings are linked to the real economy as is implicit in the method of Day & McKelvey), corporate index-linked bonds issues are avoided by companies for a number of reasons, which would need to be addressed. Firstly, U.K. taxation rules are not well disposed towards indexation and, for example, immediate tax relief on the indexation of principal would need to be allowed for index-linked bonds to be as attractive as conventional issues. Secondly, there may be a reluctance for companies to accept that they have, in practice, already accepted inflation-linked liabilities through their pension funds and, again at an aggregate level, if companies issued such debt simultaneously with the purchase of a portfolio of the same form of asset by the pension fund, the overall inflation risk exposure would remain constant.

9.3.5 *Overview of market issues*

We cannot provide any certainty that a gradual change towards a U.K. pension fund valuation methodology which hinges on index-linked gilts will not have an impact on investment markets. Nevertheless, we have sought to establish in this section that some such fears may be overstated. To those unconvinced, we believe it would be equally valid to suggest that (notwithstanding the original objectives of their method) any impact will be as much an indictment on the distortion caused by the current widespread application of the method of Day & McKelvey (as described previously) as it is a criticism of our market-related methodology.

10. CONCLUSIONS

When a problem appears too difficult to solve, it is no solution at all to claim that the problem is solved by pointing to all of the efforts that have gone into solving it.

Authanac (Nobel Physics Prize Winner)

10.1 When dealing with assets (and liabilities) of U.K. pension funds of the order of £500 billion we are reluctant to suggest radical changes to a framework which would appear to have operated successfully for over thirty years. We also recognise the importance of professional judgement (which must inevitably be subjective and flexible) in many areas of our work. However, the analysis in this paper does, we believe, raise a number of important issues in connection with the future evolution of this framework.

10.2 *Management Valuations*

10.2.1 As we have stressed repeatedly, a management valuation is ultimately a rather subjective process and we must leave open any conclusions. However, we do feel that, from an investment viewpoint, the current focus on the growth of U.K. equity income in the valuation process is unhelpful, when in reality the important issue is simply the quantum of total return. This, we believe, can lead to missed investment opportunities (in particular a reluctance to hedge actual gains) and distortion of risk perceptions, which may not be universally beneficial.

10.2.2 Furthermore, if it is acknowledged that the method of Day & McKelvey is in part a smoothing process, then other explicit market-related smoothing methods (a simple method having been proposed here) would appear to us to be equally valid. It is acknowledged that any 'realistic' market-related methodology is also likely to include a strong element of subjectivity in the assumed level of excess return and salary growth. However, we would draw particular attention to the fact that, although relatively smooth, the *dynamics* of the funding level under the Day & McKelvey method, when applied mechanically, can be quite perverse since (unlike market-related methodologies) the change over time hinges on a quantity (dividend growth) which can be (at least partly) predicted by modern markets. (That is, its *expectation* at a given point in time, given the information available, is not necessarily equal to its long-run average).

10.2.3 We acknowledge that the method of Day & McKelvey can be, and is, applied less mechanistically by the inclusion of short-term adjustments to the assumed rate of dividend growth. However, whilst we would agree with the rationale behind these, they do either add further elements of subjectivity to the process or, if they adopt implied market estimates, they may lead one to question why the market's valuation was not adopted from the outset.

10.2.4 Even with these modifications, the lack of confidence in any estimate of the long-term rate of dividend growth should, however, be noted.

10.3 *Compliance Valuations*

10.3.1 As a general principle, we consider that, for a compliance valuation, it is desirable for the degree of subjectivity to either be prescribed or kept to a minimum if the valuation is to provide a reliable and transparent basis for comparison. We suggest that great caution should be exercised when *prescribing* the method of Day & McKelvey, owing to the potential investment implications which we have described at length in this paper, since there may be less scope to ignore these implications in the case of some such applications. Furthermore, the method's focus on actual dividend growth (ignoring changes in *expectations*) is far from transparent (and indeed difficult to justify) to third parties.

10.3.2 It must also be acknowledged that, if the method requires a short-term adjustment for rational application as a management valuation technique, then it is somewhat undesirable and inequitable to apply the method mechanistically (and less rationally) in many statutory areas.

10.3.3 Accordingly, we suggest greater focus on market-related methodologies, to the exclusion of the method of Day & McKelvey, in compliance applications in the U.K. This is, in fact, no more that a suggestion that greater attention should be paid to the practice generally accepted throughout Europe and North America, as well as recognising some of the advantages associated with techniques adopted in the supervision of U.K. insurance companies.

10.3.4 It is somewhat ironic that we ignore such approaches for U.K. pension funds when we are almost unique in having a market real interest rate available (through index-linked gilts) on which to base our liability valuations.

10.4 *Capital Markets*

In terms of the macro-economic impact of our conclusions, we emphasise the need for gradual change. In the longer term it should, we believe, be beholden on the *borrowers* of these funds (rather than lenders themselves) to adapt to any changes in this framework, and we should not regard any situation as immutable given the huge flexibility of modern world capital markets. However, as a corollary to this principle, if companies are not able to issue index-linked debt (due to taxation anomalies) they should not, in our opinion, be compelled by Government to 'issue' index-linked pension liabilities, since such promises cannot be efficiently securitised within the corporate sector.

10.5 We look forward to discussing this paper more widely in the hope that it will at least prompt a debate on what we believe to be one of the most important aspects of the public perception of our profession in the 1990s.

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We do however remain ultimately responsible for the content of this paper, and the opinions expressed are not necessarily those of our colleagues or other contributors.

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APPENDIX A

DERIVATION OF A FIRST ORDER DYNAMIC MODEL FOR ASSETS AND LIABILITIES

A.1 *Basic Assumptions and Notation*

A.1.1 For convenience we define an information set I_k to represent all known information at time k, and assume that information which is received during each time period can change expectations about subsequent time periods.

A.1.2 The following parameters are then assumed to be members of I_k and are therefore taken as 'constants' under our convention (the initial period, denoted by the subscript 0, is between time k and time $k+1$)

(1) initial period spot real interest rates (r_0^k);
(2) initial period spot rate of inflation (f_0^k);
(3) initial period spot dividend growth (m_0^k); and
(4) initial period demographics (q_0^k).

A.1.3 The following expected future values for periods commencing at time $(k + 1)$, $(k + 2)$, etc. are variable (the subscript represents the start of each forward period, measured from time k):

(1) forward spot real interest rate expectations (r_1^k, r_2^k, r_3^k, ...);
(2) forward spot inflation expectations (f_1^k, f_2^k, f_3^k, ...);
(3) forward spot real dividend growth expectations (m_1^k, m_2^k, m_3^k, ...); and
(4) demographic expectations (q_1^k, q_2^k, q_3^k, ...).

A.1.4 We thus adopt the convention:

$$\boldsymbol{\delta}^k = \boldsymbol{E}\,(\boldsymbol{\delta} \mid \boldsymbol{I}_k)$$

where δ is a general series of discount rates.

A.2 *Asset Dynamics*

A.2.1 *General*

Without loss of generality, consider at time 0 an elementary stripped security offering a payment p at time $\alpha+1$ (the 'duration' of the security then being α at time $1-$) discounted at $\boldsymbol{\delta}^0$ (so, for example, in the case of a fixed income security $\boldsymbol{\delta}^0 = \boldsymbol{r}^0 + \boldsymbol{f}^0$). Then, if ${}^{\alpha}A^{k}$ is the market value at time k:

$${}^{\alpha}A^{0} = p \exp\left(-\sum_{0}^{\alpha} \delta_n^0\right); \text{ and}$$

$${}^{\alpha}A^{1} = p \exp\left(-\sum_{0}^{\alpha-1} \delta_n^1\right)$$

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and using the notation $\dot{\Delta}\,{}^{\alpha}A = \log {}^{\alpha}A^{1} - \log {}^{\alpha}A^{0}$ we have:

$$\dot{\Delta}\,{}^{\alpha}A = \delta_0^0 - \alpha\Delta^{\alpha}\bar{\delta}^{*}$$

where:
$$\Delta^{\alpha}\bar{\delta}^{*} = \frac{1}{\alpha}\sum_{n=1}^{\alpha}(\delta_{n-1}^{1} - \delta_n^0).$$

A.2.2 The simplifying assumption in this elementary model is now that:

$$\Delta^{\alpha}\bar{r}^{*} \fallingdotseq \Delta\bar{r}^{*} \quad \text{for all } \alpha;$$

$$\Delta^{\alpha}\bar{f}^{*} \fallingdotseq \Delta\bar{f}^{*} \quad \text{for all } \alpha; \text{ and}$$

$$\Delta^{\alpha}\bar{m}^{*} \fallingdotseq \Delta\bar{m}^{*} \quad \text{for all } \alpha.$$

In other words, changes in expectations over the first time period are broadly parallel, and hence, if $\Delta^{\alpha}\bar{\delta}^{*}$ is a linear function of these differences:

$$\Delta^{\alpha}\bar{\delta}^{*} \fallingdotseq \Delta\bar{\delta}^{*} \quad \text{for all } \alpha.$$

A.2.3 Finally, linearising the difference operator ($\dot{\Delta}$) for a series of such elementary securities, we have:

$$\dot{\Delta}A \fallingdotseq \delta_0 - \alpha_A\Delta\bar{\delta}^{*}$$

where α_A is the following weighted average:

$$\alpha_A = \frac{\left(\sum_{\alpha} {}^{\alpha}A^{0} \cdot \alpha\right)}{A^{0}} \text{ and } \delta_0 \in I_0.$$

A.2.4 *Example individual asset classes*

This simple model then gives the following approximate forms of $\Delta\dot{A}$ for sample asset classes (all approximations are given to first order and $\delta_0 \in I_0$ in each case):

(1) *Cash*

The total return on a one period deposit is a constant ($\alpha_c = 0$), therefore:

$$\dot{\Delta}A = \delta_0.$$

(2) *Index-linked gilts*

For index-linked gilts $\boldsymbol{\delta}^{\boldsymbol{k}} = \boldsymbol{r}^{\boldsymbol{k}}$, and hence the total return on index-linked gilts of duration α_i is given by:

$$\dot{\Delta}A \fallingdotseq \delta_0 - \alpha_i\Delta\bar{r}^{*}.$$

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(3) *Fixed-interest gilts*

For fixed-interest gilts $\boldsymbol{\delta}^k = \boldsymbol{r}^k + \boldsymbol{f}^k$, as in the example above, and the total return of current coupon fixed-interest gilts of duration α_g is given by:

$$\dot{\Delta}A \fallingdotseq \delta_0 - \alpha_g \Delta\bar{r}^* - \alpha_g \Delta\bar{f}^* .$$

(4) *Equities*

For equities we have the more complicated general functional form $\boldsymbol{\delta}^n = \boldsymbol{\delta}^n (\boldsymbol{r}^n, \boldsymbol{f}^n, \boldsymbol{m}^n)$. However, linearising, the total return on equities is given by:

$$\dot{\Delta}A \fallingdotseq \delta_0 - \hat{\alpha}_e \Delta\bar{r}^* - \hat{\beta}_e \Delta\bar{f}^* + \hat{\gamma}_e \Delta\bar{m}^*$$

where $\hat{\alpha}_e$, $\hat{\beta}_e$ and $\hat{\gamma}_e$ must be estimated empirically, but it is expected that $\hat{\beta}_e$ is small (if equities are a 'real' asset) and $\hat{\alpha}_e \fallingdotseq \hat{\gamma}_e$. (Appendix B shows the results of ordinary least squares regression of equity price movements to obtain these estimates.)

A.3 *Liability Dynamics*

In valuing the liabilities, we assume, for simplicity, that salary growth is a fixed margin above retail price inflation. With this simplifying assumption, the above approach leads to the general dynamic formula for the liabilities set out in Section 6. That is:

$$\dot{\Delta}L \fallingdotseq \delta_L - \alpha_L \Delta\bar{r}^* - \beta_L \Delta\bar{f}^* + \gamma_L \Delta\bar{q}^* \text{ where } \delta_L \in I_0.$$

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APPENDIX B

EMPIRICAL DERIVATION OF U.K. EQUITY DURATIONS BY ORDINARY LEAST SQUARES REGRESSION

B.1 *Regression Equation*

In this appendix, we set out the results of the following regression:

$$\dot{\Delta}A_n = \zeta\dot{\Delta}D_n + \alpha\Delta\bar{r}_n^* + \beta\Delta\bar{f}_n^* + u_n$$

where:

A_n = FTA All Share Index;

D_n = FTA All Share Dividend Index;

$\bar{r}_n^*$ = gross real redemption yield on IL 2% 2006; and

$\bar{f}_n^*$ = difference between the gross redemption yield on Treasury 8% 2002/06 and $\bar{r}_n^*$,

and the following notation applies:

$\dot{\Delta}X_n$ = log X_n − log X_{n-1}; and

ΔX_n = X_n − X_{n-1}.

B.2 *Orthogonality of Regressors*

The above regressors display the following characteristics:

(a) *Quarterly 1984 Q1 to 1993 Q4*

	Standard deviation	Correlations		
	(% p.q.)	$\dot{\Delta}D_n$	$\Delta\bar{r}_n^*$	$\Delta\bar{f}_n^*$
$\dot{\Delta}D_n$	1.95	1	0.29	0.21
$\Delta\bar{r}_n^*$	0.27	..	1	0.26
$\Delta\bar{f}_n^*$	0.52	..	..	1

(b) *Quarterly 1989 Q1 to 1993 Q4*

	Standard deviation	Correlations		
	(% p.q.)	$\dot{\Delta}D_n$	$\Delta\bar{r}_n^*$	$\Delta\bar{f}_n^*$
$\dot{\Delta}D_n$	2.03	1	0.39	0.42
$\Delta\bar{r}_n^*$	0.26	..	1	0.48
$\Delta\bar{f}_n^*$	0.50	..	..	1

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B.3 *Dummy Variables*

B.3.1 The following three dummy variables were adopted (zero unless otherwise stated):

$CR87_n$ = 0.01 when n = 1987 Q4 (stock market crash);

GWS_n = 0.01 when n = 1990 Q3 (Gulf War start); and

GWF_n = 0.01 when n = 1991 Q1 (Gulf War finish).

B.3.2 The use of dummy variables to remove outliers is necessary for the purposes of obtaining robust estimates of *regression coefficients* and *correlations* (Board & Sutcliffe (1993) provides another example of their use). More sophisticated methodologies can be adopted; for example, the activity rates shown in Figure 2.1 can be used to (inversely) weight the period leading to October 1987 and more realistic ('ramp-shaped') dummy variables can be adopted over a number of preceding periods, demanding that their coefficients sum to zero; likewise, it can be demanded that the coefficients of GWS and GWF sum to zero. However, these sophistications do not materially affect our conclusions.

B.3.3 It is, however, important in prospective modelling to allow for these outliers in a residual error term, either informally (equating the overall volatility of the model with that implied by option prices, for example) or more formally through the functional form of residual term. For convenience, we adopt the former, informal, approach here.

B.4 *Results*

The following results were obtained:

(a) *Quarterly 1984 Q1 to 1993 Q4*

Regressors	Coefficient	T ratio (prob)
$\dot{\Delta}D_n$	1.33 (0.24)	5.5 (0.00)
$\Delta\bar{r}_n^*$	-10.01 (3.03)	-3.3 (0.00)
$\Delta\bar{f}_n^*$	-7.69 (1.53)	-5.0 (0.00)
$CR87_n$	-40.90	
GWS_n	-17.35	
GWP_n	8.26	

Residual standard error = 0.0472 (4.72%).

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(b) *Quarterly 1989 Q1 to 1993 Q4*

Regressors	Coefficient	T ratio (prob)
$\dot{\Delta}D_n$	1.34 (0.28)	4.8 (0.00)
$\Delta\bar{r}_n^*$	-17.36 (2.98)	-5.8 (0.00)
$\Delta\bar{f}_n^*$	-4.61 (1.64)	-2.81 (0.01)
GWS_n	-17.85	
GWP_n	9.81	

Residual standard error = 0.0293 (2.93%).

B.5 *Conclusions*

B.5.1 *Volatilities of real interest rates (and correlations)*

Based on the above, we adopt the following assumptions in the paper:

SD $(\Delta\bar{r}^*)$ = 0.25% p.q.; SD $(\Delta\bar{f}^*)$ = 0.5% p.q.; and Cor $(\Delta\bar{r}^*, \Delta\bar{f}^*)$ = 0.0.

B.5.2 *Empirical real interest rate and inflation expectation durations*

We also adopt the following durations, giving weight to both prior expectations (discussed below) and the more recent data period (b):

$$\hat{\alpha} = 16; \text{ and } \hat{\beta} = 4.$$

As noted in Section 4, the coefficient value $\hat{\beta} = 4$ can be compared with Wilkie's effective inflation lag of around 3.2, while the value $\hat{\alpha} = 16$ can be compared with the implied value of around 22.7 on a par yield of 4.4% (the approximate median value) under the method of Day & McKelvey.

B.5.3 *Analysis of residual error*

Finally, equating u_n with $\hat{\gamma}\Delta\bar{m}_n^*$, we take:

$$\hat{\gamma} = \hat{\alpha} = 16$$

and hence SD $(\Delta\bar{m}_n^*)$ = 0.25% p.a.

B.5.4 *Total variance of model*

Combining the above and assuming $\Delta\bar{r}^*$ and $\Delta\bar{f}^*$ and assuming $\dot{\Delta}D_n$ is known gives:

$$\text{Var } (\dot{\Delta}A_n) = 16^2 \cdot 0.25^2 + 4^2 \cdot 0.5^2 + 16^2 \cdot 0.25^2 = 36\%^2$$

or SD $(\dot{\Delta}A_n) \fallingdotseq 0.75\%$ per day (assuming 64 working days per quarter).

B.5.5 *Expected outcome or 'intercept'*

The hypothesis that $\zeta = 1$ is not rejected by either set of results.

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APPENDIX C

MODEL FUND

C.1 *Assets, Liabilities and Funding Levels*

(a) *Liabilities : base*

Our simple model fund is assumed to be stationary on the following basis:

Investment return: 9% p.a.
Salary inflation: 7% p.a.

The base liabilities (at a constant valuation rate of interest) $\hat{L}_n$ at the end of period n and net benefit payments B_n during period $n - 1$ to n are therefore chain linked as follows:

$$\hat{r}_{n} = (1 + s_n)\, \hat{L}_{n-1}$$

and:

$$B_n = k\, \hat{L}_{n-1}$$

where:

k = 0.02 for annual data; or
= $1.09^{1/4} - 1.07^{1/4} \approx 0.00472$ for quarterly data; and

s_n = national average earnings growth (whole economy CSO Annual Supplement. Quarterly data not seasonally adjusted).

(b) *Liabilities: market-related*

For our market-related basis, the base liabilities are adjusted as follows to obtain approximate market-related liabilities $L_n(\bar{r}^*)$:

$$L_n(\bar{r}^*) = \hat{L}_n \exp\, (\alpha_L\, (\hat{r} - \bar{r}_n^*))$$

where:

$\bar{r}_L^*$ = gross real redemption yield on IL 2% 2006 for 1984 Q1 to 1993 Q4; or
= $\hat{r}$ prior to 1984 Q1;

$\hat{r}$ = median value of $\bar{r}_n^*$ = 0.0385 annual (1984-1993); or
= 0.0386 quarterly (1984 Q1-1993 Q4); and

$\alpha_L \equiv 20$ (we adopt a *stationary* duration coefficient for our model fund, the coefficient value being our estimate for a typical U.K. pension fund in the 1990s).

(c) *Assets : market value*

The market value of assets is built up as follows:

$$A_0 = L_0$$

$$A_n = (1 + \tau_n)\, A_{n-1} - B_n - E_{n-1}/\nu$$

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where:

A_n = market value of assets;
τ_n = total return on FT-A All Share Index;
E_n = past service excess or shortfall (see (e) below); and
ν = spreading period (15 years or 60 quarters).

(d) *Assets: Day & McKelvey valuation*

The actuarial value of the assets $\hat{A}_n$ under the method of Day & McKelvey is calculated as follows:

$$\hat{A}_n = A_n \times \frac{y_n}{\hat{y}}$$

where:

y_n = gross dividend yield on FTA-All Share Index; and
$\hat{y}$ = median value of y_n = 0.0471 annual (1964-1993)
= 0.0438 quarterly (1984 Q1-1993 Q4).

(e) *Surplus and funding levels*

Funding levels are calculated as follows:

$$FL_n = \frac{A_n}{L_n} \times 100 \text{ (Unsmoothed market-related method);}$$

$$\hat{FL}_n^{SM(\lambda)} = \frac{\lambda}{\nu} \left((\nu - 1)\, \hat{FL}_{n-1}^{SM(\lambda)} + 100 \right) + (1-\lambda)\, FL_n$$

(Smoothed market-related method); and

$$\hat{FL}_n^{D\&M} = \frac{\hat{A}_n}{\hat{L}_n} \times 100 \text{ (Day \& McKelvey method);}$$

and hence surplus E_n is defined as:

$$E_n = A_n - L_n \text{ (Unsmoothed market method);}$$

$$E_n^{SM(\lambda)} = \left[\frac{\hat{FL}_n^{SM(\lambda)}}{100} - 1 \right] . L_n \text{ (Smoothed market-related method); and}$$

$$E_n^{D\&M} = \hat{A}_n - \hat{L}_n \text{ (Day \& McKelvey method).}$$

(f) *Initial conditions*

The market value of the fund and the base liability value were set equal as at the *start* of the first quarter of the analysis. For smoothed market-related methods, the smoothed funding level was also set equal to 100% at this

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start time (continuity being achieved in the case of the market-related basis by assuming the index-linked gilt yield to be equal to $\hat{r}$ at the end of 1983 Q4 and all previous periods, as explained above).

C.2 *Real IRR calculations and normalisation of contribution rates*

The real internal rate of return μ is calculated as follows:

$$A_N = A_0' (1 + \mu)^N - \sum_{n=1}^{N} (B_n' + E_{n-1}'/v + hL_n') (1 + \mu)^{N-n}$$

where primes denote indexation to period N by reference to average earnings growth s_n and h is a contribution rate normalisation factor calculated as follows:

$$\sum_{n=0}^{N} \frac{E_{n-1}^{D\&M}}{vL_n} = \sum_{n=0}^{N} \left(\frac{E_{n-1}^{SM(\lambda)}}{vL_n} + h \right)$$

using the superscript D&M to denote the excess or shortfall calculated under the Day & McKelvey method and $SM(\lambda)$ to denote a smoothed market-related method with parameter λ. Therefore, assuming that the salary roll equals the unadjusted liabilities, h represents the contribution rate normalisation to ensure that the mean contribution rate (as a percentage of salaries) over the period is the same under all methods for the internal rate of return calculations.

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APPENDIX D

NON-NUMERICAL SOLUTION TO FIRST ORDER MATCH PROBLEM

D.1 *Definition of Problem*

In this appendix, we derive formally the solution to the following minimisation introduced in Section 8:

$$\text{Minimise} \quad \left[(x_1\alpha_1 + x_2\hat{\alpha}_2 - \alpha_L)^2 \, \sigma^2_{\Delta\bar{r}\cdot} + (x_1\alpha_1 + x_2\hat{\beta}_2)^2 \, \sigma^2_{\Delta\bar{f}\cdot} + x_2^2\hat{\alpha}_2^2\sigma^2_{\Delta\bar{m}\cdot}\right] \quad \text{(D.1)}$$

$$\text{subject to} \qquad x_0 + x_1 + x_2 = 1$$

where:

- x_0 = allocation to cash deposits;
- x_1 = allocation to long gilt portfolio;
- x_2 = allocation to equity portfolio;
- α_1 = duration of long gilt portfolio;
- $\hat{\alpha}_2$ = empirical duration of equity portfolio with respect to real interest rates;
- α_L = duration of liabilities with respect to real interest rates;
- $\hat{\beta}_2$ = empirical duration of equity portfolio with respect to inflation expectations;
- $\sigma^2_{\Delta\bar{r}\cdot}$ = variance of changes in real interest rate expectations;
- $\sigma^2_{\Delta\bar{f}\cdot}$ = variance of changes in inflation expectations; and
- $\sigma^2_{\Delta\bar{m}\cdot}$ = variance of changes in dividend growth expectations.

D.2 *Undetermined Multiplier Method (Lagrange)*

Using ψ as the undetermined multiplier, application of the method of Lagrange, as described by Wise, then gives:

$\psi = 0$ (differentiating equation D.1 with respect to x_0); and hence:

$$\alpha_1 (x_1\alpha_1 + x_2\hat{\alpha}_2 - \alpha_L) \, \sigma^2_{\Delta\bar{r}\cdot} + \alpha_1 (x_1\alpha_1 + x_2\hat{\beta}_2) \, \sigma^2_{\Delta\bar{f}\cdot} = 0; \text{ and}$$

$$\hat{\alpha}_2 (x_1\alpha_1 + x_2\hat{\alpha}_2 - \alpha_L) \, \sigma^2_{\Delta\bar{r}\cdot} + \hat{\beta}_2 (x_1\alpha_1 + x_2\hat{\beta}_2) \, \sigma^2_{\Delta\bar{f}\cdot} + \hat{\alpha}_2^2 x_2 \sigma^2_{\Delta\bar{m}\cdot} = 0.$$

Therefore:

$$\boldsymbol{mx} = \boldsymbol{y} \text{ and hence } \boldsymbol{x} = \boldsymbol{m}^{-1}\boldsymbol{y} \qquad \text{(D.2)}$$

where:

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$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix};$$

$$\boldsymbol{y} = \begin{bmatrix} \alpha_1 \alpha_L \sigma^2_{\Delta \bar{r}.} \\ \hat{\alpha}_2 \alpha_L \sigma^2_{\Delta \bar{r}.} \end{bmatrix}; \text{ and}$$

$$\boldsymbol{m} = \begin{bmatrix} (\alpha_1^2 \sigma^2_{\Delta \bar{r}.} + \alpha_1^2 \sigma^2_{\Delta \bar{f}.}) & (\alpha_1 \hat{\alpha}_2 \sigma^2_{\Delta \bar{r}.} + \alpha_1 \hat{\beta}_2 \sigma^2_{\Delta \bar{f}.}) \\ (\alpha_1 \hat{\alpha}_2 \sigma^2_{\Delta \bar{r}.} + \alpha_1 \hat{\beta}_2 \sigma^2_{\Delta \bar{f}.}) & (\hat{\alpha}_2^2 \sigma^2_{\Delta \bar{r}.} + \hat{\beta}_2^2 \sigma^2_{\Delta \bar{f}.} + \hat{\alpha}_2^2 \sigma^2_{\Delta \bar{m}.}) \end{bmatrix}.$$

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ABSTRACT OF THE DISCUSSION HELD AT STAPLE INN HALL

Mr C. J. Exley, F.I.A. (introducing the paper): Our motivation for embarking on this paper in the autumn of 1993 was that the traditional valuation method under discussion was first proposed over 30 years ago and a reappraisal was long overdue. More specifically, we viewed our concern from two angles.

Looking from the perspective of the valuation process, it seems unreasonable that the return on assets between two valuations could exceed the actuarial assumptions, while long-term interest rates remained stable, and yet a scheme could fall into deficit simply because actual U.K. dividend growth had failed to materialise over a particular period.

On the other side of the coin, the assumption of constant inflation expectations, typically 5% to $5\frac{1}{2}$%, based on long-term historic averages, is questionable for fixed liabilities in a declining inflation environment, and represented, in some instances, a non-trivial market call.

However, from an investment perspective we are primarily concerned about the way that the traditional method seems to thwart attempts at rational quantitative analysis of on-going funding development. In this context, we are concerned that the method being proposed for the calculation of cash equivalents and minimum solvency liabilities has potentially undesirable implications for investment in assets other than U.K. equities, notably international equities and property.

Leading on from these concerns, we reiterate two issues raised in our paper. First, in terms of the actual disclosed valuation result on a particular day, our objection to the traditional method is not necessarily its subjectivity as such. We acknowledge that realistic application of our alternative method probably requires a subjective margin. Rather, we object to the way the traditional method focuses on a subjective assumption for real dividend growth when all the indications are that it is difficult to have any particular confidence in a figure for this. As shown in the paper, overlaying any inflation linkage, there are many sources of variation which combine to make estimates unreliable. Our alternative approach is more straightforward, more transparent, and, in practical application, is really no more subjective than the traditional method.

Secondly, in terms of the actual cost of the pension scheme, it is the return on market values that matters, not actuarial values. This should be reflected in investment and investment strategy. One danger of the traditional approach is that it can obscure this reality. Pursuing this investment issue at a more technical level, we find that the dynamics of the traditional method can be quite perverse. While it is effective at smoothing the funding level, this is achieved only by ignoring new information — that is, changes in expectations. If you are prepared to ignore new information, or place weight on prior expectations, you can just as easily smooth our alternative approach. The benefits of the market-related method is that it avoids the perversity and does not have the same potential to affect investment policy adversely.

We believe that equities are not really a match for pension fund liabilities in the traditional actuarial sense, although we acknowledge that, in practical terms, they are the biggest asset in which U.K. funds can invest, and we would not necessarily argue against high levels of equity investment. By recognising the mis-match involved, trustees should be encouraged to lock-in gains rather than ignore them, and investment markets should adapt gradually to meet any demand for better matched asset classes — a demand which should not, perhaps, be suppressed artificially by valuation methodologies.

Mr G. J. Clark, F.I.A. (opening the discussion): This paper is an important contribution to the actuarial literature on the valuation of U.K. pension fund assets, particularly in the context of the debate that is taking place within the pension fund industry about short-term solvency considerations versus long-term funding ones.

The authors review the changes that have taken place since the early 1960s when Day & McKelvey set out their discounted income method for valuing pension fund assets in the triennial ongoing

valuation (*J.I.A.*, **90**,104). In the authors' opinion, an elegant and simple method is now outdated. The authors draw attention to, among other factors, the current size of pension fund assets, debates as to their adequacy to meet the pension promises, the introduction of index-linked securities and the wealth of information now available in investment markets.

In Section 3 the authors suggest a number of important attributes for an asset valuation technique: smoothness and realism or prudence (depending on the context) are familiar. What is required is a method which removes short-term market sentiment and pitches the neutral asset valuation at a level consistent with the valuation of the liabilities.

The authors consider the difficulty of making an objective assessment of the level of future dividend growth based on past experience, and hence in deriving an appropriate par yield assumption under the method of Day & McKelvey. This difficulty will not be lost on those who, for example, have sought to quantify and explain the impact of the 1993 ACT changes, in the context of a valuation approach adopting a notional U.K. equity portfolio. Some might wish to take issue with the suggestion, in ¶4.6.2, that compliance valuations might, at the current time, be based on a real return of no more than 4% p.a.

In Section 5.4 the authors express their concern that the Day & McKelvey method may distort the perceived risk associated with various asset classes, and, therefore, that it may be driving investment strategy. This is a point to which the authors return elsewhere in the paper. I believe that the practice may be somewhat different. Often the sponsor or the trustees set out to adopt an investment strategy which maximises the return whilst stabilising the funding level, but how do they set about doing that in practice? The money may be given to an investment manager to be managed so as to *maximise the return, subject to an acceptable degree of risk*. However, I have not heard of an investment manager, in that situation, asking the actuary how the assets are to be valued before making an asset allocation decision.

Alternatively an asset and liability study might be carried out in order to design a customised benchmark to achieve the objectives. Those providing advice in this area are, I think, familiar with the possible distortion, and would address that issue. Perhaps there is a distortion of the perceived risk, but I am not sure anyone is being fooled. On the other hand, for a long-term fund there is a risk associated with *not* investing in equities. Concentration on market values, I fear, may understate this!

The potential anomalies of applying a discounted income approach to each asset class are illustrated in ¶5.5.2.3. While switching the investment strategy immediately prior to a management valuation would serve little purpose (since there is no one to delude), in the context of a compliance valuation there are real dangers (or opportunities). This aspect will be familiar to those who have considered the attraction of index-linked investments as a means of extracting surplus under the Finance Act 1986 regulations. We must be careful of the impact of prescription in legislation in this area.

In Section 6 the authors outline their alternative market-related methodology. Under their proposed approach, the liabilities are valued by reference to the spot interest rates obtained from the yield curves of nominal and index-linked gilt-edged securities. The assets are taken at market value. I have not had the opportunity to examine the historical pattern of the rates defined by these yield curves; but one area of interest must be the stability, and indeed the credibility, of these estimates at high durations. This would be particularly true at times of high nominal interest rates, when the profile of the cash flows, weighted by present value from index-linked and fixed-interest, will be very different.

The topics raised in Section 6.3 require further investigations. The idea, in ¶6.3.1(1), that salary increases could be implicitly franked against expected excess equity returns in compliance valuations, could give rise to the same types of anomalies that the authors caution against when the discounted income approach is prescribed in such circumstances. Under the second method, whereby subjective explicit assumptions are made for salary increases and the excess return on equities, should one use long-term assumptions or those underlying market valuations?

Section 7 seeks to compare the market-related method with that of Day & McKelvey. In ¶7.2.1 (and again in ¶7.5.2.1) the authors make reference to the problems associated with back testing any quantitative investment techniques. They introduce a smoothing term to the funding level to deal explicitly with volatility, commenting that this may be necessary for practical applications. Figures 7.3a and 7.3b illustrate the impact on funding levels of various degrees of smoothing. The graphs

raise questions: a parameter of λ=0.788 is demonstrated to produce the same degree of smoothing as the Day & McKelvey method, but the authors need to place a lot of weight on the average historic funding levels in order to achieve a similar degree of smoothing. With λ=0.788, nearly half of the information on funding levels is three or more years out of date.

In Section 8 the authors look at a number of practical applications of their model in different situations. They consider the optimum allocation between cash, equities and bonds as a match for index-linked liabilities of increasing duration, and are able to reproduce the 2:1 ratio, U.K.:overseas, seen in many U.K. pension funds. The analysis includes some significant assumptions, and it would be interesting to expand the analysis to test the robustness of the results to variations in these assumptions. In particular, the algebra assumes that no smoothing is applied. If a smoothing coefficient were to be used for management valuations, as suggested, then how different would the appropriate policy be? The inclusion of commutations, LPI linked benefits, non-increasing or fixed-increasing benefits (such as GMPs) and long periods of deferment (making matching impossible) leads to the conclusion that index-linked gilts are by no means the perfect match for liabilities, as is implicitly assumed in this analysis.

Section 9 looks at the wisdom of moving to a valuation basis which is based on a matched index-linked holding; a market currently totalling some £30bn in comparison with pension fund assets of £500bn. A useful summary of data on the size and liquidity of the index-linked market is provided.

Section 9.3.4 seeks to view the pension fund as part of the larger corporate entity which is acting as sponsor. A corporation is both a supplier and consumer of risk capital. If risks can be aggregated and viewed at this level, then this may be to the greater benefit of the corporate whole. Attention is drawn to the currently unfavourable tax treatment of index-linked corporate debt *vis-à-vis* conventional fixed-interest stock.

In concluding, the authors return to the distinction between management and compliance valuations, first drawn in Section 3. The management valuation determines the long-term position; the compliance valuation is required by statute to determine over or under funding. I believe that, in the future, these will come together. The proposed statutory minimum funding standard raises real and important management issues regarding contribution rates and publicly disclosed funding levels.

There is more information in the current investment market levels than is currently being made use of in our valuation basis. This should be taken into account, but with care.

This paper concentrates on trying to obtain a more accurate snapshot of the funding level. The Day & McKelvey method has served its term. This paper provides several valuable pointers for a credible replacement.

Finally, we must not allow ourselves to become so mesmerised by market information that we exclude one very valuable source of information — the professional judgement of the actuary.

Mr R. B. Colbran, F.I.A.: The authors go back to the origins of the discounted income method, to papers written in a climate when fixed-interest investment was the norm and to the paper by Day & McKelvey, which was the first to address the valuation of equities seriously. That paper had a tentative feel to it, with a few alternative methods also put forward; yet the discounted income method came into general use.

One weakness of the discounted income method not covered by the authors is its treatment of new money. Gilley & Funnell dealt with that in their paper (*J.S.S.*, **15**, 43), and it was touched on in Thornton & Wilson's paper (*J.I.A.*, **119**, 229). I have long thought that the treatment of new money by the discounted income method makes the assumption that the market will rapidly come into line with the asset valuation basis. If there is any other assumption, it is not normally stated.

The authors consider some alternative methods. The formulae used in those methods adjust equities against other markets levels, on the assumption of some underlying relationship on relative valuations. However, we are dealing with international markets, not markets used only by U.K. pension schemes. I believe that they are unlikely to have a logical pattern to them sufficient to form a reliable scientific base.

The authors also refer to the use of judgement. The search for a scientific basis for asset valuation

may be like the search for the philosopher's stone. Nevertheless, in the course of searching, useful discoveries may be made which will lead to improved methods.

Mr R. F. Russell, F.I.A.: The authors draw attention, in Section 5, to the difficulties of valuing overseas equities using the discounted income approach. Given that up to 25% of a U.K. fund may be invested in such assets, this is a material disadvantage. One approach of valuing these assets involves assuming reinvestment in U.K. equities according to a model portfolio. This is unsatisfactory. Taking a long-term view, this may not be particularly significant, but, in terms of what the authors refer to as the 'dynamics' of their valuation method, it is relevant.

My other point relates to smoothing. One of the advantages of the asset-related method put forward by the authors is that it allows the actuary to control the degree of smoothing. A discounted income approach does not. To this extent, the authors' method is more flexible.

Mr R. C. Urwin, F.I.A.: As an investment actuary, I am considerably in sympathy with this paper. It has quite a radical conclusion which is iconoclastic to current actuarial practice. It is, perhaps, easier for those who do not focus on actuarial valuations, but on assets, to recognise that there are significant benefits in the approach put forward. I agree that there should be more integrated asset liability management; given that funding policy and investment policy are intertwined, they should clearly be looked at together.

In asset liability studies, I am concerned by the lack of transparency in actuarial valuation methodology. This has brought about a compromise in asset liability modelling between actuarial reality and economic reality, in that we usually model economic reality for a period of time and then introduce the actuarial reality at a horizon date.

There are two main difficulties. First, there is the question of what assessed value model to use to get the actuary's view of the future balance sheet. The benchmark or model asset allocation approach does very well in this regard, particularly if you use some form of discounted income approach for all the assets. Pension fund investment has taken very well to the introduction of fund-specific customised benchmarks. Problems still relate to the way that actuaries continue to value funds on artificial bases like the 100% U.K. equity model. Clearly they should not do that any more.

The second problem comes in setting the actuarial valuation model at the horizon time. The most obvious position to take is one in which the same basis is used as the current basis. However, this, as the authors point out, is very suspect to changes in expectations for interest rates and for inflation rates. The authors' approach on this definitely gives greater transparency. That is welcome, but the practical consequences are not clear. You could argue that there is a topsy turvy argument here. The authors stress the benefits of a market-related approach, but it confers an importance on one asset class — index-linked gilts — and it is a very substantial importance which is not credible, given that asset class's status. Do we really want to make index-linked gilts so central to the methodology, when currently only a small fraction of the assets of most funds are given over to that asset class, and, indeed, it is very unlikely that that will change, at least for the foreseeable future? This is a matter of perception more than reality, but, on balance, I am still happy to work with discounted income benchmarks, although these have to take on a greater level of transparency. Perhaps we can have these benchmarks covering other asset classes as well as U.K. equities and U.K. gilts. This would address the current bias that we see against international equities, an issue both for on-going funding levels as well as for minimum solvency.

Market consensus information on dividend income in the short term can be built explicitly into these models. In many areas this should be done. In addition, the overall discount rate might be adjusted properly to relate more accurately to the actual benchmark.

Those points would all help investment policy to be shaped properly by actuarial methodology and to bring about more consistency with the funding strategies. In summary, the index-linked gilt as a long-term illiquid tap stock, not perfectly matched to U.K. liabilities, does not really get us off the hook.

Mr A. J. Wise, F.I.A.: The authors have made an impressive analysis of pension fund asset

valuation, and invite us to debate the proposition that an actuarial value which is not a market value is not a proper value.

Starting with investment policy, it becomes ever more clear that this must be determined by the trustees acting on appropriate advice. The authors raise some potential criticisms of actuarial advice in this area, which have already been noted in the discussion, and I agree with them. However, the reference in ¶9.3.5 to "the distortion caused by the current widespread application of the method of Day & McKelvey" is very wide of the mark. The implication that the bulk of asset liability work is unsoundly based is unwarranted and wrong.

In Section 8 the authors describe their own resolution of these points, and they kindly acknowledge that their methodology follows elements of my own work which dates back to 1984. However, they refer to the wrong paper. My 1987 paper on portfolio selection and matching described a mathematical approach which was carefully evolved into the asset liability modelling system which my firm uses today. The 1987 paper concentrated on the principle of mis-matching in the determination of investment policy; in other words, balancing risk against expected return. The authors seem to miss this principle when they outline their own approach to the matter.

Referring now to the actuarial valuation of assets, a pension fund valuation is all about asset and liability cash flow projections, in which the assumed rate of return on future new investment is the discount rate. Discounted values of future cash flows are mainly to do with presentation, and are secondary to the primary concept of cash flows.

The authors give a new definition of liability valuation, based on discounting at forward implied inflation and interest rates, but they have not established a sound conceptual framework for doing so. Their explanation appears to be given in terms of a risk-free discount rate, but, in the real world, pension fund liabilities are not simply correlated with inflation, and there is no risk-free discount rate.

The paper skimps on discussion of the discount rate, which features in determining the authors' parameter u, in ¶6.3.2. Even if the authors do not have in mind an actuarial definition of the discount rate, they would agree that the risk premium u should depend on the nature of the liabilities. However, how do they choose their discount rate, what does it really represent, and is their choice any less subjective than the established actuarial approach?

The paper is out of balance when the discount rate is conveniently skipped over, but the actuary's dividend growth assumption is called unreliable. The important point that pension actuaries should, and do, emphasise is that these are two among several uncertain, but correlated, financial factors.

Do I think — as the authors seem to suspect — that my discounted value of assets is some sort of superior smoothed market value? When I consider that I have nothing to add to the combined knowledge of the market, then my assumptions will be strongly influenced by market valuation. When I consider that the current market is particularly influenced by investors with short time horizons, then my assumptions will be more influenced by long-term economic factors. I am justified in taking such views from time to time, for reasons of long practical experience and investment analysis. For example, in Wilkie's paper to the 1993 AFIR conference he asks "Can dividend yields predict share price changes?" Long live subjectivity within professional work! Nevertheless, the authors make another good point when they discuss compliance valuations, where there is no room for subjectivity.

An actuarial value which is not a market value is not a proper value, but converting discounted asset and liability values to market values is fine so long as both are converted and the funding ratio remains unaltered. Some valuations are done this way. Similarly, it would be equally acceptable to express values in roubles if that made the presentation any better, but the advice will be the same.

The authors have produced an audacious paper, which contains a wealth of interesting new material. They offer a unified approach to actuarial valuation and investment policy, which I must applaud. However, they miss the full significance of mis-matching and of the choice of discount rate. It is a pity that they dwell so much on a paper of so long ago — albeit an important one. Consulting actuaries have not all been frozen in 1963; indeed they have resolved most, if not all, of the issues which are highlighted by this paper, without abandoning important actuarial principles.

Mr S. J. Green, F.I.A.: In ¶2.5 the authors criticise Day & McKelvey for assuming that the reverse yield gap might not be a permanent feature. In an era of low inflation, statutory minimum solvency

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levels incorporating high fixed-interest content and possible punitive taxation under a new administration — the old yield gap may well reappear. It is imprudent to ignore the possibility.

In ¶2.12 the authors also criticise Day & McKelvey for supposing that negative inflation could recur. There are good historic, demographic and technical reasons for believing that it will. In the last few years we have already seen negative inflation for two periods totalling more than a year between them. It would be a foolish actuary who, when valuing a pension scheme with young active members, ignored the possibility of negative inflation in their lifetime.

While on the subject of foolishness, could I caution all actuaries that they should spend a minimum of six months investing or trading in derivatives before waxing too lyrical about them. It is true that almost all paper assets can be replicated by a derivative or a combination of derivatives; but derivatives invariably add additional counterparty risk, which is not present in the assets which they are mimicking. Furthermore, most textbooks on the subject of derivatives — written as they are by financial economists who either ignore expenses or treat them as noise — gloss over dealing costs. Also, unless the timing is immaculate, derivatives usually have to be rolled over. I thus state as 'Green's Law': "when rolling over a derivative, the expenses are always much higher than one expects, and the price is always moved against the investor".

In ¶5.4.3.4. the authors have missed the point about September 1987. The main reason that trustees and their managers did not sell equities at that time had nothing to do with the valuation bases. Managers were afraid that, if they sold, but the market held up for only three more months, their performance would be affected. The consultants would naturally point out that their bad performance was entirely due to their being underweight in equities. The trustees would sack the managers and, of course, the consultant would be delighted to help them set up a beauty parade for a 'small fee'.

Incidentally, what new piece of information became known on 19 October 1987 which was not known three days before? It must have been very significant to lead to falls of more than 25% in a few days, and that is the answer to the authors' point in ¶6.5.1.2, where they say that the market provides rational prospective expectations. The authors also say that the market uses best forecasts available at the time, and they are implicit in market pricing of assets. However, are they the best forecasts? Just over a year ago, fifty leading currency specialists were invited to forecast the $/yen and $/DM rates one quarter ahead. Three months later the closest estimate was out by a mere 7.3%. Not very surprising; but what might surprise those actuaries who place so much store on financial economists and modern techniques is that these top financial economists, using the most modern techniques, all made the same error, and all 100 forecasts overrated the strength of the dollar. All the errors were on the same side, and the market, of course, reflected these erroneous forecasts.

The authors recognise that marketability is an essential part of their methodology. Unfortunately they do not realise that, for all but the tiniest pension fund, the average market size of £54,000, given in ¶9.3.2.1, is totally inadequate. In ¶2.21 they say that "the existence of marginal activity simply reflects that those who regard the price as too high are balanced by those who regard it as too low". This is not true. Many who regard it as too high cannot sell because they cannot find buyers for enough of their stock at prices which are *too high,* but only at prices which they themselves would consider attractive; that is that the market price is not a true reflection of the market. Even if the market was invariably accurate, the market is forecasting over a different time scale to the actuary. Not even the most academic of economists pretends that the market is forecasting more than two years ahead. Do the authors suggest that actuaries should base their valuations on 18-month forecasts?

In ¶8.3.3.2 the optimal ratio of 2:1 for domestic equities to overseas equities is quoted. The mathematics is missing, but the ratio seems to have been obtained irrespective of the value of Δx — does the same ratio apply in the U.S.A., Japan, Germany and Mexico? If so, the authors should look again at their assumptions, or their mathematics, or both.

Since the authors state that their methodology hinges on index-linked gilts, they should note that it is not possible to immunise index-linked pensions with index-linked securities. This is a mistake which a number of consultants make. It would not be possible, even if there were more than £30bn of index-linked gilts and even if there were a full range of maturities. Index-linked securities are only truly index-linked if two conditions are fulfilled:

(1) they are purchased at par at issue or later at their index-linked equivalent value; and

(2) they are held to maturity or sold at the index-linked equivalent of par — otherwise they are not index-linked.

Those who believe otherwise are invited to read the prospectuses and then go back and re-read their compound interest textbooks. Also, index-linked interest payments may rise or fall, but occupational pension payments are not allowed to fall. Within the last two years we have already had at least one occasion where one half-yearly payment on an index-linked gilt was less than the previous payment!

Mr Geoffrey Heywood, M.B.E., F.F.A., F.I.A.: I am a great supporter of the discounted cash flow method and must counteract the remarks of Mr Colbran. I must also correct his statement that, when the method emerged in the early 1960s, pension funds were invested mainly in gilts. This is not true; the paper actually says that in 1964 some 45% of pension fund assets were invested in equities.

It might be appropriate to go back a little further than the 1960s. In the paper which Max Lander & I submitted in 1961 (*J.I.A.***87**,314), we quoted from two earlier papers which are at the heart of this problem. As long ago as 1925, C.R.V. Coutts, in his paper on life office investments (*J.I.A.***56**,121) — and I emphasise life office investments, not pension fund investments — said, "the adequacy therefore of the assets to meet these liabilities does not depend on their realisable capital value at the time the balance is struck. It is the interest earning power of the assets which determines their value for this purpose. It follows therefore from this argument that the proper method of valuing the assets in a life assurance balance sheet as far as they represent deferred liabilities calculated on an interest basis is to value also these assets on an interest basis". That was 70 years ago.

The other quotation was at the Faculty, where C.M. Douglas, in the discussion on a paper by A.C. Murray (*T.F.A.***13**,120) in 1930, said, "To the one side of liabilities there was directed the most devoted care and attention. In contrast to this the asset side appeared to me at that time to receive an almost casual regard and when it came to placing a value on these assets from year to year, the middle market price was chosen in the sort of indifferent air that one price was as good as another."

The consistent treatment of liabilities and assets is the fundamental concept of the method we are now discussing, and, together with it, the overthrow of the old conventional method of taking the lesser of book value and market value. Book values value individual investments according to the date of their purchase, whereas looking forward, they are all of equal actuarial value. Market values can fluctuate very much from day to day — in Japan today the equity market fell by as much as 4% in one session. Therefore, market values cannot be regarded as a reliable basis for valuing assets.

As a result, the move to the Day & McKelvey method in the 1960s was a step in the right direction, but the fundamental problem was how to determine the three parameters — the rate of interest, the rate of inflation and the rate of dividend increase — all on a long-term basis. More sophisticated methods of doing this have been developed during the last 30 years, and this paper endeavours to take the process considerably further. It seeks to show that the use of what is called 'market methodology' is better than results obtained by the actuary using subjective methods. This is very much debatable.

I disagree with the authors where they say that the Day & McKelvey paper was responsible, to some extent, for the increase in the proportion of equities held in pension funds during the last 30 years. The reason for the increase is the realisation by trustees that, in order to meet the long-term liabilities of a pension fund, of all the available investment classes, ordinary shares and property were the best and the most remunerative to meet their liabilities.

In their conclusion the authors, while naturally supporting their own methods, do not rule out Day & McKelvey. Over the next 5-10 years it will be interesting to see which of all these methods becomes the one most universally adopted. Whichever it may be, I am convinced it will be based on the fundamental concept outlined at the beginning of my remarks.

A final thought; while I appreciate that life office valuations are essentially different from pension fund valuations, is there any possibility that, at some time in the future, the proposed method, perhaps adapted in some way, might also be used in this area?

Professor A. D. Wilkie, F.F.A., F.I.A.: I congratulate the authors on some of their pertinent remarks,

such as those in ¶¶ 4.3.1 to 4.3.5, about the factors that influence dividend growth, although I cannot agree with their conclusion, in ¶4.3.6, that "the level of confidence in any estimate of m is likely to be very low indeed". I agree that the range of possibilities of estimates of m — that is the rate of dividend growth — may be uncomfortably wide, but, as I discussed in Section 9 of my recent paper, "The Risk Premium on Ordinary Shares" (*B.A.J.*,**1**,251) the plausible range is from about -1% to +2%. This may be uncomfortably wide in relation to the valuation of a pension fund, but this range does, at least, exclude a large number of other wholly implausible values for m.

One begins to appreciate the authors' fundamental approach only obliquely, through certain remarks made along the way. It is not until we come to the analysis in Appendix B that we see their real position. They appear to believe in the random walk model for share price changes, or at least that the market is always efficient in making the best estimate of the future at any time. Consider their remarks in ¶6.5.1.2: "We assume that the market gives rational prospective expectations at any point in time" and also in ¶2.24 ("the need for the actuarial profession to take a position at odds with the market has also declined"), in ¶4.1. ("How justified is the actuary in departing from the implicit values of these parameters suggested by investment markets?") and in ¶4.5.12 ("this introduces a strong element of subjectivity into this process which may be at odds with the consensus defined by the market"). This approach underlies the authors' statistical investigation in Appendix B. In the first equation in Section B.1: $\dot{\Delta}A$ — that is the change in the logarithm of the price over the quarter — is shown as equal to $\zeta\dot{\Delta}D$ plus something else. $\dot{\Delta}$ means difference in logarithms, and since the dividend yield is D divided by A, if ζ were to be put equal to unity we could bring both these terms to the same side to put ΔY, the change in the yield is equal to roughly the same something, but with the signs reversed.

What the authors make the yield depend on is the change in the yield on an index-linked stock, and the change in the differential between index-linked and fixed-interest — in a sense another type of yield. However, all these yields are stationary series, which means that they wander up and down around some fixed level, tracking back towards that level when they get too far away from it. Taking differences of such a series produces a series that appears to be more random, especially when there are not many observations. In this context, the authors' 40 or 60 quarterly observations are rather few. Thus, the authors make the assumption that successive changes are independent, and then transform the data in such a way that their point will be proved by the transformed data.

The coefficient of $\dot{\Delta}D$ in both regressions is about 1.33, implying that, if dividends go up by 10%, share prices go up by 13.3%. It so happens that over their observation period dividend yields fell, so share prices did, indeed, rise by more than share dividends, but they cannot go on doing this. I suppose that the authors would claim that this was the result of rational expectations in the market. I would say that it was because of irrational expectations at the end of 1993, when dividend yields were too low, and should have been expected to rise to more like their average level, which is precisely what they have done over the past year.

Now, in fact, regressing log yield on log yield in the previous quarter and the yield on index-linked and the implied inflation rate, produces a residual standard deviation of 4.25% without using any intervention variables at all. This is lower than the authors' 4.72%, so they have not even got the best answer.

In Appendix D, 'Non-Numerical Solution' is a reasonable title; x_0 does not appear in what is to be minimised in formula (D.1), so the constraint that $x_0+x_1+x_2=1$ is irrelevant, since the values of all the xs are unconstrained in this part. It is, therefore, quite unnecessary to use the Lagrange multiplier method, and all that the authors needed to do was to differentiate formula (D.1) with respect to x_1 and x_2, giving the results which the authors then correctly derive in the latter half of Section D.2.

The way in which the authors separate out the variances of different factors in their minimisation methodology in Section 8 is informative. If they had not done this, the minimum solution could readily be found using the methods described in Appendix C of the recent paper on 'Capital Projects' (*B.A.J.***1**,155), but to re-express the variance in terms of separate factors, analogous to the factors of the Arbitrage Pricing Model, is useful. However, as has been noted, the authors are seeking the minimum variance portfolio, regardless of expected return, rather than seeking the set of efficient portfolios that give a trade-off between variance and return.

Their main objective is to criticise the Day & McKelvey method, but one can develop a statistical justification of the Day & McKelvey method by arguing that the 'par yield' to be used is an estimate of the long-run average yield, consistent with the autoregressive model for the dividend yield on shares, or, for that matter, for the long-term real rate of interest. Using this methodology, the actuary is, indeed, saying that the market has temporarily got it wrong, but that, in due course, it will get it right, especially by the time the liabilities fall due, and that this estimate of the value is the best estimate, conditional on information to-date, of the value, after adjusting for dividends, that the assets will have by the time they have to be sold.

The authors are quite right to point out, in ¶7.2.1, the problems of 'data snooping', by which they mean assuming that you knew the average over some observation period at all times during that period, instead of just at the end. I do not think it is actually so serious as they make out, because we have got a much longer run of information than just the 15 years they have used, including 70 years of data on share dividend yields, and 2,000 years of data on interest rates that one can use. Nevertheless, I quite like their way of continuously updating their estimate of the average by using an exponential smoothing method, and this is something that requires more consideration.

Mr S. J. B. Mehta, F.I.A.: The potential contribution of this paper to pension fund asset/liability management should not be underestimated.

First of all, the authors propose to revert to the market value based approach to pension fund asset valuation, typically used by actuaries up to the early 1980s, and used as a standard in other areas of actuarial work, including life office and property/casualty work. I hope that the many arguments presented will persuade pension fund actuaries to switch away from the use of notional asset values. Market value related approaches are used outside the actuarial profession, for example by financial economists and researchers and by the investment banks. It is desirable to adopt common approaches and to work jointly with these other professions.

Just as importantly, on the subject of valuing the liabilities, a market value based method is suggested. Liability cash flows are projected, allowing for the rate of inflation implicit in the relationship between the yields on index-linked and conventional gilts. There are, of course, other approaches to the selection of an inflation assumption, and, therefore, the methodology proposed does not depend on the existence of an index-linked market. These liability cash flows are then discounted, using a discount rate which corresponds to the riskiness of the liability cash flows, based on the yield of government bonds. A similar methodology, with a few refinements, was suggested in my March 1992 Institute paper (*J.I.A.***119**,385), in relation to the valuation of life office liability cash flows.

Mr D. J. Parsons, F.I.A.: The market-related methodology proposed by the authors results in market adjustments being applied to liabilities rather than to assets. This means that the disclosed values of liabilities, as well as asset values, are volatile.

At present, most pension fund actuaries are able to present a fairly smooth progression of liability values from year to year, and, because of the method we use to value the assets, there is also a relatively smooth progression of asset values. Our clients have come to expect this.

There are presentational difficulties when using a market value method. For example, if a valuation surplus derived from a market-related method is used to provide benefit improvements which take effect 12 months later, the cost of the improvements identified at the valuation date could be £1,000,000; this figure is equally likely to have moved to £800,000 or to £1,200,000 at the date of implementation. We can justify and explain this very easily, but to a layman — and many of the trustees that we talk to are laymen — this perversity looks suspiciously like actuarial sleight of hand.

If the method is applied, it is likely only to be to larger pension schemes with financially sophisticated sponsoring employers. These probably comprise considerably fewer than 5% of the total number of pension schemes in the U.K, whatever the weight of assets or membership may be. The actuaries dealing with the other 95% of schemes, which have minimal amounts of accrued index-linked liabilities, will probably not have the time or the desire to consider changing their current approaches. The method appears, therefore, to be of minority interest. I can guess the reaction of some

clients if I tried to introduce it for them. It is likely to be along the lines "If you do use this method, will my scheme be less likely to fail the Minimum Solvency Requirement test?"

Mr P. N. Thornton, F.I.A.: We should welcome any paper which challenges accepted wisdom and proposes alternatives for us to consider seriously. The discounted income method has stood the test of time — not without changes, but for good reasons. Among those changes, it is now quite common to adjust for short-term dividend growth expectations, and these can be based on market indications. I do not see that that is any worse than making assumptions about the excess of equity returns over index-linked gilt returns, as suggested in ¶6.3.1.

The income-based approach to valuation is fundamentally closer to the real situation of an on-going pension scheme that we are trying to model. There have been quite a lot of developments in the Day & McKelvey method — for example, examining how to extend the principles to overseas equity markets as well as to U.K. markets. As other speakers have commented, it is dangerous to base too much on the index-linked gilts market, even if the index-linked gilts market becomes much more substantial, because the market conditions at any particular date would still reflect short-term market factors, which might not be at all relevant to the on-going pension fund.

However, I agree that, for compliance valuations, one needs to look much more closely at market-related methods, but I do not necessarily agree with the authors that that excludes the Day & McKelvey type of method.

I welcome the authors' attempt to relate valuation methods to market conditions; but I am not convinced that using one particular asset class is a step forward. We have to find a way of relating the valuation much more closely to the actual asset distribution of the particular schemes with which we are dealing, and covering a wider range of asset classes.

Mr R. C. Ross, F.I.A.: The authors are correct in identifying the danger of the cart coming before the horse when it comes to their relationship between funding policy and investment policy. The Day & McKelvey methodology has not actually caused the heavy allocation to equities in U.K. pension funds, particularly U.K. equities, but it has certainly accommodated it. In the 1980s, the biggest business risk that a balanced fund manager could take was to sell U.K. equities, and the fact that the triennial valuation would smooth out any down-side volatility on the reporting date made this investment policy apparently very comfortable. Clearly, there is some danger that the U.K. market will not continue to out-perform all the other markets indefinitely.

I agree that the Day & McKelvey methodology puts too much emphasis on U.K. dividend growth, both in volatility and in return, as emphasised in ¶¶5.4.3.4. and 5.4.3.5. The Day & McKelvey methodology effectively means that the volatility of all assets is measured relative to U.K. dividend growth, so that assets which follow the pattern of U.K. dividends, such as U.K. equities, look quite attractive, but other assets, such as bonds and overseas equities, look rather unattractive.

If we consider an optimisation exercise: assumptions are a return of 11% for U.K. and international equities; 8% for index-linked gilts; and for volatility, say, 17% for U.K. equities, 17% for international equities and 8% for index-linked gilts. These are broadly reasonable. If we re-express those volatilities in actuarial terms, we arrive at something like 6%, 14% and 12%. You would not need to run a model to understand that this introduces quite a distortion, and, implicitly, this is what is happening with U.K. pension funds.

One of the reasons for drawing attention to this point is that it is relevant to the minimum solvency standard. When the legislation is finally agreed, the modified closed fund will assume much significance in the industry. This will be the basic fund benchmark against which the strategic debt inherent in particular funds will be measured.

It should be apparent that to have 60% of one's assets in a single volatile asset class, in a market value environment, which is what we are looking at in solvency valuations, would be quite risky. By the same token, a modified closed fund, which assumes that all equity exposure is in the U.K., would not be minimum risk, and I hope that this myth does not get as far as the statute book.

I wonder why the discounted income method is not used in any other country for valuing assets. Perhaps one of the reasons is that it does not actually work. If you look at the U.S. market, for

example, the discounted income value is itself almost as volatile as the market value, so we do not get the smoothing quality. In fact, U.S. actuaries generally use a market average. That is interesting, because it says something about the way in which the two markets value dividends. Would anybody like to suggest why this emphasis on dividend growth is predominant in the U.K?

I have a lot of sympathy with the authors' approach, and am pleased that two investment actuaries have brought these important issues to the attention of the wider profession.

Mr D. P. Hager, F.I.A. (closing the discussion)**:** For over 20 years I have been mystified by the way that actuaries value assets in an on-going valuation. I have wondered why it was necessary to find processes which turned the world into smooth lines; why little thought for the customer's need to understand the valuation process has been given; why the marketplace and market information was apparently irrelevant; why income was said to be the most important item in investment; and why it did not seem to matter that huge surpluses could be built up in a pension fund until they became embarrassingly large. As an investment actuary, I accepted that the pension fund actuary knew best. A great deal became clearer on reading this paper. Congratulations to the authors for having the courage to publish a paper which exposes the shortcomings of the current commonly used basis of valuation and its unfortunate effects on investment policy.

It was a delight to see the emphasis on the marketplace and the use of the wealth of information contained therein, not only for assets, but also for liabilities, in the use of spot interest rates. With modern computing power we can now use time-dependent variables, and the future for actuarial valuations may well be to use spot interest rates for liability values and market values for the assets. Market prices have a clear value to the client as an easy reference point which he or she readily understands, and this is, perhaps, a different point to the one that Mr Parsons raised. Actuarial values appear to clients to come from an ivory tower, using a Utopian set of stable conditions, to which the client does not easily relate.

I had hoped that the main debate on this paper would be about whether the proposed method was the correct one, and whether members could suggest refinements to it. However, there is strong support for the actuary's judgement and for the status quo. There has been no discussion on the interesting Figure 7.4, on the pattern of contributions. Actuaries have something to answer for in the way in which pension funding tended to put money into pension funds in the 1970s and 1980s, and the proposed method seems, at least to me, to give a more sensible profile from the point of view of the pension fund trustee and from corporate management, given the economic background at the time the contributions were made.

There has been no discussion on the impact of any change on SSAP24, or of the experience of the U.S.A. and FAS, and there has been little discussion on the subject of risk. The perceived risk of investing in equities is seen as low by both investment managers and pension fund trustees. That is one of the reasons why we have had such a concentration on these values. Asset liability studies have not necessarily taken these risks fully into account. Dissatisfaction was expressed in the discounted income method on the treatment of new money and overseas equities. This is something which it has been very difficult to resolve, and, unless the method is seen by pension fund trustees to get round some of these problems, then a discounted income method has little credibility. The smoothing, which is necessarily under the actuary's control in the new method, is said to be important, but I cannot understand why smoothing is needed at all. The real commercial world is not smooth, so why should pension funding be?

Several speakers have referred to index-linked gilts being central to the new process, and how this could, in some circumstances, be undesirable. It is dangerous to draw conclusions from any small sector of the market, but equally it is an important guide that ought not to be ignored.

Some speakers commented on asset liability studies, and the fact that they had now overcome most of the problems which certain people were suggesting they had. In my experience, I have heard several actuaries from different firms talk about asset liability studies, and I have listened to concepts such as 'minimising the ultimate surplus at infinity'. I still think that some improvement is needed in the content and presentation so far as clients are concerned.

Mr Wise believes in subjectivity in actuarial valuations; but the issue is how much subjectivity, and

how much objectivity. Why is his view better than that of the marketplace? Mr Green reinforced this point by his examples of the marketplace being clearly wrong. Although we can point to instances of the market being wrong, we cannot turn our backs on the derivatives market and on the information contained therein, because it may be that it has a better long-term forecasting record than that of actuaries. I do not think that our experience during the 1980s in dealing with pension fund surplus is anything to be particularly proud of under the current method.

Mr Heywood's points on the assets being consistently valued with the liabilities is vital, and the fluctuations of market values in a few days can make important differences if you use an unadjusted valuation method. If the liabilities are allowed to fluctuate to take into account the changes which are induced in the market values, it may well be that you also have liabilities which fluctuate appreciably. It is not clear to me why this is necessarily wrong. However, we need further research to establish if valuations which use spot interest rates which change by duration for calculation of liability values, plus market values for assets, produce answers which fluctuate or are reasonably stable.

Mr Parsons raised some useful points about client reaction; the problem is that we do not know what the client reaction is to various forms of valuation. As a profession, we should seek our clients' views on our services and the sorts of methods that they would prefer us to use.

Mr Wise mentioned that actuaries are not stuck in the 1960s, and the authors have pointed out how much progress we have made since that time. Mr Urwin aptly summarised the debate when he said that it is a question of the compromise between the economic and actuarial reality.

The debate will serve as an important milestone in the move away from discounted income approaches to valuations, and in adopting valuation processes which clients understand. It should also assist in the formulation of more practical investment policies which optimise return for given risks. As the opener suggested, professional judgement is still needed, but, perhaps, in a more market-related context.

The President (Mr C. D. Daykin, C.B., F.I.A.): This paper goes to the heart of the historic difference between financial economists, who focus on price, and actuaries, who focus on value. The authors have done a very good job in bringing this to our attention and in bringing out into the open an active debate on a subject which has been simmering for some time.

The classic papers of Heywood & Lander and Day & McKelvey did an enormously important job in their time in focusing our attention on the importance of cash flows in the pension fund valuation process. I agree with Mr Wise that we are concerned here with comparing cash flows; the discount rate is essentially irrelevant to that process, except in terms of putting a value on the future increments to the cash flow.

The problem of the Day & McKelvey type of method is the number of judgemental assumptions that have to be made, and there are some genuine reasons why that can be criticised. However, it does not follow at all from the use of that method that it has any direct influence on the investment policy. The question of asset allocation and investment strategy should be considered as a quite separate issue from establishing the funding rate.

The problem in addressing the issue the other way round has always been the need to achieve consistency between the value of the assets and the value of the liabilities. If you value the assets at market value, you clearly have to value the liabilities at market value as well; and what does that mean? Our authors have gone a long way in the direction of helping us to understand how one might approach that problem in a way which I have not seen propounded within the actuarial profession up to now. They still leave a lot of open questions about determining the real market expectation of the real rate of return. It has been indicated by several people that the real return on index-linked gilts is not necessarily a very good indicator. The introduction of index-linked gilts has helped, in giving us some idea of how the market views the real rate of return, but the low liquidity and the way in which the yields are influenced by a particular sector of the investment market means that they are of less value than they might be in determining the market value of liabilities.

There is no reason why the methodology of Day & McKelvey should imply a preference for income rather than return. The valuation approach under that methodology assumes an overall return

on equities and other investments in the ultimate discount rate, which is based on the global rate of return.

The authors present an interesting alternative viewpoint. We are much indebted to them for bringing some new ideas to the profession. We shall see a continuation of this debate until we can find some way of reconciling, to a greater extent, these different theologies of valuation. Maybe the minimum solvency requirement will have some impact, because that throws the light more heavily on a compliance valuation at a particular point of time, where the concept of market value may be more important.

I should like you to join with me in thanking the authors for the work involved in putting this paper together and in bringing it to us.

Mr A. C. L. Dyson, F.I.A. (replying): When we embarked on this project we did not anticipate converting every actuary to a new method of valuation. We were hoping to start a debate and not to conclude it. The diversity of views expressed vindicate us in that. Although we talk constantly about Day & McKelvey as a convention, it is right to recognise and make sure that everyone understands what we say in ¶1.2 about the equally valid contribution made by Heywood & Lander.

I have one point of general correction. We are not saying that there is no scope for professional judgement and subjectivity, but there are some areas, which we refer to under the heading of compliance, where one has to look at that notion rather more carefully.

We threw down two gauntlets at the start. One was to say, how comfortable are we with the very crucial dividend growth assumption in terms of the overall result? The second area highlighted is the potential impact of the method on the investment strategy and the way that the assets are valued. That has particularly come to the fore with the debate on minimum solvency. Although there were one or two exceptions in the discussion, the great majority of speakers have not chosen to pick that up. Those who did have tended to support the line that we are taking.

Mr Clark suggested that the potential distortion of risk by using actuarial values for, say, asset liability modelling, is known to all. I am not convinced that that is true. He once devoted a whole paper to looking at the potential impact of changing that model fund on the investment strategy that is adopted.

Mr Urwin emphasised the asset liability management being a joint discipline affecting the assets and the liabilities. That is the key point we should like to see picked up. You cannot divorce what happens on the liability side.

We echo comments regarding the importance of cash flow, and this is where the dividend growth comes in, but just how confident can you be when you are projecting cash flows from the assets, if you are investing in line with a real fixed dividend growth assumption? Professor Wilkie said that the range was not so wide, and perhaps between -1% and +2%. This represents a 60% swing in the result you might get on the valuation. Unfortunately, if you look at the individual decades in that section, it would appear that, although this -1% to +2% is right, it does not actually fit any of the four individual decades set out. You might have to wait a very long time to be vindicated.

Many of the comments of Mr Green are not particularly relevant to the asset/liability methodology we are talking about. We are talking about something which combines the assets and the liabilities. I daresay that the liability side is not something that the average investment manager would fancy taking on too readily. We are talking about the best way to discount the cash flows that arise. You cannot ignore the fact that there exists in the marketplace a term structure of interest rates, both real and fixed potentially for discounting. That may not be the end of the solution, but a paper that criticised without putting forward something as an alternative to promote discussion would be wrong. Actuaries need to be careful these days, and should not blindly assume that a stable discounting mechanism, based on a particular fixed rate, is the right one to use when there is so much extra information around.

Mr Parsons referred to the presentational difficulties of the proposed method. I suggest that there is a large amount of difficulty involved with presenting the actual impact over a valuation period at actuarial value when markets have gone up, yet the actuarial value appears to go down.

Mr Thornton correctly said that a lot of short-term patching is used. I think he answered his point

and ours by saying, how does one do that from a compliance perspective? That is an area that will come to be increasingly important. He was among other speakers who suggested that we can cater now for overseas equities. We welcome seeing that sort of allowance incorporated in the basis for a minimum solvency standard. Like Mr Ross, we will be nervous if it is omitted.

Minimum solvency has given importance to a lot of issues which were already problematic. It is clear that the way in which actuaries value assets and liabilities can have potentially very significant investment impact. As a profession, we need to recognise this, analyse it, and decide how we want to respond to it.

WRITTEN CONTRIBUTION

The authors subsequently wrote: Our previous reply, together with many valuable contributions from the floor and, in particular, the address by the closer, responded to the broader criticisms raised during the discussion, and it is not our intention to repeat here those same remarks. Rather, we would prefer to take the opportunity to consider, in our written response, the technical criticisms made by Mr Wise and Professor Wilkie, whose combined work in the area of asset and liability matching (and mismatching) requires no introduction.

Having spoken subsequently, it is now clear that Mr Wise was not suggesting he had been misquoted, but that his later 1987 paper (*J.I.A.* **114,** 113-133), with its concept of mismatching, would have been a better starting point for our work. We disagree. We emphasise at length in our paper that any non-trivial choice of risk premium $\boldsymbol{u}$ is ultimately subjective. Our assertion is only that it is more rational to focus on this parameter rather than focusing on long-term real income growth from a particular asset class (the demonstrable lack of any stable relationship between U.K. dividend growth and U.K. price and earnings inflation, highlighted in our paper, being largely ignored by several contributors). Mr Wise's 1987 paper may well provide a framework for determining a value for $\boldsymbol{u}$, but it certainly does not eliminate subjectivity (which appears in the choice of risk premiums on different asset classes and in the choice of utility function). We would, of course, have no hesitation in acknowledging that his later work may be used to extend many of the investment principles which we discuss, although, in fact, we have in mind an alternative process, which we hope will form the basis for a subsequent paper.

By way of support for our reference to the earlier 1984 paper, it may, however, be appropriate to develop here, more formally, the link between our matching philosophy and that proposed by Mr Wise in his seminal paper of 1984 (*J.I.A.* **111,** 375-402). In doing so, it should first be noted how, in ¶¶6.5.1.3 and 8.3.1.6, we emphasise that a more complete model would permit non-parallel changes in our yield curves (in respect of both nominal and real rates). With this further generalisation, we can immediately consider Mr Wise's celebrated 'Worked Example' in Section 3 of the 1984 paper (see also Wilkie, 1985, *J.I.A.* **112,** 112-229 Sections 5 to 20) to illustrate the similarities (and differences) between the approaches.

Using i_n to denote the forward spot interest rate over the period n to $n+1$, with:

$$E(i_0) = E(i_1) = E(i_2) = 0.09$$

as in Mr Wise's example, and taking $^qA(\bar{t})$ and $L(\bar{t})$ as the (market related) present values of his two available assets ($q = 1$ and 2) and liabilities respectively, we have opening values and *expected* closing (time $t = 1-$) values of :

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	Assets		Liabilities
	$^1A(i)$	$^2A(i)$	$L(i)$
Opening Value	0.9334231	0.9480946	2.5312947
Closing Value	1.0174312	1.0334231	2.7591112

and

	Assets		Liabilities
	$^1\alpha_{i_n}$	$^2\alpha_{i_n}$	$^L\alpha_{i_n}$
i_1	-0.8272599	-0.8286552	-0.5849215
i_2	-	-0.7472094	-0.2798667

where $^q\alpha_{i_n}$ denotes our 'duration' with respect to the spot interest rate i_n, that is:

$$^q\alpha_{i_n} = \frac{\partial}{\partial i_n} \log {}^qA(i).$$

(We carry an excessive number of decimal places to establish the degree of discrepancy with the method of Mr Wise, using what is essentially a first order approximation in Δi_n).

More precisely, the $^q\alpha_{i_n}$ here could be referred to as 'partial' durations, since their sum gives the conventional (parallel) duration at the close of the first time period. In full application, the $\{i_n\}$ would represent pivot points defining the structure of interest rates δ over intermediate periods, decomposed into separate real interest $\{r_n\}$ and inflation $\{f_n\}$ pivots. (The collinearity of short and long interest rate movements complicates any empirical derivation of these partial durations for equity-type assets using the simple methodology of our Appendix B).

Now, using x_q as the proportional allocation to each asset and assuming independence of Δi_1 and Δi_2 we seek to minimise:

$$\text{Var}\,(\dot{\Delta}A - \dot{\Delta}L) = (x_1\,{}^1\alpha_{i_1} + x_2\,{}^2\alpha_{i_1} - {}^L\alpha_{i_1})^2\,\sigma^2_{\Delta i_1} + (x_1\,{}^1\alpha_{i_2} + x_2\,{}^2\alpha_{i_2} - {}^L\alpha_{i_2})^2\,\sigma^2_{\Delta i_2}$$

subject to $x_1 + x_2 = 1$.

More generally, these changes Δi_n (or Δr_n and Δf_n) in pivot points would not be completely independent (our simpler model in Section 8 assumes, alternatively, that Cor $(\Delta i_1, \Delta i_2) = 1$), nor would the $\sigma^2_{\Delta i_n}$ necessarily be equal. However, adopting Mr Wise's assumption also that $\sigma^2_{\Delta i_1} = \sigma^2_{\Delta i_2}$, after some algebra, this minimisation yields:

$$x_1 = 0.6260576; \text{ and}$$
$$x_2 = 0.3739424.$$

Hence, the number m_q of units of each stock is given, from the opening values, by (Mr Wise's result in parentheses):

$$m_1 = 0.6260576 \times \frac{2.5312947}{0.9334231} = 1.6978 \quad (1.698)$$

$$m_2 = 0.3739424 \times \frac{2.5312947}{0.9480946} = 0.9984 \quad (0.998).$$

So, in this simple example, our approximate methodology appears to agree with Mr Wise's 1984

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paper to the third decimal place, confirming our original reference. (It will be noted that the first term in the objective function is largely independent of the allocation between x_1 and x_2 (since ${}^1\alpha_{i_1} \fallingdotseq {}^2\alpha_{i_1}$) and therefore an even more approximate solution is just $x_2 = {}^L\alpha_{i_2}/{}^2\alpha_{i_2}$. However, this solution agrees less closely, giving $m_2 \fallingdotseq 1.00$ only).

Professor Wilkie made several comments on the detail of the appendices. Although few had any real bearing on the main body of the paper, we would offer the following responses.

The 'duration' (α) in A.2.1 of Appendix A must be understood to be measured at the *end* of the finite step ('closing duration') for the purposes of the summation limits in the subsequent formulae. We feel that this is a rather convenient notation (a one-period deposit having zero duration, see A.2.4(1) and our reply to Mr Wise above), and this definition of α is now emphasised in the text. With this convention understood, the final equation of A.2.1 is correct. We would, however, acknowledge that the original notation lacked clarity, and it is clear that the time dependence of the discount rates needs to be described very carefully if the concepts are to be universally understood.

The regression equation which headed the Appendix B presented on 23 January did not include the explicit regression coefficient ζ before $\dot{\Delta}D_n$. Without constraint this coefficient takes the values (standard errors) 1.33 (0.24) in 1984Q1 to 1993Q4, and 1.34 (0.28), 1989Q1 to 1993Q4, as shown. It is clear that neither of these unconstrained results violates the hypothesis that ζ=1 (a hypothesis which was implicit in the original equation). However, for the avoidance of confusion, we have added a note in new B.5.5 to state this unambiguously. As expected, reworking the regression with the $\dot{\Delta}D_n$ coefficient *constrained* to unity has no material impact on the results (for example $\hat{\alpha}$ in 1989Q1 to 1993Q4 changes from -17.36 to -17.07, and the residual error rises from 2.93% to 2.97%).

We would naturally agree with Professor Wilkie that an additional decade of data for the index-linked gilt market would be preferable for such quarterly regression analysis, although, as we stress in ¶4.5.2, we must be careful that we also look forward and not just a long way backwards. In terms of expanding the data set, of much greater interest and relevance to us than the '70 years of data' on equity yields (or '2,000 years of data on interest rates') suggested by Professor Wilkie, would be a test of our model against other modern liquid international index-linked security markets, as and when these develop in the future.

On a less trivial note, Professor Wilkie's comments concerning the inclusion or otherwise of an intercept term and his preferred alternative form of regression equation harnessed to the idea that market yields track "back towards...(some fixed) level when they get too far away from it" would appear to violate one of the main themes in our paper. In this context, the poignant quotations heading Sections 7 and 8 should not be regarded as mere ornaments, and these principles are germane to much of the discussion elsewhere. Professor Wilkie suggested that we had over-emphasised these allied problems of data snooping and information structure. On the contrary, from many of the comments made on 23 January, the most striking observation was that these two endemic issues should have been emphasised more stridently in the paper.

Finally, Professor Wilkie's observation that equation (D.1) in Appendix D is independent of x_0 is really just equivalent to our statement that $\psi = 0$ in D.2. We accept that there are easier ways of solving the minimisation problem in the central region of Figure 8.3. Nevertheless, the method of Lagrange is adopted to give a more general solution. Not only does it permit the inclusion of further constraints (along the lines suggested by Mr Wise in his 1987 paper, for example), but it can also be extended to cover the whole range of solutions. For example, on the left hand side of Figure 8.3 we have $x_0 = 0$ and hence:

$$\boldsymbol{m}\boldsymbol{x} = \boldsymbol{y} + \psi\boldsymbol{l} \text{ or } \boldsymbol{x} = \boldsymbol{m}^{-1}(\boldsymbol{y}+\psi\boldsymbol{l})$$

where:

$$\boldsymbol{l} = \begin{pmatrix}1\\1\end{pmatrix}$$

and therefore:

$$\psi = (1 - \boldsymbol{l}^T \boldsymbol{m}^{-1} \boldsymbol{y}) / \boldsymbol{l}^T \boldsymbol{m}^{-1} \boldsymbol{l}.$$

(For completeness, on the right hand side of Figure 8.3 we have $x_1 = 0$ *and* $\psi = 0$, which gives immediately:

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$$x_2 = \hat{\alpha}_2\sigma^2_{\Delta r} / (\hat{\alpha}^2_2\sigma^2_{\Delta r} + \hat{\beta}^2\sigma^2_{\Delta e} + \hat{\alpha}^2_2\sigma^2_{\Delta m}).$$

In conclusion, whilst we are grateful to Mr Wise and Professor Wilkie for having considered our paper in such detail, and whilst we respect the criticisms expressed, we do not feel that any of our views were undermined by their comments. Indeed, in several cases they have given us scope, and support, to expand on some of the detail which was previously only sketched in rather broad terms. More generally, we would like to thank all of the contributors for their comments, many of which have provided motivation for further development of our ideas.

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