Journal of Glaciology, Vol. 18, No. 78, 1977

ON WIND-INDUCED CRACKING OF SEA-ICE SHEETS

By M. D. COON *and* R. J. EVANS

(Arctic Ice Dynamics Joint Experiment, University of Washington, Seattle, Washington 98195, U.S.A.)

ABSTRACT. It is shown that when the elastic properties of sea-ice sheets are taken into account, wind-induced tilt is insufficient to cause cracking. These results are contrary to those of Weber and Erdelyi (1976) who considered the ice to be rigid.

RÉSUMÉ. *Craquement dus au vent dans la banquise.* On montre que si l'on tient compte des propriétés élastiques dans la banquise, les oscillations dues au vent sont insuffisantes pour provoquer des ruptures. Ces résultats sont en contradiction avec ceux de Weber et Erdelyi (1976) qui considéraient la glace comme rigide.

ZUSAMMENFASSUNG. *Über den Bruch von Meereisdecken durch Windeinfluss.* Es wird gezeigt, dass unter Berucksichtigung der elastischen Eigenschaften von Meereisdecken deren Verbiegung durch Windkräfte nicht ausreicht, um zum Bruch zu fuhren. Diese Ergebnisse stehen im Gegensatz zu denen von Weber und Erdelyi (1976), die das Eis als starr betrachteten.

IN a recent *Journal of Glaciology* article, "Ice and ocean tilt measurements in the Beaufort Sea", Weber and Erdelyi (1976) postulate that wind-induced tilt could cause ice sheets to break at right angles to the direction of drift. They consider the floes to be rigid and to be acted on by a horizontal-surface wind force and by an opposing bottom-surface drag force; the resulting moment tilts the floe until it is equilibrated by buoyancy forces of the water. Then for a given angle of tilt, the maximum bending moment depends on the cube of the floe length; hence, the maximum floe length can be found if the tensile strength of the ice is known. For 3 m ice and a tilt of 30 μrad, Weber and Erdelyi find maximum floe lengths to be 328 m for a strength of 10^5 N m^{-2} and 890 m for a strength of 2×10^6 N m^{-2}.*

The assumption that the ice is rigid, however, is unreasonable. A first approximation to its actual behavior is that of a floating elastic plate, and there is overwhelming evidence to support such an assumed behavior. As will now be shown, the incorporation of elastic flexural behavior into the analysis of the ice considerably changes the conclusions to be drawn.

Referring to the infinitesimal element shown in Figure 1, the differential equations of equilibrium are

$$q - \rho v = -\frac{\mathrm{d}V}{\mathrm{d}x}, \tag{1}$$

$$V + (\tau_a + \tau_w)\frac{a}{2} = \frac{\mathrm{d}M}{\mathrm{d}x}, \tag{2}$$

where ρ is the density of the ice, M is the bending moment, q is the applied vertical load, v is the displacement of the ice sheet normal to its surface, V is the shearing force, τ_a is the wind stress, τ_w is the water stress, and a is the thickness of the ice. In the above, motion in the x-direction is assumed to be uniform and τ_a and τ_w are assumed to be constant; they must therefore be equal unless some horizontal force exists due to action of an adjacent floe.

Using the plate-bending constitutive law

$$M = D\frac{\mathrm{d}^2 v}{\mathrm{d}x^2}, \tag{3}$$

where

$$D = \frac{Ea^3}{12(1-\nu^2)} \tag{4}$$

is the flexural stiffness (E is Young's modulus, ν is Poisson's ratio), the equation of equilibrium in terms of displacement becomes

$$D\frac{\mathrm{d}^4 v}{\mathrm{d}x^4} + \rho v = q. \tag{5}$$

* Our calculations show $M_{\max}$ to be one half the value given by Weber and Erdelyi in their equation (9).

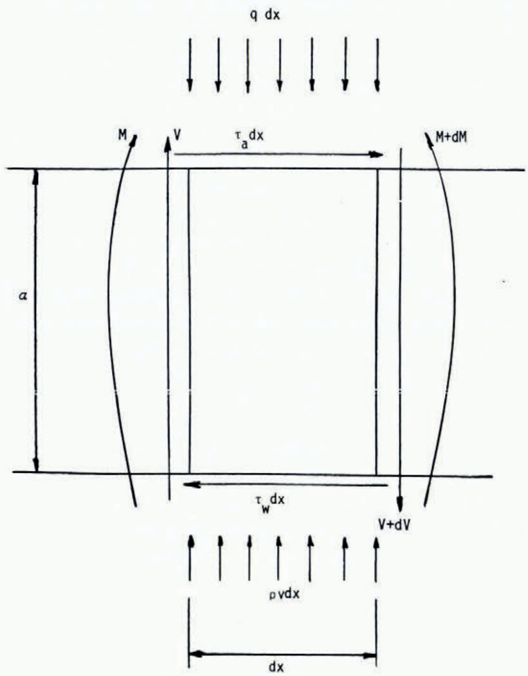

Fig. 1. Equilibrium element.

For $q = 0$, the solution to Equation (5) is

$$v = \exp(-\lambda x)(A\cos\lambda x + B\sin\lambda x) + \exp(\lambda x)(C\cos\lambda x + D\sin\lambda x), \tag{6}$$

where

$$\lambda = [\rho/4D]^{\frac{1}{4}}, \tag{7}$$

($1/\lambda$ is the characteristic plate length).

Consider first a long ice sheet, floating free at $x = 0$. Regularity conditions require that

$$C = D = 0,$$

while boundary conditions at $x = 0$ require the moment and shear there to be zero, i.e.

$$\frac{d^2v}{dx^2} = 0 \qquad \text{at } x = 0, \tag{8}$$

$$D\frac{d^3v}{dx^3} - (\tau_a + \tau_w)\frac{a}{2} = 0 \qquad \text{at } x = 0. \tag{9}$$

The corresponding solution for the displacement is

$$v = \frac{(\tau_a + \tau_w)\,a}{4\lambda^3 D}\exp(-\lambda x)\cos\lambda x. \tag{10}$$

The maximum bending moment occurs at $x = \lambda\pi/4$ and is given by

$$M_{\max} = \frac{\sqrt{2}a}{4\lambda}(\tau_a + \tau_w)\exp(-\lambda\pi/4). \tag{11}$$

The maximum bending stress $\sigma_{\max}$ is obtained directly from the maximum bending moment and is given by

$$\sigma_{\max} = \frac{6M_{\max}}{a^2}. \tag{12}$$

https://doi.org/10.3189/S0022143000021638 Published online by Cambridge University Press

Of importance here are the wind stress values required to cause fracture. Using a tensile strength value of 10^5 N m^{-2}, $E/(1-\nu^2) = 10^9$ N m^{-2}, and $\rho = 1$ Mg m^{-3} and setting $\tau_a = \tau_w$ one obtains, from Equations (11) and (12), a wind stress to cause fracture of 5040 N m^{-2} for 3 m ice and 2150 N m^{-2} for 10 cm ice. These values are so much larger than those occurring (on the order of 0.5 N m^{-2}) that fracture of ice sheets due to wind loading must be ruled out. Although the above calculations were made for long (semi-infinite) plates, the results for plates of finite length do not differ significantly. The response of the ice sheet to the surface load given by Equation (10) amounts to an edge disturbance which rapidly dies out (as do flexural stresses); subsequently, the wind stress is equilibrated by the shearing force in the plate. The addition of an axial force at one end of the ice sheet resulting from the action of a contiguous floe has virtually no effect on the above solution; wind reaches of several hundred kilometers are required before interaction of axial and flexural effects becomes significant.

Finally, it is of interest to compare typical tilt values with those measured by Weber and Erdelyi (1976). According to the elastic theory used here, tilt will be significant only near the end of the ice sheet. The maximum slope occurs at the free end and is given by

$$\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{\tau_a a}{2\lambda^2 D} \qquad \text{at } x = 0. \tag{13}$$

For a wind stress of 0.5 N m^{-2} and other data as above, the tilt is 3.16 μrad for 3 m ice and 17.3 μrad for 10 cm ice. If tilt is interpreted as the difference in level between two points (as measured by Weber and Erdelyi) and the points are chosen to be $x = 0$ and $x = \pi/2\lambda$, then the tilt is 2 μrad for 3 m ice and 11 μrad for 10 cm ice. These values are of the same order of magnitude as those measured by Weber and Erdelyi using points 120 m apart. It would be interesting if tilt measurements could somehow be used to check Equation (10).

MS. received 14 June 1976

REFERENCE

Weber, J. R., *and* Erdelyi, M. 1976. Ice and ocean tilt measurements in the Beaufort Sea. *Journal of Glaciology*, Vol. 17, No. 75, p. 61–71.