JOURNAL OF FINANCIAL AND QUANTITATIVE ANALYSIS Vol. 58, No. 6, Sept. 2023, pp. 2692–2731

doi:10.1017/S0022109022001090

Enhanced Global Asset Pricing Factors

Lukas Zimmermann
University of Mannheim Business School
lukas_zimmermann@outlook.com

Abstract

This article constructs and examines enhanced global return factors. I focus on three different enhancement approaches. First, I incorporate information about the covariance structure in the cross-section of stock returns. Second, I employ volatility-reducing techniques in the time series. Third, I exploit diversification benefits. I form six categorical factors by aggregating information from 214 characteristics. Further, I diversify across factors. The enhancement mechanisms are largely successful and when jointly applied increase the optimal Sharpe ratio on average by a factor of 1.96 compared to the traditional factors. My results point to the importance of employing efficient factors in asset pricing studies.

I. Introduction

One of the fundamental assumptions in finance is the existence of a stochastic discount factor that prices all assets. If a linear relationship between returns and their sources of common variation is assumed, the stochastic discount factor can be interpreted as a set of factor portfolios that span the mean–variance efficient frontier and that should be able to explain asset returns. In this line of reasoning, various multifactor models have been proposed to explain the cross-section of returns. A string of literature, prominently represented by Daniel, Mota, Rottke, and Santos (2020), questions whether these models correctly capture sources of common variation as implied by economic theory. Specifically, a concern is that the included factors might not be efficient with respect to how they capture these sources of common variation.

With this article, I aim at examining whether I can create more efficient factors internationally by constructing enhanced global factor versions. I build on the

This article greatly benefited from significant contributions of Sebastian Müller. The idea of employing cross-sectional enhancement and volatility scaling on return factors in international markets has been developed in many common discussions for which I am very thankful, as they made this project possible. In that regard, I kindly thank Sebastian Müller for providing the international stock market data and data on international return predictors. I further thank an anonymous referee, Hendrik Bessembinder (the editor), Scott Cederburg (a referee), Ralph Koijen, Simon Rottke, Oliver Spalt, Erik Theissen, Jiri Tresl, and seminar participants at the University of Mannheim for very helpful comments which greatly helped to improve the article. I acknowledge support by the state of Baden-Württemberg through the bwHPC high-performance computing cluster and the German Research Foundation (DFG) through grant INST 35/1134-1 FUGG.

existing literature that suggests different approaches for enhancing return factors with respect to their efficiency. Factor efficiency is judged on the (squared) Sharpe ratio. I primarily focus on three different enhancement approaches. First, I enhance factors by incorporating information about the covariance structure in the cross-section of returns following Daniel and Titman (1997) and Daniel et al. (2020). They suggest that factors that are solely sorted on characteristics are not efficient, as they load on common sources of variation in expected returns *and* on unpriced risk. Second, I employ volatility-reducing techniques in the time series. Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) suggest that volatility-scaling of a momentum strategy attenuates crashes and improves the Sharpe ratio. Similar to Moreira and Muir (2017) and Cederburg, O'Doherty, Wang, and Yan (2020), I extend the approach to further factors.

My third enhancement approach is constituted by the exploitation of diversification effects. I make use of the fact that factors and anomalies proposed in the financial literature can be categorized into different groups. Barillas and Shanken (2018) suggest a categorical factor model that consists of a market, size, value, investment, profitability, and momentum factor. The factors are referred to as categorical as each factor represents a specific category of firm characteristics.[1] Accordingly, I create composite categorical factors by aggregating 214 firm characteristics into 6 categories, while following the categorization of Hou et al. (2020): value, investment, profitability, momentum, intangibles, and frictions. To obtain the corresponding trading signal for each category, I thus diversify over multiple characteristics to improve the measurement accuracy for each category and to reduce the noise of each signal. By employing international return and accounting data, I make use of the opportunity to study the enhanced factors in different geographical areas. I create global factors for each of 5 regions according to classifications of Morgan Stanley Capital International (MSCI): North America, Europe, Japan, Asia-Pacific, and Emerging Markets. I then examine how the first two enhancement approaches, which essentially constitute ways of risk management, work when applied to the composite categorical factors and when compared across regions.

To examine whether the studied enhancement approaches add value with respect to the explanatory power of factor models, I test how well the 241 international anomalies employed in Jacobs and Müller (2018) and Jacobs and Müller (2020) can be explained by four versions of a categorical factor model consisting of a regional market factor and a value, investment, profitability, momentum, intangibles, and frictions factor. The four model versions I study are representative of different examined types of enhanced factors: traditional factors, cross-sectionally enhanced factors, volatility-scaled factors, and factors on which both enhancement approaches are applied. I thus start with a factor model that aggregates similar information as studied in the academic literature. The traditionally constructed factors, however, might be inefficient and although important sources of common variation might be captured, conclusions drawn from analyses based on these

[1]These categories are similar to those used in various anomaly studies (see Hou, Xue, and Zhang (2015), Harvey, Liu, and Zhu (2016), Cederburg et al. (2020), Hou, Xue, and Zhang (2020), and Jacobs and Müller (2020)), which jointly draw conclusions for all factor and anomalies within each category.

factors might be impaired. While one could object that the focus on only one specific factor model might be rather arbitrary, I believe that my factors represent a convenient choice for testing the effect of the cross-sectional and time-series enhancement approach and of corresponding diversification effects in an international setting, instead of preselecting one of the multiple proposed factor models.

Generally, for most factors in the studied regions, either the cross-sectional enhancement approach or volatility-reducing enhancement techniques in the time series lead to an improvement in Sharpe ratios and thus in factor efficiency. While cross-sectional enhancement on average proves successful for all factors except for the composite momentum factor, volatility management, in contrast, works consistently well for the momentum factor only, consistent with the findings of Cederburg et al. (2020). Combining the cross-sectional and time-series enhancement approaches does often lead to further, albeit only mild improvements in Sharpe ratios. Only for the market factor, I can observe that both approaches work consistently well, both individually and in combination. I, therefore, conclude that if characteristics-based factors are not mean–variance efficient, then this is most likely the case because either a relatively large fraction of cross-sectional *or* time-series information is not incorporated in the construction process. Results do generally not indicate that both types of information have on average similar effects on most factors.

The findings suggest that the source of common variation that affects momentum might be inherently different from the source affecting the other factors. If the exposure to this source can be related to past volatility, then it can be accounted for by volatility management. If the exposure to this source, however, is correlated with some unpriced factor, then it can be accounted for by cross-sectional enhancement. I do not further investigate into a theoretical explanation, as my objective lies on showing the empirical consequences of these findings. Further, I point out that the examined factors themselves are observable and not latent.[2] Practical implementation is thus feasible, as the composition of all underlying portfolios is known. Portfolio rebalancing and the leverage needed for implementation, however, will likely incur significant transaction costs. Similar to the previous point, I do not investigate their impact further, as my objective lies on understanding the economic effects of factor enhancement.

In the light of my results, the question might come forward why it should be important to obtain more efficient factors. I, therefore, investigate their implications for traditionally constructed factor models. Return factors and factor models are an important input in many applications of financial research. They are used as benchmark models in anomaly studies to calculate the alpha of a newly proposed factor, in event studies to account for abnormal returns, as expected return model for cost of equity in cost of capital calculations, or to conduct performance evaluation of mutual fund managers. I argue that it is important to employ a factor model that is as close as possible to spanning the mean–variance efficient frontier, in order to derive proper conclusions from these analyses. If this is not the case, one might run

[2]This point for instance holds for papers, which obtain their factors from principal component analysis. See Kozak, Nagel, and Santosh (2018), (2019), Kelly, Pruitt, and Su (2019), Lettau and Pelger (2020b), Cooper, Ma, Maio, and Philip (2021), and Kim, Korajczyk, and Neuhierl (2021).

into danger of encountering a false positive because the factor model is either incorrectly specified or the employed factors are inefficient. This would also imply that the power of the asset pricing test is negatively affected.

Indeed, when studying the explanatory power of different factor model versions for 241 international anomalies, I find that across all regions the enhanced factor models can explain a significantly larger fraction of anomalies than the traditional models. The model that combines enhancement approaches by applying volatility-scaling on the cross-sectionally enhanced factors yields the lowest number of significant anomaly alphas across all regions, accompanied by lower average *t*-statistics. Across all regions, the enhanced model can explain an average of 15 more anomalies than the traditional model, which corresponds to an average increase in explained anomalies of 35%. This finding is not contradictory to the statement that either cross-sectional or time-series information is important for enhancing factors. By employing both enhancement approaches, it is ensured that on each factor a method is applied that effectively improves its efficiency. Further, I observe that the ex post optimal maximum Sharpe ratio achievable by combining the factors, and thus by targeting efficiency by reducing volatility through diversification across factors, is always quantitatively larger with enhanced factors than with traditional factors. In most cases this difference is statistically significant, supporting the premise that the set of enhanced factor is closer to spanning the mean–variance efficient portfolio.

I contribute to the literature in multiple ways. Generally, I contribute to the global factor literature, as I study the efficiency of return factors in international stock markets. My study is the first to employ the methodology of Daniel et al. (2020) in an international setting. The exercise thus enables me to assess whether the procedure can successfully be replicated in international equity markets and can successfully be implemented for different groups of characteristics. Further, I combine the cross-sectional enhancement approach of Daniel et al. (2020) with the time-series enhancement approach of Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) by performing volatility-scaling on the cross-sectionally enhanced factors. This enables us to determine whether volatility-reducing techniques work across different groups of characteristics, similar to Cederburg et al. (2020), and to assess how much value volatility-scaling adds in improving factor efficiency compared to cross-sectional enhancement. Moreover, this study investigates the implication of enhancing factors for the application of factor models in asset pricing studies and shows the consequences of using enhanced instead of traditional factors. The findings have important implications for anomaly studies, as they suggest to be more careful in assessing results. By showing that enhancement techniques improve the efficiency of return factors, I present a way to improve factors such that conclusions based on factor model analyses can be more robust.

The remainder of this article is organized as follows: Section II describes the data and the construction of the categorical factors. Section III focuses on the benefits of the cross-sectional enhancement approach and Section IV focuses on the benefits of employing volatility-scaling in the time series. Section V first compares the Sharpe ratios of traditional and enhanced factors and the maximum Sharpe ratios of different factor models and then compares the explanatory power of traditional and enhanced factor model versions. Section VI concludes the article.

II. Data and Factor Construction

The main empirical exercises of this article apply cross-sectional and time-series factor enhancement on international return factors and test the implications of these enhancement approaches for the application of the return factors on a global scale. As multiple factor models have been proposed in the literature, I could start by choosing one arbitrary model out of the set of available factor models and test the enhancement approaches on this model. However, I do not want to take a stance on a specific factor model. Moreover, my objective lies on determining whether an enhanced model generally is closer to spanning the mean–variance efficient portfolio. I, therefore, seek to start with a model that represents dimensions which are independent of each other and which are relevant to explain the cross-section of returns. I start by observing which group of characteristics have been studied independently of each other in the literature and thus constitute independent categories of interest. I choose this approach to improve the measurement accuracy and reduce the noise of the trading signal for each category. In the following, the construction of these international factors is explained in detail.

The data employed are from three different databases. I employ global stock market data, where I measure returns based on values in US-Dollars, from Refinitiv Datastream. The global accounting data is from Worldscope. Moreover, analyst-related data is from Institutional Brokers' Estimate System (I/B/E/S). The final data period for which there are sufficient stock market data available spans from Jan. 1989 to June 2019. The period eventually studied in the empirical analyses is determined by the required amount of daily data in the cross-sectional enhancement approach. The approach comprises two steps and each consists of computing specific types of betas from daily data. For the Frazzini and Pedersen (2014) method as employed by Daniel et al. (2020), each step requires at least 3 years of daily data. Therefore, the corresponding analyses employ data spanning from Jan. 1995 until June 2019. As a robustness check, I also implement the approach using Dimson (1979) betas, which only require 1 year of data in each step. However, as I employ many variables based on quarterly accounting data and this data is only available starting in 1992, the corresponding analyses with Dimson betas extend from July 1992 until June 2019.

Filters and screens generally follow Jacobs and Müller (2018), (2020), and Huber, Jacobs, Müller, and Preissler (2021). I use all firms that have a nonmissing Datastream and Worldscope identifier and that are listed in Datastream for the stock market of a specific country. In the case of US firms, I also require stocks to be listed at one of the three major exchanges NYSE, NASDAQ, or AMEX. Further, the generic filter rules proposed by Griffin, Kelly, and Nardari (2010) to exclude noncommon equity are used, to include delisted stocks only up to the point of their actual delisting the methodology of Ince and Porter (2006) is used, and to eliminate remaining data errors returns are screened as proposed in Hou, Karolyi, and Kho (2011).

I conduct my analysis on the regional level, similar to Fama and French (2012) and Jacobs and Müller (2018). I start with country-level stock returns and assign 44 countries to regions based on the MSCI global investable market indexes (GIMI) methodology country classification.[3] Eventually, I examine 5 different regions: North America, Europe, Japan, Asia-Pacific, and Emerging Markets. I exclude Frontier Markets due to weak data coverage, especially in the earlier years. To determine the firm characteristics to include for the construction of the factors, I start with the same 241 characteristics examined by Jacobs and Müller (2018). They explore which anomalies based on firm characteristics examined with US data can be reconstructed with international data. I exclude all binary indicator variables that assign a dummy variable to stocks depending on whether they fulfill a certain condition or not. This leaves us with 214 firm characteristics. The construction and adjustment of all variables with Datastream and Worldscope data follow Jacobs and Müller (2018).

To aggregate the predictive information in firm characteristics, I sort firms into deciles on the country-level based on each of the 214 characteristics and aggregate the country-level decile ranks on the regional level. For each firm characteristic, I compute the regional top minus bottom portfolio return and reverse the ranking if the long–short return is significantly negative.[4] Subsequently, for each firm, I calculate a composite category score by computing the average adjusted decile rank overall firm characteristics assigned to a specific category. My approach thus resembles the approach of Stambaugh and Yuan (2017) to aggregate information to build portfolios. They obtain a composite mispricing measure by ordering individual firm characteristics and computing an aggregate rank. I also construct portfolios based on aggregated ranks.[5]

To use a starting point grounded in the literature, I follow Hou et al. (2020) and use the same 6 categories as these authors: value, investment, profitability, momentum, intangibles, and frictions. Each firm characteristic in my list is assigned to categories as suggested in this article. If a characteristic in my list is not in the list of Hou et al. (2020), I assign the variable by checking to which category the reference paper assigns the characteristics or to which category similar variables have been assigned to.[6]

My assignments might be perceived as ad hoc. For instance, one could further subcategorize certain groups. I argue that by choosing to study 6 categories I include a reasonable number of factors that encompass enough relevant dimensions. Many factor models proposed in the literature that form factors on single characteristics encompass 5 or 6 factors and represent similar categories as included in my model. These models include the Fama and French (2015), (2018) 5- and 6-factor model, the Hou et al. (2015) q- and Hou, Mo, Xue, and Zhang (2021) q5-model, or the Barillas and Shanken (2018) categorical factor

[3]The assignments are obtained from the MSCI website.

[4]If a return premium is negative, then a high value in the underlying characteristic is indicative of a low expected return. By reversing the ranking I ensure that such firms get assigned to the bottom portfolio.

[5]Novy-Marx (2016) argues to be careful when using multiple signals to form strategies with the objective to generate strong backtesting results. My objective, however, is not on suggesting a superior factor model or strategy, but on improving the efficiency of representative factors.

[6]A list of all firm characteristics that are employed and the corresponding category assignments can be found in the Supplementary Material.

model.[7] Although the practical implementation of the enhanced factors I examine is relatively complex compared to the traditional factors, the enhanced factors are observable as well. My article thus fits into the literature on factors based on investable portfolios.

Furthermore, there is a string of literature that uses different approaches based on principal component analysis (PCA) to undertake a structured search for latent factors. Kim et al. (2021) employ projected principal components analysis (PPCA) to form an arbitrage portfolio based on the component that represents the return that cannot be attributed to the predictive power of characteristics for factor loadings. When more than six eigenvectors, representing return factors, are used, the monthly performance of the arbitrage portfolio does not improve. In a related paper, Kelly et al. (2019) use instrumented principal component analysis (IPCA) that allows factor loadings to dynamically depend on firm characteristics. A model with 5 IPCA factors explains a large fraction of variation in average stock returns. Cooper et al. (2021) use asymptotic principle component analysis on 42 equity anomalies. They eventually arrive at a model with six factors of economic meaning. Lettau and Pelger (2020b) employ risk-premium PCA (RP-PCA) proposed in Lettau and Pelger (2020a), which in contrast to PCA incorporates information in the first and second moments of the data and takes cross-sectional pricing errors into account. They conclude that there are five significant factors that capture most of the time-series variation in the data. Although I do not have to deal with the issues related to latent and not directly observable factors, the number of factors I employ is thus largely consistent with the results of these studies.

Next, I obtain categorical factor returns. If no enhancement method is applied to these factors, I refer to them as "traditional." I follow a standard way of factor construction. First, I sort firms into quintile portfolios on the country-level, based on each composite score. I then aggregate all firms within a region into regional category quintiles, for which I compute the respective value-weighted portfolio level return, where I transform all stock market data into US-Dollars first for better comparability.[8] Moreover, I only focus on value-weighted portfolio returns as suggested by Green, Hand, and Zhang (2017) or Hou et al. (2020), in order to mitigate issues with respect to micro-cap stocks.

Many characteristics used to construct the trading signals are updated monthly, which accordingly applies to the category scores. The categorical factors are thus also rebalanced each month. I slightly deviate from the construction of Fama and French (1993)-type factors by refraining from using size as sorting dimension and by employing quintile sorts instead of sorts on the 30th and 70th percentile. Size is one of the firm characteristics in the frictions category. Further, I use quintiles, as I want my factors to depend more strongly on the extreme portfolios. My final factors are long the top quintile and short the bottom quintile, such that the expected return for each categorical factor is positive. Effectively, the factors are normalized

[7]See Ahmed, Bu, and Tsvetanov (2019) or Hou, Mo, Xue, and Zhang (2019) for a model comparison.

[8]I do not follow Griffin (2002) in computing factors as dollar-denominated market capitalization-weighted averages of country-level factors, as I would not obtain sufficiently diversified portfolios for many countries when computing hedge factors in the cross-sectional enhancement approach.

to a positive mean, as the top quintile does not always contain the firms with the highest values of the characteristics underlying the corresponding score. For instance, the investment and frictions factors are on average long low investment and frictions characteristics and short high investment and frictions characteristics, as most underlying anomalies have a negative long–short mean return. Table 1 shows monthly summary statistics for the market factor and for each categorical factor by region.

Most factors are of a significant economic magnitude and almost all factors are statistically significant at the 10%-level or lower. The only exceptions are the value factor in North America, the investment factor in Japan, the intangibles factor in Europe, and the friction factor in Emerging Markets. Interestingly, I observe a significant momentum factor return in Japan, which can be attributed to the fact that

TABLE 1

Summary Statistics for Traditional Factors

Table 1 displays summary statistics for traditional composite categorical factors and the market factor. First, firms are sorted into deciles on the country-level, on each of 214 firm characteristics. Subsequently, for each firm, a composite category score is computed as the average decile to which a firms has been assigned to within a category. Next, firms are sorted into quintiles on the country-level based on the composite category score. The country-level quintile assignments are then aggregated to regional quintile portfolios. Regional assignments of countries are based on the MSCI global investable market indexes (GIMI) methodology country classification. Value-weighted quintile portfolio returns are then computed on the regional level, with market capitalization weights based on values transferred to US-Dollars. The respective categorical factors go long the regional top quintile portfolio and short the regional bottom quintile portfolio. The table shows the mean, median, standard deviation, standard error, minimum, maximum, and skewness of monthly factor returns. The sample period extends from Jan. 1995 to June 2019.

Factor	Region	Mean	Median	Std. Dev.	SE	Minimum	Maximum	Skewness
MARKET	NA	0.927	1.501	4.411	0.257	−18.867	11.781	−0.782
	EU	0.779	1.105	5.006	0.292	−21.783	14.746	−0.607
	JA	0.281	0.426	4.895	0.285	−13.087	15.197	0.075
	PA	0.848	1.192	5.785	0.337	−25.118	19.216	−0.507
	EM	0.580	0.862	5.944	0.347	−28.171	15.486	−0.777
VALUE	NA	0.424	−0.041	6.096	0.356	−28.925	31.996	0.421
	EU	0.598	0.632	4.627	0.270	−24.175	17.944	−0.785
	JA	0.957	1.007	4.951	0.289	−17.847	26.416	0.137
	PA	1.168	1.185	5.992	0.349	−41.984	27.752	−0.807
	EM	0.693	0.527	3.690	0.215	−15.267	15.997	0.155
INVESTMENT	NA	0.397	0.422	3.142	0.183	−10.328	18.188	0.816
	EU	0.451	0.340	2.514	0.147	−8.134	11.189	0.512
	JA	0.145	0.173	3.333	0.194	−12.110	11.427	−0.293
	PA	0.535	0.387	4.457	0.260	−20.720	22.890	0.179
	EM	0.379	0.238	2.209	0.129	−7.319	13.421	0.486
PROFITABILITY	NA	0.818	0.625	5.490	0.320	−22.371	25.472	0.081
	EU	0.550	0.790	4.964	0.289	−26.224	15.482	−0.860
	JA	0.469	0.692	3.852	0.225	−12.260	18.326	0.084
	PA	1.163	1.030	6.325	0.369	−40.121	24.909	−0.858
	EM	0.582	0.403	2.987	0.174	−11.946	11.509	−0.069
MOMENTUM	NA	0.914	0.834	5.491	0.320	−22.572	22.814	−0.267
	EU	1.267	1.366	5.541	0.323	−27.320	21.872	−0.699
	JA	0.711	0.771	5.694	0.332	−23.911	19.267	−0.155
	PA	1.595	1.367	5.035	0.294	−18.579	14.539	−0.288
	EM	1.160	1.396	3.245	0.189	−9.348	12.049	−0.168
INTANGIBLES	NA	0.690	0.459	3.336	0.195	−12.600	16.529	0.225
	EU	0.309	0.172	2.916	0.170	−12.447	12.185	0.203
	JA	0.347	0.254	3.229	0.188	−10.865	9.541	−0.204
	PA	0.670	0.538	3.520	0.205	−10.007	15.168	0.383
	EM	0.664	0.433	2.506	0.146	−6.964	12.285	0.853
FRICTIONS	NA	0.561	0.317	6.064	0.354	−31.263	26.130	−0.101
	EU	1.349	1.130	5.680	0.331	−37.761	51.287	1.537
	JA	0.809	0.847	4.020	0.234	−14.635	16.788	0.006
	PA	1.147	1.111	5.098	0.297	−17.815	16.770	−0.233
	EM	0.833	0.889	3.339	0.195	−10.744	12.294	−0.047

https://doi.org/10.1017/S0022109022001090 Published online by Cambridge University Press

the momentum score combines price-based and earnings-based momentum measures, as well as the effects of earnings announcements. While pure price momentum is also not priced in my Japanese sample (mean 0.2% with *t*-stats 0.5), a factor combining the different types of momentum, in contrast, is priced. Moreover, momentum returns are negatively skewed in all regions. Except for the market factor, the other factors do not exhibit any specific relation to skewness. Overall, my factors thus generate significant return spreads, which I require for the factors to capture information about the return cross-section.

III. The Benefits of Cross-Sectional Enhancement

The rationale behind factor pricing is that the expected return of a stock should depend on the stock's exposure to a set of risk factors that price the return cross-section. However, there is evidence that a higher loading on a return (risk) factor is not rewarded with higher returns. Specifically, Daniel and Titman (1997) investigate the questions of whether there is a compensation for firms with higher loadings on factors as introduced by Fama and French (1993) in their seminal paper, and what the respective findings imply for risk premia associated with these factors. They show that sorting on factor betas does not lead to excess returns of corresponding long–short portfolios, suggesting that factor loadings do not explain returns, but the characteristics themselves. These results suggest that factor loadings do not seem to represent the true factor exposure.

Subsequent papers revisit the relation of covariances and returns for further factors and factor models. Chen, Liu, Wang, Wang, and Yu (2020a) study portfolios obtained by sorting on the loadings of 13 popular factors. They find that for none of the examined factors a significant relation between the respective factor loadings and returns can be determined. In a further paper, Chen, Liu, Wang, Wang, and Yu (2020b) argue that if low and high sentiment periods are studied separately, a significant return spread between high and low loadings portfolios can be observed. While Chen et al. (2020a) are more concerned with the implication for performance evaluation, I are interested in the general implications for the efficiency of factors on an international level.

A further paper related to ours is Murray (2020). Motivated by the arbitrage-pricing theory (APT) he creates "arbitrage" portfolios that profit from the flatness of the security market plane, reflected in the return difference of portfolios with different factor betas. The article is similar to Daniel et al. (2020) in that it creates hedge factors from portfolios obtained from preformation betas and that it uses those portfolios to lower the factor risk exposure while maintaining the expected return. The author, however, employs a different way to construct and obtain the hedge factors and hedged portfolios.

I implement the cross-sectional enhancement approach as suggested by Daniel et al. (2020) to improve factor efficiency, who make use of the fact that there is cross-sectional variation in loadings that is unrelated to average returns. The authors develop a theoretical concept for why characteristics, but not loadings are related to returns. I first summarize the rational of the approach and then explain how I specifically implement it with international data.

The traditional factors, as proposed by Fama and French (1993), are sorted on characteristics as book-to-market or size. However, Daniel et al. (2020) argue that these factors are not mean–variance efficient (MVE), as they do not incorporate the full information about the covariance structure of returns. Let β_i be the true future loading and λ the corresponding risk price, such that $E[r_{i,t}] = \mu_{i,t} = \beta_i \times \lambda$. According to economic theory, it thus should be expected that higher loadings are awarded with higher returns. By sorting on characteristics, I assume that these characteristics c_i are proportional to expected returns $\mu_{i,t}$, such that $\mu_{i,t} = c_{i,t} \times \rho$, where ρ is the return premium associated with $c_{i,t}$. If the firm characteristic c_i is supposed to capture risk, that is if c_i is a perfect proxy for future loadings, then the true beta β_i needs to be proportional to the characteristic: $\beta = (\rho/\lambda) \times c \Leftrightarrow \beta \propto c$. However, this will not hold true if a factor is exposed to unpriced risk. In that case, the estimated loading need not be equivalent to the true risk exposure. This can be demonstrated by observing the following relation for realized excess returns:

(1) $$r_{i,t} = \beta_{i,t-1}(f_t + \lambda_{t-1}) + \beta^v_{i,t\text{-}1} f^v_t + \varepsilon_i.$$

According to the above relation, a factor f^c, obtained from a standard sorting procedure on the characteristic c, with return r_i, can thus load on i) common source of variation ($\widehat{=}$risk) $f + \lambda$ and ii) an unpriced factor f^v. f^v is a factor with zero expected return and positive variance and is thus exposed to unpriced risk. The factors will be rendered inefficient if they are exposed to unpriced factor risk, which will be the case if there is a significant correlation between the characteristic c and the loadings on the unpriced factor β^v. The reason is that the same expected return with a lower exposure to risk is achievable. The factor portfolio will accordingly not be mean–variance efficient.

From the framework, it follows that a characteristics-based factor f^c can be enhanced by hedging out unpriced risk. Daniel et al. (2020) suggest to combine the factor portfolio with a hedge portfolio h that has zero exposure to the characteristic c and conditional on the null exposure the maximum correlation with the characteristic sorted portfolio. If we assume that characteristics proxy for the true beta, we would need that characteristics line up with the loadings on the characteristics sorted portfolios in order for f^c to be MVE, as shown above. If this is not the case and firms with the same levels of characteristics would encounter different loadings on f^c, loadings sorted portfolios can be used to construct the hedge factor. Differences in factor loadings should then stem from different exposure to the *unpriced risk* factor. Hedge factors can be constructed in a double-sort procedure as a combination of portfolios long in low-loading stocks and short in high-loading stocks, where the respective portfolio stocks have similar characteristic values. As the hedge portfolio should capture exposure to unpriced risk, its expected return will be zero.

To compute the factor loadings that are subsequently used to construct the hedge portfolios, I employ the current portfolio method, which builds upon the initial idea of Daniel and Titman (1997). Daniel et al. (2020) also refer to the procedure as high power methodology. I make use of the fact that characteristics and factor loadings can be observed ex ante. I start with the market factors and the six categorical factors introduced in Section II. Each regional factor is enhanced separately. First, I compute the factor portfolio returns for each month t and store the corresponding long and short portfolio assignments, as well as the portfolio weights. The current portfolio method traces-back the current month t portfolio assignments and portfolio weights to each of the past 60 months. Subsequently, those assignments and weights are used to calculate backward-looking past *daily* returns for the current portfolio. I thus obtain the returns the current portfolio would have yielded if an investor would have invested in it with month t assignments and weights.

In my main analyses, these backward-looking portfolios are used to obtain factor loadings similar to Frazzini and Pedersen (2014) (FP). Specifically, I compute those loadings as $\beta_i = \rho_{i,f^c}\frac{\sigma(i)}{\sigma(f^c)}$. Correlation coefficients ρ_{i,f^c} are from 5-year windows with overlapping 3 day log-returns and variances are from 1-year windows with daily log-returns. The different time windows are used based on the argument that correlations are more persistent than volatilities (e.g., De Santis and Gerard (1997)). For correlation coefficients, at least 3 years of data are required, which constitutes the first constraint for the final sample period.

There might be concerns whether results employing FP betas are robust or whether this approach to compute loadings might cause issues. Novy-Marx and Velikov (2021) demonstrate that the FP estimate of the stock market's beta (which should be one at all times by definition) exhibits significant fluctuations over time. They also show that Dimson (1979) betas generate (though only slightly) larger beta spreads than FP betas. While I only follow one of the three points criticized by Novy-Marx and Velikov (2021) (I do not use rank weighting or hedging by leveraging), I want to ensure that my results are robust to the beta estimation procedure. For that reason, in addition, I implement all relevant analyses with Dimson (1979) betas, calculated by combining the loadings on contemporaneous, one-day lagged, and one-day forward daily market returns. Further, as betas are estimated they are subject to estimation error. By applying the Dimson estimation procedure, I can also control for and mitigate a potential errors-in-variables problem. Last, the Dimson (1979) betas only require 1 year of daily data in each estimation step. The approach can thus be regarded as an additional robustness check considering the time window to sharpen inferences in some tests that potentially could have relatively lower power. In that case, the analyses can be extended (restricted by the availability of quarterly accounting data) to start in July 1992 instead of Jan. 1995.

To construct the hedge portfolios, I first sort firms into quintiles based on the composite category scores on the country-level. Within each quintile, I sort firms into terciles based on estimated preformation factor loadings. The portfolio assignments are then aggregated on the regional level. This double-sort procedure results in a total of 15 portfolios within each region. For each portfolio, I calculate value-weighted returns. The hedge portfolio h is long an equal-weighted combination of all low-loading portfolios and short an equal-weighted combination of all high-loading portfolios. I deviate from Daniel et al. (2020) in the construction of my hedge portfolios for practical reasons. They rely on a $3 \times 3 \times 3$ sort on the respective firm characteristic, size, and the factor loading. However, I sort firms into portfolios on the country-level, before aggregating the assignments to regional levels. If my approach would be based on 27 portfolios similar to Daniel and Titman

(1997) and Daniel et al. (2020), I would be at risk to obtain a poor assignment quality. By eventually basing my hedge portfolios on fewer portfolios, I ensure that there are sufficient assignments on the country levels.

An important precondition for the application of the hedge portfolios is a significant spread in loadings for firms with similar characteristic values. Table 2 shows postformation factor loadings for each of the 15 portfolios obtained with loadings based on the FP procedure. Results are shown for each of the six categorical factors. I obtain the loadings by running full-sample time-series regressions of each of the 15 portfolio returns on a combination of the six traditional categorical factors and the market factor. The reported loading is the coefficient on the corresponding categorical factor. The table presents the global average values of the regression coefficients and the respective *t*-statistics. The bottommost row for each panel eventually shows the average postformation loading across all 5 low, medium, or high preformation beta portfolios.

TABLE 2

Postformation Factor Loadings

Table 2 presents global postformation factor loadings for 15 portfolios obtained from sorting on the composite category scores and on the preformation βs. First, categorical regional factors are computed and the portfolio assignments and weights are stored. Subsequently, preformation βs are obtained by applying the current portfolio method on the categorical factors. Backward-looking daily portfolio returns for the current portfolios are computed back to 60 months in the past and the returns are used to compute the preformation βs in a similar manner as Frazzini and Pedersen (2014). Within each score quintile, firms are sorted into preformation β terciles on the country-level and then aggregated into regional portfolios. Value-weighted portfolio returns for each of the 15 portfolios are then calculated on the regional level. The postformation factor loadings are obtained by regressing the respective portfolio return on a combination of the corresponding traditional categorical factor, the market factor, and the five remaining categorical factors. The table shows the full sample beta coefficients on the traditional factor and the corresponding *t*-statistics below in parentheses.

Score	Preformation β			
	1	2	3	1–3
Panel A. Value Score				
1	−0.777	−0.276	0.142	−0.919
	(36.393)	(12.553)	(5.535)	(26.070)
2	−0.280	−0.096	0.153	−0.433
	(12.644)	(5.221)	(7.536)	(13.682)
3	−0.144	0.044	0.231	−0.376
	(7.239)	(2.610)	(13.139)	(13.411)
4	−0.013	0.060	0.292	−0.305
	(0.614)	(3.510)	(17.253)	(10.710)
5	0.149	0.192	0.523	−0.374
	(6.084)	(11.371)	(34.710)	(11.715)
$\bar{\beta}$	−0.213	−0.015	0.268	−0.481
Panel B. Investment Score				
1	0.114	0.279	0.719	−0.605
	(4.220)	(13.248)	(34.092)	(16.007)
2	−0.075	0.180	0.401	−0.476
	(3.420)	(8.468)	(15.725)	(12.861)
3	−0.175	0.119	0.357	−0.532
	(7.376)	(5.526)	(14.127)	(13.641)
4	−0.276	0.044	0.289	−0.565
	(11.507)	(1.981)	(11.005)	(14.300)
5	−0.709	−0.044	0.083	−0.792
	(34.858)	(1.898)	(2.988)	(21.727)
$\bar{\beta}$	−0.224	0.116	0.370	−0.594

(continued on next page)

https://doi.org/10.1017/S0022109022001090 Published online by Cambridge University Press

TABLE 2 (continued)

Postformation Factor Loadings

Score	Preformation β 1	2	3	1–3
Panel C. Profitability Score				
1	−1.083	−0.352	0.017	−1.100
	(46.623)	(12.343)	(0.560)	(26.337)
2	−0.526	−0.077	0.217	−0.743
	(19.184)	(3.757)	(8.410)	(18.410)
3	−0.367	−0.082	0.254	−0.621
	(16.011)	(4.465)	(11.431)	(16.985)
4	−0.284	0.062	0.277	−0.561
	(13.309)	(3.938)	(14.287)	(16.762)
5	−0.168	0.081	0.368	−0.535
	(7.426)	(4.810)	(23.484)	(16.899)
$\bar{\beta}$	−0.486	−0.074	0.227	−0.712
Panel D. Momentum Score				
1	−0.774	−0.435	−0.375	−0.398
	(46.643)	(26.269)	(16.746)	(12.862)
2	−0.400	−0.190	−0.092	−0.309
	(20.698)	(11.469)	(4.758)	(10.581)
3	−0.208	−0.067	0.013	−0.221
	(12.073)	(4.537)	(0.800)	(8.423)
4	−0.058	0.031	0.143	−0.201
	(3.484)	(2.245)	(9.308)	(8.153)
5	0.186	0.211	0.389	−0.203
	(10.555)	(14.546)	(29.350)	(8.545)
$\bar{\beta}$	−0.251	−0.090	0.016	−0.266
Panel E. Intangible Score				
1	−0.831	−0.266	0.108	−0.939
	(33.009)	(11.336)	(3.642)	(21.149)
2	−0.477	−0.132	0.261	−0.739
	(16.888)	(5.939)	(8.856)	(16.393)
3	−0.398	−0.111	0.361	−0.759
	(15.419)	(5.278)	(15.026)	(18.953)
4	−0.272	−0.086	0.447	−0.719
	(10.753)	(4.161)	(18.822)	(18.355)
5	−0.250	0.111	0.764	−1.014
	(9.080)	(5.126)	(38.478)	(26.305)
$\bar{\beta}$	−0.446	−0.097	0.388	−0.834
Panel F. Frictions Score				
1	−0.037	0.094	0.396	−0.433
	(1.737)	(5.818)	(25.139)	(14.759)
2	−0.149	0.043	0.238	−0.386
	(6.966)	(2.663)	(13.176)	(12.459)
3	−0.272	0.003	0.225	−0.498
	(11.875)	(0.174)	(11.149)	(14.720)
4	−0.372	−0.080	0.224	−0.596
	(14.295)	(3.863)	(8.582)	(14.905)
5	−0.871	−0.295	−0.058	−0.814
	(34.138)	(14.081)	(2.471)	(21.494)
$\bar{\beta}$	−0.340	−0.047	0.205	−0.545

Spreads in postformation loadings are generally significant. However, the magnitude of the spreads differs across categories. The comparatively lowest spread is obtained for the momentum factor. The average difference in loadings is only 0.26, compared to values in the range of 0.48 to 0.83 in case of the other factors. In this respect, I would expect the weakest effect of cross-sectional enhancement for the momentum factor. Further, it can be seen that the coefficients on the 15 portfolios mostly have magnitudes in the range between -0.8 and 0.8 and are approximately equally distributed across positive and negative values. Remember that the factors are normalized to a positive mean. While the notation 5 is long and 1 is short for every category, the long portfolios for the frictions and investment score mostly consist of firms with low values in the underlying characteristics. Correspondingly, beta estimates are monotonically decreasing in scores for these two categories, while they are increasing for the other categories. Table A1 reports loadings and beta spreads for the market factor and with preformation loadings obtained as Dimson (1979) betas. Spreads are generally similar for both estimation procedures. Overall, the results support the use of Frazzini and Pedersen (2014) type loadings as baseline case.

The preceding analysis has shown that hedge portfolios capture differences in factor loadings, while having the same exposure to characteristics. I make use of these hedge factors h to construct enhanced characteristic-efficient factors. The objective is to construct the enhanced factor f^{h_c} such that is has no exposure to the unpriced factor f^{v}. The perfect hedge will be obtained by combining f^{c} and h such that the variance of the enhanced factor f^{h_c} is minimized. I hedge by forming a linear combination of f^{c} and h:

$$f_t^{h_c} = f_t^{c} - \gamma_{t-1}^{c}{}' h_t. \tag{2}$$

To minimize the variance of f^{h_c}, the variances and covariances of the characteristic sorted factor f^{c} and the hedge factor h are taken into account. The hedge ratio γ is thus defined corresponding to a loading of f^{c} on the hedge factor h: $\widehat{\gamma}_{t-1}^{c} = \rho_{f^c,h} \frac{\sigma(f_t^c)}{\sigma(h_t)}$. To obtain the hedge ratio γ, I use the current portfolio method in the same manner as in case of preformation loadings. In the main analysis, I again employ the Frazzini and Pedersen (2014) procedure for computation of these loadings. I compute backward-looking daily returns using the current hedge portfolio assignments and weights and use a 5-year window to compute correlations, requiring at least 3 years of data, and a 1-year window to compute volatilities. The analyses that are based on preformation factor loadings obtained by the Dimson (1979) method also use γ obtained by the Dimson procedure, employing 1 year of daily data for the corresponding regressions.

In the first step, I hedge each categorical factor with respect to its own hedge factor only. Afterward, I consider enhanced factors that are hedged with respect to the hedge factors of the market factor and all six categorical factors.[9] In this case, I need to obtain a separate γ for each categorical factor – hedge factor combination. Table 3 shows time-series averages of monthly returns of the traditional factors, hedge factors, and enhanced factors and corresponding t-statistics, by category and region. In this case, enhanced factors are only hedged with respect to their own hedge factor.

[9]The market hedge factor is computed as equal-weighted mean of six hedge factors obtained from double sorts on the market beta and the six corresponding composite scores.

TABLE 3

Cross-Sectionally Enhanced Factor Returns

Table 3 shows monthly average returns of traditional factors, hedge-portfolios, and factors cross-sectionally enhanced according to Daniel et al. (2020). Traditional factors f^c are obtained from quintile sorts on the respective category score. Preformation βs used to obtain the hedge portfolios and γs used for hedging are obtained by the current portfolio method, which employs backward-looking factor and hedge-portfolio returns to compute covariances and volatilities. Hedge portfolios are based on 15 portfolios that are obtained from sorting firms within each score quintile into preformation β terciles. The hedge factor is long an equal-weighted combination of the low-loading portfolios and short an equal-weighted combination of the high-loading portfolios. The hedged factor f^{h_c} is obtained by hedging the traditional factor with respect to the hedge factor as in equation (2). Enhanced factors are only hedged with respect to their own hedge factor. Newey and West (1987) adjusted t-statistics are in parentheses. ***, **, and * indicate statistical significance at 1%, 5%, and 10% level, respectively.

Category	Factor	Region				
		North America	Europe	Japan	Asia Pacific	Emerging Markets
VALUE	F^c traditional	0.424 (0.973)	0.598* (1.907)	0.957*** (2.874)	1.168*** (2.924)	0.693*** (3.582)
	Hedge-portfolio h	0.182 (0.797)	−0.074 (−0.302)	0.043 (0.216)	−0.027 (−0.100)	0.533* (1.955)
	F^{h_c} enhanced	0.542* (1.825)	0.560** (2.215)	0.978*** (4.035)	1.129*** (3.489)	0.927*** (4.730)
INVESTMENT	F^c traditional	0.397* (1.954)	0.451*** (2.621)	0.145 (0.737)	0.535* (1.782)	0.379*** (2.673)
	Hedge-portfolio h	0.183 (0.800)	0.041 (0.352)	−0.070 (−0.268)	−0.059 (−0.263)	−0.111 (−0.481)
	F^{h_c} enhanced	0.530*** (3.261)	0.489*** (3.298)	0.117 (0.945)	0.520** (1.990)	0.325** (2.556)
PROFITABILITY	F^c traditional	0.818** (2.330)	0.550** (1.995)	0.469* (1.801)	1.163*** (2.800)	0.582*** (3.392)
	Hedge-portfolio h	−0.296 (−0.786)	−0.124 (−0.526)	−0.049 (−0.189)	−0.064 (−0.238)	−0.096 (−0.245)
	F^{h_c} enhanced	0.731*** (3.248)	0.476** (2.474)	0.450** (2.500)	1.120*** (3.359)	0.553*** (3.810)
MOMENTUM	F^c traditional	0.914*** (2.977)	1.267*** (3.859)	0.711* (1.912)	1.595*** (5.161)	1.160*** (5.192)
	Hedge-portfolio h	−0.233 (−1.380)	−0.124 (−0.723)	−0.140 (−0.750)	−0.023 (−0.124)	−0.390 (−1.624)
	F^{h_c} enhanced	0.790*** (2.621)	1.200*** (4.101)	0.645** (1.981)	1.590*** (5.357)	0.992*** (4.697)
INTANGIBLES	F^c traditional	0.690*** (3.222)	0.309 (1.483)	0.347* (1.852)	0.670*** (3.056)	0.664*** (3.937)
	Hedge-portfolio h	0.078 (0.265)	0.199 (0.786)	−0.196 (−1.040)	−0.124 (−0.447)	−0.016 (−0.067)
	F^{h_c} enhanced	0.763*** (5.083)	0.444*** (3.022)	0.238 (1.358)	0.569*** (3.006)	0.673*** (4.716)
FRICTIONS	F^c traditional	0.561 (1.593)	1.349*** (4.718)	0.809*** (2.834)	1.147*** (3.343)	0.833*** (4.416)
	Hedge-portfolio h	0.003 (0.009)	−0.142 (−0.737)	0.004 (0.021)	−0.175 (−0.708)	0.219 (0.561)
	F^{h_c} enhanced	0.605*** (3.301)	1.244*** (3.967)	0.801*** (4.056)	1.008*** (3.941)	0.891*** (4.689)

Hedge factors are supposed to capture unpriced risk. Expected returns of the hedge factors should thus be insignificant. The second row for each category in Table 3 shows that the average return of most regional hedge factors is indeed close to zero. Table 5, which examines the market factor separately, shows that it also holds true for the market hedge factors. No hedge factor, except the value hedge factor in Emerging Markets, exhibits a statistically significant return. Also, the economic significance stays within reasonable limits. Mostly, the monthly mean return is below 0.2%. Especially in Emerging Markets, the magnitude tends to be higher, as indicated by the value (0.53%) and momentum hedge factor (−0.39%).[10]

I proceed by comparing traditional and enhanced factors. I take two aspects into account: First, the general magnitude of returns, and second the significance level, represented by the *t*-statistics. The objective is to improve upon the efficiency of factors and those statistics can be first indicators to determine whether a potential effect is due to an increase in returns or due to reduction in volatility. Based on the underlying theory, one would expect that primarily significance levels are affected. Indeed, on average the enhanced versions of the value, investment, profitability, intangibles, and frictions factors exhibit returns relatively similar to their traditional counterparts, while their *t*-statistics are larger. This finding indicates an improvement in efficiency, although there are exceptions, especially within North America and Japan. The cross-sectional enhancement approach does generally not work for the momentum factor. Either returns or *t*-statistics decrease, leading to an overall worse performance.

For further comparison of performance, I conduct spanning tests of the enhanced factor portfolios. For each region and each factor, I use two different specifications. In the first specification, I regress the enhanced factor on its traditional factor version. In the second specification, I regress the enhanced factor on a combination of the traditional factor version, the enhanced market factor, and the five other enhanced categorical factors. I present the alphas for the categorical factors in Table 4 and for the market factor in Table 5. I make use of two versions of enhanced factors as dependent variables in the spanning regressions. The factors in Panel A are enhanced with respect to their own hedge factor only. Panel B instead uses factors enhanced with respect to the hedge factors of *all* factors.

A significant alpha implies that the enhanced factor is different from the traditional factor version and generates an excess return. For factors enhanced with respect to their own hedge factor, I can observe significant factor alphas in at least four regions for the value, profitability, frictions, and market factor. The regions that have the lowest number of significant alphas are Japan and Asia-Pacific and they contribute to the weaker results of the investment and intangibles factor. North America is the region with the highest number of significant alphas. The benchmark consisting of a factor combination is strictest to the profitability factor, as most factor alphas are no longer significant when it is employed. Alphas of the momentum factor are significant only in Europe or with the factor combination as benchmark, further supporting the finding that cross-sectional enhancement is not effective in this case. When using factors enhanced with respect to all hedge factors, results are weaker and a considerably lower number of significant alphas can be observed, indicating that the additional hedging does not add value. However, some alphas tend to be of larger economic magnitude.

I further conduct a large-scale analysis and comparison of all potential methods and procedures to construct hedge factors and cross-sectionally hedged factors. These procedures encompass the following choices: Returns from current

[10]In the Supplementary Material, I show that hedge factors based on Dimson (1979) betas provide relatively similar results.

TABLE 4

Spanning Tests of the Cross-Sectionally Enhanced Factor Portfolios

Table 4 presents the results for spanning tests of the cross-sectionally enhanced composite factor portfolios with respect to their traditional factor version (F^c) or to a combination of the traditional factor (F^c), the enhanced market factor, and the other enhanced composite categorical factors ($F^{H_{C\setminus\{c\}}}$) by running monthly time-series regressions of the enhanced factor returns on the reference factor sets. Regressions are conducted for each region separately. The table shows regression alphas for two versions of enhanced factors. Panel A shows results when the traditional factor is enhanced with respect to its own hedge factor only. Panel B shows results when the traditional factor is enhanced with respect to the hedge factors of all categorical factors and the market hedge factor. Newey and West (1987) adjusted t-statistics are in parentheses. ***, **, and * indicate statistical significance at 1%, 5%, and 10% level, respectively.

	FACTOR	VALUE		INVESTMENT		PROFITABILITY		MOMENTUM		INTANGIBLES		FRICTIONS	
Region	Factor Set	F^c	$\text{MKT} + F^c + F^{H_{C\setminus\{c\}}}$	F^c	$\text{MKT} + F^c + F^{H_{C\setminus\{c\}}}$	F^c	$\text{MKT} + F^c + F^{H_{C\setminus\{c\}}}$	F^c	$\text{MKT} + F^c + F^{H_{C\setminus\{c\}}}$	F^c	$\text{MKT} + F^c + F^{H_{C\setminus\{c\}}}$	F^c	$\text{MKT} + F^c + F^{H_{C\setminus\{c\}}}$
Panel A. Dependent Variable = Return to Factor Portfolio Cross-Sectionally Enhanced With Respect to the Factor's Hedge Factor Only													
North America	$\hat{\alpha}$	0.328**	0.411***	0.291***	0.165*	0.368***	0.291**	0.035	0.161*	0.466***	0.206**	0.391***	0.308*
	$t(\hat{\alpha})$	(2.395)	(2.616)	(3.149)	(1.877)	(2.647)	(2.223)	(0.364)	(1.650)	(4.473)	(2.027)	(2.711)	(1.893)
Europe	$\hat{\alpha}$	0.151	0.501***	0.166***	0.152**	0.148	0.004	0.161**	0.240***	0.276***	0.128	0.087	−0.072
	$t(\hat{\alpha})$	(1.162)	(4.056)	(2.897)	(2.372)	(1.420)	(0.040)	(1.968)	(2.911)	(2.962)	(1.238)	(0.535)	(−0.454)
Japan	$\hat{\alpha}$	0.424***	0.343***	0.045	−0.075	0.206**	0.174	0.095	0.313***	0.011	0.054	0.288***	0.402***
	$t(\hat{\alpha})$	(3.610)	(2.723)	(0.467)	(−0.692)	(2.109)	(1.588)	(0.830)	(2.615)	(0.101)	(0.506)	(2.888)	(3.858)
Asia Pacific	$\hat{\alpha}$	0.286*	0.335**	0.092	0.127	0.278*	−0.018	0.120	0.208**	0.125	0.063	0.242**	0.137
	$t(\hat{\alpha})$	(1.819)	(2.166)	(0.971)	(1.379)	(1.672)	(−0.110)	(1.217)	(2.240)	(1.118)	(0.527)	(2.166)	(1.185)
Emerging Markets	$\hat{\alpha}$	0.471***	0.600***	0.045	0.037	0.224**	−0.078	−0.053	−0.155	0.198**	0.276***	0.381***	0.164
	$t(\hat{\alpha})$	(4.011)	(5.038)	(0.650)	(0.504)	(1.960)	(−0.680)	(−0.550)	(−1.534)	(2.456)	(3.073)	(2.583)	(1.111)
Panel B. Dependent Variable = Return to Factor Portfolio Cross-Sectionally Enhanced With Respect to the Hedge Factors of All Factors													
North America	$\hat{\alpha}$	0.985	−0.221	1.054**	0.379	1.766***	0.824***	0.779*	0.371	1.973***	0.320	1.359**	0.142
	$t(\hat{\alpha})$	(1.384)	(−0.602)	(2.529)	(1.295)	(3.344)	(2.635)	(1.707)	(0.786)	(3.708)	(1.006)	(2.155)	(0.435)
Europe	$\hat{\alpha}$	0.023	0.218	0.499***	0.507***	0.490	−0.089	0.901***	0.712***	0.937**	1.143***	0.085	−0.796
	$t(\hat{\alpha})$	(0.053)	(0.809)	(2.645)	(2.587)	(1.074)	(−0.325)	(2.637)	(2.589)	(2.497)	(4.584)	(0.125)	(−1.343)
Japan	$\hat{\alpha}$	0.748	0.701*	0.152	0.339	0.465	0.488**	0.712	0.630	0.000	−0.208	−0.058	−0.198
	$t(\hat{\alpha})$	(1.630)	(1.729)	(0.360)	(0.844)	(1.192)	(2.314)	(1.397)	(1.375)	(0.001)	(−0.711)	(−0.121)	(−0.586)
Asia Pacific	$\hat{\alpha}$	1.097*	−0.104	0.110	−0.109	0.648	−1.187***	−0.068	−0.242	0.450	−0.348	1.262**	0.185
	$t(\hat{\alpha})$	(1.708)	(−0.310)	(0.242)	(−0.384)	(1.148)	(−4.356)	(−0.217)	(−0.861)	(1.101)	(−1.501)	(2.283)	(0.550)
Emerging Markets	$\hat{\alpha}$	0.780***	0.902***	−0.015	−0.449***	0.679*	−0.692***	0.149	−0.289	0.289	0.375**	0.799	−0.080
	$t(\hat{\alpha})$	(3.339)	(5.361)	(−0.090)	(−2.597)	(1.850)	(−3.053)	(0.564)	(−1.378)	(1.510)	(2.182)	(1.615)	(−0.261)

TABLE 5
Market Factor Analysis

Table 5 conducts the analyses concerning cross-sectional end time-series enhancement for the market factor. Panel A shows monthly average returns of the traditional regional market factors, their hedge portfolios, their cross-sectionally enhanced versions, and their volatility-managed market factor versions, as in Tables 3 and 6. Panel B shows results for spanning tests of the cross-sectionally enhanced or volatility-managed market factors with respect to their traditional factor version (F^c) or to a combination of the traditional market factor (F^c), and the other enhanced composite categorical factors ($F^{H_{C \setminus \{c\}}}$), as in Tables 4 and 7, by running monthly time-series regressions of the enhanced factor returns on the reference factor sets. ***, **, and * indicate statistical significance at 1%, 5%, and 10% level, respectively.

Panel A. Time-Series Statistics

Factor	Region				
	North America	Europe	Japan	Asia Pacific	Emerging Markets
F^c traditional	0.927***	0.779**	0.281	0.848**	0.580
	(3.359)	(2.298)	(0.838)	(2.210)	(1.332)
Cross-sectionally enhanced factors					
Hedge-portfolio h	0.082	0.137	0.274	−0.040	−0.069
	(0.286)	(0.600)	(1.014)	(−0.155)	(−0.178)
F^{h_c} enhanced	0.955***	0.915***	0.495**	0.812***	0.505
	(5.666)	(3.802)	(2.328)	(3.014)	(1.470)
Time-series managed factors					
12%	0.875***	0.770***	0.143	0.771***	0.538
	(4.029)	(3.016)	(0.657)	(2.893)	(1.486)
TS-Vol	0.400***	0.420***	−0.004	0.487***	0.405*
	(3.319)	(2.984)	(−0.037)	(2.789)	(1.731)
Dynamic	1.129***	1.106***	0.174	1.401***	1.156*
GJR	(4.130)	(2.859)	(0.366)	(2.745)	(1.794)
Dynamic	0.985***	0.871**	0.117	1.021***	0.624
OOS	(3.652)	(2.592)	(0.613)	(2.826)	(1.497)

Panel B. Factor-Spanning Regressions

Region	Factor Set	F^c	MKT + F^c + $F^{Sc_{C \setminus \{c\}}}$	F^c	MKT + F^c + $F^{Sc_{C \setminus \{c\}}}$
Dependent Variable = Cross-Sectionally Enhanced Market Portfolio					
		ONLY_MARKET_HEDGE_FACTOR		ALL_HEDGE_FACTORS	
North America	$\hat{\alpha}$	0.662***	0.691***	2.429***	1.303***
	$t(\hat{\alpha})$	(4.417)	(4.117)	(3.552)	(2.913)
Europe	$\hat{\alpha}$	0.583***	0.461***	2.240***	1.641***
	$t(\hat{\alpha})$	(3.785)	(2.850)	(3.450)	(4.487)
Japan	$\hat{\alpha}$	0.367**	0.334*	1.168*	1.304***
	$t(\hat{\alpha})$	(2.369)	(1.880)	(1.842)	(3.333)
Asia Pacific	$\hat{\alpha}$	0.448***	0.386**	1.101**	0.845**
	$t(\hat{\alpha})$	(2.615)	(2.035)	(2.018)	(1.966)
Emerging Markets	$\hat{\alpha}$	0.182	0.112	0.283	1.050***
	$t(\hat{\alpha})$	(0.924)	(0.541)	(0.701)	(2.873)
Dependent Variable = Volatility-Scaled Market Portfolio					
		CONSTANT_SCALING		DYNAMIC_SCALING	
North America	$\hat{\alpha}$	0.348**	0.231**	0.310**	0.189*
	$t(\hat{\alpha})$	(2.336)	(2.085)	(1.976)	(1.646)
Europe	$\hat{\alpha}$	0.385**	0.257**	0.304	0.110
	$t(\hat{\alpha})$	(2.074)	(2.351)	(1.520)	(0.958)
Japan	$\hat{\alpha}$	−0.192	−0.026	−0.148	−0.003
	$t(\hat{\alpha})$	(−1.393)	(−0.382)	(−0.996)	(−0.050)
Asia Pacific	$\hat{\alpha}$	0.408*	0.208*	0.320	0.072
	$t(\hat{\alpha})$	(1.713)	(1.849)	(1.237)	(0.609)
Emerging Markets	$\hat{\alpha}$	0.363	0.083	0.440	0.065
	$t(\hat{\alpha})$	(1.415)	(0.606)	(1.544)	(0.436)

method versus regular historical returns for computation of β and γ loadings, FP versus Dimson procedure for computation of loadings, and hedging with respect to own hedge factor only versus hedging with respect to all seven hedge factors. Results are reported in Table A2. Taken all together, hedging with respect to the factor's own hedge factor outperforms hedging with respect to all hedge factors, as suggested by results in Table 4. The current portfolio method delivers better results than the regular approach, especially when FP type loadings are employed. Lastly, FP type loadings generate slightly better results than Dimson-type loadings. The difference is most pronounced in the Asia-Pacific region. Therefore, in the main analysis, I focus on one specification consisting of cross-sectionally enhanced factors hedged with respect to their own hedge factor that use the current method and the FP procedure to estimation loadings.

IV. The Benefits of Time-Series Enhancement

The preceding chapter has shown that cross-sectional information can have significant implications for returns. In this chapter, I show that this can likewise hold for time-series variation and information. To start with, it is a well-established fact that the momentum strategy as introduced by Jegadeesh and Titman (1993) exhibits strong crashes. These crashes are characterized by strongly negative returns within a short time period, implying a negatively skewed distribution of returns. Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) introduce volatility-managing time-series techniques in order to minimize the crash risk associated with the momentum strategy. I am interested in understanding whether these approaches might also help in enhancing international categorical factors.

Volatility-management studies do not only have investigated and focused on the momentum factor. Moreira and Muir (2017) investigate the effect on 9 further return factors and conclude that for most other factors volatility management improves Sharpe ratios as well. While they employ time-series regressions to estimate the effect on the Sharpe ratio (similar to me in Tables 4 and 7), Cederburg et al. (2020) investigate 103 managed strategy returns directly and conclude in contrast that volatility management only works consistently for characteristics in the momentum category.[11] Analyzing whether time-series volatility management enhances international categorical factors can thus give me an indication for which group of characteristics volatility management works and whether I can thus support either the results of Moreira and Muir (2017) or Cederburg et al. (2020) internationally.

An important observation concerning momentum crashes is that they occur when volatility was recently high. Correspondingly, Barroso and Santa-Clara (2015) show that past realized volatility predicts momentum returns. I confirm this result by running pooled OLS regressions across regions of traditional and cross-sectionally enhanced factor returns on past 6- and 1-month realized volatility, presented in Table A3. For all momentum factor versions, the regression coefficients are significantly negative. Apart from that, negative coefficients can only be observed for the market factor, but not for the other factors. The meaning of a

[11]Further papers that implement volatility management successfully are Barroso, Detzel, and Maio (2021) for the betting-against beta strategy and the Hou et al. (2015) ROE factor, or Eisdorfer and Misirli (2020) for the financial distress measure of Campbell, Hilscher, and Szilagyi (2008) and the corresponding healthy-minus-distress strategy.

significant and negative coefficient for time-series enhancement is similar to the meaning of a large postformation beta spread for cross-sectional enhancement. In this respect, I would expect the strongest effect of time-series enhancement for the momentum and market factor.

The general idea of volatility management is to either take a levered or conservative position in the factor, depending on past volatility. Investments are thus reduced when volatility has recently been high and scaled up in low-volatility periods. Barroso and Santa-Clara (2015) suggest a constant scaling strategy. They use monthly realized volatility $\widehat{\sigma}$ from past 6 month daily returns and scale the factor to a target level of volatility of 12%:

$$f_t^{\sigma*_c} = \frac{c}{\widehat{\sigma}_{t-1}} f_t^c \quad \text{with} \quad c = \sigma^{\text{target}}. \tag{3}$$

In subsequent studies, Moreira and Muir (2017) and Cederburg et al. (2020) employ a similar scaling approach. However, their approach differs in that they chose c such that the traditional factor f_t^c and the managed factor $f_t^{\sigma*_c}$ have the same variance. They also diverge in how they compute past volatility, as they only use daily returns of the preceding month.

Daniel and Moskowitz (2016) take a different approach by employing a dynamic scaling strategy using both volatility *and* predicted returns as input parameters for scaling. Predicted returns are obtained by first conducting an OLS regression of monthly returns on an interaction term between a bear market indicator, equal to 1 if the cumulative market return over the past 24 months is negative, and the market variance computed from past 6 months daily returns. Parameter estimates are then used to compute the expected returns μ_{t-1}. Together with an estimate of expected variance, the strategy is scaled such that the conditional Sharpe ratio is maximized[12]:

$$f_t^{\sigma*_c} = w_{t-1}^* f_t^c \quad \text{with} \quad w_{t-1}^* = \left(\frac{1}{2\lambda}\right) \frac{\mu_{t-1}}{\widehat{\sigma}_{t-1}}. \tag{4}$$

Daniel and Moskowitz (2016) use both an in-sample and out-of-sample approach. For the in-sample version, they use a GJR-GARCH model to forecast the volatility as proposed by Glosten, Jagannathan, and Runkle (1993) (GJR).[13] The GJR estimate and past 6 months realized volatility are then fitted on realized volatility of daily returns in the subsequent month to obtain the final estimate. The out-of-sample approach uses data from the start of the sample up to the preceding month in predictive OLS regressions and only realized volatility from past 6 month daily returns as input for expected volatility.[14]

[12] λ in this equation is a time-constant scalar.

[13] The variance process is described as $\sigma_t^2 = \beta_0 + \beta_1 \sigma_{t-1}^2 + \gamma_1 \varepsilon_{t-1}^2 + \gamma_2 I(\varepsilon_{t-1} < 0)\varepsilon_{t-1}^2$ and parameters are estimated by maximum likelihood on the full sample.

[14] Ehsani and Linnainmaa (2021) suggest time-series efficient factors based on the exploitation of factor autocorrelation. They thus include a (factor) momentum effect in their construction method, while I examine momentum separately. Further, their way of estimating means, volatilities, and autocorrelations imposes a further time constraint. Results that are reported in the Supplementary Material suggest that on average there is no further improvement when their method is employed.

https://doi.org/10.1017/S0022109022001090 Published online by Cambridge University Press

In a first step, I compare raw returns of traditional factors to returns obtained by the constant scaling approaches as employed in Barroso and Santa-Clara (2015) and Moreira and Muir (2017), and to returns obtained by the dynamic in-sample and dynamic out-of-sample approach as proposed by Daniel and Moskowitz (2016). Tables 5 and 6 show monthly average returns and the corresponding *t*-statistics for these 5-factor versions for the market and the categorical factors, respectively.

Similar as in the case of cross-sectional enhancement, again I take both the magnitude and significance levels of returns into account. The only composite factor for which there is a consistent and significant improvement is the momentum factor. Although the factor signal is not only composed of price-based measures, I can observe an increase in both returns and especially significance levels within all regions. The picture for the other factors is much less clear. For the value, profitability and intangibles factors there is a mixture of slightly lower and less significant and slightly higher and more significant returns across regions. For the investment and frictions factors, there is mostly no real effect observable. However, there is a tendency for an improvement in the market factor. Whether an effect is due to an increase in returns or due to reduction in volatility could potentially also depend on how the factor is scaled. Both the constant and dynamic scaling approaches generally lead to similar enhancement effects. In the following, I will therefore focus on one constant scaling approach, where I scale to the time-series average volatility level, and the dynamic scaling out-of-sample approach, as it better represents the real-time experience of an investor.

Table 7 conducts factor-spanning regressions, analogous to Table 4. Regressions are run with respect to the traditional factor version or to a combination of the traditional factor version, the scaled market factor, and the other five scaled categorical factors. For the momentum factor, I can observe a significant alpha for almost all regional specifications. Only the specification using the benchmark consisting of a factor combination on the regional Asian-Pacific factor that employs constant scaling yields statistically insignificant results. The effect of the constant and dynamic scaling approach is relatively similar. The dynamic scaling approach mostly comes with a higher economic magnitude of the alphas, except for North America. The other categorical factors do mostly not show significant alphas. There are only a few exceptions that are consistent over scaling approaches (the investment factor in Emerging Markets and the profitability factor in Europe and Emerging Markets). For the market factor, significant alphas across scaling approaches can only be observed for North America.

Generally, there is pervasive evidence that volatility scaling leads to an improvement in performance for the categorical momentum factor internationally. Based on the results, I can also conclude that volatility-scaling is not consistently successful for all categories except momentum, supporting the conclusion of Cederburg et al. (2020).

V. Comparing Factor Performance

In the two previous sections, I have introduced and implemented two different factor enhancement techniques: Cross-sectional enhancement of factors by incorporating information about the covariance structure in the cross-section of stock

TABLE 6

Time-Series Managed Factor Returns

Table 6 shows monthly average returns of traditional and volatility-managed categorical factors. The raw returns of the unmanaged traditional factors and of four types of managed factors are shown, were the approaches of Barroso and Santa-Clara (2015), Moreira and Muir (2017), Cederburg et al. (2020), and Daniel and Moskowitz (2016) are employed, respectively. The first approach scales the strategy to an annualized target volatility of 12%, following Barroso and Santa-Clara (2015) (12%). The second approach scales the annualized target volatility to the time-series average volatility of the respective factor as in Moreira and Muir (2017) and Cederburg et al. (2020) (TS-Vol). Dynamic scaling according to Daniel and Moskowitz (2016) additionally includes an estimate of expected returns in scaling the strategy. The first dynamic approach combines an in-sample volatility estimate based on the Glosten et al. (1993) GJR-GARCH with realized volatility (GJR). The second dynamic out-of-sample approach only uses realized volatility based on past information as volatility estimate (OOS). Newey and West (1987) adjusted *t*-statistics are in parentheses. ***, **, and * indicate statistical significance at 1%, 5%, and 10% level, respectively.

Category	Factor	Region					Category	Factor	Region				
		North America	Europe	Japan	Asia-Pacific	Emerging Markets			North America	Europe	Japan	Asia-Pacific	Emerging Markets
VALUE	Trad	0.424	0.598*	0.957***	1.168***	0.693***	MOMENTUM	Trad	0.914***	1.267***	0.711*	1.595***	1.160***
		(0.973)	(1.907)	(2.874)	(2.924)	(3.582)			(2.977)	(3.859)	(1.912)	(5.161)	(5.192)
	12%	0.047	0.424	0.671**	0.703**	0.886***		12%	1.045***	1.635***	0.744**	1.677***	1.842***
		(0.194)	(1.393)	(2.508)	(2.509)	(3.568)			(4.057)	(5.444)	(2.582)	(6.487)	(6.057)
	TS-Vol	0.093	0.637*	1.243***	1.367***	0.977***		TS-Vol	1.488***	2.315***	1.473***	2.389***	2.164***
		(0.208)	(1.896)	(3.283)	(2.907)	(3.367)			(3.718)	(6.538)	(2.882)	(5.640)	(6.068)
	Dynamic	−0.046	0.343	0.633**	0.941**	0.825***		Dynamic	1.335***	1.948***	1.152***	2.136***	1.589***
	GJR	(−0.145)	(1.194)	(2.358)	(2.508)	(3.514)		GJR	(4.394)	(5.763)	(3.072)	(7.194)	(6.497)
	Dynamic	−0.096	0.208	0.455**	0.749**	0.961***		Dynamic	0.887***	1.363***	1.230***	2.598***	1.965***
	OOS	(−0.426)	(0.988)	(2.247)	(2.269)	(3.220)		OOS	(3.808)	(5.149)	(2.635)	(6.735)	(6.265)
INVESTMENT	Trad	0.397*	0.451***	0.145	0.535*	0.379***	INTANGIBLES	Trad	0.690***	0.309	0.347*	0.670***	0.664***
		(1.954)	(2.621)	(0.737)	(1.782)	(2.673)			(3.222)	(1.483)	(1.852)	(3.056)	(3.937)
	12%	0.318	0.540**	0.150	0.429*	0.851***		12%	0.893***	0.405	0.434*	0.570**	0.950***
		(1.446)	(2.364)	(0.596)	(1.672)	(3.025)			(4.025)	(1.541)	(1.749)	(2.479)	(4.352)
	TS-Vol	0.324	0.492***	0.356	0.564	0.659***		TS-Vol	0.908***	0.273	0.449*	0.656***	1.010***
		(1.485)	(2.734)	(1.216)	(1.639)	(3.086)			(3.539)	(1.402)	(1.678)	(2.627)	(3.934)
	Dynamic	0.267	0.340**	0.071	0.425	0.565***		Dynamic	0.821***	0.291	0.301	0.572***	0.759***
	GJR	(1.534)	(2.337)	(0.395)	(1.408)	(3.213)		GJR	(4.482)	(1.476)	(1.589)	(2.663)	(4.253)
	Dynamic	0.095	0.245*	0.095	0.445	0.750***		Dynamic	0.745***	0.186	0.173	0.385*	0.825***
	OOS	(0.599)	(1.826)	(0.592)	(1.277)	(2.941)		OOS	(4.327)	(1.148)	(1.196)	(1.691)	(4.153)
PROFITABILITY	Trad	0.818**	0.550**	0.469*	1.163***	0.582***	FRICTIONS	Trad	0.561	1.349***	0.809***	1.147***	0.833***
		(2.330)	(1.995)	(1.801)	(2.800)	(3.392)			(1.593)	(4.718)	(2.834)	(3.343)	(4.416)
	12%	0.717***	0.829***	0.573*	0.942***	0.879***		12%	0.339	1.107***	0.670***	0.924***	0.918***
		(2.608)	(2.945)	(1.957)	(2.774)	(3.673)			(1.593)	(4.983)	(2.712)	(3.364)	(4.329)
	TS-Vol	1.072**	0.742**	0.663*	1.805***	0.928***		TS-Vol	0.409	1.307***	0.979***	1.452***	0.930***
		(2.542)	(2.255)	(1.922)	(2.889)	(3.746)			(1.058)	(3.679)	(3.197)	(3.565)	(3.610)
	Dynamic	0.826**	0.959***	0.456**	1.282***	0.739***		Dynamic	0.536*	1.424***	0.638**	1.222***	0.835***
	GJR	(2.354)	(3.418)	(1.981)	(3.219)	(3.746)		GJR	(1.755)	(4.789)	(2.276)	(3.829)	(4.150)
	Dynamic	0.696**	0.668***	0.347**	1.278***	0.942***		Dynamic	0.261	1.235***	0.589**	1.103***	0.888***
	OOS	(2.255)	(3.208)	(2.050)	(2.666)	(3.702)		OOS	(1.084)	(4.653)	(2.515)	(2.976)	(3.797)

TABLE 7

Spanning Tests of the Volatility-Scaled Factor Portfolios

Table 7 presents the results for spanning tests of the volatility-scaled composite factor portfolios with respect to the traditional factor (F^c) or to a combination of the traditional factor (F^c), the volatility-scaled market factor, and the other volatility-scaled composite categorical factors ($F^{H_{C\setminus\{c\}}}$) by running monthly time-series regressions of the enhanced factor returns on the reference factor sets. Regressions are conducted for each region separately. The table shows regression alphas for two versions of volatility-scaled factors. Panel A shows results when constant scaling is applied on the factors, by scaling the factors to their time-series average volatility as in Moreira and Muir (2017) and Cederburg et al. (2020). Panel B shows results with dynamic out-of-sample volatility scaling as in Daniel and Moskowitz (2016). Newey and West (1987) adjusted *t*-statistics are in parentheses. ***, **, and * indicate statistical significance at 1%, 5%, and 10% level, respectively.

	FACTOR	VALUE		INVESTMENT		PROFITABILITY		MOMENTUM		INTANGIBLES		FRICTIONS	
Region	Factor Set	F^c	MKT + F^c + $F^{Sc_{C\setminus\{c\}}}$	F^c	MKT + F^c + $F^{Sc_{C\setminus\{c\}}}$	F^c	MKT + F^c + $F^{Sc_{C\setminus\{c\}}}$	F^c	MKT + F^c + $F^{Sc_{C\setminus\{c\}}}$	F^c	MKT + F^c + $F^{Sc_{C\setminus\{c\}}}$	F^c	MKT + F^c + $F^{Sc_{C\setminus\{c\}}}$
Panel A. Dependent Variable = Return to Factor Portfolio Constant Volatility-Scaled to Time-Series Average Volatility													
North	$\hat{\alpha}$	−0.310	−0.048	−0.026	−0.074	0.387	0.311	0.592**	0.474**	0.154	0.026	−0.056	−0.020
America	$t(\hat{\alpha})$	(−1.224)	(−0.173)	(−0.190)	(−0.541)	(1.354)	(1.154)	(2.507)	(2.053)	(1.137)	(0.180)	(−0.238)	(−0.081)
Europe	$\hat{\alpha}$	0.052	0.112	0.061	0.113	0.269*	0.196	1.191***	1.160***	0.009	0.075	−0.146	−0.257
	$t(\hat{\alpha})$	(0.321)	(0.597)	(0.673)	(1.123)	(1.666)	(1.128)	(5.988)	(5.524)	(0.095)	(0.722)	(−0.858)	(−1.355)
Japan	$\hat{\alpha}$	0.225*	0.220	0.161	0.090	0.090	−0.014	0.633**	0.459*	0.038	−0.079	0.113	0.083
	$t(\hat{\alpha})$	(1.685)	(1.589)	(1.442)	(0.744)	(0.606)	(−0.103)	(2.365)	(1.703)	(0.327)	(−0.663)	(0.781)	(0.539)
Asia	$\hat{\alpha}$	0.108	−0.182	0.059	−0.081	0.288	−0.178	0.520**	0.338	−0.087	−0.225**	0.230	0.025
Pacific	$t(\hat{\alpha})$	(0.493)	(−0.863)	(0.346)	(−0.446)	(0.974)	(−0.618)	(2.393)	(1.523)	(−0.870)	(−2.151)	(1.166)	(0.136)
Emerging	$\hat{\alpha}$	0.185	0.007	0.163*	0.049	0.208**	0.052	0.540***	0.559***	0.010	0.043	−0.062	−0.197
Markets	$t(\hat{\alpha})$	(1.346)	(0.047)	(1.796)	(0.506)	(2.408)	(0.546)	(3.531)	(3.472)	(0.111)	(0.436)	(−0.559)	(−1.617)
Panel B. Dependent Variable = Return to Factor Portfolio Volatility Scaled With Dynamic Out-of-Sample Approach													
North	$\hat{\alpha}$	−0.275	−0.073	−0.112	−0.184	0.422	0.076	0.572***	0.380**	0.309**	0.255*	0.053	0.215
America	$t(\hat{\alpha})$	(−1.449)	(−0.431)	(−0.889)	(−1.477)	(1.492)	(0.337)	(2.917)	(2.535)	(2.238)	(1.889)	(0.257)	(1.163)
Europe	$\hat{\alpha}$	−0.128	−0.119	−0.043	−0.051	0.556***	0.058	0.987***	0.618***	0.013	−0.046	0.361*	0.174
	$t(\hat{\alpha})$	(−0.863)	(−0.706)	(−0.453)	(−0.474)	(2.918)	(0.341)	(4.112)	(3.231)	(0.117)	(−0.414)	(1.949)	(0.870)
Japan	$\hat{\alpha}$	0.029	−0.148	0.029	0.044	0.146	0.103	0.796*	0.793**	0.021	−0.198**	0.106	0.066
	$t(\hat{\alpha})$	(0.183)	(−1.101)	(0.236)	(0.352)	(1.363)	(1.265)	(1.917)	(2.001)	(0.193)	(−2.064)	(0.606)	(0.400)
Asia	$\hat{\alpha}$	−0.048	−0.378*	0.114	−0.326	0.352	−0.494	1.444***	1.000***	−0.208	−0.430***	0.156	−0.188
Pacific	$t(\hat{\alpha})$	(−0.206)	(−1.818)	(0.372)	(−1.066)	(0.953)	(−1.621)	(4.262)	(3.207)	(−1.477)	(−3.606)	(0.540)	(−0.823)
Emerging	$\hat{\alpha}$	0.292	0.057	0.400**	0.094	0.336*	−0.039	0.883***	0.687***	0.091	0.223	0.103	−0.078
Markets	$t(\hat{\alpha})$	(1.498)	(0.289)	(1.996)	(0.443)	(1.917)	(−0.205)	(3.872)	(2.904)	(0.643)	(1.415)	(0.671)	(−0.492)

returns and volatility-management techniques in the time series. As outlined, the main objective of enhancement is to improve the efficiency of factors. In the following, I thus want to determine which effective efficiency gains can be achieved by implementing factor enhancement.

First, I focus on the Sharpe ratio as a measure for efficiency. There is a clear intuition behind this choice. Optimally, I observe a set of factors that span the mean–variance efficient frontier. Factor enhancement is supposed to make factors more efficient, such that the optimal combination of the enhanced factors will be closer to the efficient frontier. This effect should eventually be reflected in an increase in the Sharpe ratio, both of the individual enhanced factors, as well as of the optimal in-sample Markowitz portfolio.

Subsequently, I focus on explanatory power. If researchers find a significant alpha with respect to a benchmark model in an anomaly study, they usually interpret this as evidence that there might be a further factor that explains the cross-section of returns beyond the factors already known. Hou et al. (2020) show that many of those anomalies are not significant out-of-sample, even without controlling for any benchmark model. I hypothesize that with more rigorous testing by relying on a more efficient benchmark factor model, there might be less anomalies that survive. I, therefore, test the explanatory power of different factor model versions for a multitude of anomalies.

A. Comparing Sharpe Ratios

To compare the efficiency of factors, I consider six versions of each categorical factor and the market factor, obtained by applying both introduced enhancement approaches. First, I distinguish between unhedged and hedged factors. For the hedged factors, I apply cross-sectional enhancement as in Daniel et al. (2020) by hedging the categorical factors with respect to their *own* hedge factor only. Next, I perform volatility-scaling on both the unhedged and hedged factor versions. By scaling the cross-sectionally hedged factors, I combine both enhancement approaches. For volatility management, I employ both constant scaling as in Moreira and Muir (2017) and Cederburg et al. (2020) and dynamic out-of-sample scaling as in Daniel and Moskowitz (2016). I compute the compounded annualized factor return and the corresponding annualized Sharpe ratio for all factor versions. Table 8 reports the results for the market factor and each categorical factor by region.[15]

After cross-sectional enhancement, a clear quantitative improvement in the Sharpe ratio of the value and profitability factor in all regions can be observed. The Sharpe ratio of the investment factor improves in all regions except for Emerging Markets. There, however, the factor profits from a combination of enhancement approaches. The effect on the intangibles and friction factor is more regionally diverse. In each case, cross-sectional enhancement leads to an increases in the Sharpe ratio in 3 out of 5 regions. For these five categorical factors in the majority of cases, an increase in the Sharpe ratio of significant economic magnitude can

[15]Here, I use annually compounded returns for computation of Sharpe ratios, unlike various papers in the literature, which multiply the monthly average return by 12 for annualization. A comparison of both annualization approaches and their resulting Sharpe ratios can be found in the Supplementary Material.

TABLE 8

Sharpe Ratio Improvement

Table 8 compares Sharpe ratios of traditional factors with Sharpe ratios of enhanced factors. For each region and each composite categorical factor, as well as the market factors, the compounded annualized returns and the corresponding Sharpe ratio are shown. Six different factor versions are compared: The unhedged traditional factors, the cross-sectionally hedged factors, the unhedged volatility-managed factors, where I apply both the constant scaling approach of Moreira and Muir (2017) and Cederburg et al. (2020) and the dynamic scaling out-of-sample approach of Daniel and Moskowitz (2016), and the cross-sectionally hedged volatility-managed factors, where I apply the same constant and dynamic scaling approach as for the unhedged factors.

CATEGORY		MARKET				VALUE				INVESTMENT			
Region	Factor	Unhedged		Hedged		Unhedged		Hedged		Unhedged		Hedged	
		Ann. Return	Sharpe Ratio	Ann. Return	Sharpe Ratio	Ann. Return	Sharpe Ratio	Ann. Return	Sharpe Ratio	Ann. Return	Sharpe Ratio	Ann. Return	Sharpe Ratio
North America	Unscaled	11.715	0.676	11.648	1.014	7.746	0.194	8.312	0.323	5.170	0.358	6.955	0.608
	Scaled TS-Vol	15.184	0.774	16.558	0.974	2.163	0.068	9.430	0.304	4.166	0.316	5.547	0.627
	Scaled dynamic	13.244	0.628	9.611	0.920	−1.193	−0.093	6.106	0.350	1.201	0.134	3.696	0.451
Europe	Unscaled	10.006	0.491	11.977	0.772	7.920	0.379	7.084	0.457	5.395	0.492	5.865	0.596
	Scaled TS-Vol	15.649	0.569	19.043	0.855	8.289	0.389	6.534	0.390	5.786	0.508	6.337	0.670
	Scaled dynamic	14.109	0.590	15.279	0.843	2.624	0.227	2.332	0.227	2.476	0.338	3.045	0.505
Japan	Unscaled	4.241	0.188	5.926	0.443	12.630	0.628	12.916	0.839	2.104	0.167	1.451	0.192
	Scaled TS-Vol	3.953	0.124	5.980	0.294	16.941	0.694	18.752	0.938	5.105	0.273	3.114	0.285
	Scaled dynamic	6.305	0.319	7.171	0.537	6.178	0.471	7.349	0.695	1.453	0.153	0.885	0.164
Asia Pacific	Unscaled	10.935	0.445	10.635	0.571	15.945	0.531	14.952	0.662	7.263	0.360	6.847	0.412
	Scaled TS-Vol	19.527	0.520	23.709	0.662	18.995	0.526	23.865	0.627	8.023	0.363	9.048	0.471
	Scaled dynamic	12.121	0.553	17.463	0.642	9.966	0.445	14.229	0.618	6.547	0.294	6.903	0.385
Emerging markets	Unscaled	8.437	0.299	7.108	0.319	8.940	0.723	12.411	0.928	4.674	0.512	4.027	0.503
	Scaled TS-Vol	18.592	0.387	20.100	0.386	13.356	0.657	18.234	0.895	8.297	0.593	7.125	0.590
	Scaled dynamic	8.963	0.352	9.235	0.365	13.415	0.676	15.805	0.865	9.813	0.577	8.283	0.610

(continued on next page)

TABLE 8 (continued)

Sharpe Ratio Improvement

Category		PROFITABILITY				MOMENTUM				INTANGIBLES				FRICTIONS			
Region	Factor	Unhedged		Hedged		Unhedged		Hedged		Unhedged		Hedged		Unhedged		Hedged	
		Ann. Return	Sharpe Ratio	Ann. Return	Sharpe Ratio	Ann. Return	Sharpe Ratio	Ann. Return	Sharpe Ratio	Ann. Return	Sharpe Ratio	Ann. Return	Sharpe Ratio	Ann. Return	Sharpe Ratio	Ann. Return	Sharpe Ratio
NA	Unsc.	11.171	0.426	9.864	0.582	11.326	0.588	10.228	0.505	9.301	0.607	10.006	0.968	6.731	0.273	7.278	0.570
	Sc^{Vol}	15.479	0.506	8.942	0.548	20.139	0.712	16.590	0.614	12.365	0.709	11.794	1.092	4.865	0.203	6.510	0.383
	Sc^{Dyn}	10.134	0.448	6.105	0.479	11.612	0.701	10.924	0.612	9.784	0.855	9.474	1.251	2.847	0.197	5.015	0.377
EU	Unsc.	6.238	0.381	5.657	0.484	15.526	0.806	14.738	0.859	4.138	0.318	5.855	0.576	16.458	0.864	15.287	0.756
	Sc^{Vol}	9.088	0.428	6.140	0.427	30.858	1.231	29.607	1.227	3.678	0.291	5.260	0.547	15.643	0.664	20.290	0.699
	Sc^{Dyn}	8.455	0.564	5.996	0.553	18.338	0.832	15.313	0.839	2.865	0.249	4.489	0.552	14.719	0.918	13.978	0.894
JA	Unsc.	5.927	0.350	5.552	0.455	9.094	0.416	8.556	0.430	4.212	0.365	2.947	0.308	10.703	0.523	10.373	0.790
	Sc^{Vol}	8.345	0.381	4.817	0.456	20.489	0.572	17.297	0.546	5.906	0.326	4.153	0.350	12.864	0.623	11.313	0.837
	Sc^{Dyn}	4.535	0.371	2.912	0.431	18.439	0.450	15.983	0.468	2.471	0.246	1.237	0.237	8.389	0.465	6.539	0.565
PA	Unsc.	14.819	0.497	14.022	0.610	20.643	1.106	20.739	1.183	8.605	0.573	7.264	0.585	15.203	0.641	12.975	0.749
	Sc^{Vol}	25.017	0.517	25.473	0.618	33.228	1.098	37.402	1.136	8.330	0.521	7.025	0.558	19.421	0.684	17.122	0.750
	Sc^{Dyn}	16.577	0.475	17.503	0.551	36.940	1.172	35.921	1.123	4.770	0.352	5.711	0.546	14.479	0.592	12.286	0.604
EM	Unsc.	7.063	0.620	6.668	0.740	15.257	1.039	12.819	0.946	8.388	0.725	8.508	0.913	10.476	0.829	11.041	0.884
	Sc^{Vol}	11.813	0.670	8.767	0.743	30.679	1.119	29.487	1.022	12.610	0.715	13.180	0.950	12.193	0.698	17.640	0.865
	Sc^{Dyn}	11.978	0.644	8.552	0.585	27.523	1.110	27.331	0.979	10.295	0.747	10.808	1.076	11.468	0.712	17.893	0.917

thus be achieved by incorporating information about the return cross-section. There are only few cases in which volatility management improves their Sharpe ratio, such as the profitability factor in Europe and the intangibles factor in North America. Combining both enhancement approaches does often lead to further, albeit mostly only mild improvements in their Sharpe ratios.

The market factor is the only factor where a combination of enhancement approaches is consistently and markedly successful. While cross-sectional enhancement achieves strong results for the market factor in all regions except for Emerging Markets, also volatility-management improves the market factor's Sharpe ratio, however, the effect is less pronounced. In contrast, combining the enhancement approaches leads to the strongest increases in the Sharpe ratio of the market factor.

The momentum factor can be regarded as the most unique among the six studied categorical factors. By employing volatility-scaling, the Sharpe ratio can be significantly improved in North America, Europe, Japan, and Emerging Markets. For the Asian-Pacific momentum factor, I do not observe a clear effect, however, the Sharpe ratio of the traditional factor is already comparatively high (1.1). Generally, it holds that the constant and dynamic scaling approach generate relatively similar results, with each having an advantage in different regions.[16] Cross-sectional enhancement mostly deteriorates the momentum factor's Sharpe ratio or leaves it unchanged at best.

The comparison of efficiency of traditional and enhanced factors has so far relied on scrutinizing the economic magnitude of Sharpe ratios. To obtain a more complete picture of the significance and consistency of the effects of factor enhancement, I conduct a formal Sharpe ratio analysis following Barillas, Kan, Robotti, and Shanken (2020). My formal comparison analysis is carried out for squared Sharpe ratios, assuming that the studied factors are observable. The enhanced factors are based on traded portfolios (traditional and hedge factors) and employ leverage (implied by scaling). Both features are implementable, which makes my factors realistically tradable. Barillas et al. (2020) show how to compare the squared Sharpe ratio $\widehat{\theta}^2$ of two nonnested factor models i and j and propose an asymptotic distribution for the difference in sample squared Sharpe ratios $\widehat{\theta}_i^2 - \widehat{\theta}_j^2$. I implement the procedure based on the assumption that I compare non-nested 1-factor models.

Table 9 reports the results of tests of equality of squared Sharpe ratios of traditional factors and their corresponding enhanced factor versions. I report both the difference in squared Sharpe ratios and the corresponding t-statistics based on the asymptotic variance from proposition (1) in Barillas et al. (2020). For the sake of brevity, in this analysis, I only focus on the constant volatility-scaling approach. Generally, the results confirm the conclusion that cross-sectional enhancement works well in case of international factors, except for the momentum factor. Corresponding to the preceding Sharpe ratio analysis, the findings suggest significant

[16]A comparison of results for the constant scaling approach of Barroso and Santa-Clara (2015) and the in-sample dynamic scaling approach of Daniel and Moskowitz (2016) can be found in the Supplementary Material.

TABLE 9

Tests of Equality of Squared Sharpe Ratios

Table 9 shows pairwise test of equality of squared Sharpe ratios of traditional and enhanced factors following Barillas et al. (2020). The left column for each region reports the difference $\widehat{\theta}_i^2 - \widehat{\theta}_j^2$ between the sample squared Sharpe ratio of the enhanced factor version i (applied enhancement method as indicated in each row) and the traditional factor version j. The right column for each region reports the associated *t*-statistics for the test of $H_0: \widehat{\theta}_i^2 = \widehat{\theta}_j^2$. ***, **, and * indicate statistical significance at 1%, 5%, and 10% level, respectively.

		North America		Europe		Japan		Asia-Pacific		Emerging Markets	
		$\widehat{\theta}_i^2 - \widehat{\theta}_j^2$	t	$\widehat{\theta}_i^2 - \widehat{\theta}_j^2$	t	$\widehat{\theta}_i^2 - \widehat{\theta}_j^2$	t	$\widehat{\theta}_i^2 - \widehat{\theta}_j^2$	t	$\widehat{\theta}_i^2 - \widehat{\theta}_j^2$	t
MARKET	Hedged	0.572***	4.470	0.355***	4.575	0.160***	4.459	0.128***	3.702	0.012	0.660
	Scaled	0.142***	3.124	0.082***	2.967	−0.020**	−2.484	0.072***	3.251	0.061***	3.884
	Combined	0.492***	4.254	0.489***	7.297	0.051**	2.295	0.241***	5.752	0.060***	2.888
VALUE	Hedged	0.067***	5.633	0.065***	2.590	0.310***	4.884	0.156***	4.118	0.338***	5.290
	Scaled	−0.033***	−3.294	0.008	0.391	0.087***	2.662	−0.005	−0.215	−0.091***	−2.594
	Combined	0.054***	4.215	0.009	0.341	0.486***	6.198	0.110***	2.656	0.278***	4.091
INVESTMENT	Hedged	0.241***	6.327	0.113***	4.327	0.009	0.552	0.040**	2.240	−0.008	−0.217
	Scaled	−0.028*	−1.709	0.015	0.825	0.047***	2.828	0.002	0.118	0.090***	3.829
	Combined	0.265***	3.990	0.207***	4.877	0.053**	2.228	0.092***	2.751	0.086**	1.979
PROFITABILITY	Hedged	0.157***	4.370	0.088**	2.292	0.085***	3.379	0.124***	3.459	0.163**	2.048
	Scaled	0.074***	2.695	0.037*	1.905	0.023	1.545	0.020	0.870	0.064***	3.377
	Combined	0.119**	2.555	0.037	1.088	0.086**	2.550	0.134***	3.315	0.167**	2.041
MOMENTUM	Hedged	−0.090***	−3.475	0.088***	2.738	0.011	0.658	0.177**	2.514	−0.186***	−3.934
	Scaled	0.161***	4.268	0.866***	7.808	0.153***	6.069	−0.018	−0.155	0.173***	2.629
	Combined	0.032	0.864	0.856***	9.085	0.125***	4.734	0.068	0.530	−0.034	−0.394
INTANGIBLES	Hedged	0.568***	8.244	0.231***	5.676	−0.038	−1.525	0.014	0.368	0.309***	3.365
	Scaled	0.133***	2.783	−0.016	−1.149	−0.027*	−1.722	−0.057***	−3.226	−0.013	−0.339
	Combined	0.824***	7.563	0.198***	3.933	−0.011	−0.398	−0.017	−0.369	0.378***	3.786
FRICTIONS	Hedged	0.250***	5.705	−0.174**	−2.540	0.350***	6.336	0.149***	4.011	0.093	0.834
	Scaled	−0.033**	−2.393	−0.305***	−6.021	0.114***	3.173	0.057*	1.651	−0.201***	−4.397
	Combined	0.072*	1.921	−0.258***	−4.224	0.427***	5.428	0.151**	2.569	0.060	0.597

efficiency gains for the value and profitability factor in all regions (largest $\widehat{\theta}_i^2 - \widehat{\theta}_j^2$ of 0.57 and 0.16) and for the investment, intangibles, and frictions factor in 3 out of 5 regions. Similarly, there are also some cases in which volatility scaling leads to improvements in efficiency, albeit their economic significance is rather low ($\widehat{\theta}_i^2 - \widehat{\theta}_j^2$ between 0.04 and 0.13).

As suggested by previous results, volatility scaling works consistently well for the momentum factor only. There are significant efficiency gains in all regions ($\widehat{\theta}_i^2 - \widehat{\theta}_j^2$ between 0.16 and 0.87), besides Asia-Pacific (here the dynamic-scaling approach, however, would lead to an improvement). In comparison, no other categorical factor can be systematically enhanced by volatility-scaling. For the market factor both the cross-sectional and time-series enhancement approach give significant efficiency gains in 4 out of 5 regions, while the factor versions based on a combination of both enhancement approaches achieve these gains in all regions. This finding confirms the important role of the combination for the market factor.

Overall, results are mostly consistent across regions and suggest that it depends on the category a factor belongs to which enhancement approach is appropriate to improve upon the Sharpe ratio and to increase factor efficiency.

B. Benefits of Diversification

I have stated that it is important to employ a factor model that is as close as possible to spanning the mean–variance efficient frontier when studying the significance of a new anomaly or of a profitable trading strategy in order to derive sound conclusions. After having studied the individual effect of cross-sectional and time-series enhancement on different return factors, I thus formally examine whether there are benefits of diversification over factors by computing the in-sample Markowitz optimal combination of the different versions of the 7-factor portfolios. The Sharpe ratio should be higher if the factors are closer to spanning the mean–variance efficient frontier. Table 10 reports the maximum ex post squared Sharpe ratio and the corresponding annualized Sharpe ratio. I report results when including all factors and when excluding the market factor. As before, I test the significance in the difference in squared Sharpe ratios following Barillas et al. (2020).

The optimal combination of cross-sectionally enhanced factors mostly generates a higher ex post Sharpe ratio than the combination of traditional factors and the improvement is quantitatively stronger if the market factor is excluded. The only exception is Emerging Markets. In contrast, ex post Sharpe ratios of the volatility-scaled factor combination are similar to those of the traditional combination. In North America, Europe, and Emerging Markets, the factor combination consisting of factors on which both enhancement approaches have been applied obtains the quantitatively strongest and most significant improvement in ex post Sharpe ratios compared to its traditional counterpart. The improvement is also positive and economically substantial in the other two regions, however only statistically significant in Asia-Pacific and when the market factor is excluded. This effect could potentially be improved by a preselection of enhancement approaches on specific factors, such that only the method is applied that has been shown to be successful in

TABLE 10
Sharpe Ratios of Optimal Factor Portfolios

Table 10 shows ex post Sharpe ratios of Markowitz optimal factor portfolios. I compute the in-sample Markowitz optimal combination of the different versions of the 7-factor portfolios and report the maximum ex post squared Sharpe ratio and the corresponding annualized Sharpe ratio. Results are reported when including all factors and when excluding the market factor. Significance in the difference in squared Sharpe ratios is tested following the methodology of Barillas et al. (2020) and associated *t*-statistics are shown in the third column of the respective enhancement approaches. ***, **, and * indicate statistical significance at 1%, 5%, and 10% level, respectively.

Enhancement		Traditional		Cross-Sectional			Time Series			Combination		
Region	Factors	SR^2	Ann SR	SR^2	Ann SR	SR^2 Test	SR^2	Ann SR	SR^2 Test	SR^2	Ann SR	SR^2 Test
NA	All	0.251	1.734	0.318	1.952	1.368	0.278	1.825	0.567	0.360	2.078	2.064**
	Excl. MKT	0.134	1.267	0.171	1.434	1.884*	0.184	1.487	1.354	0.215	1.605	2.285**
EU	All	0.286	1.852	0.354	2.062	1.973**	0.369	2.105	1.353	0.498	2.444	3.160***
	Excl. MKT	0.170	1.429	0.231	1.667	2.068**	0.305	1.913	2.435**	0.369	2.105	3.145***
JA	All	0.185	1.490	0.259	1.761	1.791*	0.141	1.303	−0.929	0.250	1.732	1.180
	Excl. MKT	0.130	1.247	0.221	1.628	2.168**	0.105	1.125	−0.643	0.195	1.532	1.334
PA	All	0.256	1.751	0.294	1.878	0.827	0.243	1.706	−0.284	0.299	1.894	0.752
	Excl. MKT	0.150	1.343	0.205	1.567	1.929*	0.210	1.589	1.471	0.257	1.757	2.370**
EM	All	0.442	2.302	0.441	2.300	−0.010	0.484	2.410	0.695	0.588	2.656	1.850*
	Excl. MKT	0.331	1.993	0.384	2.148	1.049	0.400	2.190	1.285	0.570	2.615	3.102***
GL	All	0.647	2.787	0.805	3.109	2.222**	0.658	2.810	0.126	1.030	3.515	3.540***
	Excl. MKT	0.461	2.353	0.616	2.719	2.664***	0.526	2.512	0.856	0.881	3.251	3.877***

a preceding analysis. However, I do not further focus on such an approach, as I want my comparison to treat all factors equally across regions.

Eventually, I compute a global portfolio GL, which consists of the optimal combination of seven global factors that each invests an equal share in the corresponding regional categorical or market factors. In this case, I can observe an ex post optimal Sharpe ratio of up to 3.515, corresponding to an average increase by a factor of 1.96 compared to the optimal Sharpe ratio of the regional traditional factors. In total, results thus suggest that the enhanced set of factors is closer to spanning the mean variance efficient portfolio, especially if enhancement approaches are combined.

C. Comparing Traditional and Enhanced Factor Models

While I have shown that the Markowitz optimal ex post portfolios consisting of enhanced factors generate higher Sharpe ratios, enhanced factors that are more efficient should also perform better when studying the significance of a new anomaly or trading strategy implementable in *real-time*. In this regard, there is the possibility to run into the issue of encountering a false positive. A new strategy would then be considered a novel anomaly, although it possibly is not. One reason could be that the factor model might be incorrectly specified, but I do not consider this option. I focus on the issue that the employed factors might be inefficient.

I, therefore, test how many anomalies can be explained by different versions of the categorical factor model that consists of the market factor and of the categorical value, investment, profitability, momentum, intangibles, and frictions factor. I start with the 214 anomalies I use to construct the categorical factors. I further add 27 anomalies that are based on intermediate past returns, which I neither attribute to

TABLE 11

Comparative Performance of Traditional and Enhanced Factor Models

Table 11 reports comparative performance in the explanatory power for anomalies across regions for the categorical factor model based on different versions of enhanced factors. As comparative statistics, the number of alphas significant at the 5% level and the average absolute *t*-values are used. The table shows the aggregate performance for all anomalies and for each category of anomalies separately. The values for all anomalies includes 24 anomalies based on dummy variables that have not been employed previously. The table shows results for the CAPM based on the regional market factor and for four versions of the categorical factor model that consists of the market factor and the six categorical factors. A factor model version is included with the unhedged traditional factors, the cross-sectionally hedged factors, the unhedged volatility-scaled factors and the factors that combine both enhancement approaches, respectively.

Category		ALL	VALUE	INV	PROF	MOM	INT	FRIC	ALL	VALUE	INV	PROF	MOM	INT	FRIC
Region	Enhancement	$\#_{p_\alpha<5\%}$							$\overline{\lvert t \rvert}$						
North America	Market	103	14	21	17	16	10	14	1.91	1.99	2.41	2.15	2.19	1.43	1.64
	Traditional	34	3	5	3	2	7	9	1.15	1.15	1.13	0.88	1.07	1.38	1.17
	Cross-sectional	59	7	7	7	5	15	13	1.44	1.37	1.22	1.31	1.28	1.80	1.52
	Time series	37	6	6	6	3	4	8	1.15	1.35	1.40	1.04	0.93	1.04	1.22
	Combined	18	2	2	2	3	4	3	0.91	0.71	0.94	0.90	0.95	1.06	0.86
Europe	Market	88	9	13	18	15	6	16	1.73	1.42	1.73	2.35	2.32	1.13	1.62
	Traditional	31	2	2	3	4	10	7	1.04	0.69	0.88	1.15	1.11	1.31	1.03
	Cross-sectional	41	2	5	5	3	14	8	1.18	0.84	1.05	1.13	1.24	1.44	1.35
	Time series	25	3	2	4	2	5	6	0.97	0.78	0.83	1.13	0.96	0.94	1.05
	Combined	20	1	0	2	3	7	5	0.94	0.76	0.78	1.04	0.97	1.07	1.01
Japan	Market	30	5	2	3	5	4	8	1.03	1.12	0.80	0.89	1.19	0.96	1.23
	Traditional	31	4	3	6	0	9	7	1.03	1.08	0.95	1.11	1.02	1.04	1.03
	Cross-sectional	34	6	4	5	1	7	9	1.09	1.05	0.92	0.99	1.18	1.17	1.20
	Time series	36	8	6	6	2	5	8	1.03	1.20	1.04	1.13	0.91	1.02	1.06
	Combined	13	1	1	0	1	4	5	0.81	0.77	0.70	0.82	0.84	0.98	0.85
Asia Pacific	Market	87	14	13	18	15	7	14	1.69	1.62	1.72	2.02	2.73	1.19	1.65
	Traditional	46	2	3	3	9	13	13	1.17	0.87	0.72	0.83	1.48	1.56	1.58
	Cross-sectional	40	2	2	1	10	8	15	1.14	0.91	0.70	0.79	1.52	1.40	1.62
	Time series	35	5	5	2	12	4	7	1.19	0.92	1.21	1.13	1.77	1.13	1.35
	Combined	32	1	5	2	12	4	7	1.10	0.83	1.00	1.02	1.85	1.01	1.27
Emerging markets	Market	67	17	6	9	9	5	13	1.50	1.99	1.20	1.50	1.72	1.14	1.55
	Traditional	32	7	1	6	5	4	7	1.07	1.14	0.76	1.25	1.17	0.92	1.24
	Cross-sectional	24	3	1	4	2	3	6	0.98	0.81	0.84	0.79	0.94	0.90	1.30
	Time series	22	6	0	3	1	2	7	1.05	1.24	0.77	1.07	1.12	0.91	1.29
	Combined	19	2	0	2	1	4	8	0.94	0.78	0.68	0.78	0.95	1.05	1.26

the value nor momentum category or on dummy variables. The dummy variables indicate whether a firm is exposed to a specific event or feature. The corresponding value-weighted strategies go long a portfolio of firms to which the signal applies and short a portfolio of firms to which the signal does not apply. In total, there are 241 left-hand side anomalies.[17] For each anomaly and each region, I conduct a time-series regression of the anomaly return on the categorical factor model. I then test the significance of all regression alphas employing Newey and West (1987) adjusted standard errors. Table 11 shows the number of anomalies significant at the 5%-level and the average absolute *t*-statistic of the alphas, in aggregate over all anomalies and separately for each category.

For comparative reasons I also include the CAPM as a benchmark model, denoted market. The CAPM clearly performs worst. The traditional categorical factor model version works significantly better and cuts the number of significant anomalies in North America, Europe, the Asia-Pacific region, and in Emerging Markets more than in half. Cross-sectional enhancement of factors yields better results compared to the traditional model only in the Asia-Pacific region and in Emerging Markets. In North America, the number of significant alphas is even

[17]The Supplementary Material contains the full list of the 214 variables used to construct the factors and the additional 27 variables used in the regressions.

inflated. This might seem odd at first, but can be most likely attributed to the fact that cross-sectional enhancement does not improve efficiency equally across categories. Similar observations can be made for volatility-scaling, where only in Europe, the Asia-Pacific region, and Emerging Markets the corresponding factor model works slightly better than its traditional counterpart. The model that delivers the lowest number of significant alphas within all regions is the categorical factor model employing factors on which both cross-sectional and time-series enhancement is applied. With this factor model version across all regions on average 15 additional anomalies (only 11 in Europe) can be explained compared to the traditional version.

I conclude that by employing both enhancement approaches, on each factor a method is applied that effectively improves efficiency. For instance, in case of the momentum factor volatility-scaling of the cross-sectionally enhanced factors cancels out the negative effect of the later procedure and leads to an overall improvement in efficiency. To contribute to the big picture, I conduct the same analysis with cross-sectional enhancement based on Dimson-type loadings for an extended time period. Results, presented in Table A4, are generally similar across regions. On average 11 additional anomalies (interestingly only 2 in Emerging Markets) can be explained compared to the traditional factor model.

Further, I note that there is some difference in improvement in explanatory power of the factor models across categories and regions. For the value, investment, and profitability category the improvement is relatively similar across regions, ranging between 1 and 6 additionally explained anomalies. In case of the intangibles and frictions category, the enhanced model significantly improves the number of explained anomalies in all regions except for Emerging Markets. In the momentum category, the number of explained anomalies differs between 3 less (Asia-Pacific) and 4 more (Emerging Markets). Interestingly, while the enhanced factor model works less well in Emerging Markets for explanation of anomalies in the intangibles and frictions categories, it works better in the momentum category.

Generally, the enhanced factor versions improve the explanatory power with respect to a large number of anomalies. The results thus suggest that enhancement techniques improve efficiency of return factors, which correspondingly implies that factor enhancement can help to improve the validity of conclusions drawn based on factor model analyses.

VI. Conclusion

This article investigates enhancement approaches for return factors and examines whether they improve factor efficiency on an international scale. Three distinct enhancement mechanism are employed: cross-sectional enhancement as suggested by Daniel et al. (2020), volatility-scaling based on time-series information as suggested by Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016), and the exploitation of diversification effects. Specifically, I construct enhanced global versions of categorical factors formed from 214 firm characteristics across 44 countries. These characteristics are aggregated to six category scores (based on

https://doi.org/10.1017/S0022109022001090 Published online by Cambridge University Press

the classification of Hou et al. (2020)), from which regional factor portfolios are constructed.

Generally, cross-sectional enhancement is successful in improving factor efficiency for all factors, except the momentum factors. Volatility management, in contrast, only works consistently well for the momentum factors, consistent with the findings of Cederburg et al. (2020). Results thus indicate that *either* cross-sectional *or* time-series information matter for the improvement of factor efficiency. Further, I show that a model that makes use of enhanced factors has a higher optimal ex post Sharpe ratio and is able to explain a larger number of anomalies than a traditional model.

I consider the findings as support for my suggestion to be more careful in assessing results of anomaly studies, for both conducted on the US and international level. Taken all together, I demonstrate that factor efficiency can be improved to point out that results where factor models are used as benchmark models need to be treated with care.

Appendix

TABLE A1

Postformation Factor Loadings

Table A1 presents global postformation factor loadings for portfolios obtained from sorting on the composite category scores and on the preformation market βs and for portfolios obtained from sorting on the composite category scores and on the preformation Dimson βs. First, the categorical regional factors are computed and the portfolio assignments and weights are stored. Subsequently, preformation βs are obtained by applying the current portfolio method. Backward-looking daily portfolio returns for the current portfolios are computed back to 60 months in the past and the returns are used to compute the preformation βs. In case of preformation market βs, they are computed in the manner of Dimson (1979) and Frazzini and Pedersen (2014). In case of preformation βs on categorical factors, they are computed in the manner of Dimson (1979) only. Within each score quintile, firms are sorted into preformation β terciles on the country level and then aggregated into regional portfolios. In case of the market βs, value-weighted portfolio returns are calculated for each of the 15 portfolios for each score/beta combination on the regional level. The portfolio returns are then averaged across score portfolios. In case of βs on categorical factors, returns are calculated for each of the 15 portfolios obtained from combining the specific category score with this beta. The postformation factor loadings are obtained by regressing the respective portfolio return on a combination of the traditional market factor and the six categorical factors. The table shows the full sample beta coefficients on the corresponding traditional factor and the corresponding *t*-statistics below in parentheses.

Panel A. Market – Frazzini/Pedersen Betas

Score	Preformation β			
	1	2	3	1–3
1	0.619	0.867	1.233	−0.614
	(48.379)	(80.960)	(91.362)	(28.382)
2	0.619	0.862	1.208	−0.589
	(50.840)	(83.361)	(97.922)	(28.956)
3	0.625	0.853	1.198	−0.574
	(50.333)	(83.584)	(104.237)	(28.808)
4	0.623	0.867	1.211	−0.588
	(51.806)	(88.733)	(105.791)	(30.472)
5	0.639	0.863	1.197	−0.558
	(52.526)	(89.086)	(110.601)	(28.838)
$\bar{\beta}$	0.625	0.862	1.209	−0.585

(continued on next page)

TABLE A1 (continued)

Postformation Factor Loadings

Score	Preformation β			
	1	2	3	1–3
Panel B. Market – Dimson Betas				
1	0.631 (49.376)	0.864 (101.634)	1.215 (112.631)	−0.584 (29.372)
2	0.624 (51.865)	0.865 (99.461)	1.203 (118.038)	−0.579 (30.507)
3	0.649 (54.662)	0.862 (96.895)	1.203 (121.719)	−0.555 (29.617)
4	0.652 (57.083)	0.864 (101.839)	1.215 (122.643)	−0.563 (30.806)
5	0.662 (55.195)	0.869 (105.298)	1.202 (122.760)	−0.540 (28.481)
$\bar{\beta}$	0.644	0.865	1.208	−0.564
Panel C. Value Score				
1	−0.868 (46.140)	−0.271 (15.232)	0.045 (2.001)	−0.914 (28.078)
2	−0.306 (15.105)	−0.070 (4.386)	0.174 (9.333)	−0.480 (16.164)
3	−0.143 (7.809)	0.058 (3.905)	0.249 (14.733)	−0.392 (14.284)
4	0.020 (1.040)	0.119 (8.390)	0.291 (17.589)	−0.271 (9.770)
5	0.176 (8.080)	0.222 (14.490)	0.581 (38.767)	−0.405 (13.654)
$\bar{\beta}$	−0.224	0.012	0.268	−0.492
Panel D. Investment Score				
1	0.081 (3.349)	0.300 (16.751)	0.746 (36.567)	−0.666 (18.455)
2	−0.109 (5.211)	0.143 (7.747)	0.360 (15.915)	−0.468 (13.973)
3	−0.196 (9.335)	0.100 (5.664)	0.318 (14.239)	−0.515 (14.536)
4	−0.286 (12.724)	−0.004 (0.232)	0.244 (10.571)	−0.530 (14.902)
5	−0.763 (43.004)	−0.177 (9.131)	0.028 (1.176)	−0.791 (23.792)
$\bar{\beta}$	−0.255	0.072	0.339	−0.594
Panel E. Profitability Score				
1	−1.132 (54.821)	−0.335 (16.937)	−0.045 (1.713)	−1.086 (27.887)
2	−0.564 (24.142)	−0.112 (6.143)	0.204 (8.545)	−0.768 (20.802)
3	−0.395 (19.054)	−0.035 (2.175)	0.251 (11.975)	−0.646 (19.196)
4	−0.296 (15.350)	0.090 (6.044)	0.279 (16.046)	−0.575 (18.691)
5	−0.203 (10.138)	0.117 (7.945)	0.434 (28.448)	−0.637 (21.174)
$\bar{\beta}$	−0.518	−0.055	0.225	−0.742

(continued on next page)

TABLE A1 (continued)

Postformation Factor Loadings

Score	Preformation β			
	1	2	3	1–3
Panel F. Momentum Score				
1	−0.796 (52.500)	−0.429 (28.816)	−0.319 (16.553)	−0.477 (17.346)
2	−0.380 (21.622)	−0.161 (11.256)	−0.059 (3.397)	−0.321 (12.470)
3	−0.198 (12.277)	−0.048 (3.764)	0.053 (3.399)	−0.251 (10.401)
4	−0.052 (3.104)	0.047 (3.883)	0.175 (12.573)	−0.227 (9.923)
5	0.249 (14.945)	0.219 (18.064)	0.439 (35.293)	−0.190 (8.391)
$\bar{\beta}$	−0.235	−0.074	0.058	−0.293
Panel G. Intangible Score				
1	−0.859 (43.347)	−0.289 (16.279)	0.068 (2.619)	−0.927 (24.664)
2	−0.483 (18.785)	−0.146 (7.636)	0.209 (8.327)	−0.691 (16.919)
3	−0.386 (15.419)	−0.088 (4.717)	0.339 (14.205)	−0.725 (18.289)
4	−0.285 (12.541)	−0.012 (0.682)	0.450 (19.736)	−0.735 (20.244)
5	−0.130 (5.327)	0.181 (9.872)	0.788 (41.149)	−0.918 (25.432)
$\bar{\beta}$	−0.429	−0.071	0.371	−0.799
Panel H. Frictions Score				
1	−0.034 (1.779)	0.151 (10.734)	0.419 (28.079)	−0.453 (16.935)
2	−0.136 (6.521)	0.063 (4.284)	0.261 (15.297)	−0.397 (13.080)
3	−0.298 (13.789)	0.004 (0.267)	0.225 (11.872)	−0.524 (16.528)
4	−0.372 (15.262)	−0.072 (4.199)	0.203 (8.385)	−0.575 (15.622)
5	−1.054 (48.501)	−0.274 (14.891)	−0.095 (4.524)	−0.959 (27.028)
$\bar{\beta}$	−0.379	−0.026	0.203	−0.582

TABLE A2

Sharpe Ratios Across Different Types of Cross-Sectionally Enhanced Factors

Table A2 shows annualized returns and Sharpe ratios for cross-sectionally enhanced factors, where the factors differ in the choice of their respective enhancement method and procedure. I distinguish between Dimson (1979) and Frazzini and Pedersen (2014) factor loadings and hedge factor gammas, between hedging with respect to the categorical factor's own hedge factor only or with respect to all seven hedge factors, and in the choice of the computation method for returns used to obtain loadings (current method with back-tracing of current portfolio returns or regular method with historical returns). The corresponding returns are presented for each region and for the market factor and all six categorical factors. The time period extends from Jan. 1995 until June 2019.

Category	Factor Loading	Hedge Factor Covariance	Portfolio Loadings Method	North America		Europe		Japan		Asia Pacific		Emerging Markets	
				Ann. Ret	SR	Ann. Ret	SR	Ann. Ret	SR	Ann. Ret	SR	Ann. Ret	SR
MARKET			Unhedged	11.124	0.728	9.343	0.539	3.371	0.199	10.170	0.508	6.956	0.338
	FP	Only own	Current	11.462	1.117	10.985	0.996	5.935	0.447	9.745	0.755	6.062	0.387
			Regular	11.885	1.163	11.261	1.020	6.598	0.498	10.156	0.785	10.316	0.669
		All hedge factors	Current	11.667	0.256	13.329	0.291	11.780	0.258	12.491	0.421	5.645	0.244
			Regular	13.812	0.265	13.550	0.257	14.829	0.327	13.416	0.430	14.060	0.605
	Dimson	Only own	Current	10.029	1.006	7.534	0.705	4.825	0.366	8.050	0.614	3.268	0.186
			Regular	10.852	1.120	8.422	0.802	5.983	0.453	9.864	0.775	8.360	0.513
		All hedge factors	Current	1.673	0.032	3.082	0.062	5.603	0.144	11.105	0.261	−2.101	−0.062
			Regular	2.939	0.055	2.645	0.049	12.561	0.275	15.914	0.355	11.653	0.363
VALUE			Unhedged	5.087	0.241	7.173	0.448	11.478	0.669	14.022	0.676	8.315	0.651
	FP	Only own	Current	6.500	0.473	6.718	0.504	11.738	0.950	13.546	0.775	11.119	1.056
			Regular	5.946	0.418	4.230	0.296	9.715	0.644	12.256	0.695	7.838	0.694
		All hedge factors	Current	3.088	0.052	1.097	0.043	5.574	0.205	10.600	0.281	12.210	0.873
			Regular	−5.394	−0.072	−5.730	−0.230	−2.013	−0.065	6.140	0.162	3.941	0.246
	Dimson	Only own	Current	6.884	0.504	5.339	0.421	12.054	1.002	9.440	0.592	9.649	0.962
			Regular	4.378	0.304	3.831	0.295	8.078	0.587	7.036	0.402	8.832	0.872
		All hedge factors	Current	6.209	0.100	−10.538	−0.324	10.011	0.285	−8.768	−0.198	13.397	0.853
			Regular	−5.401	−0.061	−16.270	−0.403	−6.655	−0.132	−18.283	−0.345	6.748	0.274
INVESTMENT			Unhedged	5.406	0.621	5.406	0.621	1.738	0.150	6.419	0.416	4.547	0.594
	FP	Only own	Current	5.780	0.494	5.870	0.826	1.399	0.164	6.239	0.462	3.899	0.581
			Regular	2.345	0.149	4.796	0.628	1.369	0.141	5.873	0.430	4.337	0.596
		All hedge factors	Current	6.864	0.267	5.780	0.494	0.115	0.004	2.783	0.116	2.435	0.225
			Regular	5.439	0.144	2.345	0.149	1.434	0.051	2.450	0.102	4.061	0.514
	Dimson	Only own	Current	2.991	0.184	5.545	0.775	1.267	0.148	5.247	0.396	3.997	0.569
			Regular	2.841	0.105	5.157	0.719	1.657	0.177	5.475	0.441	4.275	0.623
		All hedge factors	Current	8.783	0.280	2.991	0.184	−1.924	−0.063	−4.312	−0.167	3.088	0.052
			Regular	9.489	0.210	2.841	0.105	−1.841	−0.054	−3.093	−0.078	−5.394	−0.072

(continued on next page)

TABLE A2 (continued)

Sharpe Ratios Across Different Types of Cross-Sectionally Enhanced Factors

Category	Factor Loading	Hedge Factor Covariance	Portfolio Loadings Method	North America		Europe		Japan		Asia Pacific		Emerging Markets	
				Ann. Ret	SR	Ann. Ret	SR	Ann. Ret	SR	Ann. Ret	SR	Ann. Ret	SR
PROFITABILITY			Unhedged	9.814	0.516	6.594	0.383	5.630	0.422	13.958	0.637	6.983	0.675
	FP	Only own	Current	8.770	0.792	5.708	0.487	5.398	0.574	13.435	0.743	6.641	0.781
			Regular	7.199	0.696	6.323	0.530	6.505	0.602	13.112	0.709	6.484	0.752
		All hedge factors	Current	8.457	0.207	2.305	0.087	2.353	0.092	9.691	0.310	5.291	0.242
			Regular	−3.599	−0.071	5.949	0.196	5.707	0.188	8.921	0.361	7.464	0.514
	Dimson	Only own	Current	8.166	0.749	6.293	0.536	5.695	0.603	7.309	0.459	7.468	0.859
			Regular	8.165	0.794	6.978	0.586	6.912	0.698	9.221	0.530	5.899	0.725
		All hedge factors	Current	6.159	0.105	4.202	0.127	4.870	0.156	−8.075	−0.217	8.129	0.286
			Regular	−0.535	−0.007	9.258	0.260	10.857	0.260	−2.697	−0.079	5.949	0.255
MOMENTUM			Unhedged	10.971	0.577	15.202	0.792	8.528	0.432	19.143	1.098	13.923	1.239
	FP	Only own	Current	9.480	0.573	14.395	0.883	7.736	0.468	19.076	1.108	11.909	1.081
			Regular	12.215	0.680	17.132	0.995	11.087	0.595	19.930	1.155	13.995	1.237
		All hedge factors	Current	5.347	0.170	8.287	0.426	6.101	0.203	14.805	0.637	11.098	0.697
			Regular	12.818	0.329	23.103	0.659	14.118	0.381	22.020	1.088	15.344	1.264
	Dimson	Only own	Current	9.821	0.623	13.800	0.893	7.715	0.479	17.146	1.073	11.312	0.999
			Regular	12.416	0.746	17.891	1.138	12.830	0.742	20.513	1.180	13.783	1.200
		All hedge factors	Current	4.745	0.109	7.284	0.244	1.105	0.030	9.655	0.439	15.445	0.788
			Regular	12.342	0.189	27.529	0.451	19.973	0.285	19.407	0.563	14.973	0.704
INTANGIBLES			Unhedged	8.282	0.717	3.708	0.367	4.164	0.372	8.040	0.659	7.971	0.918
	FP	Only own	Current	9.155	1.186	5.331	0.715	2.856	0.310	6.825	0.660	8.073	1.021
			Regular	8.693	1.051	5.024	0.651	2.141	0.209	8.214	0.754	7.471	0.935
		All hedge factors	Current	9.947	0.279	8.816	0.404	0.414	0.022	4.668	0.215	7.066	0.561
			Regular	11.689	0.305	9.873	0.410	0.216	0.014	8.360	0.523	6.564	0.607
	Dimson	Only own	Current	9.341	1.156	6.040	0.792	3.845	0.408	4.660	0.423	6.064	0.761
			Regular	8.227	1.018	5.585	0.723	1.812	0.194	4.729	0.478	6.599	0.863
		All hedge factors	Current	12.312	0.364	17.366	0.605	2.778	0.115	−6.349	−0.208	1.521	0.097
			Regular	11.235	0.257	17.894	0.592	−4.087	−0.183	−2.863	−0.129	4.656	0.298
FRICTIONS			Unhedged	6.729	0.320	16.192	0.823	9.705	0.697	13.766	0.780	10.001	0.865
	FP	Only own	Current	7.261	0.630	14.926	0.784	9.615	0.876	12.098	0.888	10.690	1.032
			Regular	7.329	0.631	14.431	0.685	8.606	0.671	11.850	0.789	9.815	1.011
		All hedge factors	Current	3.487	0.062	9.862	0.226	0.492	0.017	12.764	0.391	5.906	0.224
			Regular	−2.028	−0.025	12.127	0.288	−1.376	−0.062	8.586	0.312	11.027	0.566
	Dimson	Only own	Current	7.802	0.650	9.746	0.535	9.657	0.839	9.218	0.704	11.838	1.134
			Regular	5.940	0.514	11.770	0.648	8.229	0.630	9.413	0.672	11.829	1.233
		All hedge factors	Current	6.780	0.106	0.081	0.002	4.875	0.149	−4.408	−0.115	9.690	0.283
			Regular	−0.855	−0.009	2.274	0.050	−6.286	−0.153	−6.631	−0.167	10.718	0.401

TABLE A3

Performance of Factors Conditional on Volatility

Table A3 presents results of pooled OLS regressions of factor returns across regions on realized volatility in the previous 6 months and the previous month, respectively. Volatility is calculated as in Barroso and Santa-Clara (2015) and Moreira and Muir (2017. Dependent variables are traditional or cross-sectionally enhanced factor returns, where factors resulting from both enhancement with the Dimson (1979) and Frazzini and Pedersen (2014) method for betas estimation are employed. ***, **, and * indicate statistical significance at 1%, 5%, and 10% level, respectively.

Enhancement	Traditional		Frazzini/Pedersen		Dimson	
Time Window	6 Months	1 Month	6 Months	1 Month	6 Months	1 Month
MARKET	−0.008 (−0.397)	−0.006* (−1.848)	−0.045* (−1.750)	−0.011 (−1.475)	−0.030 (−1.201)	−0.003 (−0.416)
VALUE	0.107*** (4.027)	0.012*** (2.584)	0.072*** (3.061)	0.008 (1.450)	0.065 (1.567)	0.005 (1.098)
INVESTMENT	0.054** (2.412)	0.014*** (3.376)	0.017 (0.655)	0.006 (0.806)	0.005 (0.176)	0.005 (0.638)
PROFITABILITY	0.012 (0.502)	0.008* (1.783)	0.031 (1.245)	0.013** (2.519)	0.002 (0.085)	0.008 (1.266)
MOMENTUM	−0.069*** (−3.024)	−0.012** (−2.175)	−0.073*** (−2.894)	−0.012** (−2.012)	−0.101*** (−3.640)	−0.013* (−1.717)
INTANGIBLES	0.032 (1.441)	0.006** (2.415)	0.030 (1.375)	0.005** (2.032)	0.033 (1.194)	0.008*** (2.851)
FRICTIONS	0.029 (1.500)	0.003 (1.496)	0.016 (0.850)	0.002* (1.842)	0.005 (0.287)	0.001 (0.738)

TABLE A4

Comparative Performance of Traditional and Enhanced Factor Models with Dimson Beta Enhancement

Table A4 reports comparative performance in the explanatory power for anomalies across regions for the categorical factor model based on different versions of enhanced factors. In contrast to Table 11, cross-sectionally enhanced factors are based on Dimson betas. As comparative statistics, the number of alphas significant at the 5% level and the average absolute *t*-values are used. The table shows the aggregate performance for all anomalies and for each category of anomalies separately. The values for all anomalies include 24 anomalies based on dummy variables that have not been employed previously. The table shows results for the CAPM based on the regional market factor and for four versions of the categorical factor model that consists of the market factor and the six categorical factors. A factor model version is included with the unhedged traditional factors, the cross-sectionally hedged factors, the unhedged volatility-scaled factors, and the factors that combine both enhancement approaches, respectively. The time period extends from July 1992 until June 2019.

Category		All	Value	Inv	Prof	Mom	Int	Fric	All	Value	Inv	Prof	Mom	Int	Fric
Region	Enhancement	$\#_{p_\alpha<5\%}$							$\overline{\lvert t \rvert}$						
North America	Market	100	15	20	16	15	9	14	1.92	1.99	2.52	2.03	2.26	1.40	1.65
	Traditional	38	3	5	2	3	7	12	1.16	1.15	1.16	0.86	1.08	1.35	1.26
	Cross-sectional	55	8	7	7	4	13	12	1.37	1.25	1.25	1.23	1.25	1.64	1.60
	Time series	39	8	6	4	4	3	10	1.16	1.40	1.40	1.02	0.97	0.99	1.24
	Combined	18	2	2	0	4	3	4	0.92	0.74	1.02	0.90	1.00	0.95	0.91
Europe	Market	92	8	16	18	16	6	17	1.78	1.48	1.90	2.35	2.39	1.18	1.59
	Traditional	32	3	1	4	5	8	8	1.06	0.77	0.82	1.16	1.25	1.31	1.04
	Cross-sectional	33	2	2	4	5	8	9	1.09	0.78	0.90	1.03	1.22	1.29	1.28
	Time series	28	2	1	3	4	7	7	1.00	0.86	0.97	1.11	1.06	0.92	1.09
	Combined	24	2	0	2	4	6	7	0.93	0.87	0.89	0.94	0.95	0.93	0.99
Japan	Market	36	8	3	2	5	3	11	1.06	1.18	0.83	0.86	1.19	0.94	1.31
	Traditional	30	5	1	4	1	8	9	1.01	1.00	0.91	1.04	1.13	1.06	1.03
	Cross-sectional	23	1	2	1	3	8	6	1.00	0.92	0.85	0.94	1.21	1.08	1.04
	Time series	25	4	3	4	2	5	6	0.94	0.99	0.78	0.97	1.02	1.06	1.02
	Combined	18	3	1	1	2	5	5	0.84	0.84	0.68	0.85	0.97	0.94	0.82
Asia Pacific	Market	85	13	14	18	15	6	13	1.75	1.67	1.89	2.06	2.72	1.22	1.73
	Traditional	50	2	3	4	8	14	15	1.22	0.91	0.79	0.89	1.45	1.66	1.65
	Cross-sectional	40	5	2	4	6	13	10	1.16	0.97	0.79	0.95	1.46	1.50	1.45
	Time series	36	5	5	2	11	5	8	1.20	0.92	1.29	1.08	1.78	1.19	1.34
	Combined	41	7	7	3	12	4	6	1.16	0.98	1.20	1.10	1.70	1.17	1.23
Emerging markets	Market	75	18	8	12	11	3	14	1.51	2.11	1.22	1.47	1.76	1.08	1.48
	Traditional	33	4	2	6	5	4	8	1.10	1.12	0.94	1.26	1.15	0.86	1.31
	Cross-sectional	47	3	2	5	4	5	16	1.22	0.94	0.99	1.07	1.09	1.01	1.65
	Time series	28	8	3	3	3	2	8	1.10	1.25	0.98	1.08	1.12	0.94	1.27
	Combined	31	4	2	3	1	4	12	1.08	0.94	0.84	1.04	0.94	0.99	1.38

Supplementary Material

To view supplementary material for this article, please visit https://doi.org/10.1017/S0022109022001090.

References

Ahmed, S.; Z. Bu; and D. Tsvetanov. "Best of the Best: A Comparison of Factor Models." *Journal of Financial and Quantitative Analysis*, 54 (2019), 1713–1758.

Barillas, F.; R. Kan; C. Robotti; and J. Shanken. "Model Comparison with Sharpe Ratios." *Journal of Financial and Quantitative Analysis*, 55 (2020), 1840–1874.

Barillas, F., and J. Shanken. "Comparing Asset Pricing Models." *Journal of Finance*, 73 (2018), 715–754.

Barroso, P.; A. Detzel; and P. Maio. "Managing the Risk of the Beta Anomaly." Working Paper, Catolica-Lisbon School of Business and Economics (2021).

Barroso, P., and P. Santa-Clara. "Momentum Has its Moments." *Journal of Financial Economics*, 116 (2015), 111–120.

Campbell, J. Y.; J. Hilscher; and J. Szilagyi. "In Search of Distress Risk." *Journal of Finance*, 63 (2008), 2899–2939.

Cederburg, S.; M. S. O'Doherty; F. Wang; and X. S. Yan. "On the Performance of Volatility-Managed Portfolios." *Journal of Financial Economics*, 138 (2020), 95–117.

Chen, Z.; B. Liu; H. Wang; Z. Wang; and J. Yu. "Characteristics-Based Factors." PBCSF-NIFR Research Paper (2020a).

Chen, Z.; B. Liu; H. Wang; Z. Wang; and J. Yu. "Investor Sentiment and the Pricing of Characteristics-Based Factors." PBCSF-NIFR Research Paper (2020b).

Cooper, I.; L. Ma; P. Maio; and D. Philip. "Multifactor Models and Their Consistency with the APT." *Review of Asset Pricing Studies*, 11 (2021), 402–444.

Daniel, K., and T. J. Moskowitz. "Momentum Crashes." *Journal of Financial Economics*, 122 (2016), 221–247.

Daniel, K.; L. Mota; S. Rottke; and T. Santos. "The Cross-Section of Risk and Returns." *Review of Financial Studies*, 33 (2020), 1927–1979.

Daniel, K., and S. Titman. "Evidence on the Characteristics of Cross Sectional Variation in Stock Returns." *Journal of Finance*, 52 (1997), 1–33.

De Santis, G., and B. Gerard. "International Asset Pricing and Portfolio Diversification with Time-Varying Risk." *Journal of Finance*, 52 (1997), 1881–1912.

Dimson, E. "Risk Measurement when Shares Are Subject to Infrequent Trading." *Journal of Financial Economics*, 7 (1979), 197–226.

Ehsani, S., and J. Linnainmaa. "Time-Series Efficient Factors." Working Paper, Tuck School of Business (2021).

Eisdorfer, A., and E. U. Misirli. "Distressed Stocks in Distressed Times." *Management Science*, 66 (2020), 2452–2473.

Fama, E. F., and K. R. French. "Common Risk Factors in the Returns of Stocks and Bonds." *Journal of Financial Economics*, 33 (1993), 3–56.

Fama, E. F., and K. R. French. "Size, Value, and Momentum in International Stock Returns." *Journal of Financial Economics*, 105 (2012), 457–472.

Fama, E. F., and K. R. French. "A Five-Factor Asset Pricing Model." *Journal of Financial Economics*, 116 (2015), 1–22.

Fama, E. F., and K. R. French. "Choosing Factors." *Journal of Financial Economics*, 128 (2018), 234–252.

Frazzini, A., and L. H. Pedersen. "Betting Against Beta." *Journal of Financial Economics*, 111 (2014), 1–25.

Glosten, L. R.; R. Jagannathan; and D. E. Runkle. "On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks." *Journal of Finance*, 48 (1993), 1779–1801.

Green, J.; J. R. M. Hand; and X. F. Zhang. "The Characteristics that Provide Independent Information about Average U.S. Monthly Stock Returns." *Review of Financial Studies*, 30 (2017), 4389–4436.

Griffin, J. M. "Are the Fama and French Factors Global or Country Specific?" *Review of Financial Studies*, 15 (2002), 783–803.

Griffin, J. M.; P. J. Kelly; and F. Nardari. "Do Market Efficiency Measures Yield Correct Inferences? A Comparison of Developed and Emerging Markets." *Review of Financial Studies*, 23 (2010), 3225–3277.

Harvey, C. R.; Y. Liu; H. Zhu. "… And the Cross-Section of Expected Returns." *Review of Financial Studies*, 29 (2016), 5–68.
Hou, K.; G. A. Karolyi; and B.-C. Kho. "What Factors Drive Global Stock Returns?" *Review of Financial Studies*, 24 (2011), 2527–2574.
Hou, K.; H. Mo; C. Xue; and L. Zhang. "Which Factors?" *Review of Finance*, 23 (2019), 1–35.
Hou, K.; H. Mo; C. Xue; and L. Zhang. "An Augmented Q-Factor Model with Expected Growth." *Review of Finance*, 25 (2021), 1–41.
Hou, K.; C. Xue; and L. Zhang. "Digesting Anomalies: An Investment Approach." *Review of Financial Studies*, 28 (2015), 650–705.
Hou, K.; C. Xue; and L. Zhang. "Replicating Anomalies." *Review of Financial Studies*, 33 (2020), 2019–2133.
Huber, D.; H. Jacobs; S. Müller; and F. Preissler. "International Factor Models." Working Paper, University of Hamburg (2021).
Ince, O. S., and R. B. Porter. "Individual Equity Return Data from Thomsen Datastream: Handle with Care!" *Journal of Financial Research*, 29 (2006), 463–479.
Jacobs, H., and S. Müller. "…And Nothing Else Matters? On the Dimensionality and Predictability of International Stock Returns." Working Paper, University of Duisburg-Essen (2018).
Jacobs, H., and S. Müller "Anomalies across the Globe: Once Public, No Longer Existent?" *Journal of Financial Economics*, 135 (2020), 213–230.
Jegadeesh, N., and S. Titman. "Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency." *Journal of Finance*, 48 (1993), 65.
Kelly, B. T.; S. Pruitt; and Y. Su. "Characteristics Are Covariances: A Unified Model of Risk and Return." *Journal of Financial Economics*, 134 (2019), 501–524.
Kim, S.; R. A. Korajczyk; and A. Neuhierl. "Arbitrage Portfolios." *Review of Financial Studies*, 34 (2021), 2813–2856.
Kozak, S.; S. Nagel; and S. Santosh. "Interpreting Factor Models." *Journal of Finance*, 73 (2018), 1183–1223.
Kozak, S.; S. Nagel; and S. Santosh. "Shrinking the Cross Section." *Journal of Financial Economics*, 135 (2019), 271–292.
Lettau, M., and M. Pelger. "Estimating Latent Asset-Pricing Factors." *Journal of Econometrics*, 218 (2020a), 1–31.
Lettau, M., and M. Pelger. "Factors That Fit the Time Series and Cross-Section of Stock Returns." *Review of Financial Studies*, 33 (2020b), 2274–2325.
Moreira, A., and T. Muir. "Volatility-Managed Portfolios." *Journal of Finance*, 72 (2017), 1611–1644.
Murray, S. "Betting Against Other Betas." Working Paper, Georgia State University (2020).
Newey, W., and K. West. "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix." *Econometrica*, 55 (1987), 703–708.
Novy-Marx, R. "Backtesting Strategies Based on Multiple Signals." NBER Working Paper Series (2016).
Novy-Marx, R., and M. Velikov. "Betting Against Betting Against Beta." *Journal of Financial Economics*, 111 (2021), 1–25.
Stambaugh, R. F., and Y. Yuan. "Mispricing Factors." *Review of Financial Studies*, 30 (2017), 1270–1315.