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ARTICLES

DISCOUNT WINDOW POLICY, BANKING CRISES, AND INDETERMINACY OF EQUILIBRIUM

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We study how discount window policy affects the frequency of banking crises, the level of investment, and the scope for indeterminacy of equilibrium. Previous work has shown that providing costless liquidity through a discount window has mixed effects in terms of these criteria: It prevents episodes of high liquidity demand from causing crises but can lead to indeterminacy of stationary equilibrium and to inefficiently low levels of investment. We show how offering discount window loans at an above-market interest rate can be unambiguously beneficial. Such a policy generates a unique stationary equilibrium. Banking crises occur with positive probability in this equilibrium and the level of investment is suboptimal, but a proper combination of discount window and monetary policies can make the welfare effects of these inefficiencies arbitrarily small. The near-optimal policies can be viewed as approximately implementing the Friedman rule.

Keywords: Discount Window Lending, Liquidity Crisis, Investment, Friedman Rule

1. INTRODUCTION

A fundamental characteristic of banks is that their assets and liabilities have different maturity structures. A bank's liabilities are largely short-term deposits, whereas a substantial part of its assets are typically held in long-term, less liquid investments. This maturity transformation activity produces substantial social benefits, but it also exposes banks to the possibility of a liquidity shortage. If

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total withdrawal demand exceeds the value of the liquid assets in the banking system, a "crisis" will occur in which some withdrawal demand cannot be met.[1] How should a central bank react to such a situation? One natural response is to open a discount window, where short-term loans of currency are offered to banks that are illiquid but otherwise in sound financial condition. Such lending will certainly ease a liquidity-induced banking crisis, but it will also have important general equilibrium effects on the economy. During noncrisis times, for example, the terms at which central-bank credit will be available if a crisis occurs will influence the amount of liquidity that each bank chooses to hold in its portfolio. This choice, in turn, determines (i) the likelihood that the entire banking system will run out of liquidity and slide into a crisis and (ii) the quantity of resources placed into (illiquid) investment and hence the future wealth of the economy. In addition, discount window lending has been shown, in a certain class of models, to be potentially destabilizing in that it leads to indeterminacy of equilibrium allocations. We analyze optimal discount window policy in a general equilibrium environment in which all of the above concerns arise. We study how the interest rate charged on discount window loans affects the frequency and severity of banking crises, the quantity of real investment, and the scope for indeterminacy of equilibrium allocations. We show how discount window lending can be unambiguously beneficial in this environment and that desirable policies can be interpreted as approximately implementing the Friedman rule.

A number of previous papers have shown, in general equilibrium settings, how discount window lending can facilitate the smooth functioning of the banking system and lead to better equilibrium allocations. In Sargent and Wallace (1982), for example, the demand for credit fluctuates deterministically and there is a legal restriction on the issue of private credit instruments. Having a discount window offer loans at a zero nominal interest rate leads to the existence of a Pareto optimal equilibrium, whereas closing the discount window does not. A zero nominal interest rate on discount window loans is optimal in this setting for two reasons. First, because the central bank can costlessly print and lend fiat currency, this rate equates the private cost of liquidity to the social cost of creating it. Second, the market nominal interest rate that supports the Pareto optimal allocation in this environment is also zero. The discount window loans are therefore being made at the market rate of interest and, thus, do not distort savings decisions away from the optimal level. The benefits of providing discount window loans at a zero nominal interest rate have since been established in a variety of environments; examples include Freeman (1996), which focuses on the role of liquidity in the payments system, and Williamson (1998), which examines a model with moral hazard.[2] In each of these papers, having a discount window provide costless liquidity improves equilibrium allocations by eliminating or minimizing the impact of liquidity shortages.

Several recent papers study monetary policy in environments where the social return on real investment is always higher than the social return on money, and hence a market nominal interest rate of zero cannot lead to an optimal allocation.

[See, for example, Bhattacharya, Haslag, and Russell (in press), Haslag and Martin (2003), and Schreft and Smith (2002).] The proper operation—and even the desirability—of a discount window in such settings is not well understood. Smith (2002) presented one such environment. When there is no discount window, a fundamental tension arises between the stability of the banking system and the efficiency of equilibrium allocations. If monetary policy generates a positive nominal interest rate, banks perceive an opportunity cost of holding cash reserves and therefore economize on such holdings. As a result, the banking system is relatively illiquid, and there are recurrent crises in which bank reserves are exhausted and agents in need of liquidity suffer losses in consumption. These crises can be avoided entirely if monetary policy instead conforms to the Friedman rule and generates a zero nominal interest rate. In this case, there is no opportunity cost of holding cash and banks therefore hold sufficient reserves to meet any possible level of liquidity demand. However, banks are then no longer performing their maturity-transformation function and the level of investment is far below optimal.

This environment seems to represent precisely the type of situation where discount window lending would be a useful policy tool. If after observing withdrawal demand banks could obtain loans of currency ("discounting" their real investments to the central bank), they could continue to serve their intermediary function and at the same time crises caused by a shortage of liquidity might be averted. Smith (2002) showed that this intuition is correct: Granting banks access to discount window loans at a zero nominal interest rate eliminates liquidity-induced banking crises. In this respect, his results are in line with the previous literature. However, he also showed that this policy leads to a "massive" indeterminacy of equilibrium, which he interprets as a form of macroeconomic instability.[3] The intuition for this result is as follows: Discount window borrowing is a perfect substitute for cash reserves in this environment, and therefore in equilibrium the discount window cannot be a cheaper source of liquidity than the market. In other words, offering discount window loans at a zero nominal interest rate forces monetary policy to follow the Friedman rule. When both interest rates are zero, however, a bank is completely indifferent between holding liquid assets and making real investments. As a result, any division of banks' portfolios between currency and investment is consistent with equilibrium and there are infinitely many stationary equilibria, each with a different level of real investment. Some of these equilibria generate higher welfare than closing the discount window, but others generate lower welfare. The model does not give clear guidance as to whether or not a discount window should be opened; perhaps the best statement that can be made based on this analysis is that opening a discount window is "dangerous."

We show, in this same environment, how opening a discount window can be unambiguously beneficial. We do so by broadening the set of policies under consideration. When the nominal interest rate charged on discount window loans is *positive* and higher than the market interest rate, there is a unique stationary equilibrium in which money has value. In other words, free access can be granted to discount window loans without generating indeterminacy as long as borrowing from

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the discount window is more expensive than holding cash reserves. In addition, charging a positive interest rate at the discount window allows monetary policy to generate a positive market interest rate and thereby encourage real investment. There are downsides to this approach, however: Banking crises will occur in equilibrium and real investment will remain below the efficient level. Nevertheless, we show that a proper combination of monetary and discount window policies can make the welfare cost of these inefficiencies arbitrarily small. The near-optimal policies entail having low (but positive) market nominal interest rates and nearly costless liquidity at the discount window, and thus approximately implement the Friedman rule.

The remainder of the paper is organized as follows: In the next section, we present the basic model and describe in detail the optimal behavior of banks. In Section 3 we describe the equilibria of the model under different policy regimes. In Section 4 we present the corresponding welfare and optimal policy analysis, and in Section 5 we offer some concluding remarks.

2. THE MODEL

We begin by presenting the environment of Smith (2002) and then introduce a discount window that offers loans at a positive nominal interest rate. The majority of the section is devoted to deriving the optimal behavior of competitive banks as a function of the monetary and discount window policies.

2.1. The Environment

The economy consists of an infinite sequence of two-period lived, overlapping generations of agents, plus an initial old generation. In each period $t = 0, 1, 2, \ldots,$ a continuum of identical agents with unit mass is born in each of two locations. There is a single consumption good; each agent is endowed with $w > 0$ units of this good when young and none when old. Agents only care about consumption in the second period of life and have the utility function $u(c) = \ln(c)$.[4] In the initial period there is a continuum of old agents with unit mass in each location, and each of these agents is endowed with M_{-1} units of fiat currency.

At the beginning of a period, young agents receive their endowment and, possibly, a transfer of currency. At this point, neither agents nor banks can move between or communicate across locations, and therefore trade can only occur within each location. Young agents can trade with old agents and can deposit resources in a bank. Banks can also trade with old agents in this market. After trade takes place and deposits have been made, there is an opportunity to invest goods in a storage technology. This technology transforms one unit of the period t good into $R > 1$ units of the period $t + 1$ good, and it is the only form of real investment available. Goods that are neither consumed nor placed into this technology will perish once the investment opportunity has passed. With the deposits it receives, a bank first

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engages in trade to achieve the desired allocation of its portfolio between money and goods and then invests the goods in the storage technology.

After the investment opportunity has passed, a fraction π_t of young agents in each location discover that they will be moved to the other location.[5] Goods invested in the storage technology cannot be transported between locations unless the investment is first liquidated. A unit of investment that is liquidated yields a return of $r < 1$. Limited communication prevents privately issued liabilities, such as checks, from being verifiable in the other location. Currency, on the other hand, is universally recognizable and noncounterfeitable, and is therefore accepted in both locations. In other words, money facilitates transactions made difficult by spatial separation and limited communication [as in Townsend (1987)]. Movers contact their bank in a decentralized manner and withdraw resources in the form of currency and/or liquidated investment. Immediately afterward, movers are relocated and the next period begins. In this new period, movers use the currency they received from the bank to buy consumption in their new location, and nonmovers contact their bank and withdraw any remaining currency together with the proceeds of matured investment. At this point, all old agents consume and end their life cycle.

The relocation probability π_t is a random variable in each period. Because there is a continuum of young agents, it represents both the probability of relocation for each agent and the fraction of all agents who move. That is, π_t gives the size of the aggregate liquidity shock in period t; higher realizations of π_t correspond to higher demand for liquid assets. It is publicly observable and is independently and identically distributed over time. Let G represent the distribution function, which is assumed to be smooth and strictly increasing on [0,1], and g the associated density function. We should emphasize that the market where goods are exchanged for money in period t meets *before* the realization of π_t. After π_t is realized, no trade occurs until the following period. As a result, the general price level in period t will not depend on the realization of liquidity demand in that period.

2.2. Monetary and Discount Window Policies

The central bank has two policy variables, both of which are chosen once and for all in the initial period. First, it sets a (gross) growth rate σ for the money supply, that is,

$$M_{t+1} = \sigma M_t. \tag{1}$$

Monetary injections/withdrawals take place through lump-sum transfers to young agents. Let τ_t denote the real value of the transfer given to young agents at time t; a negative value of τ_t denotes a tax that must be paid in currency. These transfers take place at the beginning of each period, and hence a state-contingent policy where σ depends on the realization of π_t is infeasible. We assume that $\sigma R \geq 1$ holds. In a stationary equilibrium, σR will be the market nominal interest rate, and we are therefore ruling out policies that would lead money to have a strictly

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higher return than investment. The qualitative properties of equilibrium under such a policy would be very similar to the case where $\sigma R = 1$ holds. Excluding these policies simplifies the presentation without any loss of economic insight.

Second, the central bank sets a (gross) nominal interest rate $\phi \geq 1$ on discount window loans.[6] For most of the analysis, we will focus on the case where $\phi > 1$ holds and the interest rate on discount window loans may be above the market rate of interest. To our knowledge, this is the first paper to study this case in a general equilibrium setting. Interestingly, this case closely resembles the policy adopted by the Federal Reserve System in January 2003. Previously, discount window loans were typically granted at below-market rates, with stringent requirements employed to limit access to this credit. Under the new Primary Credit Program, however, banks in good standing can borrow freely from the discount window at slightly above-market rates.

If, in period t, a bank demands a loan of λ_t (measured in real terms, per unit of deposits), it receives $\lambda_t p_t$ dollars from the discount window, where p_t is the general price level in period t.[7] In the following period, the bank must pay back $\phi\lambda_t p_t$ dollars. We assume that the central bank destroys $\lambda_t p_t$ of these dollars and uses the remaining $(\phi - 1)\lambda_t p_t$ to purchase goods. In this way, the stock of currency in circulation continues to obey (1) and the quantity of discount window lending in period t will not affect the price level in subsequent periods. We assume that agents derive no utility from the revenue earned by the central bank through this lending policy. As will become clear later, if instead the revenue were rebated to agents as a state-contingent, lump-sum payment, our main results would not change. Such rebates complicate the derivations substantially, so we present the simpler case here.

2.3. Banks

A bank offers to pay a return $d_t^m(\pi_t)$ to a depositor if she is relocated and a return $d_t(\pi_t)$ if she is not. As the notation indicates, both of these returns can depend on the size of the (observable) aggregate liquidity shock. It is assumed that banks behave competitively in the sense that they (i) take the real return on assets as given and (ii) choose the deposit return schedules d_t^m and d_t to maximize the expected utility of young lenders. A young lender will therefore deposit her entire income $w + \tau_t$ in a bank; there is no incentive to hold assets directly in this environment. Without any loss of generality, we consider a representative bank that holds all deposits in the economy.

Per unit of deposits, the bank acquires an amount γ_t of real money balances and invests the remaining $1 - \gamma_t$. Let δ_t denote the fraction of this investment that is liquidated early and given to movers, and $(1 - \delta_t)$ the fraction held until maturity and given to nonmovers. Let $\lambda_t \geq 0$ denote the real value of the bank's borrowing from the discount window. The bank faces two constraints on the return schedules it can offer. First, relocated agents must be given currency or liquidated investment. Let $\alpha_t(\pi_t)$ denote the fraction of the bank's cash reserves given to

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movers. Because the real return to holding money between periods t and $t+1$ is given by (p_t/p_{t+1}), the return offered to movers must satisfy

$$\pi_t d_t^m(\pi_t) = \alpha_t(\pi_t)\gamma_t \frac{p_t}{p_{t+1}} + \delta_t(\pi_t)(1-\gamma_t)r + \lambda_t(\pi_t)\frac{p_t}{p_{t+1}}. \quad \textbf{(2)}$$

The second constraint is that payments to nonmovers cannot exceed the value of the bank's residual portfolio—remaining cash reserves plus matured investment minus the repayment of the discount window loan. This constraint can be written as

$$(1-\pi_t)d_t(\pi_t) = [1-\alpha_t(\pi_t)]\gamma_t \frac{p_t}{p_{t+1}} + [1-\delta_t(\pi_t)](1-\gamma_t)R$$
$$-\lambda_t(\pi_t)\phi\frac{p_t}{p_{t+1}}. \quad \textbf{(3)}$$

The bank will therefore choose the functions d_t^m and d_t to maximize

$$\int_0^1 \left\{\pi \ln\left[d_t^m(\pi)(w+\tau_t)\right] + (1-\pi)\ln\left[d_t(\pi)(w+\tau_t)\right]\right\} g(\pi)\,d\pi$$

subject to (2), (3), and non-negativity constraints. Let $I_t \equiv R(p_{t+1}/p_t)$ denote the (gross) market nominal interest rate. That is, I_t reflects the additional return that investment offers over currency and hence represents the opportunity cost of holding cash reserves. Substituting in the two constraints and performing some manipulations, the bank's problem can be written as maximizing

$$\int_0^1 \pi \ln[\alpha_t(\pi)\gamma_t + \delta_t(\pi)(1-\gamma_t)I_t(r/R) + \lambda_t(\pi)]g(\pi)\,d\pi$$
$$+ \int_0^1 (1-\pi)\ln\{[1-\alpha_t(\pi)]\gamma_t + [1-\delta_t(\pi)](1-\gamma_t)I_t$$
$$-\ \lambda_t(\pi)\phi\}g(\pi)d\pi, \quad \textbf{(4)}$$

subject to

$$0 \le \gamma_t \le 1,\quad 0 \le \alpha_t(\pi) \le 1,\quad 0 \le \delta_t(\pi) \le 1,\quad \text{and}$$
$$0 \le \lambda_t(\pi) \le \frac{1}{\phi}\{[1-\alpha_t(\pi)]\gamma_t + [1-\delta_t(\pi)](1-\gamma_t)I_t\} \ \text{ for all } \pi.$$

The final inequality represents the constraint that borrowing at the discount window in period t cannot exceed the bank's ability to repay the loan in period $t+1$.

The fractions of currency reserves and investment paid out to movers, as well as the amount of discount window borrowing, are chosen after the realization of π_t, whereas the fraction of currency in the bank's asset portfolio is chosen before the realization of π_t. Hence, we can solve the problem backwards, by first finding the optimal values of α_t, δ_t, and λ_t as functions of γ_t and π_t. That is, we can first choose $(\alpha_t,\ \delta_t,\ \lambda_t)$ to

$$\begin{aligned}\text{maximize}\quad & \pi_t \ln[\alpha_t\gamma_t + \delta_t(1-\gamma_t)I_t(r/R) + \lambda_t] \\ & +(1-\pi_t)\ln[(1-\alpha_t)\gamma_t + (1-\delta_t)(1-\gamma_t)I_t - \lambda_t\phi],\end{aligned} \tag{5}$$

subject to the constraints above. We begin the process of solving this problem by showing that, outside of one knife-edge case, the bank may respond to high liquidity demand by either liquidating investment or borrowing from the discount window, but not both.

PROPOSITION 1. *If $\phi < R/r$ holds, the solution to* (5) *has $\delta_t = 0$ for all values of γ_t and π_t. If $\phi > R/r$ holds, the solution to* (5) *has $\lambda_t = 0$ for all values of γ_t and π_t.*

The proof of this proposition is contained in the Appendix, but the intuition is straightforward. Borrowing from the discount window and liquidating investment are both ways of generating additional consumption for movers as a group (at the expense of nonmovers). The bank will only use the less costly of the two methods. If the interest rate at the discount window is low, borrowing is less costly and the bank will never liquidate investment. If the interest rate at the discount window is high enough, however, liquidation is less costly and the discount window will be inactive. In the latter case, our model reduces to that presented in Smith (2002). Because we are interested in optimal discount window policies, we focus on the case where $\phi < R/r$ holds. In Section 4, we show how such policies can always increase welfare relative to having an inactive discount window.[8]

In this case, the solution to (5) sets δ_t to zero for all values of π_t and sets

$$\alpha_t = \begin{Bmatrix} \pi_t[1 + I_t(1-\gamma_t)/\gamma_t] \\ 1 \\ 1 \end{Bmatrix} \text{ and } \lambda_t = \begin{Bmatrix} 0 \\ 0 \\ \pi_t(1-\gamma_t)(I_t/\phi) - (1-\pi_t)\gamma_t \end{Bmatrix} \tag{6}$$

$$\text{for } \pi_t \in \begin{Bmatrix} [0, \pi^*) \\ [\pi^*, \pi^{**}) \\ [\pi^{**}, 1) \end{Bmatrix},$$

where we have

$$\pi^* = \frac{\gamma_t}{\gamma_t + (1-\gamma_t)I_t} \tag{7}$$

and

$$\pi^{**} = \frac{\gamma_t}{\gamma_t + (1-\gamma_t)(I_t/\phi)}. \tag{8}$$

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When demand for liquidity is fairly low (i.e., the relocation shock is below a critical value π^*), the bank is able to give movers and nonmovers the same return by paying out only a fraction of its reserves to movers. When the realization of the relocation shock is greater than π^*, however, this approach is no longer feasible. In this case, there are so many movers that even if all of the bank's cash reserves are given to them, they will receive a lower return than the (relatively few) nonmovers. The bank then has an incentive to borrow currency from the discount window so that it can transfer resources from nonmovers to movers. However, such borrowing is costly and, as a result, the bank only undertakes it if the number of movers is above a second critical level π^{**}. For values of π_t above π^*, a banking crisis occurs: $d_t^m(\pi_t) < d_t(\pi_t)$ holds and depositors who need liquidity suffer losses in consumption.

Some intuition for the range of inaction $[\pi^*, \pi^{**}]$ can be gained by thinking about the set of feasible ways for the bank to divide resources between movers (as a group) and nonmovers (as a group), given that γ_t is already fixed. One action that is always feasible is to give all cash reserves to movers and the return from all investment to nonmovers. If instead the bank wants to give fewer total resources to movers and more to nonmovers, perhaps because there are very few movers this period, it can do so on a one-for-one basis. That is, for every unit of future consumption (in the form of currency) that is taken away from movers as a group, exactly one unit is given to nonmovers as a group. Now suppose that instead the bank wants to give more resources to movers and fewer to nonmovers, perhaps because there is a large number of movers. In this case the bank must either liquidate investment or obtain a loan from the discount window, so that for every unit of additional consumption given to movers, nonmovers must give up either R/r or ϕ units. This difference in the rates of transformation is what leads to the range of inaction $[\pi^*, \pi^{**}]$ in the optimal levels of α_t and λ_t. When there are very few movers, the optimal action is to give almost all of the resources to nonmovers, and hence we are in the region where the rate of transformation is unity. As we examine larger and larger realizations of π_t, the solution gives more and more of the bank's currency reserves to movers. At $\pi_t = \pi^*$, the optimal action reaches the kink in the constraint set where all currency reserves are given to movers. This point remains the optimal choice for a range of values of π_t; only when the realization is greater than π^{**} is it optimal to move to the steeper-sloped part of the boundary. In conjunction with (8), this reasoning also demonstrates how the interest rate on discount window loans determines the potential severity of banking crises. The more costly it is to borrow, the larger π_t must be (and therefore the larger the gap between the returns of movers and nonmovers must be) for a bank to be willing to borrow to ease the crisis.

We now proceed to solve for the optimal value of γ_t. To do so, we substitute the optimal values of α_t and λ_t into the bank's objective function in (4) so that the only remaining variable to be determined is γ_t. The problem can then be written

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as

$$\max_{0\leq\gamma_t\leq 1}\int_0^{\pi^*}\ln[\gamma_t+(1-\gamma_t)I_t]g(\pi)\,d\pi$$
$$+\int_{\pi^*}^{\pi^{**}}\{\pi\ln(\gamma_t/\pi)+(1-\pi)\ln[(1-\gamma_t)I_t/(1-\pi)]\}g(\pi)\,d\pi$$
$$+\int_{\pi^{**}}^{1}\{\pi\ln[\gamma_t+(1-\gamma_t)(I_t/\phi)]$$
$$+(1-\pi)\ln[\gamma_t\phi+(1-\gamma_t)I_t]\}g(\pi)\,d\pi. \tag{9}$$

Because borrowing is costly, the solution to this problem will be interior as long as $1 < I_t < \phi$ holds. The first-order condition is given by

$$\frac{I_t-1}{\gamma_t+(1-\gamma_t)I_t}G(\pi^*)+\frac{I_t/\phi-1}{\gamma_t+(1-\gamma_t)(I_t/\phi)}[1-G(\pi^{**})]$$
$$=\frac{1}{\gamma_t}\int_{\pi^*}^{\pi^{**}}\pi g(\pi)d\pi-\frac{1}{1-\gamma_t}\int_{\pi^*}^{\pi^{**}}(1-\pi)g(\pi)\,d\pi,$$

which can be reduced to

$$\gamma_t=\pi^{**}-\int_{\pi^*}^{\pi^{**}}G(\pi)d\pi.$$

This equation implicitly defines (recall that γ_t appears in the expressions for π^* and π^{**} above) the solution to the bank's portfolio allocation problem as a function of the variable $I_t \in (1,\phi)$. Let $\gamma_\phi(I_t)$ denote this solution, where the ϕ subscript indicates that the solution (i) applies in the region of the parameter space where the discount window is active and (ii) depends on the interest rate charged on discount window loans. The next proposition establishes some properties of this solution.

PROPOSITION 2. *For any given $\phi \in (1, R/r)$ and any $I_t > 0$, the bank's problem has a unique solution. The reserve–deposit ratio γ_ϕ in this solution is a continuous function of I_t and satisfies*:

$$\begin{aligned}&(a)\quad \gamma_\phi(I_t)=1 \quad \textit{for}\quad I_t\leq 1,\\ &(b)\quad \gamma_\phi(I_t)=0 \quad \textit{for}\quad I_t\geq \phi,\quad \textit{and}\\ &(c)\quad \gamma_\phi'(I_t)<0 \quad \textit{for}\quad I_t\in(1,\phi).\end{aligned}$$

To see the intuition for this result, suppose that $I_t \leq 1$ holds. Then the return on currency is at least as high as the return on investment. Because currency offers the additional advantage of being liquid, the bank will hold only currency. If $I_t \geq \phi$ holds, on the other hand, then borrowing from the discount window costs no more

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than holding cash reserves. Because the quantity of borrowing can be tailored to the realization of liquidity demand, the bank will hold no cash reserves. For intermediate values of I_t, the bank will hold both types of assets, with the fraction of resources placed in currency being a decreasing function of I_t. A formal proof of the proposition is given in the Appendix. Having solved the optimization problem of the bank, we now turn to an analysis of general equilibrium.

3. EQUILIBRIUM

An equilibrium consists of sequences for the price level $\{p_t\}$ and for bank decision rules $\{\gamma_t, \alpha_t, \delta_t, \lambda_t\}$ such that (i) given $\{p_t\}$, the decision rules solve the bank's problem (4) in each period; (ii) the market where money is traded for goods at the beginning of each period clears; and (iii) the government's budget constraint

$$\tau_t = \frac{M_t - M_{t-1}}{p_t} = \frac{\sigma - 1}{\sigma}\frac{M_t}{p_t}$$

holds in each period. Because young agents deposit all of their income in the bank, the market-clearing condition for period t can be written as

$$\frac{M_t}{p_t} = (w + \tau_t)\gamma_t. \tag{10}$$

Define $z_t \equiv M_t/p_t$ to be the (per-capita) level of real balances in the economy. Then we have

$$\frac{p_t}{p_{t+1}} = \frac{1}{\sigma}\frac{z_{t+1}}{z_t},$$

and hence the market nominal interest rate is given by

$$I_t = \sigma R \frac{z_t}{z_{t+1}}.$$

We can then rewrite (10) as

$$z_t = \left(w + \frac{\sigma - 1}{\sigma} z_t\right)\gamma_\phi\left(\sigma R \frac{z_t}{z_{t+1}}\right).$$

As this expression shows, the behavior of real money balances is governed by a deterministic difference equation. Because markets meet before the realization of the liquidity shock, ex post liquidity demand cannot affect the current-period price level. In addition, currency borrowed from the discount window is removed from circulation when these loans are repaid at the beginning of the following period and hence has no effect on future price levels. For these reasons, the price level and the level of real money balances both follow deterministic paths. It is important to keep in mind, however, that equilibrium consumption is potentially stochastic. Depending on the bank's level of cash reserves and the discount window policy, the returns received by depositors may depend on the realization of π_t.

Following Smith (2002), we focus on stationary equilibria, where $z_t = z$ for all t. In such equilibria, the nominal interest rate is given by $I_t = \sigma R$ and the level of real balances by

$$z^* = \frac{\sigma w \gamma_\phi(\sigma R)}{\sigma - (\sigma - 1)\gamma_\phi(\sigma R)}. \qquad \textbf{(11)}$$

This expression demonstrates that a stationary equilibrium with valued fiat currency exists if and only if $\gamma_\phi(\sigma R) > 0$ holds. In other words, the central bank must set its two policies in such a way that banks demand a positive amount of reserves. Using Proposition 2, a necessary and sufficient condition for this to be the case is that

$$\phi > \sigma R \qquad \textbf{(12)}$$

holds. If this inequality were reversed, borrowing from the discount window would be a strictly better source of liquidity than holding cash reserves. The demand for cash reserves would then be zero and money would have no value. The expression for z^* in (11) also demonstrates that, when (12) holds, there is a *unique* positive level of real money balances consistent with stationary equilibrium. We summarize these results in the following proposition:

PROPOSITION 3. *If* (12) *holds, there exists a unique stationary monetary equilibrium.*

In other words, discount window lending does *not* lead to indeterminacy of stationary equilibrium in this environment if the loans carry a "penalty" rate of interest.[9] This result contrasts strongly with that reported in Smith (2002) for the case where $\phi = \sigma R = 1$ holds. In that case, the bank is completely indifferent between holding reserves and borrowing from the discount window, and this indeterminacy in the solution to the bank's problem translates into an indeterminacy of stationary equilibrium prices and allocations. When $\phi > \sigma R$ holds, however, borrowing from the discount window is more costly than holding cash reserves. We have shown that this implies a unique level of demand for cash reserves, which in turn generates a unique stationary monetary equilibrium. Banking crises may or may not occur in this equilibrium, depending on the exact policies followed by the central bank. We investigate the properties of this equilibrium as well as the optimal policy question in the next section.

4. BANKING CRISES AND WELFARE

We now examine the welfare properties of the equilibrium described above, continuing to assume that (12) holds. Recall that a banking crisis occurs whenever movers receive a lower return on their deposits than do nonmovers. We begin by establishing that, for all but one choice of monetary policy, banking crises will occur in equilibrium.

PROPOSITION 4. *If* $\sigma R = 1$ *holds, banking crises never occur in equilibrium. If* $\sigma R > 1$ *holds, however, banking crises occur with positive probability in each period.*

This result follows directly from Proposition 2, using the fact that $I_t = \sigma R$ holds in a stationary equilibrium. With $I_t = 1$, the bank will set γ_t to unity and therefore will have sufficient cash reserves to meet any level of liquidity demand. With $I_t > 1$, on the other hand, the bank will set γ_t less than unity, and therefore with positive probability the realized value of π_t will be greater than π^*. Because $\phi > 1$ holds, it follows from (7) and (8) that such a value of π_t will necessarily lead to a crisis.

The welfare cost of a banking crisis depends on two factors: the size of the wedge between the returns given to the two types of depositors and the amount of revenue the discount window collects and removes from the economy. Both of these distortions clearly depend on the realized value of π_t as well as on the interest rate at the discount window. We can calculate the largest possible wedge between the two returns using (6). For values of π_t greater than π^{**}, we have

$$d_t^m = \gamma_t \frac{1}{\sigma} + (1 - \gamma_t)\frac{R}{\phi} \quad \text{and} \quad d_t = \gamma_t \frac{\phi}{\sigma} + (1 - \gamma_t)R.$$

The difference between these two expressions is strictly increasing in ϕ. In other words, a lower interest rate on discount window loans implies a better "worst-case scenario" in terms of the gap between d_t^m and d_t. Using (6) again, we can write the revenue collected by the discount window when π_t is greater than π^{**} as

$$\frac{\phi - 1}{\phi}\pi_t(1 - \gamma_t),$$

which is also strictly increasing in ϕ. Thus, lowering the interest rate increases the total resources available to the bank—and hence the average return given to depositors—during times of crisis. Minimizing the welfare loss due to banking crises therefore clearly requires charging a low interest rate at the discount window.

The analysis of the previous sections, however, has shown how the choice of discount window policy restricts the set of monetary policies that can be followed. Together, these two policies determine the level of real investment and hence the total wealth of the economy in the following period. How should a benevolent central bank set the policy pair (σ, ϕ)? Following Smith (2002), we take the objective to be the steady-state utility of a young agent.[10] We begin by describing the first-best allocation in this environment. Consider the problem of a social planner who directly controls investment and allocation decisions in both locations and who therefore is essentially unaffected by the relocation friction. It should be clear that this planner has no use for money and will therefore place the total endowment into storage in each period. When the stored goods mature, the planner will divide the proceeds equally among the (now-old) agents in each location, regardless of their place of birth. The utility level of a young agent in

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this allocation is given by $\ln(Rw)$. No policy implements this allocation in the decentralized economy with relocation and information frictions. However, as we now show, there are policies that implement arbitrarily nearby allocations without introducing indeterminacy of stationary equilibrium.

PROPOSITION 5. *For any* $\varepsilon > 0$, *there exists a policy* (σ, ϕ) *such that steady-state welfare in the unique stationary equilibrium generated by* (σ, ϕ) *is within* ε *of the first-best value* $\ln(Rw)$.

Getting very close to the first-best allocation requires having nearly all of the economy's total endowment placed into storage (and very little held as cash reserves). The bank will be willing to hold very little currency only if borrowing from the discount window is relatively inexpensive, that is, if ϕ is very close to unity. In order for a stationary monetary equilibrium to exist and be unique, we need $1 < \sigma R < \phi$ to hold, and hence for σR to be very close to unity as well. In the proof in the appendix, we show that a sequence of policies can be constructed so that, along this sequence, the allocation in the unique stationary monetary equilibrium converges uniformly to the first-best allocation described above. This policy sequence converges to $\sigma = 1/R$ and $\phi = 1$, so that in the limit the nominal interest rate would be zero both in the market and at the discount window. The near-optimal policies can therefore be viewed as approximating the Friedman rule. We should emphasize that this result is not driven by our assumption that agents derive no utility from the revenue made by the central bank on discount window loans. Because a low interest rate policy can bring the economy very close to the first-best allocation, it is better than a high interest rate policy *regardless* of how this revenue is used.

Note that the first-best allocation itself cannot be implemented. Achieving the first-best allocation requires that *all* of the economy's resources be placed into storage, which implies that there must be zero demand for cash reserves and hence money cannot have value. When money has no value, discount window lending is clearly ineffective. This fact points to an important distinction between money and liquidity. Money is an asset that is inherently liquid, but a demand for liquidity does not necessarily imply a demand for money. Indeed, the benefit of having a discount window in this environment derives precisely from the fact that it helps meet the liquidity needs of relocated agents in a way that does not prevent socially productive investments from being undertaken.[11] What Proposition 5 shows is that a "good" policy in this environment is to make the demand for cash reserves very small and to use the discount window to provide nearly costless liquidity.

5. CONCLUDING REMARKS

Studying environments where the social return on investment is always higher than the social return on money is important for understanding the aggregate tradeoff between liquidity and real investment. Previous work has indicated that opening a discount window in such an environment may lead to macroeconomic instability. We show that this need not be the case. By charging a positive interest rate on discount window loans, a central bank *can* open a discount window without generating indeterminacy of equilibrium. Furthermore, by carefully coordinating this interest rate with the growth rate of the money supply, allocations arbitrarily close to the first-best can be implemented.

The specific policy prescriptions of our analysis obviously depend on the specific features of the environment we consider. In particular, the nonexistence of an exact optimal policy stems from the fact that discount window loans are a perfect substitute for cash reserves. This feature would most likely change if money played another role in the model, such as facilitating investment or market transactions. The demand for currency would then not be completely undermined by a costless-liquidity policy at the discount window, and an exact optimal policy would likely exist. It would be interesting to know under what conditions this policy would or would not correspond to the Friedman rule. Similarly, the only uncertainty in our model is about liquidity demand. There is no uncertainty about the real value of a bank's assets or a bank's ability to repay a discount window loan. It would be interesting to know how the optimal discount window policy would change in the presence of solvency shocks[12] and, in particular, whether or not such shocks would make a strictly positive penalty rate optimal. If so, the optimal money growth rate might well differ from the Friedman rule. Another important friction that is absent from our framework is asymmetric information. It is well known that the presence of a lender of last resort can generate moral hazard, resulting in excessive risk taking by banks and therefore greater uncertainty about a bank's ability to repay the loan.[13] Again, it would be interesting to see how such a change in the environment would affect the optimal monetary and discount window policies. We leave these questions for future research.

NOTES

1. We use the term "banking crisis" to indicate a situation where withdrawal demand is larger than total cash reserves and depositors in need of liquidity suffer consumption losses. Champ, Smith, and Williamson (1996) argued that this definition describes the crises of the nineteenth-century U.S. banking system very well. A comprehensive discussion of the features of modern banking crises around the world is provided by Boyd, Kwak, and Smith (in press).

2. In a later paper, Freeman (1999) studied an environment with aggregate financial shocks and compared zero-nominal-rate discount window lending with other ways of providing an elastic currency. See also Schreft and Smith (2002), which showed how discount window lending can be superior to open market operations as a policy tool, and Williamson (2003), which focused on the role of overnight lending in the payments system.

3. Smith and Weber (1999) studied a related environment and showed how having an elastic currency generated by private banknote issue can lead to a similar indeterminacy of equilibrium.

4. As in Champ, Smith, and Williamson (1996) and others, the assumption of logarithmic utility here permits the solution to the bank's problem to be characterized analytically.

5. The stochastic relocations in this model play a role similar to that of the portfolio-preference shocks commonly used in the literature on bank runs. Diamond and Dybvig (1983) is the classic reference; see also the recent papers by Peck and Shell (2003) and Ennis and Keister (2003) and

the references therein. However, we should emphasize that a crisis in our model is caused by a high realization of liquidity demand, not a self-fulfilling bank run.

6. In principle, the interest rate ϕ could be made contingent on the realization of π_t. However, we show below that policies where ϕ is fixed can achieve allocations arbitrarily close to the first-best, and hence there is no gain in looking at more general policy rules.

7. By symmetry, p_t will be the same in both locations. Throughout the analysis, we only consider equilibria where money has value and hence p_t is finite for all t.

8. There is one borderline case that the proposition does not cover, when ϕ is exactly equal to R/r. In this case the solution to (5) is not unique, because the bank is indifferent between liquidating investment and borrowing from the discount window. In what follows, we ignore this knife-edge case in order to simplify the presentation. The results in Section 4 show that setting $\phi = R/r$ cannot be part of an optimal policy.

9. Interestingly, this type of policy was advocated by Bagehot (1873), who stated that in times of crisis the monetary authority should act as a lender of last resort and lend freely to the banking system, but "at a penalty rate." Martin (2002) argued that Bagehot's prescription applies in a commodity money regime where total reserves are scarce, but not in a fiat money regime where currency can be freely printed. Proposition 3 gives an example of how a penalty-rate policy can be useful in a fiat money system.

10. That is, we ignore the initial old generation in our welfare calculations. See footnote 9 in Smith (2002) on this issue.

11. In this way, our Proposition 5 is closely related to Proposition 5 in Haslag and Martin (2003). They studied a model where the fraction of agents relocated is the same in every period and showed how discount window lending can allow banks to hold fewer reserves and make more productive investment.

12. In a different environment, Freeman (1999) studied this problem and found that a zero-nominal-rate lending policy should be combined with quantity restrictions at the discount window. See also Antinolfi, Huybens, and Keister (2001) on the role of quantity restrictions.

13. Williamson (1998) studied a model in which there is moral hazard and showed that in this setting, discount window lending at a zero nominal interest rate is welfare improving.

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APPENDIX

Proof of Proposition 1. The derivatives of the objective function (5) with respect to the three choice variables are

$$\frac{\partial}{\partial \alpha_t} : \frac{\pi_t \gamma_t}{\alpha_t \gamma_t + \delta_t (1-\gamma_t) I_t (r/R) + \lambda_t} - \frac{(1-\pi_t)\gamma_t}{(1-\alpha_t)\gamma_t + (1-\delta_t)(1-\gamma_t) I_t - \lambda_t \phi}, \tag{13}$$

$$\frac{\partial}{\partial \delta_t} : \frac{\pi_t (1-\gamma_t) I_t (r/R)}{\alpha_t \gamma_t + \delta_t (1-\gamma_t) I_t (r/R) + \lambda_t} - \frac{(1-\pi_t)(1-\gamma_t) I_t}{(1-\alpha_t)\gamma_t + (1-\delta_t)(1-\gamma_t) I_t - \lambda_t \phi}, \tag{14}$$

$$\frac{\partial}{\partial \lambda_t} : \frac{\pi_t}{\alpha_t \gamma_t + \delta_t (1-\gamma_t) I_t (r/R) + \lambda_t} - \frac{(1-\pi_t)\phi}{(1-\alpha_t)\gamma_t + (1-\delta_t)(1-\gamma_t) I_t - \lambda_t \phi}. \tag{15}$$

Suppose that $\phi < R/r$ holds. For the solution to have $\delta_t > 0$, we need (14) to be non-negative. However, this would imply that both (13) and (15) are strictly positive, and therefore that the solution must have α_t and λ_t at their maximum possible values. In other words, it must be the case that no resources are kept for nonmovers, so that $d_t = 0$ holds. Given this, it is straightforward to show that d_t^m is strictly decreasing in δ_t (because an increase in liquidation implies a corresponding decrease in discount window borrowing and liquidation is more costly). Therefore, the solution cannot have $\delta_t > 0$.

Now, suppose that $\phi > R/r$ holds. For the solution to have $\lambda_t > 0$, we need (15) to be non-negative. This would imply that both (13) and (14) are strictly positive, and therefore that the solution must have $\alpha_t = 1$ and $\delta_t = 1$. However, the bank would then have no resources left at time $t+1$ with which to repay the loan, and therefore the upper bound on λ_t would be zero. Hence, the solution cannot have $\lambda_t > 0$. ■

Proof of Proposition 2. Let $M(\gamma_t, I_t)$ denote the objective function in (9). Because this function is strictly concave and the constraint set is compact, we know that there is a unique solution to the bank's problem for any $I_t > 0$. The first derivative of the objective can be written as

$$M_1(\gamma_t, I_t) = \frac{1 - I_t}{\gamma_t + (1-\gamma_t)I_t} G(\pi^*) + \frac{1 - I_t/\phi}{\gamma_t + (1-\gamma_t)(I_t/\phi)}[1 - G(\pi^{**})]$$

$$+ \int_{\pi^*}^{\pi^{**}} \left(\frac{\pi}{\gamma_t} + \frac{1-\pi}{1-\gamma_t} \right) g(\pi)\, d\pi. \tag{16}$$

Using L'Hôpital's rule, one can show

$$\lim_{\gamma_t \to 1} M_1(\gamma_t, I_t) = 1 - I_t.$$

If $I_t \leq 1$ holds, then this limit is non-negative and the concavity of the objective function implies that the solution must be at the boundary point $\gamma_t = 1$. This establishes part (a) of the proposition. Part (b) is established in a similar manner. Again using L'Hôpital's rule, we have

$$\lim_{\gamma_t \to 0} M_1(\gamma_t, I_t) = \frac{\phi}{I_t} - 1.$$

If $I_t \geq \phi$ holds, this limit is non-positive and, thus, the solution must be $\gamma_t = 0$.

For values of I_t strictly between unity and ϕ, the solution to the problem is defined by the first-order condition $M_1(\gamma_t, I_t) = 0$. The effect of a change in I_t on the optimal value of γ_t is then determined by implicit differentiation of the identity

$$M_1(\gamma_\phi(I_t), I_t) \equiv 0,$$

which yields

$$\gamma_\phi'(I_t) = -\frac{M_{12}(\gamma_t, I_t)}{M_{11}(\gamma_t, I_t)}.$$

Differentiating (16) with respect to I_t yields

$$M_{12}(\gamma_t, I_t) = -\frac{G(\pi^*)}{[\gamma_t + (1-\gamma_t)I_t]^2} - \frac{[1 - G(\pi^{**})]}{\phi\,[\gamma_t + (1-\gamma_t)(I_t/\phi)]^2} < 0.$$

By strict concavity we know that M_{11} is also negative and, therefore, we have

$$\gamma_\phi'(I_t) < 0,$$

which establishes part (c) of the proposition. Finally, the limits displayed above show that the function $\gamma_\phi(I_t)$ is continuous at the points $I_t = 1$ and $I_t = \phi$, and hence is continuous over the whole domain. This completes the proof of the proposition. ■

Proof of Proposition 5. Fix any sequence $\{\phi_j\}_{j=1}^{\infty}$ of discount window interest rates such that $1 < \phi_j < R/r$ holds for all j and

$$\lim_{j \to \infty} \phi_j = 1.$$

Next, fix a sequence $\{\overline{\gamma}_j\}_{j=1}^{\infty}$ of reserve-deposit ratios such that $0 < \overline{\gamma}_j < 1$ holds for all j and

$$\lim_{j \to \infty} \overline{\gamma}_j = 0.$$

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For each j, find the money growth rate σ_j that would lead the bank to choose $\overline{\gamma}_j$ as the reserve-deposit ratio. That is, choose σ_j to satisfy

$$\gamma_{\phi_j}(\sigma_j R) = \overline{\gamma}_j.$$

From Proposition 2, we know that such a value of σ_j always exists. Furthermore, because $\overline{\gamma}_j$ is strictly between zero and unity, we must have

$$1 < \sigma_j R < \phi_j$$

for all j, and therefore the sequence $\{\sigma_j\}$ must converge to $1/R$.

Proposition 3 implies that each policy (σ_j, ϕ_j) generates a unique stationary monetary equilibrium. From (6) it is straightforward to show that in such an equilibrium, both

$$\gamma_t \frac{1}{\sigma} + (1-\gamma_t)\frac{R}{\phi} \leq d_t^m(\pi_t) \leq \gamma_t \frac{1}{\sigma} + (1-\gamma_t)R$$

and

$$\gamma_t \frac{1}{\sigma} + (1-\gamma_t)R \leq d_t(\pi_t) \leq \gamma_t \frac{\phi}{\sigma} + (1-\gamma_t)R$$

hold for all $\pi_t \in [0, 1]$. These two expressions show that as $\overline{\gamma}_j$ converges to zero and ϕ to unity, the functions d_t^m and d_t both converge uniformly to the constant function R. The expected utility level of a young agent along the sequence of policies (σ_j, ϕ_j) must therefore converge to $\ln(Rw)$, which establishes the result. ■

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