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RESEARCH ARTICLE

Modeling and base parameters identification of legged robots

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Abstract

This paper first uses a decoupling modeling method to model legged robots. The method groups all the degrees of freedom according to the number of limbs, regarding each limb as a manipulator with serial structure, which greatly reduces the number of dynamic parameters that need to be identified simultaneously. On this basis, a step-by-step identification method from the end-effector link to the base link, referred to as "E-B" identification method, is proposed. Simulation verification is carried out on a quadruped robot with 16 degrees of freedom in Gazebo, and the validity of the method proposed is discussed.

1. Introduction

With the expansion of robot application scenarios, it is of great practical significance to study how to enable robots to perform tasks in the rugged and complex unstructured environment. Compared with caterpillar or wheeled mobile robots, legged robots with strong flexibility and terrain adaptability are more likely to perform tasks excellently in complex terrain environments. The dynamic model of a legged robot is usually complex. Some literatures, for example, ref. [1] took an eight-legged walking robot as a whole to establish the dynamic model. References [2, 3] proposed the control method ignoring mass of legs in the robot model to reduce model complexity. In the case of heavy load and high-speed leg movement, there are two reasons for that a legged robot model without mass of legs cannot meet application requirements. One is the weight of joint actuators generally increased in order to provide more power, and the other is the dynamic effect of the legs' mass becoming non-negligible at high-speed movement. With the legs' mass, it is necessary to know the parameters in the dynamic equations whether to solve the interaction force between the robot and the environment or to boost model-based control. Many literatures have studied the identification of dynamic parameters. References [4, 5] and [6] showed that not all inertial parameters participate in the calculation of dynamic equations, and the symbolic expression of the minimum inertial parameters set that contributed to the dynamic equations is discussed. Reference [7] proposed an approach of base parameters identification using neural network. Reference [8] divided the identification process into two steps. In the first step, inverse dynamics and pseudo-inverse matrix are used to solve the base parameters of the model. In the second step, neural network compensator is used to improve the accuracy of the parameters. Reference [9] proposed the load inertial parameters identification method of industrial manipulator, which is significant in that the control law can be adjusted to improve motion performance of the manipulator. Reference [10] systematically summarized the identification methods of robot dynamic parameters and listed the practical applications of dynamic parameters identification. Reference [11] studied the identification problem of the open-loop robot system and discussed the influence of temperature on base parameters. Reference [12] indicated that control laws based on careful identification procedures improve transparency compared to classical closed-loop position control laws.

Dynamic parameters can be divided into three categories: geometric parameters, inertial parameters, and other dynamic parameters. Geometric parameters are determined at design stage, but other parameters in practical systems are difficult to obtain accurately. Although the dynamic equations are nonlinear, they are linear about the inertial parameters. The linear combination of inertial parameters called "base parameters" [6] can be obtained by the least square method. References [13] and [14] proposed an approach of the numerical solution of the base parameters set. Reference [15] studied the issue of base parameters identification under the condition of noise interference. Reference [16] proposed a base parameters calculation method for general multi-body systems, and ref. [17] studied the base parameters symbolic expressions for planar open-loop and closed-loop mechanisms.

The solution sensitivity of the linear equations to the input error can be measured by the condition number. [18] The smaller the condition number, the higher the accuracy of the result. Condition number of regression matrix is related to the form of dynamic equations, the excitation trajectory, and so on. Many literatures studied the generation of excitation trajectory. Reference [19] proposed a method of excitation trajectory generation, which can reduce the influence of model error and noise on least squares solution, and the excitation trajectory is composed of a series of interpolation points in space. References [20] and [21] suggested that the excitation trajectories are composed of Fourier series; they thought an analytical solution of velocity and acceleration can be obtained in this way. Reference [22] proposed a method for excitation trajectory generation using both Fourier series and polynomial. A predetermined trajectory composed of Fourier series with inertial parameters provided by Computer Aided Design (CAD) is used to generate motion data of a manipulator in ref. [23]. Reference [24] made use of the Fourier series trajectory generation method to identify dynamic parameters in robot system with complaint actuators, but the specific excitation trajectory parameters are not given. In fact, without the knowledge of dynamic parameters, there is no way to use force control to let the manipulator move along the desired trajectory. If using position control, no matter how close the distance between any two interpolation points, velocity and acceleration will change with time and not equal to the first- and second-order derivative of the desired trajectory, which limits the practical application of excitation trajectory research.

The methods introduced above have a common point that all parameters are identified simultaneously. To the best of our knowledge, the performance of the step-by-step identification method has not been fully discussed.

Based on the analysis of the equivalence principle of Einstein's general theory of relativity and general relativity principle, regarding the body fixed with inertial measurement unit (IMU) of legged robot as base link, each limb is equivalent to a manipulator with serial structure. Using the acceleration data provided by IMU, the robot dynamics model is decomposed into mutually decoupled dynamic models of each limb. Thus, the model decoupling between robot limbs is realized without any simplification of the model, and the number of degrees of freedom coupled to each other and the number of inertial base parameters to be identified simultaneously are greatly reduced. In this paper, our main contribution is to propose a new identification method which is different from the traditional method of identifying all the parameters simultaneously. Our method only needs to identify base parameters of one link at one step, so as to improve the accuracy of the identification results. In order to compare identification results with real values, verification with a quadruped robot built in Gazebo simulation environment is provided.

This paper is organized as follows. Section 2 discusses decoupling modeling method of legged robots. Section 3 introduces the "E-B" identification method and points out that two kinds of different forms of dynamic equations can be applied to obtain base parameters. In Section 4, simulation verification is carried out in Gazebo, giving the regression matrix condition numbers with different identification methods and comparison between base parameters obtained by the least square method and real values. Finally, the validity of the method proposed in this paper is verified by estimating foot contact forces using the dynamic equations.

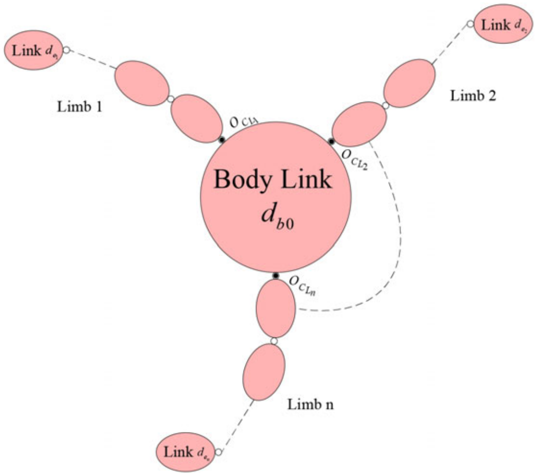

Figure 1. *Schematic diagram of a legged robot with n limbs.*

2. Decoupling modeling of legged robot

In order to give full play to the flexibility of legged robot and make it achieve good quality of control effect, it is important to establish dynamic equations of the robot and obtain the parameters in the equations. In general, the number of degrees of freedom of legged robot is far more than that of a manipulator, which leads to the complexity of dynamic equations and the number of dynamic parameters to be identified also increasing. Since the limbs of a legged robot are similar in structure, one natural idea is whether the limbs can be modeled separately from each other. Fortunately, this can be achieved with Einstein's general relativity and data provided by IMU.

The two basic principles of Einstein's general theory of relativity can be briefly described as follows:

- Equivalence principle: Dynamic effects of the inertial and gravitational fields are locally indistinguishable;
- Principle of generalized relativity: All laws of physics take the same form in any frame of reference.

As shown in Fig. 1, assuming the number of limbs of a legged robot is n, each limb is a serial structure, and a IMU is fixed to the body link. The cartesian coordinate frame is established on the origin of measurements of the IMU, which is called the body frame. Denoting end-effector links of limbs as $d_{e_i}(i=1,2,\ldots,n)$, all the links in the path from link d_{b0} to link d_{e_i}, excluding link d_{b0}, form a set, and there are altogether n of them. Each of them has a link connected to d_{b0} by a joint, and a cartesian coordinate frame is established by taking the fixed point $o_{C_{L_i}}$ connected to d_{b0} on the axis of the joint as the origin, which is called the limb frame. The acceleration $\mathbf{a}_{o_{C_{L_i}}}$ at the origin of the limb frame can be expressed as

$$\mathbf{a}_{o_{C_{L_i}}} = \mathbf{a}_{\bar{o}} + \boldsymbol{\omega}_{\bar{o}} \times (\boldsymbol{\omega}_{\bar{o}} \times \mathbf{r}_{o_{C_{L_i}}}) + \mathbf{w}_{\bar{o}} \times \mathbf{r}_{o_{C_{L_i}}}$$

where $\mathbf{a}_{\bar{o}}$, $\boldsymbol{\omega}_{\bar{o}}$, and $\mathbf{w}_{\bar{o}}$, are, respectively translational acceleration, angular velocity, and angular acceleration of the origin of measurements inside the IMU. $\mathbf{r}_{o_{C_{L_i}}}$ is the position vector of point $o_{C_{L_i}}$ in body frame $\bar{o}$. The position of a point in the limb frame $o_{C_{L_i}}$ is expressed as $\mathbf{r}_j$. The acceleration $\mathbf{a}_j$ at this point is determined by the origin translational acceleration $\mathbf{a}_{o_{C_{L_i}}}$ of the limb frame, the rotation acceleration $\mathbf{w}_{\bar{o}}$ of the limb frame, the rotation angular velocity $\boldsymbol{\omega}_{\bar{o}}$, and the linear velocity $\dot{\mathbf{r}}_j$ of this point relative to the limb frame. Thus,

$$\mathbf{a}_j = \mathbf{a}_{o_{C_{L_i}}} + f(\mathbf{r}_j, \dot{\mathbf{r}}_j, \boldsymbol{\omega}_{\bar{o}}, \mathbf{w}_{\bar{o}})$$

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where $f(\mathbf{r}_j, \dot{\mathbf{r}}_j, \boldsymbol{\omega}_{\bar{o}}, \mathbf{w}_{\bar{o}})$ consists of centripetal acceleration, Coriolis acceleration, and angular acceleration. According to the basic principle of general relativity, gravitational field and inertial force field are locally equivalent. Only if we just apply a force $m_j\mathbf{a}_j$ (m_j is mass at point j) in the opposite direction to the acceleration at any point j in frame $o_{C_{L_i}}$, all laws of physics are true in this frame, which allows dynamic equations of legged robot to be set up in n groups. The inertial force generated by the translational acceleration $\mathbf{a}_{o_{C_{L_i}}}$ directly acts on the center of mass of the link, and the inertial force generated by the rotation of the limb frame is related to the position. In practical application, if the angular velocity of the limb frame is not large, the rotation can be ignored to simplify the dynamic equations.

The key to decoupling modeling is to obtain the translational acceleration $\mathbf{a}_{o_{C_{L_i}}}$ of the limb frame and the acceleration $f(\mathbf{r}_j, \dot{\mathbf{r}}_j, \boldsymbol{\omega}_{\bar{o}}, \mathbf{w}_{\bar{o}})$ generated by the rotation. This is the reason why IMU plays a role in decoupling modeling. It can directly give the data of the acceleration $\mathbf{a}_{\bar{o}}$ and figure out $\boldsymbol{\omega}_{\bar{o}}$ and $\mathbf{w}_{\bar{o}}$ from attitude angle. With Einstein's general theory of relativity and IMU sensor, the decoupling modeling of legged robot is realized without any simplification. By decoupling modeling, the dynamic parameters to be identified are also divided into n groups, which reduces the number of parameters to be identified at one data set and consequently improve the accuracy of identification results. The robot dynamic equations after decoupling modeling can be expressed as

$$\begin{cases} \mathbf{M}_1(\mathbf{q}_1)\ddot{\mathbf{q}}_1 + \mathbf{C}_1(\mathbf{q}_1, \dot{\mathbf{q}}_1)\dot{\mathbf{q}}_1 + \mathbf{G}_1(\mathbf{q}_1) = \boldsymbol{\tau}_1 + \mathbf{f}_{ext,1}, \\ \mathbf{M}_2(\mathbf{q}_2)\ddot{\mathbf{q}}_2 + \mathbf{C}_2(\mathbf{q}_2, \dot{\mathbf{q}}_2)\dot{\mathbf{q}}_2 + \mathbf{G}_2(\mathbf{q}_2) = \boldsymbol{\tau}_2 + \mathbf{f}_{ext,2}, \\ \qquad\qquad\qquad \cdots \\ \mathbf{M}_n(\mathbf{q}_n)\ddot{\mathbf{q}}_n + \mathbf{C}_n(\mathbf{q}_n, \dot{\mathbf{q}}_n)\dot{\mathbf{q}}_n + \mathbf{G}_n(\mathbf{q}_n) = \boldsymbol{\tau}_n + \mathbf{f}_{ext,n}. \end{cases} \tag{1}$$

where $\mathbf{q}_i (i = 1, 2, \ldots n)$ is generalized coordinates vector, $\mathbf{M}_i(\mathbf{q}_i)(i = 1, 2, \ldots n)$ is the mass matrix which is symmetric and positive definite, $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)(i = 1, 2, \ldots n)$ represents the Coriolis force and centripetal force terms, $\mathbf{G}_i(\mathbf{q}_i)(i = 1, 2, \ldots n)$ is the gravity term, $\boldsymbol{\tau}_i(i = 1, 2, \ldots n)$ represents joint generalized force, and $\mathbf{f}_{ext,i}(i = 1, 2, \ldots, n)$ represents all other external generalized forces. The dynamic equations for every group (or every limb) are similar. More importantly, they are independent of each other. Compared with solving all the dynamic equations, several independent sets of dynamic equations allow us to solve some task-related equations as needed, which can reduce the computation amount and increase the execution speed of the program. Furthermore, the direct result by decoupling modeling method is to reduce the number of subsequent dynamic parameters that need to be identified simultaneously.

An important issue is how to obtain the angular velocity and the angular acceleration. In practice, the angular velocity can usually be obtained by difference of angles and proper filtering. As for the angular acceleration, it cannot usually be obtained by the difference method. We can ignore the relevant terms and use other robust control methods to compensate. Related modeling work can also refer to refs. [25, 26].

3. Base parameters identification

3.1. Two kinds of dynamic equations

The dynamic equations of the set C_{L_i} is

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \boldsymbol{\tau} + \mathbf{f}_{ext} \tag{2}$$

The inertia tensor matrix of link $d_i \in C_{L_i}$ with mass m_i about the link frame is

$$\mathbf{I}_i = \begin{bmatrix} I_{ixx} & I_{ixy} & I_{ixz} \\ I_{ixy} & I_{iyy} & I_{iyz} \\ I_{ixz} & I_{iyz} & I_{izz} \end{bmatrix} \tag{3}$$

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where $\mathbf{I}_i$ is the symmetric and positive definite matrix. The first moment-of-inertia vector of link d_i is

$$\mathbf{L}_i = m_i\mathbf{r}_i = \begin{bmatrix} m_i r_{x_i} \\ m_i r_{y_i} \\ m_i r_{z_i} \end{bmatrix} \tag{4}$$

where $\mathbf{r}_i = \begin{bmatrix} r_{x_i} \\ r_{y_i} \\ r_{z_i} \end{bmatrix}$ is the position vector of center of mass of link d_i about the link frame. Inertial parameters of link d_i are denoted as

$$\boldsymbol{\delta}_i = \left[I_{ixx}\ I_{iyy}\ I_{izz}\ I_{ixy}\ I_{ixz}\ I_{iyz}\ m_i r_{x_i}\ m_i r_{y_i}\ m_i r_{z_i}\ m_i \right]^T \tag{5}$$

Then, all inertia parameters of a limb with N links are

$$\boldsymbol{\delta} = \left[\boldsymbol{\delta}_1^{\mathrm{T}}\ \boldsymbol{\delta}_2^{\mathrm{T}} \cdots \boldsymbol{\delta}_N^{\mathrm{T}} \right]^T \tag{6}$$

By arranging Eq. (2) according to the parameters in vector $\boldsymbol{\delta}$, the form of dynamic equation often used in identification can be obtained,

$$\mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\boldsymbol{\delta} = \boldsymbol{\tau} \tag{7}$$

Although Eq. (2) is nonlinear, it is a linear equation about $\boldsymbol{\delta}$. So, the least square method can be used to get the dynamic parameters. Here we point out that, in addition to Eq. (2), there is a second form of the dynamic equations which is linear about $\boldsymbol{\delta}$. According to Lagrange's equations of the second kind, the dynamic equations of the multi-degree-of-freedom system can be expressed as [30]

$$\sum_{k=1}^{n} M_{ks}\ddot{q}_k + \sum_{k=1}^{n}\sum_{m=1}^{n} [k, m;s]\dot{q}_k\dot{q}_m = Q_s \quad (s = 1, 2, \ldots, n) \tag{8}$$

where $q_k(k = 1, 2, \ldots, n)$ is the generalized coordinate, M_{ks} is the component of the mass matrix $\mathbf{M}$, Q_s is a generalized force (including gravity term) relative to generalized coordinates q_s, and $[k, m;s]$ denotes Christoffel's symbols of the first kind,

$$[k, m;s] = [m, k;s] = \frac{1}{2}\left(\frac{\partial M_{ks}}{\partial q_m} + \frac{\partial M_{ms}}{\partial q_k} - \frac{\partial M_{km}}{\partial q_s} \right) \tag{9}$$

Let $f(s, m) = \sum_{k=1}^{n} [k, m;s]\dot{q}_k$, since

$$\sum_{k=1}^{n}\sum_{m=1}^{n} [k, m;s]\dot{q}_k\dot{q}_m = \sum_{m=1}^{n} f(s, m)\dot{q}_m \tag{10}$$

comparing with Eq. (2) to get

$$\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} f(1, 1) & f(1, 2) & \cdots & f(1, n) \\ f(2, 1) & f(2, 2) & \ldots & f(2, n) \\ \vdots & \vdots & \vdots & \vdots \\ f(n, 1) & f(n, 2) & \cdots & f(n, n) \end{bmatrix}_{\mathbf{n}\times\mathbf{n}} \tag{11}$$

Considering

$$f(s, m) = \sum_{k=1}^{n} [k, m;s]\dot{q}_k = \sum_{k=1}^{n} \frac{1}{2}\left(\frac{\partial M_{ks}}{\partial q_m} + \frac{\partial M_{ms}}{\partial q_k} - \frac{\partial M_{km}}{\partial q_s} \right)\dot{q}_k \tag{12}$$

and

$$f(m, s) = \sum_{k=1}^{n} [k, s;m]\dot{q}_k = \sum_{k=1}^{n} \frac{1}{2}\left(\frac{\partial M_{km}}{\partial q_s} + \frac{\partial M_{sm}}{\partial q_k} - \frac{\partial M_{ks}}{\partial q_m} \right)\dot{q}_k \tag{13}$$

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then adding the two equations, we get

$$f(m,s)+f(s,m)=\sum_{k=1}^{n}\frac{\partial M_{ms}}{\partial q_k}\dot{q}_k=\frac{dM_{ms}}{dt} \tag{14}$$

By writing the above equation into matrix form, the relation between mass matrix $\mathbf{M}(\mathbf{q})$ and matrix $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ is derived,

$$\dot{\mathbf{M}}(\mathbf{q})=\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})+\mathbf{C}^{\mathrm{T}}(\mathbf{q},\dot{\mathbf{q}}) \tag{15}$$

The second form of dynamic equations can be obtained using Eq. (15). Integrating both sides of Eq. (2) and integration by parts applied on the left side of that with Eq. (15), we derive dynamic equations of the following form:

$$\mathbf{M}(\mathbf{q}(t))\dot{\mathbf{q}}(t)=\mathbf{M}(\mathbf{q}(t_0))\dot{\mathbf{q}}(t_0)+\int_{t_0}^{t}(\boldsymbol{\tau}+\mathbf{C}^{\mathrm{T}}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}-\mathbf{G}(\mathbf{q}))dt' \tag{16}$$

where $\mathbf{M}(\mathbf{q}(t))\dot{\mathbf{q}}(t)$ is the generalized momentum at time t. We refer to Eq. (16) as "acceleration-free dynamic equations." By arranging Eq. (16), a new dynamic equation form linear about vector $\boldsymbol{\delta}$ can be obtained,

$$\mathbf{K}(\mathbf{q},\dot{\mathbf{q}})\boldsymbol{\delta}=\int_{t_0}^{t}\boldsymbol{\tau}\,dt' \tag{17}$$

In the case of only position sensor, velocity and acceleration can be solved by first- and second-order difference of the position, respectively. In a real system with noise, the accuracy of acceleration signal obtained by second-order difference is low. The low-precision acceleration signal will reduce the data quality of regression matrix $\mathbf{H}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})$. Because the regression matrix $\mathbf{K}(\mathbf{q},\dot{\mathbf{q}})$ has no acceleration term, we can predict that the data quality will be better. The identification results of dynamic Eq. (17) are given in Section 4.

3.2. Base parameters

The reason why all inertial parameters cannot be identified by dynamic equations is analyzed below from the perspective of matrix theory.

No matter which dynamic equation form is adopted, the corresponding regression matrix $\mathbf{W}$ and joint generalized force vector $\mathbf{Y}$ can be obtained by sampling motion data from the identification trajectory and torque sensor. Their relationship satisfies the following equation:

$$\mathbf{W}_{m\times n}\boldsymbol{\delta}_{n\times 1}=\mathbf{Y}_{m\times 1} \tag{18}$$

where $\mathbf{W}_{m\times n}$ is an $m\times n$ regression matrix($m\gg n$), $\mathbf{Y}_{m\times 1}$ is an m dimensional column vector, and $\boldsymbol{\delta}_{n\times 1}$ is an n dimensional inertial parameters to be identified (matrix dimension subscripts are omitted for convenience hereafter). Since the regression matrix is not column full rank, [27] the least square method cannot be directly used to solve Eq. (18). In fact, there is a nonsingular matrix $\mathbf{K}$ that satisfies

$$\mathbf{W}\mathbf{K}=\left[\,\mathbf{W}_b\ \mathbf{0}\,\right] \tag{19}$$

Right multiplying a matrix by a nonsingular matrix is equivalent to performing a column elementary transformation, [28] so matrix $\mathbf{W}$ is decomposed into a column full rank matrix $\mathbf{W}_b$ and a null matrix. Furthermore, the following relations are obtained by transforming Eq. (18),

$$\mathbf{W}\boldsymbol{\delta}=\mathbf{W}\mathbf{K}\mathbf{K}^{-1}\boldsymbol{\delta}=\left[\,\mathbf{W}_b\ \mathbf{0}\,\right]\left[\,\boldsymbol{\delta}_b^{\mathrm{T}}\ \mathbf{0}\,\right]^{\mathrm{T}}=\mathbf{Y}$$

$$\Rightarrow \mathbf{W}_b\boldsymbol{\delta}_b=\mathbf{Y} \tag{20}$$

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For Eq. (20), the least square method can be directly used to solve the vector $\boldsymbol{\delta}_b$, $\boldsymbol{\delta}_b$ and $\boldsymbol{\delta}$ satisfy the following relationship:

$$\mathbf{K}^{-1}\boldsymbol{\delta} = \left[\, \boldsymbol{\delta}_b^{\mathrm{T}} \; \mathbf{0} \,\right]^{\mathrm{T}} \tag{21}$$

Left multiplying a vector by a nonsingular matrix is equivalent to performing a row elementary transformation, so elements of vector $\boldsymbol{\delta}_b$ are a linear combination of that of vector $\boldsymbol{\delta}$. The linear dependent of the columns of the regression matrix $\mathbf{W}$ makes it impossible to identify the inertial parameters vector directly, and only the linear combination of inertial parameters vector $\boldsymbol{\delta}_b$ can be obtained. Generally, elements of vector $\boldsymbol{\delta}_b$ are called base parameters. Base parameters are the minimum set to generate a dynamic equation. [29]

3.3. Other dynamic parameters identification

Other dynamic factors such as friction often exist in a real system. For each joint, friction model is usually modeled as [23]

$$f_k = f_{vk}\dot{q}_k + f_{ck}\mathrm{sgn}(\dot{q}_k) + f_{ok} \tag{22}$$

where sgn represents the sign function, f_{vk} and f_{ck} are the viscous friction and coulomb friction coefficient, respectively, and f_{ok} is the Coulomb friction offset generated by the static bias of the driver such as a motor current offset. When identifying friction parameters, substitute Eq. (22) to dynamic equations, then expansion vector $\boldsymbol{\delta}$ with f_{vk}, f_{ck}, and f_{ok}. Similar to friction parameters, a general method for identifying other dynamic parameters can be summarized as follows:

- Establish relevant dynamic model, and the model is linear with respect to parameters to be identified;
- Expand vector $\boldsymbol{\delta}$ with new parameters provided by the model.

3.4. E-B identification method

By using the decoupling modeling method in Section 2, dynamic model of each limb is separated from each other. Consequently, inertial parameters can be identified in groups. Assuming that set C_{L_i} containing n links, the base parameters vector $\boldsymbol{\delta}_b$ can be obtained by solving Eq. (20) with the least square method directly. In case of noise, the number of parameters, the data set quality of regression matrix, and the condition number are important factors which affect identification results. Some literatures try to design excitation trajectories to improve identification results, but without knowledge of inertial parameters, there is no way to adopt force control to let the robot move along the desired trajectory. If using position control, no matter how close the distance between any two interpolation points, velocity and acceleration will change with time and not equal to the first- and second-order derivative of the desired trajectory. In view of this situation, this paper proposes an identification method to improve identification results without specified excitation trajectories.

Under the condition that the leg is a serial structure, along from the base link to the end link, take n joint angles as generalized coordinates, corresponding to the dynamic equations $1 \sim n$. To make the statement clearer, we give the following definitions and theorems. In the identification process, the link to be identified usually does not collide with its surroundings, so the following descriptions assume that the external force is zero.

Definition 1. Along from the base link to the end link, label the links from 1 to n. If a certain base parameter is composed of a linear combination of inertial parameters of several links, and the smallest labeled link of these inertial parameters is k, then the base parameter is said to be the base parameter about link k.

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Theorem 1. The s-th dynamic equation contains only base parameters about the link $s, s+1, \ldots, n$.

Proof: According to Lagrange's equations of the second kind, the s-th dynamic equation is [30]

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_s} - \frac{\partial T}{\partial q_s} = Q_s \tag{23}$$

where q_s is a generalized coordinate, T is the total kinetic energy of n links, and Q_s is the generalized force of all external forces including gravity about generalized coordinate q_s. For serial structure, the contribution of link $s, s+1, \ldots, n$ to $\frac{\partial T}{\partial q_s}$, $\frac{\partial T}{\partial \dot{q}_s}$ and Q_s is not zero, but the contribution of link $1, 2, \ldots, s-1$ to $\frac{\partial T}{\partial q_s}$, $\frac{\partial T}{\partial \dot{q}_s}$ and Q_s is zero, that is, the s-th dynamic equation does not contain inertial parameters of link $1, 2, \ldots, s-1$. Hence, the s-th dynamic equation only contains base parameters about link $s, s+1, \ldots, n$.

Theorem 2. When the selection of the base parameters is determined, the s-th dynamic equation contains the maximum set of the base parameters about the link s.

Proof: Since the base parameters are the minimum set to generate dynamic equations, [29] when the selection of the base parameters is determined, expressions of base parameters for link s that appear in dynamic equation $1 \sim s$ are the same. For the dynamic equation s, the base parameters about link s are composed of linear combinations of its inertial parameters, which means that all base parameters about link s are contained. Whereas in equation $1 \sim s-1$, more column vectors of the regression matrix are included than those in equation s, which may generate a new linear dependent term, so that the number of base parameters about link s in dynamic equation $1 \sim s-1$ is not more than those in dynamic equation s. So the set of base parameters about link s in dynamic equations $1 \sim s-1$ is a subset of that in the dynamic equation s, and that proves Theorem 2.

Based on the above definitions and properties, let the base parameters vector of link s in the dynamic equation s be $\boldsymbol{\delta}_{b,s}$, and its subset in the dynamic equation $m(m \leq s)$be $\boldsymbol{\sigma}_{m,s} \subseteq \boldsymbol{\delta}_{b,s}$. Then there are n vectors composed of base parameters,

$$\begin{cases} \boldsymbol{\beta}_1 = \left[\boldsymbol{\delta}_{b,1}^T \ \boldsymbol{\sigma}_{1,2}^T \ \cdots \ \boldsymbol{\sigma}_{1,n-1}^T \ \boldsymbol{\sigma}_{1,n}^T \right]^T \\ \boldsymbol{\beta}_2 = \left[\boldsymbol{\delta}_{b,2}^T \ \cdots \ \boldsymbol{\sigma}_{2,n-1}^T \ \boldsymbol{\sigma}_{2,n}^T \right]^{\mathrm{T}} \\ \cdots \\ \boldsymbol{\beta}_{n-1} = \left[\boldsymbol{\delta}_{b,n-1}^T \ \boldsymbol{\sigma}_{n-1,n}^T \right]^T \\ \boldsymbol{\beta}_n = \boldsymbol{\delta}_{b,n} \end{cases} \tag{24}$$

satisfying the dynamic equations

$$\begin{cases} \mathbf{W}_1\boldsymbol{\beta}_1 = \boldsymbol{\tau}_1 \\ \mathbf{W}_2\boldsymbol{\beta}_2 = \boldsymbol{\tau}_2 \\ \cdots \\ \mathbf{W}_{n-1}\boldsymbol{\beta}_{n-1} = \boldsymbol{\tau}_{n-1} \\ \mathbf{W}_n\boldsymbol{\beta}_n = \boldsymbol{\tau}_n \end{cases} \tag{25}$$

Starting from the n-th dynamic equation, $\boldsymbol{\beta}_n$ is first solved by the least square method, and its subset $\boldsymbol{\sigma}_{n-1,n}$ is substituted into the $n-1$-th dynamic equation to get

$$\mathbf{W}_{n-1}\boldsymbol{\beta}_{n-1} = \left[\mathbf{W}_{n-1,n-1} \ \mathbf{W}_{n-1,n} \right] \left[\boldsymbol{\delta}_{b,n-1}^{\mathrm{T}} \ \boldsymbol{\sigma}_{n-1,n}^{\mathrm{T}} \right]^{\mathrm{T}} = \boldsymbol{\tau}_{n-1} \tag{26}$$

where $\mathbf{W}_{n-1,n-1}$ is a regression matrix whose dimension is appropriate to that of $\boldsymbol{\delta}_{b,n-1}$. Expand the above equation to get

$$\mathbf{W}_{n-1,n-1}\boldsymbol{\delta}_{b,n-1} = \boldsymbol{\tau}_{n-1} - \mathbf{W}_{n-1,n}\boldsymbol{\sigma}_{n-1,n} \tag{27}$$

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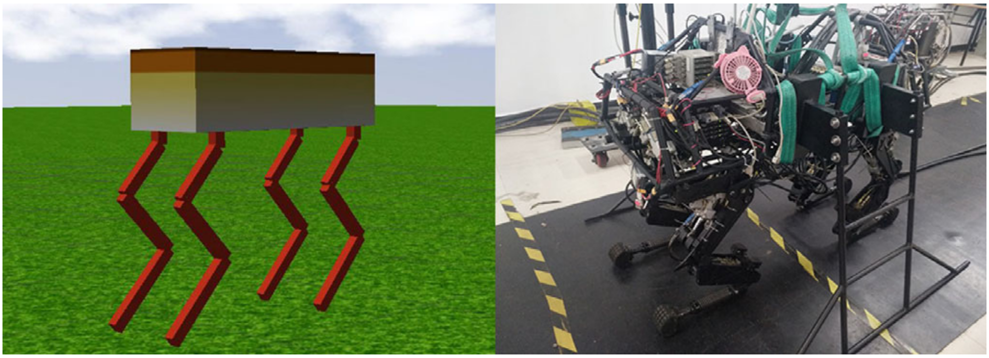

Figure 2. *Quadruped robot simulation model and its prototype.*

Continue with the least square method to solve vector $\boldsymbol{\delta}_{b,n-1}$, and its subset $\boldsymbol{\sigma}_{n-2,n-1}$ is substituted into the $n-2$-th dynamic equation to get

$$\mathbf{W}_{n-2,n-2}\boldsymbol{\delta}_{b,n-2} = \boldsymbol{\tau}_{n-2} - \mathbf{W}_{n-2,n-1}\boldsymbol{\sigma}_{n-2,n-1} - \mathbf{W}_{n-2,n}\boldsymbol{\sigma}_{n-2,n} \tag{28}$$

and so on until the first dynamic equation is solved,

$$\mathbf{W}_{1,1}\boldsymbol{\delta}_{b,1} = \boldsymbol{\tau}_{1} - \mathbf{W}_{1,2}\boldsymbol{\sigma}_{1,2} - \mathbf{W}_{1,3}\boldsymbol{\sigma}_{1,3} - \cdots - \mathbf{W}_{1,n}\boldsymbol{\sigma}_{1,n} \tag{29}$$

The step-by-step identification method from the end link to the base link is called "E-B"(The end link to the base link) identification method. In "E-B" identification method, the base parameters are only related to one link each step, which avoids identifying all the base parameters at one step. A smaller number of parameters will increase the accuracy of the identification results. Comparison results among three methods (directly using the least square method, the E-B method, and the E-B method with no acceleration term) are given in Section 4.

4. Simulation verification

Generally, it is hard to obtain real values of dynamical parameters in a real robot. In order to compare the identification results with real values, simulation is carried out in Gazebo which offers the real robot simulation environment. As shown in Fig. 2, a quadruped robot with 16 degrees of freedom is built in Gazebo according to the size and weight of a real robot in our laboratory and a IMU sensor is fixed in the body of the robot. The total weight of the robot is 136.68 kg, and each limb, which is composed of four links and four revolute joints, weighs 14.04 kg. The friction model of the joints includes viscous friction and Coulomb friction, and the Coulomb friction offset is 0. The geometric parameters, inertial parameters, and friction parameters of each limb are the same. As described in Section 2, through decoupling modeling, the dynamic equations for every limb are similar and independent of each other. We can choose only one limb to study, and the other three limbs can be identified separately in the same way when we want to obtain the parameters of them. Therefore, in the simulation, only base parameters of the left front limb are identified. The links and joints along the direction from the body to the ground are labeled as $1 \sim 4$ in sequence. The specific expressions of the base parameters are shown in Table I. It should be noted that there is no need to perform identification in the condition of movement of the robot body. In the identification stage, fixing the robot body does not affect the parameters of leg identification results. The robot body should be lifted and fixed so that the foot does not touch the ground within the range of motion, and ensure that the left front limb is not affected by external forces during the motion. The driving torque of each joint is applied in a sinusoidal form,

$$F_k = a_k \sin(\omega_k t) + b_k$$

The values for a_k, ω_k, and b_k are listed in Table II.

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Table I. *Base parameters of the left front limb*

δ_b	Corresponding linear combination of inertial parameters
$\delta_{b,1}$	$I_{1xx}+I_{2zz}+I_{3zz}+I_{4zz}+l_1^2(m_2+m_3+m_4)$
$\delta_{b,2}$	$m_1y_{c1}+m_2y_{c2}+m_3y_{c3}+m_4y_{c4}$
$\delta_{b,3}$	$m_1z_{c1}+l_1(m_2+m_3+m_4)$
$\delta_{b,4}$	f_{v1}
$\delta_{b,5}$	f_{c1}
$\delta_{b,6}$	$I_{2xx}-I_{2zz}+l_2^2(m_3+m_4)$
$\delta_{b,7}$	$I_{2yy}+l_2^2(m_3+m_4)$
$\delta_{b,8}$	I_{2xy}
$\delta_{b,9}$	I_{2xz}
$\delta_{b,10}$	$I_{2yz}-l_2(m_3y_{c3}+m_4y_{c4})$
$\delta_{b,11}$	m_2x_{c2}
$\delta_{b,12}$	$m_2z_{c2}+l_2(m_3+m_4)$
$\delta_{b,13}$	f_{v2}
$\delta_{b,14}$	f_{c2}
$\delta_{b,15}$	$I_{3xx}-I_{3zz}+l_3^2m_4$
$\delta_{b,16}$	$I_{3yy}+l_3^2m_4$
$\delta_{b,17}$	I_{3xy}
$\delta_{b,18}$	I_{3xz}
$\delta_{b,19}$	$I_{3yz}-l_3m_4y_{c4}$
$\delta_{b,20}$	m_3x_{c3}
$\delta_{b,21}$	$m_3z_{c3}+l_3m_4$
$\delta_{b,22}$	f_{v3}
$\delta_{b,23}$	f_{c3}
$\delta_{b,24}$	$I_{4xx}-I_{4zz}$
$\delta_{b,25}$	I_{4yy}
$\delta_{b,26}$	I_{4xy}
$\delta_{b,27}$	I_{4xz}
$\delta_{b,28}$	I_{4yz}
$\delta_{b,29}$	m_4x_{c4}
$\delta_{b,30}$	m_4z_{c4}
$\delta_{b,31}$	f_{v4}
$\delta_{b,32}$	f_{c4}

Table II. *Parameter values of driving torque* $F_k=a_k\sin(\omega_k t)+b_k$

Joint k	a_k	ω_k	b_k
1	6	π	0
2	6	π	7.5
3	−3	π	−9
4	5	π	5.5

There is no need to consider the specific form of the excitation trajectory, but the sinusoidal torque signal of a single frequency is applied, which is the advantage of the "E-B" identification method. The amplitude, frequency, and bias of the torque can be determined by several attempts, which satisfy the need to make joints movements cover the workspace as much as possible.

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Table III. *Comparison between identification results and real values. The blue numbers represent the results closest to the real values when measured by absolute error*

δ_b	Real value	E-B method	No-Acc dynamic equations using E-B method	Direct identification
$\delta_{b,1}$	0.0514	1.2543	3.1146	1.5660
$\delta_{b,2}$	0	−0.0229	0.0517	−0.0246
$\delta_{b,3}$	0.8038	0.3655	1.5395	3.6128
$\delta_{b,4}$	1	0.8668	1.2139	1.4466
$\delta_{b,5}$	1	0.0788	−0.9823	0.1621
$\delta_{b,6}$	0.6597	0.5525	2.6177	0.1566
$\delta_{b,7}$	1.5334	1.5294	1.8208	–
$\delta_{b,8}$	0	0.009	−0.1372	0.5845
$\delta_{b,9}$	0	−0.0233	−1.0949	0.1819
$\delta_{b,10}$	0	0.027	−0.2681	−0.2286
$\delta_{b,11}$	0	0.0637	−0.0603	1.2700
$\delta_{b,12}$	2.797	2.8316	3.8579	1.4795
$\delta_{b,13}$	1	0.9926	2.0525	1.5515
$\delta_{b,14}$	1	−0.079	−1.4671	0.2162
$\delta_{b,15}$	0.6008	0.8664	0.6686	0.2187
$\delta_{b,16}$	1.1305	1.1059	1.0938	–
$\delta_{b,17}$	0	0.059	−0.0094	0.3997
$\delta_{b,18}$	0	0.4075	0.1381	−0.3819
$\delta_{b,19}$	0	0.0624	−0.0114	−0.3593
$\delta_{b,20}$	0	−0.0062	−0.0138	−1.1099
$\delta_{b,21}$	2.06	2.0668	2.0434	1.0800
$\delta_{b,22}$	1	0.9296	0.9511	0.6429
$\delta_{b,23}$	1	−0.0157	0.0949	−0.4694
$\delta_{b,24}$	0.2435	0.1903	0.1799	0.7574
$\delta_{b,25}$	0.2454	0.2349	0.2393	–
$\delta_{b,26}$	0	0.0203	0.0191	0.1144
$\delta_{b,27}$	0	0.0064	0.0458	−0.1110
$\delta_{b,28}$	0	0.0195	0.0065	0.0053
$\delta_{b,29}$	0	−0.0017	0.0007	0.2672
$\delta_{b,30}$	0.91428	0.9166	0.9109	0.4931
$\delta_{b,31}$	1	0.9942	0.9921	1.1246
$\delta_{b,32}$	1	−0.0567	0.0058	−0.1625

The limb motion lasts 4 s, the sampling interval is 1ms, and the regression matrix **W** has in total $4 \times 4000 = 16,000$ rows. Joint position and joint torque are got by the corresponding sensors in Gazebo, respectively. The first and second derivatives of the position are solved by difference method, then filtered appropriately. Table III compares real values with the results of base parameters obtained by different identification methods. "E-B" method refers to the value of base parameters obtained by "E-B" identification method with the dynamic Eq. (2), and the condition numbers of regression matrices are 32, 38, 50, and 24 successively. "No-Acc dynamic equations using E-B method are the result of using the dynamic Eq. (17) and "E-B" identification method, and the condition numbers of regression matrices are 600, 551, 736, and 197, respectively. Direct identification shows the results obtained by directly using the least square method with dynamic Eq. (2), and the condition number of the regression matrix is 72. By comparison, it can be found that the results obtained by using the "E-B" method are closer to

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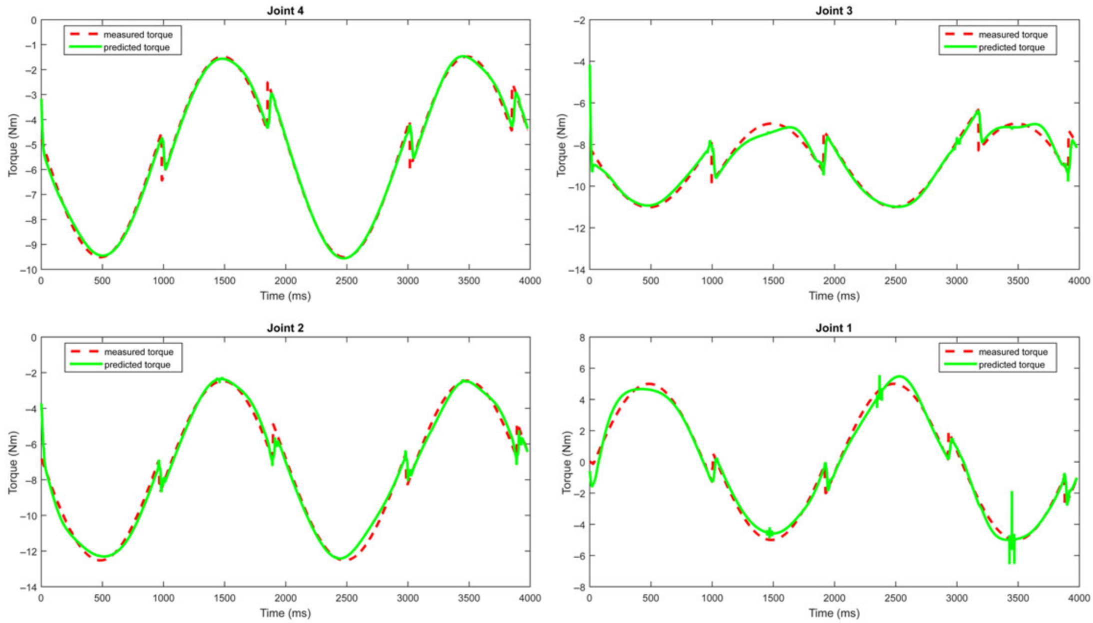

Figure 3. *Comparison between estimated torques and measured torques obtained using the "E-B" identification method.*

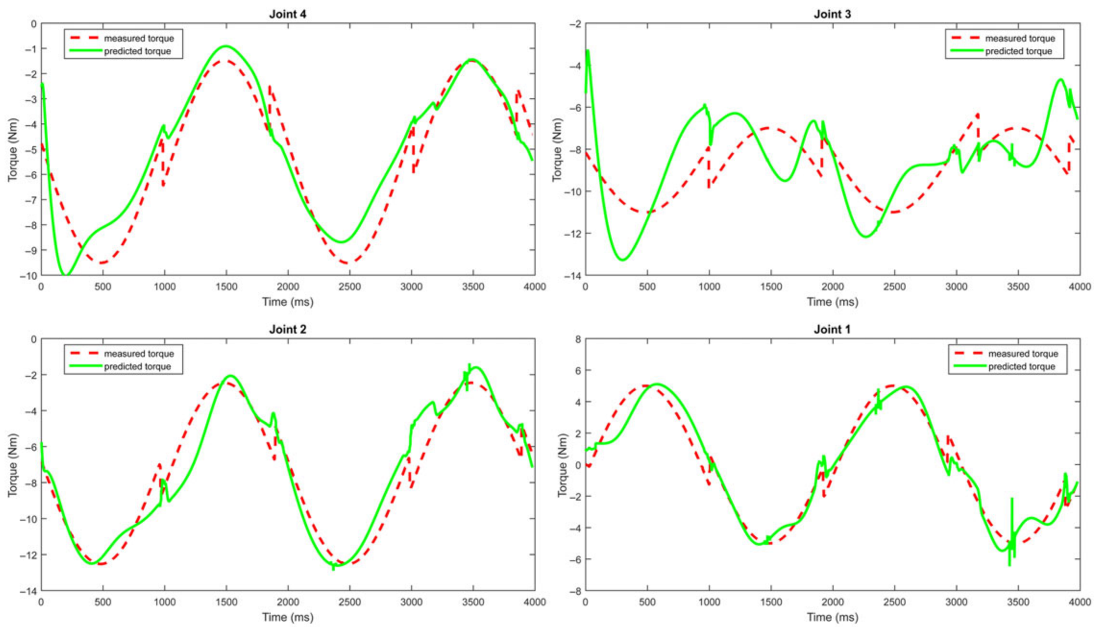

Figure 4. *Comparison between estimated torques and measured torques obtained by direct identification.*

the real values. Although the data quality of regression matrices obtained using Eq. (17) is better, the condition number of regression matrices is too high to get good result. Similar to the real environment, noise sources also exist in Gazebo, including sensor noise, numerical calculation error, joint constraint correction force, etc. This may lead to the violation of physical laws for some parameters. After using

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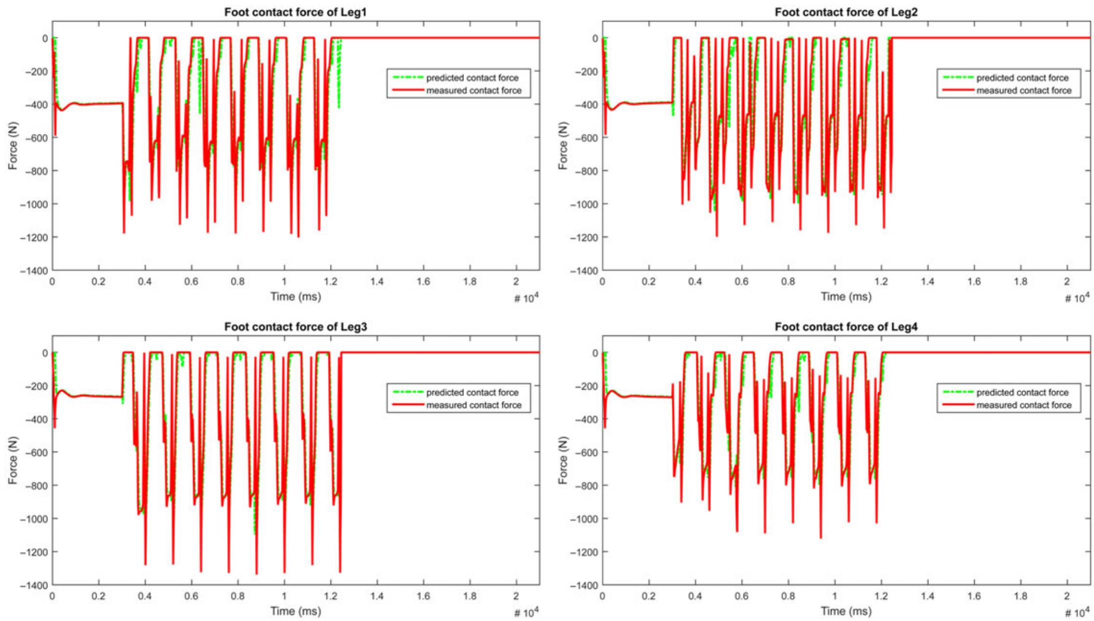

Figure 5. *Feet contact forces estimation of the quadruped robot.*

"E-B" method to obtain the original base parameters, we can use the method proposed by ref. [23] to map the parameters to a physically feasible set. With the "E-B" identification method and the direct identification method, two sets of base parameters are obtained. Figures 3 and 4, respectively, show the comparison between joint torque estimation and sensor measured torque using these two sets of base parameters. It can be seen that the "E-B" method is superior to the direct identification method in estimating torques. One way to explain it is that the direct identification method contains all parameters with the least square method in one step, and the regression matrix is made up of all data of dynamic equations, so the identification results should satisfy all dynamic equations to minimize the squared sum of errors of torques estimation. Whereas the "E-B" identification method only uses the data from one dynamic equation to identify base parameters of one link every step, and the number of parameters solved in a single step decreases. Hence, it can be predicted that the estimation of joint torque by the "E-B" identification method will be better than that by the direct identification method.

The identification process is offline. When the identification task is completed, we can use the dynamic equation to calculate external forces. A direct application of modeling and identification of legged robots is the foot forces estimation. If errors occur in either modeling or base parameters identification process, then the external generalized force calculated by dynamic equations will be wrong. Therefore, foot contact forces of the robot which are calculated by dynamic equations can be used to verify the proposed method. Figure 5 shows the foot contact force (perpendicular to the ground) curves of the quadruped robot from standing state to trotting gait. The dot dash line is the estimation value of the foot contact force from dynamic equations which are established by decoupling modeling and "E-B" identification method. The solid line is the value given by the contact force sensor in Gazebo. Both sets of values are processed through a low-pass filter. The results show that the method proposed in this paper is reliable.

5. Conclusion

By decoupling modeling, the dynamic equations of legged robot can be decomposed into several independent sets of equations, and its advantages can be summarized into the following three points:

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1. The modeling process is more concise and clearer.
2. Instead of solving all the dynamic equations, several independent sets of dynamic equations allow us to solve only task-related dynamic equations as needed, which greatly reduces the amount of computation and thus increases the execution speed of the program.
3. The mutually decoupled dynamic equations reduce the number of base parameters to be identified simultaneously, which can improve the accuracy of the identification results.

The "E-B" identification method proposed in this paper does not depend on the specific identification trajectory and only identifies base parameters of one link at one step. Compared with the direct identification method, the result of "E-B" identification method is closer to the real values. Obviously, accurate model and sensor data can improve the accuracy of identification results. The limitation of this paper is that it does not discuss the influence of various types of signals on the identification results under the condition of different sensor noises.

In this paper, base parameters identification with dynamic equations in acceleration-free form is also discussed. As can be seen from the results in Table III, although the condition number of the regression matrix is higher, parameters of link 4 and link 3 maintain certain accuracy. Due to the error accumulation, link 2 and link 1 deviate from the true value far away. The accuracy of data and condition number of the regression matrix are both important factors that influence the identification results.

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