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LETTER

The Probability of Casting a Pivotal Vote in an Instant Runoff Voting Election

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Abstract

If instant runoff voting (IRV) mitigates strategic voting, is that because the rules of the system mechanically reduce strategic opportunities, or because of other more indirect effects? In single-vote plurality elections, a voter can be pivotal if adding one vote to a candidate would cause that candidate to win. In IRV, it is more complicated to identify when one voter can be pivotal. This letter derives all the ways that a single ballot can change the result of an IRV election, for any number of candidates and voters. I obtain an expression for the probability of casting a pivotal vote in IRV by phrasing the probability that any pivotal event occurs as a function of all the rankings cast by other voters. This expression facilitates modeling vote choice in IRV, and enables the estimation of voters' strategic opportunities in IRV contests between any number of candidates. I present some illustrative simulations estimating pivotal probabilities in both IRV and Single-Member District Plurality for stylized electorates with identical preference structures. These simulations produce similar estimated pivotal probabilities in the two systems, suggesting that these systems may provide similar opportunities to strategically cast a decisive vote.

Keywords: Ranked Choice Voting; Instant Runoff Voting; pivotal probability; strategic voting; electoral systems

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1. Introduction

Instant runoff voting (IRV) has rapidly become a popular electoral system for major elections in several large democracies. A central motivation for its adoption is that it renders strategic voting practically ineffective (Gehl and Porter 2020, Chapter 5). However, the best-understood connections between IRV and strategic voting are mediated by institutions or voter psychology (Santucci 2021), and research into how the mechanical rules of IRV shape voters' strategic opportunities has required substantial original formal work (Atsusaka 2025; Bouton 2013; Eggers and Nowacki 2024). The biggest obstacle to estimating the size of voters' strategic opportunities in that system, and to modeling voting in IRV, is that we do not yet have a method to estimate the probability that a ballot will change the outcome of the election, which is one of the essential terms in the classical theory of strategic voting (Riker and Ordeshook 1968). So, what is the probability that a ballot cast in any IRV election will change the election winner?

There is widespread agreement that IRV "encourages sincere (as opposed to strategic) voting" (Drutman and Strano 2021, 68), potentially implying that "voters do not need to make complicated strategic calculations" (Tolbert and Kuznetsova 2021, 266). Many possible rationales have been proposed and studied. However, in Duverger's (1951, Book 2, Ch. 1, §1) classic division between the "mechanical effects" of an electoral system that arise automatically out of its literal vote-counting rules and the "psychological effects" caused by voters' anticipation of those rules, most research on the relationship between IRV and strategic voting has focused on psychological effects or downstream institutional changes. This includes impressive bodies of work on candidate or party behavior (Buisseret and Prato 2022; Donovan and Tolbert 2023; Santucci 2021), and on voters' actual decision-making processes under IRV (Dowling *et al.* 2024; Reilly 2021; Simmons and Waterbury 2024).

IRV would have a deeper and less contingent kind of resistance if it *mechanically* reduces voters' opportunities for strategic voting, for example by liberating voters from the pressure in Single-Member District Plurality (SMDP) to vote for the lesser of two evils (Benjamin and Burden 2021; Simmons, Gutierrez, and Transue 2022). However, while IRV walls off this strategic avenue, it opens others, and research has pointed in both directions. There can be voters, for example, who have an incentive to vote strategically under two-round runoff rules, but that incentive is eliminated by switching to IRV (Saari 2003, 543). And yet, versions of the "lesser evil" logic (Eggers and Nowacki 2024) and the spoiler effect (Graham-Squire and McCune 2022) have both happened in IRV elections to the federal governments of large democracies. A full accounting of whether IRV mechanically mitigates strategic voting would require comparing the strategic opportunities that IRV introduces to the strategic opportunities that it walls off.

This letter contributes to that foundation by deriving the probability that a vote cast in a single-winner IRV election changes the outcome of that election. I obtain the first general expression of the probability that a ballot cast in IRV is pivotal in selecting the election winner, for any number of voters and any number of candidates, and where any number of those candidates can be ranked on the ballot. This expression only requires information about the rankings that are expected to be cast in the election, so for example it could be applied to the results of a poll that asks voters how they will rank the candidates. This could facilitate comparisons of strategic opportunities under SMDP and IRV using voters' real preferences and actual numbers of candidates and lengths of ballots; as an initial example, I present stylized simulations that suggest the two systems may have broadly similar strategic opportunities.

This points to a second, methodological motivation to derive pivotal probabilities in IRV. The probability of casting a pivotal vote is a crucial ingredient in formal treatments of voting as a strategic activity (Cox 1994; Myerson 1998). Researchers studying substantive questions about IRV using formal methods, including those specifically studying the connection between IRV and strategic voting (Eggers and Nowacki 2024), have often needed to derive expressions for the pivotal probability of a ballot up to the number of candidates they consider in order to proceed with their substantive investigation (Bouton 2013). Other formal comparisons of strategic voting opportunities across electoral systems have been unable to include IRV until the pivotal probability of a ballot is known (Baltz 2022). This letter's core methodological contribution is to generalize the pivotal probability expressions of Bouton (2013) and Eggers and Nowacki (2024) from three or four candidates up to contests between any number of candidates, so that future researchers can plug any candidate number and ballot length into an equation to obtain the pivotal probability of a ballot for use in their models.[1]

2. Definitions and notation

Let C be the set of all κ candidates contesting an IRV election. For a candidate c, denote that candidate's vote total v_c. β will represent the ordering of candidates on a ranked ballot, where a voter may make $L \leq \kappa$ choices, with the positions indexed by i. When considering a list λ, the ordered sub-list from index a up to index b (inclusive, indexing from 1) will be denoted $\lambda_{a:b}$. We will represent the order in which candidates are dropped as a list of dropped candidates $S = [S_1, S_2, \ldots, S_{\kappa-1}]$. For brevity let $S_{-1} \equiv S_{\kappa-1}$. We will also consider the list of dropped candidates with the winner w concatenated to the end, denoted by A, so that $A = [S_1, S_2, \ldots, S_{\kappa-1}, w]$. To represent the number of voters who rank candidate c in any of the ballot positions between 1 and r, given that they assigned every higher ballot position to some candidate in the sub-list $S_{1:n}$, we will use $\mu_c^r | S_{1:n}$.

[1]Section 1 of the Supplementary Material discusses other important connections to existing work and defines the rules of IRV.

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I use "ballot" to mean a voter's cast ranking β, "ballot length" to mean the number L of candidates who can be ranked, and "round" for each act of comparing votes (e.g., S_1 is dropped in the "first round"). There are two mutually exclusive ways that a ballot could be pivotal:

- **Direct pivotality**: because a candidate is ranked on a ballot, they win the election.
- **Indirect pivotalty**: because candidate A is ranked on a ballot, candidate B wins the election.

When I state a pivotal probability, the implicit comparison is to abstention, as explained in Section 2 of the Supplementary Material. Section 4 of the Supplementary Material discusses the history of indirect pivotality.

3. Example

Imagine a voter in Alaska's 2022 U.S. House contest expects ballots to be cast in the following (unrealistic but illustrative) distribution.[2] For exposition, suppose Bye always loses tie-breakers and Palin always wins them.

Voters	Ballot
30,000	[Chris Bye, Nick Begich III, Sarah Palin, Mary Peltola]
30,000	[Nick Begich III, Chris Bye, Sarah Palin, Mary Peltola]
60,000	[Sarah Palin, Mary Peltola, Nick Begich III, Chris Bye]
120,000	[Mary Peltola, Chris Bye, Nick Begich III, Sarah Palin]

Default: If the voter abstains, Palin wins as follows:

	Bye	Begich	Palin	Peltola
initial	30,000	30,000	60,000	120,000
round 1	–	60,000	60,000	120,000
round 2	–	–	120,000	120,000
round 3	–	–	240,000	–

Direct pivotality: If the voter ranks Peltola first (e.g., β = [Peltola, Bye, Begich, Palin]), they cause Peltola to win:

	Bye	Begich	Palin	Peltola
initial	30,000	30,000	60,000	120,001
round 1	–	60,000	60,000	120,001
round 2	–	–	120,000	120,001
round 3	–	–	–	240,001

Indirect pivotality: Imagine the voter casts β = [Bye, Begich, Palin], leaving the last spot blank. By ranking *Bye*, they cause *Peltola* to win:

	Bye	Begich	Palin	Peltola
initial	30,001	30,000	60,000	120,000
round 1	60,001	–	60,000	120,000
round 2	60,001	–	–	180,000
round 3	–	–	–	240,000

[2] $\beta = [A,B,C,D]$ means A ranked first, B second, etc. "–" denotes an eliminated candidate.

4. Direct pivotality

After d candidates have been dropped, candidate c has the following vote total:

$$v_c = \overbrace{\mu_c^1}^{c\text{ first}} \underbrace{|S_{1:0}}_{\text{vacuous condition}} + \overbrace{\mu_c^2|S_{1:1}}^{\substack{c\text{ second,}\\ \text{any dropped}\\ \text{candidate first}}} + \cdots + \underbrace{\mu_c^{d+1}|S_{1:d}}_{\substack{c\text{ ranked in } d+1,\\ \text{all candidates ranked}\\ 1\text{ to } d\text{ dropped}}} \tag{1}$$

$$v_c = \sum_{q=0}^{d} \mu_c{}^{d+1}|S_{1:q}. \tag{2}$$

Denote the probability that candidate c has k more votes than some candidate j by

$$\mathbb{P}(v_c - v_j = k). \tag{3}$$

The probability that placing some candidate c in position i on the ballot will cause that candidate to win is the probability that, after every candidate has been eliminated except two, c is among those two remaining candidates, is ranked on the ballot above the other remaining candidate, and is either a) one vote short of winning and would not win the tie-breaker, or b) two votes short of winning and would win the tie-breaker. The probability that c and some other candidate have the same vote totals after some number of eliminations is:

$$\mathbb{P}(v_c - v_{S_{-1}} = 0). \tag{4}$$

In order to reach a pivotal contest against candidate S_{-1}, candidate c must exceed the vote total of every other candidate at the time at which they are dropped. But this is only pivotal if c also is not among the dropped candidates.

In Section 3 of the Supplementary Material, I state two independence assumptions, which I introduce only for ease of communication, so that we can multiply the probability of events without specifying their joint probabilities. By Assumption 2, the probability $p_d|S$ of a directly pivotal contest involving candidate c (for now just considering the case in which a tie can be broken), given a specific sequence S in which the other candidates are dropped, is:

$$\begin{aligned} p_d|S &= \mathbb{P}(v_c = v_{S_{-1}}) \times \Bigg[\overbrace{\left(\mathbb{P}(\mu_c^1 > \mu_{S_1}^1)\right)}^{\substack{\text{Probability } c\\ \text{not eliminated 1st}}} \underbrace{\left(\mathbb{P}(\mu_c^2|S_1 > \mu_{S_2}^2|S_1)\right)}_{\substack{\text{Probability } c\\ \text{not eliminated 2nd}}} \ldots \underbrace{\left(\mathbb{P}(\mu_c^{\kappa-2}|S_{1:\kappa-3} > \mu_{S_{\kappa-2}}^{\kappa-2}|S_{1:\kappa-3})\right)}_{\substack{\text{Probability } c \text{ not}\\ \text{eliminated 2nd-last}}} \Bigg] \\ &= \mathbb{P}(\mu_c^{\kappa-1}|S_{1:-1} = \mu_{S_{-1}}^{\kappa-1}|S_{1:-1}) \times \prod_{h=1}^{\kappa-2} \mathbb{P}\big(\mu_c^h|S_{1:h-1} > \mu_{S_h}|S_{1:h-1}\big). \end{aligned} \tag{5}$$

This equation is conditional on the candidates being dropped in the order of some list S. The probability of S occurring is the probability that candidate S_1 has fewer initial votes than any other candidate, that S_2 has the fewest votes once S_1 has been dropped, and so on. Recall that A is the list obtained by concatenating the winning candidate to the end of S. By Assumptions 1 and 2, the probability $p_d(S)$ that S is the list of candidates dropped, followed by c being in a directly pivotal contest, is:

$$
\begin{aligned}
p_d(S) = {} & \mathbb{P}\left(v_{S_1}^1 < v_{S_2}^1\right)\mathbb{P}\left(v_{S_1}^1 < v_{S_3}^1\right)\cdots\mathbb{P}\left(v_{S_1}^1 < v_{S_{-1}}^1\right)\mathbb{P}\left(v_{S_1}^1 < v_c^1\right)\times \\
& \mathbb{P}\left(v_{S_2}^2 < v_{S_3}^2\right)\cdots\mathbb{P}\left(v_{S_2}^2 < v_{S_{-1}}^2\right)\mathbb{P}\left(v_{S_2}^2 < v_c^2\right)\times \\
& \vdots \\
& \mathbb{P}\left(v_{S_{\kappa-2}}^{\kappa-2} < v_{S_{-1}}^{\kappa-2}\right)\mathbb{P}\left(v_{S_{\kappa-2}}^{\kappa-2} < v_c^{\kappa-2}\right)\times \\
& \mathbb{P}\left(v_{S_{-1}}^{\kappa-1} = v_c^{\kappa-1}\right)\times\frac{1}{2} \\
= {} & \overbrace{\left[\prod_{\ell=1}^{\kappa-2}\prod_{r=\ell+1}^{\kappa}\mathbb{P}\left(v_{A_\ell}^{\ell} < v_{A_r}^{\ell}\right)\right]}^{\text{Probability the candidates are dropped in order } A}\overbrace{\mathbb{P}\left(v_{A_{\kappa-1}}^{\kappa-1} = v_c^{\kappa-1}\right)}^{\text{Probability that } c \text{ is in a first-place tie given the drop order } A}\times\overbrace{\frac{1}{2}}^{\text{Probability after tying } c \text{ loses}} \\
= {} & \left[\prod_{\ell=1}^{\kappa-2}\prod_{r=\ell+1}^{\kappa}\mathbb{P}\left(\sum_{q=0}^{\ell-1}\mu_{A_\ell}^{q+1}|A_{1:q} < \sum_{q=0}^{\ell-1}\mu_{A_r}^{q+1}|A_{1:q}\right)\right]\mathbb{P}\left(\sum_{q=0}^{\kappa-2}\mu_{A_{\kappa-1}}^{q+1}|A_{1:q} = \sum_{q=0}^{\kappa-2}\mu_c^{q+1}|A_{1:q}\right)\times\frac{1}{2}.
\end{aligned}
\tag{6}
$$

S is mutually exclusive with any other drop sequence, so the probability that ranking candidate c first will be directly pivotal is obtained by summing over Equation 6 for every possible drop sequence:

$$
p_d = \sum_{\substack{S\in \\ \mathrm{Sym}(C\setminus c)}}\left\{\left[\prod_{\ell=1}^{\kappa-2}\prod_{r=\ell+1}^{\kappa}\mathbb{P}\left(\sum_{q=0}^{\ell-1}\mu_{A_\ell}^{q+1}|A_{1:q} < \sum_{q=0}^{\ell-1}\mu_{A_r}^{q+1}|A_{1:q}\right)\right]\frac{1}{2}\cdot\mathbb{P}\left(\sum_{q=0}^{\kappa-2}\mu_{A_{\kappa-1}}^{q+1}|A_{1:q} = \sum_{q=0}^{\kappa-2}\mu_c^{q+1}|A_{1:q}\right)\right\}
\tag{7}
$$

where $\mathrm{Sym}(C\setminus c)$ is the symmetric group on the set of other candidates.

To move beyond the first ballot position, impose a simple restriction: the ith ballot position can only be pivotal if all candidates ranked higher on the ballot have been dropped. Note also that a voter can be pivotal not just by *breaking* a first-place tie that the candidate would not have won, but also by *creating* a first-place tie that the candidate wins. The direct pivotal probability of the ballot β is obtained from summing over Equation 7 for all positions on the ballot:

$$
\begin{aligned}
p_{\mathrm{direct}}(\beta) = \sum_{i=1}^{L}\Bigg(& \sum_{\substack{S\in \\ \mathrm{Sym}(C\setminus i) \\ \beta_{1:i-1}\subset S}}\left\{\left[\prod_{\ell=1}^{\kappa-2}\prod_{r=\ell+1}^{\kappa}\mathbb{P}\left(\sum_{q=0}^{\ell-1}\mu_{A_\ell}^{q+1}|A_{1:q} < \sum_{q=0}^{\ell-1}\mu_{A_r}^{q+1}|A_{1:q}\right)\right]\times\right. \\
& \left[\frac{1}{2}\cdot\mathbb{P}\left(\sum_{q=0}^{\kappa-2}\mu_{A_{\kappa-1}}^{q+1}|A_{1:q} = \sum_{q=0}^{\kappa-2}\mu_i^{q+1}|A_{1:q}\right)+\right. \\
& \left.\left.\frac{1}{2}\cdot\mathbb{P}\left(\sum_{q=0}^{\kappa-2}\mu_{A_{\kappa-1}}^{q+1}|A_{1:q} = 1+\sum_{q=0}^{\kappa-2}\mu_i^{q+1}|A_{1:q}\right)\right]\right\}\Bigg).
\end{aligned}
\tag{8}
$$

5. Indirect Pivotality

Suppose that candidates will be dropped according to $A \equiv [S_1, S_2, \ldots, S_\kappa]$, but because a voter ranks some candidate c in position i on their ballot, instead candidates are dropped according to a different sequence A'. In Section 4 of the Supplementary Material, I identify two conditions specifying the cases in which a switch from A to A' represents an indirectly pivotal event. By the reasoning there, the probability of c being involved in a potentially pivotal tie with another candidate is the sum of the probability of c being in a tie or a near-tie with each remaining candidate t. So, where y is the index of c in A,

$$
p_{\mathrm{tie}}|A' = \sum_{t\in A_{y+1:\kappa}}\left[\frac{1}{2}\,\mathbb{P}\left(v_c^y = v_t^y\right)+\frac{1}{2}\,\mathbb{P}\left(v_c^y = v_t^y - 1\right)\right].
\tag{9}
$$

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To obtain the probability of switching from A to A', it is necessary to know not just the probability that there was a tie to create or break, but also that the end of A' will be the specific sequence that follows creating or breaking that tie involving c. That is the probability that every candidate d in $A'_{y+1:\kappa}$ defeats every candidate prior to it. For brevity denote $G \equiv A'_{y+1:\kappa}$, that is, G is the alternate ending in the hypothetical pivotal event. Recall that the candidate that c is in a last-place tie with must be A'_y, and let $t \equiv A'_y$. Then, by Assumption 1:

$$p_{\text{tie}}(A'|A) = \overbrace{\prod_{d=1}^{|G|}\left[\prod_{h=1}^{d-1}\mathbb{P}\left(v_{G_d} > v_{G_h}\right)\right]}^{\text{Probability that } G \text{ is the sequence after a tie with } c} \times \overbrace{\left[\frac{1}{2}\,\mathbb{P}\left(v_c^y = v_t^y\right) + \frac{1}{2}\,\mathbb{P}\left(v_c^y = v_t^y - 1\right)\right]}^{\text{Probability that } c \text{ enters a tie they will win, or avoids a tie they would not have won}}. \tag{10}$$

This is the conditional probability of turning some A into a specific sequence A'. But there is not just one valid A'. Let $\mathbf{A}$ denote the set of all A' that, for a given A, fulfill Conditions 1 and 2 from Section 4 of the Supplementary Material, and note that any two A' represent mutually exclusive events. Then the probability of any indirectly pivotal event arising from the ranking of c in position i given that the drop sequence would otherwise have followed A is the sum of Equation 10 over all possible A':

$$p_{\text{tie}}|A = \sum_{A'\in\mathbf{A}}\left\{\prod_{d=1}^{|G|}\left[\prod_{h=1}^{d-1}\mathbb{P}\left(v_{G_d} > v_{G_h}\right)\right] \times \left[\frac{1}{2}\,\mathbb{P}\left(v_c^y = v_t^y\right) + \frac{1}{2}\,\mathbb{P}\left(v_c^y = v_t^y - 1\right)\right]\right\}. \tag{11}$$

To obtain the probability of A' arising because of a vote for c, it is now necessary to know the probability that A occurs. By Assumptions 1 and 2, the probability $p_{\neg\text{d}}(A \to A')$ that a vote for c changes the sequence from A to A' is the product of Equation 11 with the probability of A:

$$\begin{aligned} p_{\neg\text{d}}(A \to A') = {} & \overbrace{\prod_{\ell=2}^{\kappa}\left[\prod_{h=1}^{\ell-1}\mathbb{P}\left(v_{A_\ell} > v_{A_h}\right)\right]}^{\text{Probability of } A \text{ occurring}} \times \\ & \underbrace{\sum_{A'\in\mathbf{A}}\left\{\prod_{d=1}^{|G|}\left[\prod_{h=1}^{d-1}\mathbb{P}\left(v_{G_d} > v_{G_h}\right)\right] \times \left[\frac{1}{2}\,\mathbb{P}\left(v_c^y = v_t^y\right) + \frac{1}{2}\,\mathbb{P}\left(v_c^y = v_t^y = -1\right)\right]\right\}}_{\text{Probability of any } A' \text{ arising from } A, \text{ with } c \text{ tied for being dropped at some point}}. \end{aligned} \tag{12}$$

What remains is to sum over the possible sequences A, and also conduct the calculation for every ballot location. The one important detail in the sum over different values of A is that the only sequences that should be included are those which involve dropping every candidate listed before i on the ballot, which we can obtain by summing Equation 12 over all possible A and all ballot locations:

$$\begin{aligned} p_{\text{indirect}}(\beta) = \sum_{i=1}^{L}\Bigg(& \sum_{\substack{A\in\text{Sym}(C)\\ \beta_{1:i}\subset A_{1:y}}}\left[\prod_{\ell=2}^{\kappa}\left[\prod_{h=1}^{\ell-1}\mathbb{P}\left(v_{A_\ell} > v_{A_h}\right)\right]\right. \\ & \left.\times \sum_{A'\in\mathbf{A}}\left\{\prod_{d=1}^{|G|}\left[\prod_{h=1}^{d-1}\mathbb{P}\left(v_{G_d} > v_{G_h}\right)\right]\sum_{t\in F}\left[\frac{1}{2}\,\mathbb{P}\left(v_c^y = v_t^y\right) + \frac{1}{2}\,\mathbb{P}\left(v_c^y = v_t^y - 1\right)\right]\right\}\right]\Bigg). \end{aligned} \tag{13}$$

Finally, substitute Equation 2, and account for making as well as breaking ties, to obtain the full expression for the indirect pivotal probability of β:

$$
\begin{aligned}
p_{\text{indirect}}(\beta) = \sum_{i=1}^{L}\Bigg(& \sum_{\substack{A \in \text{Sym}(C)\\ \beta_{1:i} \subset A_{1:y}}} \Bigg[\prod_{\ell=2}^{\kappa} \Bigg[\prod_{h=1}^{\ell-1} \mathbb{P}\Bigg(\sum_{q=0}^{h-1} \mu_{A_\ell}^{q+1} | A_{1:q} > \sum_{q=0}^{h-1} \mu_{A_h}^{q+1} | A_{1:q} \Bigg) \Bigg] \\
& \times \sum_{A' \in \mathbf{A}} \Bigg\{ \prod_{d=1}^{|G|} \Bigg[\prod_{h=1}^{d-1} \mathbb{P}\Bigg(\sum_{q=0}^{y+d} \mu_{G_d}^{q+1} | A_{1:q} > \sum_{q=0}^{y+d} \mu_{G_h}^{q+1} | A_{1:q} \Bigg) \Bigg] \\
& \times \Bigg[\frac{1}{2}\,\mathbb{P}\Bigg(\sum_{q=0}^{y} \mu_c^{q+1} | A_{1:q} = \sum_{q=0}^{y} \mu_t^{q+1} | A_{1:q} \Bigg) + \frac{1}{2}\,\mathbb{P}\Bigg(\sum_{q=0}^{y} \mu_c^{q+1} | A_{1:q} = \sum_{q=0}^{y} \mu_t^{q+1} | A_{1:q} - 1 \Bigg) \Bigg] \Bigg\} \Bigg] \Bigg).
\end{aligned} \tag{14}
$$

6. Full Pivotal Probability

The probability that a ballot β is pivotal is Equation 8 (direct pivotal probability) plus Equation 14 (indirect pivotal probability). We can obtain the expected utility, or the probability of causing a pivotal event times the utility obtained from that change, by multiplying by the change in the voter's utility as a result of the pivotal event. This yields the full expression for expected utility in IRV as a function of the ballots cast:

$$
\begin{aligned}
u(\beta) = & \overbrace{\sum_{i=1}^{L}}^{\substack{\text{Sum over}\\ \text{ballot}}} \Bigg(\overbrace{\sum_{\substack{S \in \\ \text{Sym}(C \setminus i)\\ \beta_{1:i-1} \subset S}}}^{\substack{\text{Sum over}\\ \text{possible drop}\\ \text{sequences}}} \Bigg\{ \Bigg[\overbrace{\prod_{\ell=1}^{\kappa-2} \prod_{r=\ell+1}^{\kappa} \mathbb{P}\Bigg(\sum_{q=0}^{\ell-1} \mu_{A_\ell}^{q+1} | A_{1:q} < \sum_{q=0}^{\ell-1} \mu_{A_r}^{q+1} | A_{1:q} \Bigg)}^{\text{Probability of that drop sequence occurring}} \Bigg] \times \\
& \Bigg[\overbrace{\frac{1}{2}\,\mathbb{P}\Bigg(\sum_{q=0}^{\kappa-2} \mu_{A_{\kappa-1}}^{q+1} | A_{1:q} = \sum_{q=0}^{\kappa-2} \mu_i^{q+1} | A_{1:q} \Bigg)}^{\substack{\text{Probability of breaking first-place tie}\\ \text{for a candidate who would not have won}}} + \overbrace{\frac{1}{2}\,\mathbb{P}\Bigg(\sum_{q=0}^{\kappa-2} \mu_{A_{\kappa-1}}^{q+1} | A_{1:q} = 1 + \sum_{q=0}^{\kappa-2} \mu_i^{q+1} | A_{1:q} \Bigg)}^{\substack{\text{Probability of creating first-place tie}\\ \text{for a candidate who then wins}}} \Bigg] \Bigg\} \times \\
& \overbrace{\Big[u(i) - u(S_{-1}) \Big]}^{\substack{\text{Net utility from causing}\\ \text{that candidate to win}}} + \overbrace{\sum_{\substack{A \in \text{Sym}(C)\\ \beta_{1:i} \subset A_{1:y}}}}^{\substack{\text{Sum over eligible}\\ \text{drop sequences}}} \Bigg[\overbrace{\prod_{\ell=2}^{\kappa} \Bigg[\prod_{h=1}^{\ell-1} \mathbb{P}\Bigg(\sum_{q=0}^{h-1} \mu_{A_\ell}^{q+1} | A_{1:q} > \sum_{q=0}^{h-1} \mu_{A_h}^{q+1} | A_{1:q} \Bigg) \Bigg]}^{\text{Probability of that drop sequence occurring}} \Bigg] \times \\
& \overbrace{\sum_{A' \in \mathbf{A}}}^{\substack{\text{Sum over alternative}\\ \text{drop sequences}}} \overbrace{\Bigg\{ \prod_{d=1}^{|G|} \Bigg[\prod_{h=1}^{d-1} \mathbb{P}\Bigg(\sum_{q=0}^{y+d} \mu_{G_d}^{q+1} | A_{1:q} > \sum_{q=0}^{y+d} \mu_{G_h}^{q+1} | A_{1:q} \Bigg) \Bigg]}^{\text{Conditional probability of alternative drop sequence}} \times \\
& \Bigg[\overbrace{\frac{1}{2}\,\mathbb{P}\Bigg(\sum_{q=0}^{y} \mu_c^{q+1} | A_{1:q} = \sum_{q=0}^{y} \mu_t^{q+1} | A_{1:q} \Bigg)}^{\substack{\text{Probability of breaking first-place tie}\\ \text{for a candidate who would not have won}}} + \overbrace{\frac{1}{2}\,\mathbb{P}\Bigg(\sum_{q=0}^{y} \mu_c^{q+1} | A_{1:q} = \sum_{q=0}^{y} \mu_t^{q+1} | A_{1:q} - 1 \Bigg)}^{\substack{\text{Probability of creating first-place tie}\\ \text{for a candidate who then wins}}} \Bigg] \Bigg\} \Bigg] \times \\
& \overbrace{\Big[u(A'_{-1}) - u(A_{-1}) \Big]}^{\substack{\text{Net utility from causing}\\ \text{that candidate to win}}} \Bigg)
\end{aligned} \tag{15}
$$

where $\mathrm{Sym}(C)$ is the symmetric group of the set C, L is the length of the ballot β such that $L \le \kappa$, $\mu_a^b|S$ denotes the number of voters expected to rank candidate a in any of the ballot positions 1 through b conditional on assigning any higher ballot position to candidates in the set of previously dropped candidates S, y is the index in the list A of the candidate ranked at position i, t is the candidate in A'_y, and $\mathbf{A}$ is the set of all ordered lists A' formed by pre-pending $A_{1:y}$ to G, for every G in the symmetric group on the set $C \setminus A_{1:y}$.

Equation 15 accounts for partially blank ballots or contests where not all candidates can be ranked. If any i in β is blank, the probability that candidate ties is vacuously zero, so add zero to the pivotal probability. When $L < \kappa$, simply sum up to L. Section 5 of the Supplementary Material provides worked examples, §6 provides pseudocode, §7 discusses the caveat that multiple ballots may have the same expected utility, and §8 introduces one idea for modeling probabilities.

Section 9 of the Supplementary Material performs an initial, illustrative comparison between the pivotal probability in IRV versus SMDP of ballots cast by hypothetical electorates with two types of preferences: uniformly distributed preferences, or preferences that follow a power law. These highly stylized simulations produce numbers of very similar orders of magnitudes, suggesting that IRV and SMDP may provide similar strategic opportunities.

7. Conclusion

This letter presented the first general expression for the probability that an IRV ballot changes the election winner, for any number of voters and candidates, when any number of candidates can be ranked. The probability is a function of the rankings that a voter expects to be cast. This equation will facilitate modeling vote choice in IRV, especially to study the kinds and amount of strategic opportunities that are mechanically baked into that system. Highly stylized simulations comparing pivotal probability in IRV and SMDP suggests that the level of pivotal opportunity may be broadly similar in those two systems.

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Data Availability Statement. Replication code for the simulations has been published in Code Ocean, a computational reproducibility platform that enables users to run the code, and can be viewed interactively at https://codeocean.com/capsule/1700151/tree/v1 (Baltz 2024).

Supplementary Material. For supplementary material accompanying this paper, please visit https://doi.org/10.1017/pan.2024.32.

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