

operational research with the disturbing name of “obnoxious facility location”, where the aim is to maximise the separation between objects. Think corona virus!

The author notes a similarity between certain types of rhythm and the need to incorporate some form of “leap day” adjustment within calendar systems.

Similarities are pointed out between repeating rhythms and the structures of lattices and crystals.

A connection is also made with radio astronomy, where one requirement for minimising interference is to arrange radio telescope dishes in a way that maximises the number of distinct distances present in the separations between the dishes.

A connection is made with tangle diagrams from knot theory.

Hamming distance, more commonly encountered in error detection, is also described.

The author finishes with a discussion of why the *son clave* is considered to be such a good rhythm, describing its history and evolution, accompanied by a list of familiar examples such as George Michael's ‘Faith’ and Bow Wow Wow's ‘I Want Candy’.

The book is very informative about a wide range of styles of music and musical instruments and their history. If you have an interest in music and in a range of areas of mathematics, you are likely to find plenty of topics of interest. Speaking as a mathematician and not a musician, I felt that some of the connections were a little tenuous – for example, the significance of the 12 apostles and the 12 days of Christmas, the calendar systems and the Pythagorean triangle. However, these items were interesting and informative in their own right as topics in recreational mathematics.

Another feature I particularly liked was the wealth of diagrams and photos included, which makes this book fun to ‘dip into’. There is also a very comprehensive bibliography at the end of the book for readers who want to follow up any of the themes.

I am grateful to my flatmate Shane Thomas (pianist and drummer) for the assistance and insight he provided from the viewpoint of a professional musician.

10.1017/mag.2022.52 © The Authors, 2022

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Where do numbers come from? by T.W. Körner, pp. 260, £24.99 (paper), ISBN 978-1-10873-838-5, Cambridge University Press (2020)

In 1930, Edmund Landau published his famous little book *Foundations of analysis*. Aimed at undergraduates and still in print, [1], it covers the construction of the natural numbers, integers, real and complex numbers from scratch in just over 100 pages. But it is written in what Landau himself described as a “merciless telegram style”, and his hope in the teacher's preface that the book could be read by students in two days has always struck me as optimistic. In his latest book, Tom Körner sets himself the challenge of retracing Landau's journey for the same audience (early undergraduates or apprentice mathematicians) and with the same equipment (naïve set theory and routine logical reasoning) and with much more in the way of commentary, historical context, motivation and applications.

Part I deals with the construction of \mathbb{Q} from \mathbb{N}^+ via \mathbb{Q}^+ as equivalence classes of equivalent fractions, and as equivalence classes of differences: this is a variant on the more usual route going from \mathbb{N}^+ to \mathbb{Z} to \mathbb{Q} . Part II looks at the construction of \mathbb{N}^+ from the Peano

axioms and, as light relief, also features some elementary number theory – prime factorisation, modular arithmetic, Fermat's little theorem, Wilson's theorem, Hamming error-correcting codes and Rabin codes. Part III contains the construction of the real numbers (via equivalence classes of Cauchy sequences in \mathbb{Q}) and the complex numbers (as ordered pairs of reals), as well as a glimpse of the quaternions and their connection with vector analysis. Polynomials are introduced with the chapter culminating in the construction of Liouville (transcendental) numbers and a proof of the fundamental theorem of algebra. As ever, the author writes with verve and wit with many characteristic "Körnerisms" among the anecdotes in the text and footnotes: even the index is amusing!

I felt the book could be enjoyed in three different ways. Reading the text and skipping the proofs provides a roadmap of the journey, the issues involved, and the challenges to be faced and overcome. Following the details of the proofs would give early undergraduates plenty of practice in standard modes of reasoning although (beware!) there are some subtler proofs that take a bit of unpacking. And, although knowledge of their other courses is not presumed, such readers would enjoy spotting which arguments derive from more general ones in the theory of groups, rings and fields. Finally, those already familiar with the subject matter can relish and admire the skill with which the author organises and presents the material. As examples, I would highlight the clear exposition of the relationship between the least member principle and the principle of induction, the very full presentation of equivalents of the fundamental (completeness) axiom for \mathbb{R} , the use of Cantor's original non-diagonalisation proof that \mathbb{R} is uncountable, and the proof of the fundamental theorem of algebra. The latter is the author's own take on the "show that $|P(z)|$ attains a minimum value $|P(z_0)|$ and then show $P(z_0) = 0$ " line of argument and involves proving that $z^m = \alpha$, $|\alpha| = 1$, has a root without introducing angles. A fitting endpoint to the book is provided by Palais's neat proof of Frobenius's famous theorem that if \mathbb{R}^n can be given the structure of a skew-field, then $n = 1, 2$ or 4 , corresponding to \mathbb{R}, \mathbb{C} and \mathbb{H} . The text is interleaved with mainly routine exercises, solutions to which are available on the author's website.

In his book *Mathematics for the general reader*, [2], the Oxford analyst E. C. Titchmarsh wrote, "I am all in favour of an intelligent theory of number. It should add to the pleasure of mathematics, just as an intelligent theory of rigid dynamics should add to the pleasure of bicycling. But it is possible to pedal along without it." *Where do numbers come from?* certainly adds to the pleasure of mathematics as well as narrating a journey that surely every mathematician should undertake at some stage. As such, I enthusiastically recommend it to all *Gazette* readers.

References

1. E. Landau, *Foundations of analysis* (translation by F. Steinhardt), Chelsea (1951).
2. E. C. Titchmarsh, *Mathematics for the general reader*, Hutchinson's University Library (1948) p.11.

10.1017/mag.2022.53 © The Authors, 2022

Published by Cambridge University Press on
behalf of The Mathematical Association

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The maths of life and death by Kit Yates, pp. 333, £20 (paper), ISBN 978-1-78747-542-7, Quercus Books (2019)

This is an interesting, useful and important book on some serious topics involving mathematics: exponential growth, medical health, the law, media statistics, number systems, algorithms, and mathematical epidemiology. Such matters are