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COINCIDENCE RESULTS FOR SUMMING MULTILINEAR MAPPINGS

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Abstract In this paper we prove coincidence results concerning spaces of absolutely summing multilinear mappings between Banach spaces. The nature of these results arises from two distinct approaches: the coincidence of two *a priori* different classes of summing multilinear mappings, and the summability of all multilinear mappings defined on products of Banach spaces. Optimal generalizations of known results are obtained. We also introduce and explore new techniques in the field: for example, a technique to extend coincidence results for linear, bilinear and even trilinear mappings to general multilinear ones.

Keywords: absolutely summing; projective tensor product; Cohen summing; multilinear mapping

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1. Introduction

The theory of absolutely summing multilinear mappings between Banach spaces has its root in the research programme designed by Pietsch [**27**] as an attempt to generalize the linear operator theory to a multilinear context. Since then a great amount of research in this direction has been carried out and applications of the theory of nonlinear summing mappings to other fields have been found; for example, it has been used in the study of the maximal domain of convergence of vector-valued Dirichlet series [**17**] and in quantum information theory [**23**, **25**].

By definition, absolutely summing multilinear mappings improve the summability of sequences in Banach spaces, and this is why many researchers have focused much of their interest on the study of these mappings. The general purpose of these studies is to obtain and to improve summability conditions for multilinear mappings. A desirable result in the theory is what has been called in the literature a *coincidence result*. This consists in finding examples of Banach spaces $E_1, \dots, E_n, F$, or general conditions on them, that ensure a good summability behaviour of every continuous n-linear mapping from $E_1 \times \dots \times E_n$ to F or improve the summability behaviour of those mappings that already enjoy some summing property. A cornerstone in the linear theory is Grothendieck's theorem, which asserts that every continuous linear operator from ℓ_1 to ℓ_2 is absolutely summing. Grothendieck's theorem has been a permanent source of inspiration in the search of linear and multilinear coincidence results. The Defant–Voigt theorem (see [**10**, Corollary 3.2], [**26**, Proposition 17.5.1] or [**6**], where an improved version can be found), which states that any continuous multilinear functional is absolutely $(1; 1, \dots, 1)$-summing, was probably the first coincidence result for multilinear mappings. We mention a few more examples. The fact that bilinear forms on either an $\mathcal{L}_\infty$-space or the disc algebra or the Hardy space are 2-dominated was proved in [**7**, Theorem 3.3] and [**9**, Proposition 2.1], respectively. In [**6**, Theorem 3.7] Blasco *et al.* used this bilinear coincidence to show that all n-linear forms defined on a product $E_1 \times \dots \times E_n$ of Banach spaces is $(1; 2, \dots, 2)$-summing whenever $E_1 = E_2$ and each E_j is either an $\mathcal{L}_\infty$-space, the disc algebra $\mathcal{A}$ or the Hardy space $\mathcal{H}^\infty$. It is worth mentioning that the problem of lifting properties from bilinear to multilinear mappings is non-trivial in many cases. Indeed, it is not true that the multilinear theory follows by induction from the linear case. Many examples of the difficulties of lifting the linear theory to the multilinear setting can be found in the literature (see, for example, [**20**]).

This paper is concerned with coincidence results in the theory of absolutely summing multilinear mappings. In § 2 we fix notation and recall some definitions and basic facts. In § 3 we investigate two notions related to that of absolutely summing multilinear mappings, namely, *weakly* $(p; p_1, p_2, \dots, p_n)$*-summing* multilinear mappings (Definition 3.1) and *Cohen* $(p; p_1, p_2, \dots, p_n)$*-nuclear* multilinear mappings (Definition 3.4). Relations between these notions and between them and the usual concept of absolutely summing multilinear mappings are established. In § 4 we use the cotype of the Banach spaces $E_1, \dots, E_n$ and find some conditions on the positive numbers $p, p_1, \dots, p_n, q, q_1, \dots, q_n$ to ensure that $(p; p_1, \dots, p_n)$- and $(q; q_1, \dots, q_n)$-summing mappings coincide. We apply the results of this section to get an optimal generalization of results in [**12**, **20**, **28**]. The results presented in this section also generalize some results of [**5**, **29**]. In § 5 we get conditions that ensure that all continuous multilinear mappings on Banach spaces $E_1, \dots, E_n$ are $(p; p_1, \dots, p_n)$-summing. We show how to lift summability properties of bilinear mappings defined on $E_{2i-1} \times E_{2i}$ to n-linear mappings defined on $E_1 \times \dots \times E_n$. We prove that if any bilinear form defined on E^2 is $(1; r, r)$-summing and any trilinear mapping on E^3 is $(1; r, r, r)$-summing, $1 \leqslant r \leqslant 2$, then any n-linear mapping on E^n is $(1; r, \dots, r)$-summing. We also characterize when all multilinear mappings are $(p; p_1, \dots, p_n)$-summing by means of projective tensor products of vector valued sequences spaces. A close connection between Cohen summability and the Littlewood–Orlicz property is also provided.

2. Notation and background

All Banach spaces are considered over the scalar field $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$. Given a Banach space E, let B_E denote the closed unit ball of E and let E' denote its topological dual.

Let $p > 0$. By $\ell_p(E)$ we denote the (p-)Banach space of all absolutely p-summable sequences $(x_j)_{j=1}^\infty$ in E endowed with its usual ℓ_p-norm (p-norm if $0 < p < 1$). Let $\ell_p^w(E)$ be the space of those sequences $(x_j)_{j=1}^\infty$ in E such that $(\varphi(x_j))_{j=1}^\infty \in \ell_p$ for every $\varphi \in E'$ endowed with the norm (p-norm if $0 < p < 1$)

$$\|(x_j)_{j=1}^\infty\|_{\ell_p^w(E)} = \sup_{\varphi \in B_{E'}} \left(\sum_{j=1}^\infty |\varphi(x_j)|^p \right)^{1/p}.$$

For $1 \leqslant p \leqslant \infty$, let p' be the conjugate of p, i.e. $1/p + 1/p' = 1$. We denote by $\ell_p\langle E\rangle$ the Banach spaces of sequences $(x_j)_{j=1}^\infty$ in E such that

$$\|(x_j)_{j=1}^\infty\|_{\ell_p\langle E\rangle} = \sup \left\{ \sum_{j=1}^\infty |\langle x_j, y_j^*\rangle| \colon \|(y_j^*)_{j=1}^\infty\|_{\ell_{p'}^w(E')} = 1 \right\} < \infty.$$

Obviously, one has for $p \geqslant 1$ that

$$\ell_p \hat{\otimes}_\pi E \subseteq \ell_p\langle E\rangle \subseteq \ell_p(E) \subseteq \ell_p^w(E).$$

The space $\ell_p\langle E\rangle$ was first introduced in [**15**] and it has been recently described in different ways (see [**4**] for a description as the space of integral operators from $\ell_{p'}$ into X, or [**14, 19**] for the identification with the projective tensor product $\ell_p\langle E\rangle = \ell_p \hat{\otimes}_\pi E$). In the particular case of dual spaces, using the weak principle of local reflexivity (see [**16**, p. 73]) one has that a sequence $(x_j^*)_{j=1}^\infty$ in E' belongs to $\ell_p\langle E'\rangle$ if

$$\|(x_j^*)_{j=1}^\infty\|_{\ell_p\langle E'\rangle} = \sup \left\{ \sum_{j=1}^\infty |\langle x_j^*, y_j\rangle| \colon \|(y_j)_{j=1}^\infty\|_{\ell_{p'}^w(E)} = 1 \right\} < \infty. \tag{2.1}$$

Recall that, for $1 \leqslant q \leqslant p < \infty$, an operator $T \in \mathcal{L}(E; F)$ is absolutely (p, q)-*summing* if there is a $C > 0$ such that

$$\left(\sum_{j=1}^m \|T(x_j)\|^p \right)^{1/p} \leqslant C \|(x_j)_{j=1}^m\|_{\ell_q^w(E)} \tag{2.2}$$

for any finite sequences $x_1, \dots, x_m \in E$. $\Pi_{(p;q)}(E; F)$ denotes the space of (p, q)-summing operators with the norm (p-norm if $0 < p < 1$) given by the infimum of the constants satisfying (2.2).

For $1 \leqslant p, q < \infty$, an operator $T \in \mathcal{L}(E; F)$ is *Cohen* (p, q)-*nuclear* (see [**3**, p. 56]) if there is a $C > 0$ such that

$$\sum_{j=1}^m |\langle T(x_j), y_j^*\rangle| \leqslant C \|(x_j)_{j=1}^m\|_{\ell_q^w(E)} \|(y_j^*)_{j=1}^m\|_{\ell_{p'}^w(F')} \tag{2.3}$$

for any finite sequences $x_1, \dots, x_m \in E$ and $y_1^*, \dots, y_m^* \in F'$. $\mathrm{CN}_{(p;q)}(E; F)$ denotes the space of Cohen (p, q)-nuclear operators with the norm given by the infimum of the constants satisfying (2.3). This notion was introduced by Cohen [**15**] for $p = q$.

Note that $T \in \mathrm{CN}_{(p;q)}(E;F)$ is equivalent to the correspondence

$$(x_j)_j \in \ell_q^w(E) \mapsto (T(x_j))_j \in \ell_p\langle F\rangle$$

being well defined (hence, linear) and bounded. Hence, $\mathrm{CN}_{(p;q)}(E;F) \subseteq \Pi_{(p;q)}(E;F)$ and, due to (2.1), $T \in CN_{(p;q)}(E;F')$ if there is a $C > 0$ such that

$$\sum_{j=1}^{m} |\langle T(x_j), y_j\rangle| \leqslant C\|(x_j)_{j=1}^m\|_{\ell_q^w(E)}\|(y_j)_{j=1}^m\|_{\ell_{p'}^w(F)}. \tag{2.4}$$

We now turn our attention to multilinear maps. Let $E_1, \dots, E_n, E, F$ be Banach spaces. The Banach space of all continuous n-linear mappings from $E_1 \times \cdots \times E_n$ to F is denoted by $\mathcal{L}(E_1, \dots, E_n; F)$ and endowed with the usual sup norm. We simply write $\mathcal{L}(^nE;F)$ when $E_1 = \cdots = E_n = E$.

For $0 < p, p_1, p_2, \dots, p_n \leqslant \infty$, we assume that $1/p \leqslant 1/p_1 + \cdots + 1/p_n$. An n-linear mapping $A \in \mathcal{L}(E_1, \dots, E_n; F)$ is *absolutely* $(p; p_1, p_2, \dots, p_n)$*-summing* if there is a $C > 0$ such that

$$\|(A(x_j^1, x_j^2, \dots, x_j^n))_{j=1}^k\|_p \leqslant C \prod_{i=1}^{n} \|(x_j^i)_{j=1}^k\|_{\ell_{p_i}^w(E_i)} \tag{2.5}$$

for all finite families of vectors $x_1^i, \dots, x_k^i \in E_i$, $i = 1, 2, \dots, n$. The infimum of such a $C > 0$ is called the $(p; p_1, \dots, p_n)$-summing norm (p-norm if $0 < p < 1$) of A and is denoted by $\pi_{(p;p_1,\dots,p_n)}(A)$. Let $\Pi_{(p;p_1,p_2,\dots,p_n)}(E_1, \dots, E_n; F)$ denote the space of all absolutely $(p; p_1, p_2, \dots, p_n)$-summing n-linear mappings from $E_1 \times \cdots \times E_n$ to F endowed with the norm (p-norm if $0 < p < 1$) $\pi_{(p;p_1\dots,p_n)}$.

For $n \geqslant 1$ and $A \in \mathcal{L}(E_1, \dots, E_n; F)$,

$$\hat{A}\colon \ell_\infty(E_1) \times \cdots \times \ell_\infty(E_n) \to \ell_\infty(F), \quad \hat{A}((x_j^1)_{j=1}^\infty, \dots, (x_j^n)_{j=1}^\infty) := (A(x_j^1, \dots, x_j^n))_{j=1}^\infty,$$

is a bounded n-linear mapping. Given subspaces $X_i \subseteq \ell_\infty(E_i)$ for $1 \leqslant i \leqslant n$ and $Y \subseteq \ell_\infty(F)$, each one endowed with its own norm, we say that $\hat{A}\colon X_1 \times \cdots \times X_n \to Y$ is bounded (or, equivalently, $\hat{A} \in \mathcal{L}(X_1, \dots, X_n; Y)$) if the restriction of $\hat{A}$ to $X_1 \times \cdots \times X_n$ is a well-defined (hence, n-linear) continuous Y-valued mapping. Clearly, $A \in \Pi_{(p;p_1,p_2,\dots,p_n)}(E_1, \dots, E_n; F)$ if and only if $\hat{A}$ maps $\ell_{p_1}^w(E_1) \times \cdots \times \ell_{p_n}^w(E_n)$ boundedly into $\ell_p(F)$.

Note also that if $T \in \mathcal{L}(E,F)$ and we write $A_T\colon E \times F' \to \mathbb{K}$ for the bilinear map

$$A_T(x, y^*) = \langle T(x), y^*\rangle, \quad x \in E,\ y^* \in E',$$

then

$$T \in \mathrm{CN}_{(p;q)}(E;F) \iff A_T \in \Pi_{(1;q,p')}(E, F'; \mathbb{K}).$$

Absolutely summing mappings fulfil the following inclusion result, which appears in [**24**, Proposition 3.3] (see also [**6**]) and will be used several times in this paper.

Theorem 2.1 (inclusion theorem). *Let $0 < q \leqslant p \leqslant \infty$, $0 < q_j \leqslant p_j \leqslant \infty$ for all $j = 1, \dots, n$. If*

$$\frac{1}{q_1} + \cdots + \frac{1}{q_n} - \frac{1}{q} \leqslant \frac{1}{p_1} + \cdots + \frac{1}{p_n} - \frac{1}{p},$$

then

$$\Pi_{(q;q_1,\dots,q_n)}(E_1, \dots, E_n; F) \subseteq \Pi_{(p;p_1,\dots,p_n)}(E_1, \dots, E_n; F)$$

and $\pi_{(p;p_1,\dots,p_n)} \leqslant \pi_{(q;q_1,\dots,q_n)}$ for all Banach spaces $E_1, \dots, E_n, F$.

If

$$\frac{1}{p} = \frac{1}{p_1} + \cdots + \frac{1}{p_n},$$

absolutely $(p; p_1, \dots, p_n)$-summing n-linear mappings are usually called $(p_1, \dots, p_n)$-*dominated*. They also satisfy a factorization result (see [**27**, Theorem 13]).

For the basic theory of type and cotype in Banach spaces we refer the reader to [**18**, Chapter 11].

A Banach space E is said to have the *Orlicz property* q, $2 \leqslant q < \infty$, if the identity operator $\mathrm{Id}\colon E \to E$ is absolutely $(q, 1)$-summing, that is, if $\ell_1^w(E) \subseteq \ell_q(E)$. Clearly, cotype q implies Orlicz property q. Some deep results by Talagrand [**32**, **33**] show that for $q > 2$ both notions actually coincide, while this is not the case for $q = 2$.

Some of the basic problems in the theory of absolutely summing operators are to analyse when $\Pi_{(p_1;q_1)}(X, Y) = \Pi_{(p_2;q_2)}(X, Y)$ or when $\mathcal{L}(X, Y) = \Pi_{(p;q)}(X, Y)$. Typical results in this direction are that $\Pi_{(1;1)}(X, Y) = \Pi_{(2;2)}(X, Y)$ for any space Y of cotype 2 or the fundamental theorem due to Grothendieck:

$$\mathcal{L}(\ell_1; \ell_2) = \Pi_{(1;1)}(\ell_1; \ell_2) \quad \text{or, equivalently,} \quad \mathcal{L}(c_0; \ell_1) = \Pi_{(2;2)}(c_0; \ell_1). \tag{2.6}$$

Besides the notion of cotype, the property that plays an important role in our development is the so-called *Littlewood–Orlicz property*. A Banach space E has the *Littlewood–Orlicz property* if $\ell_1^w(E) \subseteq \ell_2\langle E\rangle$, that is, $\mathrm{Id}\colon E \to E$ is Cohen $(2, 1)$-nuclear or the duality map $A_I\colon E \times E' \to \mathbb{K}$ is $(1; 1, 2)$-summing. The reader is referred to [**13**, § 4] for a related concept of the Littlewood–Orlicz operator. This is considered in § 5.

3. Weakly summing and Cohen nuclear multilinear mappings

The bilinear form

$$A\colon \ell_2 \times \ell_2 \to \mathbb{K}, \quad A((\alpha_j)_j, (\beta_j)_j) = \sum_j \alpha_j \beta_j,$$

is bounded, but $A \notin \Pi_{(1;1,2)}(E_1, E_2; \mathbb{K})$ because $(e_j)_j \in \ell_2^w(\ell_2)$, $(e_j/j)_j \in \ell_1^w(\ell_2)$ and $(A(e_j, e_j/j))_j = (e_j/j)_j \notin \ell_1$. This example suggests that no generalization of the Defant–Voigt theorem can be expected for general $p, p_1, \dots, p_n$ other than $p = p_1 = \cdots = p_n = 1$. However, let us see that whenever

$$\frac{1}{p_1} + \frac{1}{p_2} + \cdots + \frac{1}{p_n} - n + 1 \geqslant \frac{1}{p}$$

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we can get a coincidence result for $(p;p_1,\dots,p_n)$-summing n-linear functionals. Moreover, in order to extend such a result to the vector-valued case, we need to consider weakly $(p;p_1,\dots,p_n)$-summing operators. The essence of the concept of weakly summing multilinear operators, as far as we know, has its roots in [**31**].

Definition 3.1. For $n\in\mathbb{N}$, let $0<p,p_1,\dots,p_n<\infty$ be such that

$$\frac{1}{p_1}+\frac{1}{p_2}+\cdots+\frac{1}{p_n}\geqslant\frac{1}{p}.$$

We say that $A\in\mathcal{L}(E_1,\dots,E_n;F)$ is *weakly* $(p;p_1,\dots,p_n)$*-summing* if the induced mapping $\hat{A}\colon \ell_{p_1}^w(E_1)\times\ell_{p_2}^w(E_2)\times\cdots\times\ell_{p_n}^w(E_n)\to\ell_p^w(F)$ is well defined (hence, n-linear and bounded). The space formed by these mappings is denoted by

$$\Pi_{w(p;p_1,\dots,p_n)}(E_1,\dots,E_n;F)$$

and the weakly $(p;p_1,\dots,p_n)$-summing norm (p-norm if $0<p<1$) $\pi_{w(p;p_1,\dots,p_n)}(A)$ of A is defined as the norm (p-norm if $0<p<1$) of $\hat{A}$ as an operator from $\ell_{p_1}^w(E_1)\times\ell_{p_2}^w(E_2)\times\cdots\times\ell_{p_n}^w(E_n)$ to $\ell_p^w(F)$.

An example of a coincidence situation involving weakly summing operators can be found in [**30**, Proposition 13]: if $1<p,p_1,\dots,p_n<\infty$ are such that

$$\frac{1}{p'}=\frac{1}{p_1'}+\cdots+\frac{1}{p_n'},$$

then

$$\mathcal{L}(E_1,\dots,E_n;F)=\Pi_{w(p;p_1,\dots,p_n)}(E_1,\dots,E_n;F).$$

Proposition 3.2. *Let $n\in\mathbb{N}$, let $0<p,p_1,\dots,p_n<\infty$ be such that*

$$\frac{1}{p_1}+\frac{1}{p_2}+\cdots+\frac{1}{p_n}\geqslant\frac{1}{p},$$

and let $E_1,\dots,E_n,F$ be Banach spaces. Then the spaces

$$\Pi_{w(p;p_1,\dots,p_n)}(E_1,\dots,E_n;F')$$

and

$$\mathcal{L}(F,\Pi_{(p;p_1,\dots,p_n)}(E_1,\dots,E_n;\mathbb{K}))$$

are isometrically isomorphic.

Proof. Given $A\in\Pi_{w(p;p_1,\dots,p_n)}(E_1,\dots,E_n;F')$, consider the linear operator

$$\Phi_A\colon F\to\Pi_{(p;p_1,\dots,p_n)}(E_1,\dots,E_n;\mathbb{K}),\quad \Phi_A(y)(x_1,\dots,x_n)=\langle A(x_1,\dots,x_n),y\rangle$$

for any $x_k\in E_k$, $k=1,\dots,n$, and $y\in F$. Let $(x_j^k)_{j=1}^\infty\in\ell_{p_k}^w(E_k)$, $k=1,\dots,n$, and $y\in F$. Then

$$\begin{aligned}\|(\Phi_A(y)(x_j^1,\dots,x_j^n))_j\|_{\ell_p}&=\left(\sum_j|\langle A(x_j^1,\dots,x_j^n),y\rangle|^p\right)^{1/p}\\&\leqslant\pi_{w(p;p_1,\dots,p_n)}(A)\|y\|\|(x_j^1)_{j=1}^\infty\|_{\ell_{p_1}^w(E_1)}\cdots\|(x_j^n)_{j=1}^\infty\|_{\ell_{p_n}^w(E_n)}.\end{aligned}$$

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Hence, $\pi_{(p;p_1,\ldots,p_n)}(\Phi_A(y)) \leqslant \pi_{w(p;p_1,\ldots,p_n)}(A)\|y\|$, and therefore $\|\Phi_A\| \leqslant \pi_{w(p;p_1,\ldots,p_n)}(A)$. Now let $\Phi\colon F \to \Pi_{(p;p_1,\ldots,p_n)}(E_1,\ldots,E_n;\mathbb{K})$ be given. Consider the multilinear mapping

$$A_\Phi\colon E_1 \times \cdots \times E_n \to F', \quad \langle A_\Phi(x_1,\ldots,x_n), y\rangle = \Phi(y)(x_1,\ldots,x_n)$$

for any $x_k \in E_k$, $k = 1,\ldots,n$, and $y \in F$. From

$$\begin{aligned}\|(A_\Phi(x_j^1,\ldots,x_j^n))_j\|_{\ell_p^w(F')} &= \sup_{\varphi\in B_{F''}} \left(\sum_j |\langle A_\Phi(x_j^1,\ldots,x_j^n),\varphi\rangle|^p\right)^{1/p} \\ &= \sup_{y\in B_F} \left(\sum_j |\Phi(y)(x_j^1,\ldots,x_j^n)|^p\right)^{1/p} \\ &\leqslant \sup_{y\in B_F} \pi_{(p;p_1,\ldots,p_n)}(\Phi(y))\|(x_j^1)_{j=1}^\infty\|_{\ell_{p_1}^w(E_1)}\cdots\|(x_j^n)_{j=1}^\infty\|_{\ell_{p_n}^w(E_n)} \\ &= \|\Phi\|\|(x_j^1)_{j=1}^\infty\|_{\ell_{p_1}^w(E_1)}\cdots\|(x_j^n)_{j=1}^\infty\|_{\ell_{p_n}^w(E_n)},\end{aligned}$$

we conclude that $\pi_{w(p;p_1,\ldots,p_n)}(A_\Phi) \leqslant \|\Phi\|$. □

In general, for

$$\frac{1}{p} \leqslant \frac{1}{p_1} + \frac{1}{p_2} + \cdots + \frac{1}{p_n} < \frac{1}{p} + n - 1$$

one has that $\Pi_{w(p;p_1,\ldots,p_n)}(E_1,\ldots,E_n;F) \subsetneq \mathcal{L}(E_1,\ldots,E_n;F)$, as was shown in the example at the beginning of the section, where $n = 2$, $E_1 = E_2 = \ell_2$ and $F = \mathbb{K}$.

To analyse when

$$\mathcal{L}(E_1,E_2,\ldots,E_n;F) = \Pi_{w(p;p_1,p_2,\ldots,p_n)}(E_1,E_2,\ldots,E_n;F),$$

for some values of

$$0 \leqslant \frac{1}{p_1} + \cdots + \frac{1}{p_n} - \frac{1}{p} < n - 1$$

and some Banach spaces $E_1,\ldots,E_n,F$, the consideration of projective tensor products is quite profitable for our purposes. Recall that by $E_1\hat{\otimes}_\pi\cdots\hat{\otimes}_\pi E_n$ we mean the completed projective tensor product of the Banach spaces $E_1,\ldots,E_n$. The following lemma will be useful in this paper; part of it can be found in [**31**].

Lemma 3.3. *Let $n \in \mathbb{N}$, let $0 < p, p_1,\ldots,p_n < \infty$ be such that*

$$\frac{1}{p} \leqslant \frac{1}{p_1} + \frac{1}{p_2} + \cdots + \frac{1}{p_n} < \frac{1}{p} + n - 1,$$

and let $E_1,\ldots,E_n$ be Banach spaces. The following are equivalent.

(i) $\mathcal{L}(E_1,\ldots,E_n;F) = \Pi_{w(p;p_1,\ldots,p_n)}(E_1,\ldots,E_n;F)$ *and* $\pi_{w(p;p_1,\ldots,p_n)} \leqslant C\|\cdot\|$ *for every Banach space F.*

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(ii) *There exists a* $C > 0$ *such that*

$$\|(x_j^1 \otimes \cdots \otimes x_j^n)_{j=1}^\infty\|_{\ell_p^w(E_1 \hat{\otimes}_\pi \cdots \hat{\otimes}_\pi E_n)} \leqslant C \prod_{i=1}^n \|(x_j^i)_{j=1}^\infty\|_{\ell_{p_i}^w(E_i)}$$

for all sequences $(x_j^i)_{j=1}^\infty \in \ell_{p_i}^w(E_i)$, $i = 1, \dots, n$.

(iii) $\mathcal{L}(E_1, \dots, E_n; \mathbb{K}) = \Pi_{(p;p_1,\dots,p_n)}(E_1, \dots, E_n; \mathbb{K})$ *and there exists a* $C > 0$ *such that* $\pi_{(p;p_1,\dots,p_n)} \leqslant C\| \cdot \|$.

Proof. (i) $\Longrightarrow$ (ii) Take $F = E_1 \hat{\otimes}_\pi \cdots \hat{\otimes}_\pi E_n$ and $A\colon E_1 \times \cdots \times E_n \to E_1 \hat{\otimes}_\pi \cdots \hat{\otimes}_\pi E_n$ given by $A(x_1, \dots, x_n) = x_1 \otimes \cdots \otimes x_n$.

(ii) $\Longrightarrow$ (iii) Given $A \in \mathcal{L}(E_1, \dots, E_n; \mathbb{K})$, its linearization $T\colon E_1 \hat{\otimes}_\pi \cdots \hat{\otimes}_\pi E_n \to \mathbb{K}$ is bounded, and hence $\hat{T}\colon \ell_p^w(E_1 \hat{\otimes}_\pi \cdots \hat{\otimes}_\pi E_n) \to \ell_p$ is bounded. Now, given sequences $(x_j^i)_{j=1}^\infty \in \ell_{p_i}^w(E_i)$, $i = 1, \dots, n$,

$$\begin{aligned}
\|(A(x_j^1, \dots, x_j^n))_{j=1}^\infty\|_p &= \|(T(x_j^1 \otimes \cdots \otimes x_j^n))_{j=1}^\infty\|_p \\
&= \|\hat{T}((x_j^1 \otimes \cdots \otimes x_j^n)_{j=1}^\infty)\|_p \\
&\leqslant \|\hat{T}\|\|(x_j^1 \otimes \cdots \otimes x_j^n)_{j=1}^\infty\|_{\ell_p^w(E_1 \hat{\otimes}_\pi \cdots \hat{\otimes}_\pi E_n)} \\
&\leqslant C\|T\| \prod_{i=1}^n \|(x_j^i)_{j=1}^\infty\|_{\ell_{p_i}^w(E_i)}.
\end{aligned}$$

(iii) $\Longrightarrow$ (i) For $T \in \mathcal{L}(E_1, \dots, E_n; F)$ and $\varphi \in F'$, we have $\varphi \circ T \in \mathcal{L}(E_1, \dots, E_n; \mathbb{K}) = \Pi_{(p;p_1,\dots,p_n)}(E_1, \dots, E_n; \mathbb{K})$ and the result follows straightforwardly. □

A direct consequence of the previous lemma and the inclusion theorem is that if $p, p_k \geqslant 1$ for $k = 1, \dots, n$ are such that

$$\frac{1}{p_1} + \frac{1}{p_2} + \cdots + \frac{1}{p_n} - n + 1 \geqslant \frac{1}{p},$$

then

$$\mathcal{L}(E_1, \dots, E_n; F) = \Pi_{w(p;p_1,\dots,p_n)}(E_1, \dots, E_n; F)$$

for all $E_1, \dots, E_n, F$. So, weakly $(p; p_1, \dots, p_n)$-summing operators are a variant of the concept of $(p; p_1, \dots, p_n)$-summing operators that shed light on the summability properties for operators. Let us introduce another variant of the notion of a summing multilinear operator (for a related notion we refer the reader to [**2**]).

Definition 3.4. Let $1 \leqslant p, p_1, \dots, p_n < \infty$ and let

$$\frac{1}{p_1} + \frac{1}{p_2} + \cdots + \frac{1}{p_n} \geqslant \frac{1}{p}.$$

We say that $A \in \mathcal{L}(E_1, \dots, E_n; F)$ is *Cohen* $(p; p_1, \dots, p_n)$*-nuclear* if the induced mapping

$$\hat{A}\colon \ell_{p_1}^w(E_1) \times \ell_{p_2}^w(E_2) \times \cdots \times \ell_{p_n}^w(E_n) \to \ell_p\langle F \rangle$$

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is well defined (hence, n-linear and bounded). The space formed by these mappings is denoted by $\mathrm{CN}_{(p;p_1,\dots,p_n)}(E_1,\dots,E_n;F)$ and the Cohen $(p;p_1,\dots,p_n)$-nuclear norm $\|A\|_{CN(p;p_1,\dots,p_n)}$ is defined as the norm of $\hat{A}$ as a bounded linear operator from $\ell^w_{p_1}(E_1)\times\ell^w_{p_2}(E_2)\times\cdots\times\ell^w_{p_n}(E_n)$ to $\ell_p\langle F\rangle$.

Of course

$$\mathrm{CN}_{(p;p_1,\dots,p_n)}(E_1,\dots,E_n;F)\subseteq \mathit{\Pi}_{(p;p_1,\dots,p_n)}(E_1,\dots,E_n;F)\\ \subseteq \mathit{\Pi}_{w(p;p_1,\dots,p_n)}(E_1,\dots,E_n;F).$$

A more general concept was introduced in [**1**, Definition 2.1].

Theorem 3.5. *Let $n\in\mathbb{N}$, let $1\leqslant p,p_1,\dots,p_n<\infty$ be such that*

$$\frac{1}{p_1}+\frac{1}{p_2}+\cdots+\frac{1}{p_n}\geqslant\frac{1}{p},$$

and let $E_1,\dots,E_n$ and F be Banach spaces. Then the spaces

$$\mathrm{CN}_{(p;p_1,\dots,p_n)}(E_1,\dots,E_n;F')$$

and

$$\mathit{\Pi}_{(1;p_1,\dots,p_n,p')}(E_1,\dots,E_n,F;\mathbb{K})$$

are isometrically isomorphic.

Proof. Clearly, one has that

$$\mathcal{L}(E_1,\dots,E_n,F;\mathbb{K})=\mathcal{L}(E_1,\dots,E_n;F')$$

with the identification $A\longleftrightarrow\tilde{A}$ given by

$$A(x_1,\dots,x_n,z)=\langle\tilde{A}(x_1,\dots,x_n),z\rangle$$

for $x_k\in E_k$, $k=1,\dots,n$, and $z\in F$. Given $\tilde{A}\in\mathrm{CN}_{(p;p_1,\dots,p_n)}(E_1,\dots,E_n,F')$, take $(x^k_j)^\infty_{j=1}\in\ell^w_{p_k}(E_k)$, $k=1,\dots,n$, and $(z_j)^\infty_{j=1}\in\ell^w_{p'}(F)$. Then

$$\begin{aligned}\sum_j|A(x^1_j,\dots,x^n_j,z_j)|&=\sum_j|\langle\tilde{A}(x^1_j,\dots,x^n_j),z_j\rangle|\\ &\leqslant\|(\tilde{A}(x^1_j,\dots,x^n_j))_j\|_{\ell_p\langle F'\rangle}\|(z_j)^\infty_{j=1}\|_{\ell^w_{p'}(F)}\\ &\leqslant\|\tilde{A}\|_{CN(p;p_1,\dots,p_n)}\|(x^1_j)^\infty_{j=1}\|_{\ell^w_{p_1}(E_1)}\cdots\|(x^n_j)^\infty_{j=1}\|_{\ell^w_{p_n}(E_n)}\\ &\qquad\times\|(z_j)^\infty_{j=1}\|_{\ell^w_{p'}(F)}.\end{aligned}$$

Hence, $A\in\mathit{\Pi}_{(1;p_1,\dots,p_n,p')}(E_1,\dots,E_n,F;\mathbb{K})$ and $\pi_{(1;p_1,\dots,p_n,p')}(A)\leqslant\|\tilde{A}\|_{CN(p;p_1,\dots,p_n)}$. Assume now that $A\in\mathit{\Pi}_{(1;p_1,\dots,p_n,p')}(E_1,\dots,E_n,F;\mathbb{K})$, let $(x^k_j)^\infty_{j=1}\in\ell^w_{p_k}(E_k)$, $k=$

$1,\dots,n$, and let $(z_j)_{j=1}^{\infty} \in \ell_{p'}^w(F)$ be given. Then

$$\begin{aligned}\sum_j |\langle \tilde{A}(x_j^1,\dots,x_j^n), z_j\rangle| &= \sum_j |A(x_j^1,\dots,x_j^n,z_j)| \\ &\leqslant \pi_{(1;p_1,\dots,p_n,p')}(A)\|(x_j^1)_{j=1}^{\infty}\|_{\ell_{p_1}^w(E_1)}\cdots\|(x_j^n)_{j=1}^{\infty}\|_{\ell_{p_n}^w(E_n)} \\ &\qquad\times \|(z_j)_{j=1}^{\infty}\|_{\ell_{p'}^w(F)}.\end{aligned}$$

Hence, $\tilde{A} \in \mathrm{CN}_{(p;p_1,\dots,p_n)}(E_1,\dots,E_n;F')$ and $\|\tilde{A}\|_{CN(p;p_1,\dots,p_n)} \leqslant \pi_{(1;p_1,\dots,p_n,p')}(A)$. □

Corollary 3.6. *Let $n \in \mathbb{N}$, let $1 \leqslant p, p_1,\dots,p_n < \infty$, let*

$$\frac{1}{p_1}+\frac{1}{p_2}+\cdots+\frac{1}{p_n} \geqslant 1,$$

and let $E_1,\dots,E_n$ and F be Banach spaces. Then

$$\mathcal{L}(F;\mathrm{CN}_{(p_n';p_1,\dots,p_{n-1})}(E_1,\dots,E_n;E_n'))$$

and

$$\varPi_{w(1;p_1,\dots,p_n)}(E_1,\dots,E_n;F')$$

are isometrically isomorphic.

Proof. The proof follows from Theorem 3.5 and Proposition 3.2. □

4. Coincidences between spaces of summing multilinear mappings

It is well known that the cotype plays a fundamental role in coincidence results for linear and nonlinear operators. For example, if E has cotype 2, then $\varPi_{(1,1)}(E;F) = \varPi_{(2,2)}(E;F)$ for any Banach space F [**18**, Corollary 11.16 (a)]. The first attempt to lift this result to the multilinear setting yields the following result (see [**28**, Corollary 3.7 (a)(i)]). The polynomial version appeared in [**22**, Theorem 16].

Proposition 4.1 (Popa [28, Corollary 3.7 (a)(i)]). *Let $E_1,\dots,E_n$ be cotype 2 spaces. Then*

$$\varPi_{(1/n;1,\dots,1)}(E_1,\dots,E_n;F) = \varPi_{(2/n;2,\dots,2)}(E_1,\dots,E_n;F)$$

for every Banach space F.

Corollary 4.2. *Let $E_1,\dots,E_n$ be cotype 2 spaces. Then*

$$\varPi_{w(1/n;1,\dots,1)}(E_1,\dots,E_n;F) = \varPi_{w(2/n;2,\dots,2)}(E_1,\dots,E_n;F)$$

for every Banach space F.

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Recently, in [**20**, Theorem 3 and Remark 2], [**28**, Corollary 4.6] and [**12**, Theorem 3.8 (ii)] it was proved that if $E_1, \dots, E_n$ are Banach spaces with cotype c and $n \geqslant 2$ then:

(i) if $c = 2$, then

$$\Pi_{(q;q,\dots,q)}(E_1, \dots, E_n; F) \subseteq \Pi_{(p;p,\dots,p)}(E_1, \dots, E_n; F) \tag{4.1}$$

holds for $1 \leqslant p \leqslant q \leqslant 2$ and arbitrary Banach spaces F;

(ii) if $c > 2$, then

$$\Pi_{(q;q,\dots,q)}(E_1, \dots, E_n; F) \subseteq \Pi_{(p;p,\dots,p)}(E_1, \dots, E_n; F) \tag{4.2}$$

holds for $1 \leqslant p \leqslant q < c'$ and arbitrary Banach spaces F.

It is easy to see that the inclusions (4.1) and (4.2) are not optimal. So it is natural to ask for the best s for which, under the same assumptions,

$$\Pi_{(q;q,\dots,q)}(E_1, \dots, E_n; F) \subseteq \Pi_{(s;p,\dots,p)}(E_1, \dots, E_n; F).$$

Let us settle this question and get a much better result in this direction.

We aim to prove a more general result. The key point is to work with spaces for which we have the factorization $\ell_p^w(E) = \ell_r \ell_s^w(E)$ for $1/p = 1/r + 1/s$.

Lemma 4.3. *Let $1 < p < r < \infty$ and let E be a Banach space.*

(a) *If $\mathcal{L}(\ell_{p'}; E) = \Pi_{(r;r)}(\ell_{p'}; E)$, then $\ell_p^w(E) = \ell_r \ell_s^w(E)$, where $1/r + 1/s = 1/p$.*

(b) *If $\mathcal{L}(c_0; E) = \Pi_{(r;r)}(c_0; E)$, then $\ell_1^w(E) = \ell_r \ell_s^w(E)$, where $1/r + 1/s = 1$.*

Proof. Take $(x_j)_{j=1}^\infty \in \ell_p^w(E)$. Define $u\colon \ell_{p'} \to E$ by $u(e_j) = x_j$, $j \in \mathbb{N}$. Since $u \in \Pi_{(r;r)}(\ell_{p'}; E)$, by [**18**, Lemma 2.23] there exist $(\alpha_j)_{j=1}^\infty \in \ell_r$ and $(y_j)_{j=1}^\infty \in \ell_s^w(E)$ such that $u(e_j) = x_j = \alpha_j y_j$ for every j. The other case is similar. □

It is well known that if E has cotype 2, then $\mathcal{L}(c_0; E) = \Pi_{2;2}(c_0; E)$ (see [**18**, p. 224]). Hence, for cotype 2 space one has $\ell_1^w(E) = \ell_2 \ell_2^w(E)$. In order to understand the use of cotype in factorization, let us recall the following definitions from [**26**, §§ 16.4 and 20.1]. Let E be a Banach space, let $0 < p \leqslant s \leqslant \infty$ and let r be determined by $1/r + 1/s = 1/p$.

- A sequence $(x_j)_{j=1}^\infty$ in E is called *mixed (s,p)-summable* if it can be written in the form $x_j = \alpha_j y_j$ with $(\alpha_j)_{j=1}^\infty \in \ell_r$ and $(y_j)_{j=1}^\infty \in \ell_s^w(E)$.

- An operator $u\colon E \to F$ is called *(s,p)-mixing* if every weakly p-summable sequence in E is mapped into an (s,p)-mixed summable sequence in F. $\mathcal{M}_{(s,p)}$ denotes the ideal of (s,p)-mixing operators.

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From the definition, if Id_E is the identity operator on the Banach space E, then $\mathrm{Id}_E \in \mathcal{M}_{(s,p)}(E;E)$ if and only if $\ell_p^w(E) = \ell_r \ell_s^w(E)$, where $1/r + 1/s = 1/p$. In [**26**, Theorem 20.3.1] it is proved that, for $s \geqslant 1$, $\mathcal{M}_{(s,p)} = \Pi_{(s,s)}^{-1} \circ \Pi_{(p,p)}$, which relates, in a very strong way, mixing operators with absolutely summing operators. Actually it says that the identity Id_E is (s,p)-mixing if and only if $\Pi_{(s,s)}(E;F) = \Pi_{(p,p)}(E;F)$ for every Banach space F. In other words, $\ell_p^w(E) = \ell_r \ell_s^w(E)$ if and only if $\Pi_{(s,s)}(E;F) = \Pi_{(p,p)}(E;F)$ for every Banach space F.

Now, in order to find examples of Banach spaces E for which $\ell_p^w(E) = \ell_r \ell_s^w(E)$, we look for the cotype.

- If E has cotype 2, then $\Pi_{(s;s)}(E;F) = \Pi_{(p;p)}(E;F)$ for every Banach space F, whenever $1 \leqslant p \leqslant s \leqslant 2$. In this case $\ell_p^w(E) = \ell_r \ell_s^w(E)$.
- If E has cotype s, $2 < s < \infty$, then $\Pi_{(q;q)}(E;F) = \Pi_{(p;p)}(E;F)$ for every Banach space F, whenever $1 \leqslant p \leqslant q < s'$. In this case $\ell_p^w(E) = \ell_r \ell_q^w(E)$.

Therefore, we have the following result.

Lemma 4.4. *Let E be a Banach space of cotype $2 \leqslant s < \infty$, let $p \leqslant q$ and let $1/p = 1/r + 1/q$. Then*

(a) $\ell_p^w(E) = \ell_r \ell_q^w(E)$ *for $s = 2$ and $1 \leqslant p \leqslant q \leqslant 2$,*

(b) $\ell_p^w(E) = \ell_r \ell_q^w(E)$ *for $s > 2$ and $1 \leqslant p \leqslant q < s'$.*

Theorem 4.5. *For $i = 1, \dots, n$, let E_i be a Banach space with cotype $c_i \in [2, \infty]$. Let $1 < p_i, q_i < \infty$ with $1/r_i = 1/p_i - 1/q_i \geqslant 0$, $p \leqslant q$ and $1/p = \sum_{i=1}^n 1/r_i + 1/q$. Assume that*

$$\begin{aligned} &1 \leqslant p_i = q_i && \text{if } c_i = \infty, \\ &1 \leqslant p_i \leqslant q_i \leqslant 2 && \text{if } c_i = 2, \\ &1 \leqslant p_i \leqslant q_i < c_i' && \text{if } 2 < c_i < \infty. \end{aligned}$$

Then

$$\Pi_{(q;q_1,\dots,q_n)}(E_1, \dots, E_n; F) = \Pi_{(p;p_1,\dots,p_n)}(E_1, \dots, E_n; F)$$

for every Banach space F. In particular, if each $c_i \in [2, \infty)$ and $1 \leqslant p \leqslant q < \min_i\{c_i'\}$, then

$$\Pi_{(q;q,\dots,q)}(E_1, \dots, E_n; F) = \Pi_{(qp/(n(q-p)+p);p,\dots,p)}(E_1, \dots, E_n; F)$$

and

$$\Pi_{(q;q,\dots,q)}(E_1, \dots, E_n; F) \subseteq \Pi_{(p;p,\dots,p)}(E_1, \dots, E_n; F).$$

Proof. The inclusion

$$\Pi_{(p;p_1,\dots,p_n)}(E_1, \dots, E_n; F) \subseteq \Pi_{(q;q_1,\dots,q_n)}(E_1, \dots, E_n; F)$$

follows from the inclusion theorem (see Theorem 2.1). Let us suppose that $A \in \Pi_{(q;q_1,\dots,q_n)}(E_1, \dots, E_n; F)$ and let the sequences $(x_j^i)_{j=1}^\infty \in \ell_{p_i}^w(E_i)$, $i = 1, \dots, n$, be given. From Lemma 4.4 we know that $\ell^w_{p_i}(E_i) = \ell_{r_i}\ell^w_{q_i}(E_i)$, $i = 1, \ldots, n$. Hence, there are sequences $(\alpha^i_j)_{j=1}^\infty \in \ell_{r_i}$ and $(y^i_j)_{j=1}^\infty \in \ell^w_{q_i}(E_i)$ such that $(x^i_j)_{j=1}^\infty = (\alpha^i_j y^i_j)_{j=1}^\infty$, $i = 1, \ldots, n$. In this fashion,

$$(\alpha^1_j \cdots \alpha^k_j)_{j=1}^\infty \in \ell_{r_1} \cdots \ell_{r_n} = \ell_r,$$

where $1/r = \sum_{j=1}^n 1/r_j$, and $(A(y^1_j, \ldots, y^n_j))_{j=1}^\infty \in \ell_q(F)$. Since $1/r + 1/q = 1/p$, it follows that

$$(A(x^1_j, \ldots, x^n_j))_{j=1}^\infty = (\alpha^1_j \cdots \alpha^n_j A(y^1_j, \ldots, y^n_j))_{j=1}^\infty \in \ell_p(F).$$

□

The above theorem is a variation of some well-known results in this area proved first by Popa [**29**, Theorem 1, Corollaries 2–4] and later by Bernardino [**5**, Theorem 3.1].

5. Summability of all multilinear mappings

In § 3 we observed some coincidence results for weakly absolutely summing multilinear mappings. We are now interested in understanding when

$$\mathcal{L}(E_1, E_2, \ldots, E_n; F) = \Pi_{(p;p_1,p_2,\ldots,p_n)}(E_1, E_2, \ldots, E_n; F)$$

for some values $p, p_1, \ldots, p_n$ and some Banach spaces $E_1, \ldots, E_n, F$. First, we will show how to lift coincidence results from the bilinear and trilinear case to the n-linear setting. Afterwards, we will explore related notions, such as the cotype of a Banach space, in order to achieve our aim.

Let us now explain a procedure that allows us to lift coincidence results of bilinear maps to coincidence results of multilinear maps.

Theorem 5.1. *Given $n \in \mathbb{N}$, $n \geqslant 2$, let $m \in \mathbb{N}$ be such that $n = 2m$ if n is even and $n = 2m + 1$ if n is odd. For $j = 1, \ldots, n$, let E_j be a Banach space and let $1 \leqslant r_j < \infty$, $p_1, \ldots, p_m > 0$ be such that $1/p_1 + \cdots + 1/p_m > m - 1$ and $1/p_i \leqslant 1/r_{2i-1} + 1/r_{2i} < 1/p_i + 1$, $i = 1, \ldots, m$. Assume that*

$$\mathcal{L}(E_{2i-1}, E_{2i}; \mathbb{K}) = \Pi_{(p_i;r_{2i-1},r_{2i})}(E_{2i-1}, E_{2i}; \mathbb{K})$$

for all $i = 1, \ldots, m$.

(i) *If n is even, then*

$$\mathcal{L}(E_1, \ldots, E_n; \mathbb{K}) = \Pi_{(p;r_1,\ldots,r_n)}(E_1, \ldots, E_n; \mathbb{K})$$

whenever $1/p \leqslant 1/p_1 + \cdots + 1/p_m - m + 1$.

(ii) *If n is odd, then*

$$\mathcal{L}(E_1, \ldots, E_n; \mathbb{K}) = \Pi_{(q;r_1,\ldots,r_n)}(E_1, \ldots, E_n; \mathbb{K})$$

whenever $1/p \leqslant 1/p_1 + \cdots + 1/p_m - m + 1$ and $1/q \leqslant 1/r_{2m+1} + 1/p - 1$.

Proof. (i) Let $A \in \mathcal{L}(E_1, \dots, E_n; \mathbb{K}) = \mathcal{L}(E_1, \dots, E_{2m}; \mathbb{K})$. Using the associativity of the projective tensor norm π it is easy to see that there is an m-linear mapping

$$B \in \mathcal{L}(E_1 \hat{\otimes}_\pi E_2, \dots, E_{2m-1} \hat{\otimes}_\pi E_{2m}; \mathbb{K})$$

such that

$$B(x^1 \otimes x^2, \dots, x^{2m-1} \otimes x^{2m}) = A(x^1, x^2, \dots, x^{2m-1}, x^{2m})$$

for all $x^j \in E_j$. Using the Defant–Voigt theorem we know that B is $(1; 1, \dots, 1)$-summing and, since $1/p \leqslant 1/p_1 + \cdots + 1/p_m - m + 1$, the inclusion theorem (Theorem 2.1) tells us that

$$B \in \Pi_{(p;p_1,\dots,p_m)}(E_1 \hat{\otimes}_\pi E_2, \dots, E_{2m-1} \hat{\otimes}_\pi E_{2m}; \mathbb{K}).$$

From Lemma 3.3 it follows that

$$\begin{aligned}
&\left(\sum_{j=1}^{\infty} |A(x_j^1, \dots, x_j^{2m})|^p \right)^{1/p} \\
&\quad = \left(\sum_{j=1}^{\infty} |B(x_j^1 \otimes x_j^2, \dots, x_j^{2m-1} \otimes x_j^{2m})|^p \right)^{1/p} \\
&\quad \leqslant \pi_{(p;p_1,\dots,p_m)}(B) \|(x_j^1 \otimes x_j^2)_{j=1}^{\infty}\|_{\ell_{p_1}^w(E_1 \hat{\otimes}_\pi E_2)} \\
&\qquad\qquad \times \cdots \times \|(x_j^{2m-1} \otimes x_j^{2m})_{j=1}^{\infty}\|_{\ell_{p_m}^w(E_{2m-1} \hat{\otimes}_\pi E_{2m})} \\
&\quad \leqslant C\|B\| \prod_{i=1}^{2m} \|(x_j^i)_{j=1}^{\infty}\|_{\ell_{r_i}^w(E_i)}
\end{aligned}$$

for all $(x_j^i)_{j=1}^{\infty} \in \ell_{r_i}^w(E_i)$, $i = 1, \dots, 2m = n$. Then $A \in \Pi_{(p;r_1,\dots,r_{2m})}(E_1, \dots, E_{2m}; \mathbb{K})$.

(ii) Let $A \in \mathcal{L}(E_1, \dots, E_n; \mathbb{K}) = \mathcal{L}(E_1, \dots, E_{2m}, E_{2m+1}; \mathbb{K})$. From (i) we know that

$$\mathcal{L}(E_1, \dots, E_{2m}; \mathbb{K}) = \Pi_{(p;r_1,\dots,r_{2m})}(E_1, \dots, E_{2m}; \mathbb{K}).$$

Then, by [**10**, Corollary 3.2], we get that

$$\mathcal{L}(E_1, \dots, E_{2m+1}; \mathbb{K}) = \Pi_{(p;r_1,\dots,r_{2m},1)}(E_1, \dots, E_{2m+1}; \mathbb{K}).$$

Hence, $A \in \Pi_{(p;r_1,\dots,r_{2m},1)}(E_1, \dots, E_{2m+1}; \mathbb{K})$. Using the inclusion theorem (Theorem 2.1) once again, we conclude that $A \in \Pi_{(q;r_1,\dots,r_{2m},r)}(E_1, \dots, E_{2m+1}; \mathbb{K})$ for any $p \leqslant q$ and $1 \leqslant r < \infty$ such that $1/p - 1/q \geqslant 1/r'$. Choose $r = r_{2m+1}$ to complete the proof. □

Applying the previous result to the case in which $p_i = 1$, $E_i = E$ and $r_i = r \geqslant 1$ for any values of i, we obtain that if $\mathcal{L}(^2E; \mathbb{K}) = \Pi_{(1;r,r)}(^2E; \mathbb{K})$, then

$$\mathcal{L}(^{2n}E; \mathbb{K}) = \Pi_{(1;r,\dots,r)}(^{2n}E; \mathbb{K}) \quad \text{and} \quad \mathcal{L}(^{2n+1}E; \mathbb{K}) = \Pi_{(r;r,\dots,r)}(^{2n+1}E; \mathbb{K}).$$

We can actually slightly improve the result by imposing conditions on trilinear maps.

Theorem 5.2. *Let $1 \leqslant r \leqslant 2$ and let E be a Banach space. If $\mathcal{L}(^2E;\mathbb{K}) = \Pi_{(1;r,r)}(^2E;\mathbb{K})$ and $\mathcal{L}(^3E;\mathbb{K}) = \Pi_{(1;r,r,r)}(^3E;\mathbb{K})$, then*

$$\mathcal{L}(^nE;\mathbb{K}) = \Pi_{(1;r,\dots,r)}(^nE;\mathbb{K})$$

for every $n \geqslant 2$.

Proof. We have already proved the case in which n is even. Let us consider the case in which n is odd and proceed by induction. Suppose that the result is valid for a fixed k odd. Let us prove that it is also true for $k+2$. Given $A \in \mathcal{L}(^{k+2}E;\mathbb{K})$, let $F = E\hat{\otimes}_\pi \cdots \hat{\otimes}_\pi E$ (k times, that is $F = \hat{\otimes}_\pi^k E$) and $G = E\hat{\otimes}_\pi E$. From the associativity properties of the projective norm there is a bilinear form $B \in \mathcal{L}(F,G;\mathbb{K})$ such that

$$B(x^1 \otimes \cdots \otimes x^k, x^{k+1} \otimes x^{k+2}) = A(x^1,\dots,x^{k+2})$$

for all $x^j \in E$, $j = 1,\dots,k+2$. Using the Defant–Voigt theorem we know that B is $(1;1,1)$-summing and

$$\begin{aligned}\sum_{j=1}^{\infty} |A(x_j^1,\dots,x_j^{k+2})| &= \sum_{j=1}^{\infty} |B(x_j^1 \otimes \cdots \otimes x_j^k, x_j^{k+1} \otimes x_j^{k+2})| \\ &\leqslant \pi_{(1;1,1)}(B)\|(x_j^1 \otimes \cdots \otimes x_j^k)_{j=1}^{\infty}\|_{\ell_1^w(E\hat{\otimes}_\pi \cdots \hat{\otimes}_\pi E)} \\ &\qquad \times \|(x_j^{k+1} \otimes x_j^{k+2})_{j=1}^{\infty}\|_{\ell_1^w(E\hat{\otimes}_\pi E)}.\end{aligned}$$

From Lemma 3.3, there are positive constants C_1 and C_2 such that

$$\begin{aligned}&\|(x_j^1 \otimes \cdots \otimes x_j^k)_{j=1}^{\infty}\|_{\ell_1^w(E\hat{\otimes}_\pi \cdots \hat{\otimes}_\pi E)} \|(x_j^{k+1} \otimes x_j^{k+2})_{j=1}^{\infty}\|_{\ell_1^w(E\hat{\otimes}_\pi E)} \\ &\quad \leqslant (C_1\|(x_j^1)_{j=1}^{\infty}\|_{\ell_r^w(E)} \cdots \|(x_j^k)_{j=1}^{\infty}\|_{\ell_r^w(E)})(C_2\|(x_j^{k+1})_{j=1}^{\infty}\|_{\ell_r^w(E)}\|(x_j^{k+2})_{j=1}^{\infty}\|_{\ell_r^w(E)})\end{aligned}$$

for all $(x_j^i)_{j=1}^{\infty} \in \ell_r^w(E)$, $i = 1,\dots,k+2$. □

Next we present a technique to lift $(n-1)$-linear coincidences to n-linear coincidences. A few definitions are in order: by $\mathrm{Rad}(E)$ we denote the space of sequences $(x_j)_{j=1}^{\infty}$ in E such that

$$\|(x_j)_{j=1}^{\infty}\|_{\mathrm{Rad}(E)} := \sup_{n\in\mathbb{N}} \left\| \sum_{j=1}^{n} r_j x_j \right\|_{L^2([0,1],E)} < \infty,$$

where $(r_j)_{j\in\mathbb{N}}$ are the usual Rademacher functions.

Let $\ell_p^u(E)$ denote the closed subspace of $\ell_p^w(E)$ formed by the sequences $(x_j)_{j=1}^{\infty} \in \ell_p^w(E)$ such that $\lim_{k\to\infty} \|(x_j)_{j=k}^{\infty}\|_{\ell_p^w(E)} = 0$. It is well known that $\ell_{p_k}^w(E_k)$ can be replaced with $\ell_{p_k}^u(E_k)$ in the definition of absolutely summing mappings (see, for example, [**21**]).

According to [**8**, **11**] (see [**18**, Chapter 12] for the linear case), a multilinear map $A \in \mathcal{L}(E_1,\dots,E_n;F)$ is said to be *almost summing* if the induced map $\hat{A}\colon \ell_2^u(E_1) \times \cdots \times \ell_2^u(E_n) \to \mathrm{Rad}(F)$ is well defined (hence, n-linear and bounded). We write $\Pi_{\mathrm{as}}(E_1,\dots,E_n;F)$ for the space of all such multilinear maps.

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Remember that a *GT-space* is a Banach space E for which every bounded linear operator from E to ℓ_2 is absolutely 1-summing. The following result is a variant of [**6**, Theorem 3.7] and the same technique was used in [**10**].

Proposition 5.3. *Let $n \geqslant 2$ and let $E_1, \dots, E_n$ be Banach spaces such that*

$$\mathcal{L}(E_1, \dots, E_{n-1}; E_n') = \Pi_{\mathrm{as}}(E_1, \dots, E_{n-1}; E_n').$$

(i) *Then*

$$\mathcal{L}(E_1, \dots, E_n; \mathbb{K}) = \Pi_{(1;2,\dots,2,1)}(E_1, \dots, E_n; \mathbb{K}).$$

(ii) *If E_n' is a GT-space of cotype 2, then*

$$\mathcal{L}(E_1, \dots, E_n; \mathbb{K}) = \Pi_{(1;2,\dots,2)}(E_1, \dots, E_n; \mathbb{K}).$$

Proof. Given $A \in \mathcal{L}(E_1, \dots, E_n; \mathbb{K})$, define $A_{n-1} \in \mathcal{L}(E_1, \dots, E_{n-1}; E_n')$ in the obvious way, that is,

$$A_{n-1}(x_1, \dots, x_{n-1})(x_n) = A(x_1, \dots, x_n).$$

By assumption, $A_{n-1} \in \Pi_{\mathrm{as}}(E_1, \dots, E_{n-1}; E_n')$. Let $(x_j^i)_{j\in\mathbb{N}} \in \ell_2^u(E_i)$ for $1 \leqslant i \leqslant n-1$ and let $(x_j^n)_{j\in\mathbb{N}} \in \ell_1^u(E_n)$ be given. For any $m \in \mathbb{N}$ there exists $(\lambda_j)_{j=1}^m$ such that $|\lambda_j| = 1$, and we have

$$\begin{aligned}
\sum_{j=1}^m &|A(x_j^1, \dots, x_j^n)| \\
&= \sum_{j=1}^m A_{n-1}(x_j^1, \dots, x_j^{n-1})(\lambda_j x_j^n) \\
&= \int_0^1 \sum_{j=1}^m [A_{n-1}(x_j^1, \dots, x_j^{n-1}) r_j(t)] \biggl(\sum_{j=1}^m \lambda_j x_j^n r_j(t) \biggr) \,\mathrm{d}t \\
&\leqslant \int_0^1 \biggl\| \sum_{j=1}^m A_{n-1}(x_j^1, \dots, x_j^{n-1}) r_j(t) \biggr\| \biggl\| \sum_{j=1}^m \lambda_j x_j^n r_j(t) \biggr\| \,\mathrm{d}t \\
&\leqslant \biggl(\int_0^1 \biggl\| \sum_{j=1}^m A_{n-1}(x_j^1, \dots, x_j^{n-1}) r_j(t) \biggr\|^2 \mathrm{d}t \biggr)^{1/2} \biggl(\int_0^1 \biggl\| \sum_{j=1}^m \lambda_j x_j^n r_j(t) \biggr\|^2 \mathrm{d}t \biggr)^{1/2} \\
&\leqslant C \|A_{n-1}\| \prod_{i=1}^{n-1} \|(x_j^i)_{j=1}^m\|_{\ell_2^w(E_i)} \|(x_j^n)_{j=1}^m\|_{\ell_1^w(E_n)}.
\end{aligned}$$

Passing to the limit for $m \to \infty$ we get (i). The proof of (ii) follows easily using the characterization of E' being a GT-space of cotype 2 (see [**4**, Theorem 1] and [**19**]) in terms of the equality $\mathrm{Rad}(E') = \ell_2 \hat{\otimes}_\pi E'$ with equivalent norms. Hence, given $(x_j^i)_{j\in\mathbb{N}} \in \ell_2^w(E_i)$ for $1 \leqslant i \leqslant n$, $m \in \mathbb{N}$ and $(\lambda_j)_{j=1}^m$ such that $|\lambda_j| = 1$, as above we now can write, using that $\ell_2^w(E) \subseteq \ell_2^w(E'') = (\ell_2 \hat{\otimes}_\pi E')'$,

$$\begin{aligned}\sum_{j=1}^m |A(x_j^1, \dots, x_j^n)| &= \sum_{j=1}^m A_{n-1}(x_j^1, \dots, x_j^{n-1})(\lambda_j x_j^n) \\ &\leqslant \|(A_{n-1}(x_j^1, \dots, x_j^{n-1}))_{j=1}^m\|_{\ell_2 \hat{\otimes}_\pi E_n'} \|(\lambda_j x_j^n)_{j=1}^m\|_{\ell_2^w(E_n)} \\ &\leqslant C\|(A_{n-1}(x_j^1, \dots, x_j^{n-1}))_{j=1}^m\|_{\mathrm{Rad}(E_n')} \|(\lambda_j x_j^n)_{j=1}^m\|_{\ell_2^w(E_n)} \\ &\leqslant C\|A_{n-1}\| \prod_{i=1}^n \|(x_j^i)_{j=1}^m\|_{\ell_2^w(E_n)}.\end{aligned}$$

This finishes the proof when passing to the limit for $m \to \infty$. □

Now we recall that $\Pi_{\mathrm{as}}(\ell_1, E) = \mathcal{L}(\ell_1, E)$ if and only if E has type 2 (see [**18**, Theorem 21.10]). Therefore, we obtain the following corollary.

Corollary 5.4. *Let E be a Banach space such that E' has type 2. Then*

$$\mathcal{L}(\ell_1, E; \mathbb{K}) = \Pi_{(1;2,1)}(\ell_1, E; \mathbb{K}).$$

In particular, $\mathcal{L}(\ell_1, \ell_p; \mathbb{K}) = \Pi_{(1;2,1)}(\ell_1, \ell_p; \mathbb{K})$ for $1 < p \leqslant 2$.

An application of Lemma 3.3 yields the following result on the structure of some tensor products.

Corollary 5.5. *Let E be a Banach space such that E' has type 2. Then there exists a $C > 0$ such that*

$$\|(x_j^1 \otimes x_j^2)_{j=1}^\infty\|_{\ell_1^w(\ell_1 \hat{\otimes}_\pi E)} \leqslant C\|(x_j^1)_{j=1}^\infty\|_{\ell_2^w(\ell_1)} \|(x_j^2)_{j=1}^\infty\|_{\ell_1^w(E)}$$

for all sequences $(x_j^1)_{j=1}^\infty \in \ell_2^w(\ell_1)$ and $(x_j^2)_{j=1}^\infty \in \ell_1^w(E)$.

As we mentioned in the introduction, the fact that any operator $T\colon E_1 \to E_2'$ is Cohen (p_2', p_1)-nuclear is equivalent to the coincidence $\Pi_{(1;p_1,p_2)}(E_1, E_2; \mathbb{K}) = \mathcal{L}(E_1, E_2; \mathbb{K})$. In particular, using Grothendieck's theorem, $\Pi_{(1;2,2)}(c_0, c_0; \mathbb{K}) = \mathcal{L}(c_0, c_0; \mathbb{K})$, we get that $\mathcal{L}(c_0; \ell_1) = \mathrm{CN}_{(2;2)}(c_0; \ell_1)$. Now, using Lemma 3.3 this can be reformulated as the existence of a constant $C > 0$ such that

$$\|(x_j^1 \otimes x_j^2)_{j=1}^\infty\|_{\ell_1^w(c_0 \hat{\otimes}_\pi c_0)} \leqslant C\|(x_j^1)_{j=1}^\infty\|_{\ell_2^w(c_0)} \|(x_j^2)_{j=1}^\infty\|_{\ell_2^w(c_0)}$$

for all sequences $(x_j^1)_{j=1}^\infty, (x_j^2)_{j=1}^\infty \in \ell_2^w(c_0)$.

As pointed out before, the coincidence $\mathcal{L}(E_1, E_2; \mathbb{K}) = \Pi_{(1;p_1,p_2)}(E_1, E_2; \mathbb{K})$ is used to lift the coincidence result to $n \geqslant 2$. We shall show now that this condition is very much connected to the Littlewood–Orlicz property.

Definition 5.6. Let $2 \leqslant q < \infty$. We say that a Banach space E satisfies the *q-Littlewood–Orlicz property* if $\ell_1^w(E) \subseteq \ell_q\langle E \rangle$.

Note that, due to Talagrand's result, spaces with the q-Littlewood–Orlicz property for $q > 2$ must have cotype q.

Theorem 5.7. *Let $2 \leqslant q < \infty$ and let E be a Banach space. The following statements are equivalent.*

(i) *E' has the q-Littlewood–Orlicz property.*

(ii) *$\mathcal{L}(X, E; \mathbb{K}) = \Pi_{(1;1,q')}(X, E; \mathbb{K})$ for any Banach space X.*

(iii) *There exists a $C > 0$ such that*

$$\|(x_j^1 \otimes x_j^2)_{j=1}^{\infty}\|_{\ell_1^w(X \hat{\otimes}_\pi E)} \leqslant C\|(x_j^1)_{j=1}^{\infty}\|_{\ell_1^w(X)}\|(x_j^2)_{j=1}^{\infty}\|_{\ell_{q'}^w(E)}$$

for all sequences $(x_j^1)_{j=1}^{\infty} \in \ell_1^w(X)$ and $(x_j^2)_{j=1}^{\infty} \in \ell_{q'}^w(E)$.

Proof. (i) $\Longrightarrow$ (ii) Let $A \colon X \times E \to \mathbb{K}$ be a bounded bilinear form and let $T_A \colon X \to E'$ be the associated linear operator. For $(x_j)_{j=1}^{\infty} \in \ell_1^w(X)$ and $(y_j)_{j=1}^{\infty} \in \ell_{q'}^w(E)$,

$$\begin{aligned}\sum_{j=1}^{\infty} |A(x_j, y_j)| &= \sum_{j=1}^{\infty} |T_A(x_j)(y_j)| \\ &= \sup_{|\alpha_j|=1} \left|\sum_{j=1}^{\infty} T_A(x_j)(\alpha_j y_j)\right| \\ &\leqslant \|(T_A(x_j))_{j=1}^{\infty}\|_{\ell_q \hat{\otimes}_\pi E'}\|(y_j)_{j=1}^{\infty}\|_{\ell_{q'}^w(E)} \\ &\leqslant C\|(T_A(x_j))_{j=1}^{\infty}\|_{\ell_1^w(E')}\|(y_j)_{j=1}^{\infty}\|_{\ell_{q'}^w(E)} \\ &\leqslant C\|A\|\|(x_j)_{j=1}^{\infty}\|_{\ell_1^w(X)}\|(y_j)_{j=1}^{\infty}\|_{\ell_{q'}^w(E)}.\end{aligned}$$

(ii) $\Longrightarrow$ (i) Let $(x_j')_{j=1}^{\infty} \in \ell_1^w(E')$ be given. Consider the bounded bilinear form $A \colon c_0 \times E \to \mathbb{K}$ defined by the condition $A(e_j, x) = x_j'(x)$ for $x \in E$ and $j \in \mathbb{N}$. To show that $(x_j')_{j=1}^{\infty} \in \ell_q\langle E'\rangle$ it suffices to check that there is a $C > 0$ such that

$$\sum_{j=1}^{\infty} |x_j'(x_j)| \leqslant C\|(x_j)_j\|_{\ell_{q'}^w(E)}$$

for every $(x_j)_{j=1}^{\infty} \in \ell_{q'}^w(E)$. Using $X = c_0$ in the assumption, this follows from

$$\sum_{j=1}^{\infty} |x_j'(x_j)| = \sum_{j=1}^{\infty} |A(e_j, x_j)| \leqslant \|A\|\|(e_j)_{j=1}^{\infty}\|_{\ell_1^w(c_0)}\|(x_j)_{j=1}^{\infty}\|_{\ell_{q'}^w(E)}.$$

(ii) $\Longleftrightarrow$ (iii) This is a particular case of Lemma 3.3. □

Proposition 5.8. *Let $2 \leqslant q < \infty$ and let E be a Banach space. The following statements are equivalent.*

(i) *E has the q-Littlewood–Orlicz property.*

(ii) *$\mathcal{L}(X, E; \mathbb{K}) = \Pi_{(1;q',1)}(X, E; \mathbb{K})$ for any Banach space X.*

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(iii) *There exists* $C > 0$ *such that*

$$\|(x_j^1 \otimes x_j^2)_{j=1}^{\infty}\|_{\ell_1^w(X\hat{\otimes}_\pi E)} \leqslant C\|(x_j^1)_{j=1}^{\infty}\|_{\ell_{q'}^w(X)}\|(x_j^2)_{j=1}^{\infty}\|_{\ell_1^w(E)}$$

for all sequences $(x_j^1)_{j=1}^{\infty} \in \ell_{q'}^w(X)$ *and* $(x_j^2)_{j=1}^{\infty} \in \ell_1^w(E)$.

Proof. (i) $\Longrightarrow$ (ii) Let $A \in \mathcal{L}(X, E; \mathbb{K})$ and let $T_A\colon X \to E'$ be its associated linear map. Let $(x_j)_{j=1}^{\infty} \in \ell_{q'}^w(X)$ and let $(y_j)_{j=1}^{\infty} \in \ell_1^w(E)$. From (i) we have that $(y_j)_{j=1}^{\infty} \in \ell_q\langle E\rangle$, and hence

$$\begin{aligned}\sum_{j=1}^{\infty}|A(x_j, y_j)| &= \sum_{j=1}^{\infty}|\langle T_A(x_j), y_j\rangle| \\ &= \sup_{|\alpha_j|=1}\left|\sum_{j=1}^{\infty}\langle T_A(\alpha_j x_j), y_j\rangle\right| \\ &\leqslant \|(T_A(x_j))_{j=1}^{\infty}\|_{\ell_{q'}^w(E')}\|(y_j)_{j=1}^{\infty}\|_{\ell_q\langle E\rangle} \\ &\leqslant \|A\|\|(x_j)_{j=1}^{\infty}\|_{\ell_{q'}^w(X)}\|(y_j)_{j=1}^{\infty}\|_{\ell_1^w(E)}.\end{aligned}$$

(ii) $\Longrightarrow$ (i) Let $(x_j)_{j=1}^{\infty} \in \ell_1^w(E)$ be given. To show that $(x_j)_{j=1}^{\infty} \in \ell_q\langle E\rangle$, fix $(x'_j)_{j=1}^{\infty} \in \ell_{q'}^w(E')$ and consider the bounded bilinear form $A\colon \ell_q \times E \to \mathbb{K}$ defined by the condition

$$A((\lambda_j), x) = \sum_j \lambda_j x'_j(x)$$

for $x \in E$ and $(\lambda_j) \in \ell_q$. Clearly, $\|A\| = \|(x'_j)_j\|_{\ell_{q'}^w(E')}$ and $A(e_j, x_j) = x'_j(x_j)$. By assumption we know that $A \in \Pi_{(1;q',1)}(\ell_q, E; \mathbb{K})$, from which it follows that

$$\sum_{j=1}^{\infty}|x'_j(x_j)| \leqslant \|A\|\|(e_j)_j\|_{\ell_{q'}^w(\ell_q)}\|(x_j)_j\|_{\ell_1^w(E)} = \|(x'_j)_j\|_{\ell_{q'}^w(E')}\|(x_j)_j\|_{\ell_1^w(E)}$$

for every $(x_j)_{j=1}^{\infty} \in \ell_1^w(E)$. This shows the result. The equivalence (ii) $\Longleftrightarrow$ (iii) was shown in Lemma 3.3. □

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