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RESEARCH ARTICLE

Growth and long-run sustainability

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Abstract

From any state of economic and environmental assets, the maximin value defines the highest level of utility that can be sustained forever. Along any development path, the maximin value evolves over time according to investment decisions. If the current level of utility is lower than this value, there is room for growth of both the utility level and the maximin value. For any resource allocation mechanism (*ram*) and economic dynamics, growth is limited by the long-run level of the maximin value, which is an endogenous dynamic sustainability constraint. If utility reaches this limit, sustainability imposes growth to stop, and the adoption of maximin decisions instead of the current *ram*. We illustrate this pattern in two canonical models, the simple fishery and a two-sector economy with a nonrenewable resource. We discuss what our results imply for the assessment of sustainability in the short and the long run in non-optimal economies.

Keywords: growth; maximin value; non-optimal economies; resource allocation mechanism; sacrifice; sustainability improvement; sustainable development

JEL classification: O44; Q56

1. Introduction

Sustainability is more often associated with the idea of development than with stagnation, and certainly not with decline (Asheim *et al.*, 2001). The ability to sustain utility over time can be measured by the *maximin value*, which is the highest level of utility that can be sustained forever from the current set of productive assets in the economy, including natural resources, human and manufactured capital stocks, etc. (Solow, 1974; Pezzey, 1997; Cairns and Long, 2006; Cairns and Martinet, 2014; Fleurbaey, 2015). Society could decide to enjoy this sustainable utility forever by following the maximin path. This level of utility may, however, be considered too low, or there may be a social preference for 'sustained improvement' (Pezzey, 1997). As such, maximin, as a social objective, has often been criticized as entailing stagnation. Sustainable development, on the other hand, implies growth toward an acceptable standard of living that can be maintained over the 'very long run' (Solow, 1993). Under what conditions is growth sustainable? Toward what utility level does a development pattern lead us?

For growth to be sustainable, the development path must stay within environmental and technological limits. Utility can grow as long as it stays below the maximin value (Cairns and Martinet, 2014; Fleurbaey, 2015). So long as utility growth is meant to continue, the maximin value should grow too. In an efficient economy, growth entails the diversion of resources from consumption by the current generation to investment that will increase productivity in the future. Setting the current utility level lower than the maximin level, so that in a sense the current generation sacrifices some of its ability to consume in a sustainable way in favor of future generations, makes it possible to increase both the maximin level and current utility over time. Once utility catches up with the maximin level, growth is no longer possible and utility can be sustained only at that level. In this study, we analyze the consequences of such a growth pattern on long-run sustainability.

Sustainability is increasingly defined not only as the requirement that current utility is lower than the maximin value, but as a requirement that the current maximin value does not decline (Onuma, 1999; Martinet, 2007; Doyen and Martinet, 2012; Cairns and Martinet, 2014; Gerlagh, 2017). Even though a maximin policy may not be pursued, at any economic state the maximin value can be determined by solving the maximin problem for the levels of the stocks at that state. This value evolves over time, according to investment decisions, including the depletion of natural resources. The evolution of this maximin value along any trajectory plays a fundamental role in the sense that it is an indicator of sustainability improvement or decline. If current decisions reduce what is sustainable, the maximin value decreases, but if current decisions improve what is sustainable, the maximin value increases (Doyen and Martinet, 2012; Cairns and Martinet, 2014; Fleurbaey, 2015). The interplay between current consumption and the current evolution of the maximin value has been studied by Cairns and Martinet (2014) in a continuous-time framework, and by Fleurbaey (2015) in a discrete-time framework, both without discussing the long-run consequences of this pattern. Cairns and Martinet (2014) mention that whatever the objective of the society and the growth pattern it follows, the maximin value constitutes a dynamic limit to growth, but without studying the behavior of sustainable development paths. There remain trade-offs among present utility, growth, and long-run sustained utility that have not been studied adequately. Our first contribution is to put the insights of Cairns and Martinet (2014) and Fleurbaey (2015) into a long-run perspective.

Most of the literature focusing on the long-run aspects of sustainability relies on the definition of an optimality criterion and the study of the limiting behavior of the optimal path. In the tradition of Ramsey's model of undiscounted utility (Ramsey, 1928), some authors assume that growth leads the economy asymptotically toward an exogenous, bliss level of utility (Asheim and Buchholz, 2004; D'Autume and Schubert, 2008). The bliss level coincides with what is known as the green golden rule, which is the maximum level of utility that can ever be attained (Chichilnisky *et al.*, 1995). A sustainable development pattern may also lead to a lower level than bliss. The criterion proposed by Chichilnisky (1996) explicitly trades off discounted utility and the long-run utility of the path. Pezzey (1994) defines the 'opsustimal path,' which maximizes discounted utility subject to the constraint that utility is non-declining over time.[1] This path is efficient and

[1] Other authors studied discounted-utility optimal paths under a constraint of non-decreasing utility (see, e.g., Asheim and Buchholz, 2004). The 'sustainable discounted utilitarian' criterion (Asheim and Mitra, 2010) corresponds to the maximization of discounted utility subject to a constraint of non-decreasing consumption when technologies are stationary. On the set of non-decreasing consumption streams, discounting

corresponds either to the maximin path, or to a path with rising utility for a finite period followed by an egalitarian path (Pezzey, 1994, propositions 3.7).[2] Even if the interpretation of this result refers qualitatively to the current maximin value (in particular figure 3.4 in Pezzey, 1994, p. 154), the characterization of opsustimal paths has not formally led to studying the evolution of the maximin value, nor analyzed the interplay between the consumption and investment patterns during the (optimal) growth phase and the maximin value. Moreover, few authors have studied sustainability without assuming optimality (Fleurbaey, 2015). Our second contribution is to examine how decisions in a non-optimal economy pursuing a growth pattern induce a long-run, endogenous level of sustainability, which is imposed by the behavior of the maximin value in the long-run.

To do so, we consider that the development path is generated by a *resource allocation mechanism* (*ram*) that is not assumed to be optimal or efficient. In other frameworks, growth theorists have specified as parameters certain variables that could have been modeled as choices to be optimized. Among the parameters are a constant savings ratio, a constant capital-output ratio, or balanced growth. Holding a variable constant in this way has simplified complicated dynamic problems and has allowed for many revealing analyses. Following these tentatives in growth theory and in positive economics, we assume that the economies we study are not maximizing a specific objective but rather pursuing a parametric policy that seems plausible, for example, constant employment or constant growth. Other papers have departed from the optimality assumption in the studying of sustainability. Arrow *et al.* (2003) study genuine savings based on discounted utility along sub-optimal paths resulting from a given *ram*. But no paper thus far has departed from both optimality and discounted utility as a measure of welfare (Asheim, 2003). We offer a first step in that direction.

The paper starts by unfurling the conceptual framework (section 2) in which we discuss the trade-off between the current generation's sacrifice and the prospect for growth it offers in sub-optimal economies. On the one hand, a poor economy has to give up something to grow out of poverty. On the other hand, there may be a limit to growth. If investment is low, or growth is too fast, this limit is reached rapidly.

This endogenous limit to growth is illustrated by two examples (section 3) corresponding to two canonical models for which the maximin problem has been solved, namely, the simple fishery (a variant of the Ramsey one-sector growth model) and the Dasgupta-Heal-Solow model. These two types of model are often complementary, used to discuss different aspects of sustainability issues or to illustrate the consequences of various criteria in contrasted settings (see, for example, Asheim and Mitra, 2010; Zuber and Asheim, 2012). In both models, we characterize the trade-offs between the initial consumption level and the consumption level reached in the long run, for a given *ram*. The models appear to us to summarize the qualitative concerns that face a society contemplating how to develop sustainably. Their simplicity allows for a complete study of the long-run implications of the development rule and thus for greater insight into a sustained development policy that is close to frequently-expressed social objectives.

We then offer a discussion that emphasizes our contributions, in particular with respect to the literature on sustainability accounting; the limits of our work; and suggests future research avenues (section 4).

utility can be justified from an ethical point of view, as more weight is put on the worse-off generations, in the spirit of the 'extended rank-discounted utilitarian' criterion (Zuber and Asheim, 2012).

[2] A similar result was established by Asheim (1988, lemma 4) in a discrete-time model.

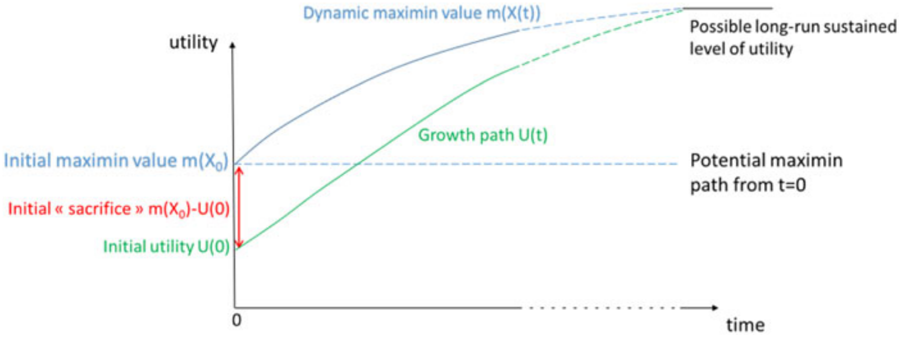

Figure 1. Illustrative pattern of sustainable growth with sustainability improvement.

2. Conceptual framework

We define sustainable growth as follows. Utility increases according to some growth pattern as long as the pattern is sustainable, in the sense that current utility is lower than the current maximin value (Pezzey, 1997), which is formally defined later on. When (if) utility catches up with the maximin value, the economy stops growing and follows the maximin path starting from the economic state reached. figure 1 illustrates such a sustainable growth pattern.

To reveal the interplay between the initial level of utility, the pursued growth pattern, and the duration of the growth period – or equivalently the level of sustained utility that is reached in the long run – we examine how the maximin value evolves along the development path.

We adopt the following formalism. Consider a comprehensive vector of capital stocks $X \in \mathbb{R}^n_+$ (encompassing all manufactured, natural, or human capital stocks that contribute to production or well-being) and decisions c (consumption, natural resources extraction, etc.) within the set $C(X) \subseteq \mathbb{R}^p$ of admissible controls at state X. The transition equation for each state variable X_i, $i = 1, \ldots, n$, is given by

$$\dot{X}_i = F_i\left(X, c\right) \ , \tag{1}$$

where the functions F_i can represent technologies or natural resources dynamics. The instantaneous utility $U(X, c)$ may depend on the state and the decisions. In our continuous-time framework, each generation is assumed to correspond to a single point in time, without overlapping. We also abstract from intragenerational equity issues. The utility of the generation living at time t is $U(X(t), c(t))$.

The highest utility level that can be sustained is given by the maximin value (Cairns and Long, 2006), which is denoted by $m(X)$ and formally defined, for any state X, as

$$m(X) = \max \bar{U} \tag{2}$$

$$\text{s.t. } U\left(X(t), c(t)\right) \geq \bar{U}, \forall t \geq 0 \tag{3}$$

$$\dot{X}_i = F_i\left(X, c\right)$$

$$X(0) = X.$$

In a *regular*, time-autonomous maximin problem,[3] if the economy pursues the maximin objective, both utility and the maximin value remain constant over the indefinite future, with $U(X(t), c(t)) = m(X(t)) = m(X(0))$ at all times. On a different path, the maximin value, which depends only on the capital stocks, evolves according to investment decisions. The evolution of the maximin value informs, along any path, on the way current decisions modify the level of utility that can be sustained (Cairns and Martinet, 2014). This evolution is given by net investment evaluated at maximin accounting prices, which is defined for a given current economic state X and any admissible vector of decisions c as follows:

$$M(X, c) = \dot{m}(X)\,|_c = \sum_{i=1}^{n} \dot{X}_i \frac{\partial m(X)}{\partial X_i} = \sum_{i=1}^{n} F_i(X, c) \frac{\partial m(X)}{\partial X_i} \,. \tag{4}$$

The maximin shadow values at state X are given by $\partial m(X)/\partial X_i$ and depend only on the current state X and not on the economic decisions c. That is to say, for the current stock levels X, shadow values are the same whatever the trajectory determined by the current decisions c and the functions $F_i(X, c)$. These shadow values are the basis of an accounting system for sustainability that does not depend on the assumptions on the path followed by the economy.

From here on, we consider any given development path, from initial state $X(0) = X_0$, and a *ram* $\gamma(X, t)$, which defines decisions at all times as $c(t) = \gamma(X(t), t) \subset C(X(t))$. The *ram* is not necessarily efficient or optimal with respect to some criterion. It just represents some decision rules. The economic trajectory is defined according to the dynamics in (1), starting from X_0, and according to the *ram* γ. At any time along this development path, the current maximin value $m(X(t))$ can be related to the initial maximin value $m(X_0)$ as follows:

$$m(X(t)) = m(X_0) + \int_0^t M(X(s), \gamma(X(s), s))ds \,. \tag{5}$$

The integral on the right-hand side of equation (5) corresponds to the sum over time of the changes in the maximin value. The very-long-run sustainability level would be captured by the limit of the maximin value, i.e., $m(X(\infty))$. This limit may or may not be reached in finite time. As soon as utility reaches the maximin value (i.e., when $U(X(t), \gamma(X(t), t)) = m(X(t))$), a positive net maximin investment is no longer possible (Cairns and Martinet, 2014, theorem 1). Growth has to stop. In our study of sustainable growth paths, we thus assume that the *ram* corresponds to some growth pattern initially, but that it changes to maximin decisions if utility reaches the maximin value.

It is important to identify when such a change has to happen, and how the current *ram* affects a sustainable development path. As an infinite number of such *ram* could be considered, we base our examples on a particular *ram* that corresponds to deviations from the maximin path. This choice is purely illustrative, with no normative basis.

A particular ram: local deviation from the maximin path The maximin program prescribes a constant utility path in regular problems, and is not consistent with a growth

[3]We shall consider only regular maximin problems (Burmeister and Hammond, 1977) in our illustrations. For a discussion on *regularity* and non-regularity in maximin problems, see Doyen and Martinet (2012); Cairns and Martinet (2014); Cairns *et al.* (2019).

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pattern. Rawls (1971) acknowledged that economic growth may be necessary to reach a state of the economy in which material resources are sufficient to implement a 'just society.' He pointed out that the maximin principle must be modified to allow for economic growth. We propose local deviations from the maximin program that allow for growth. These deviations are based on a reduced utility that results in extra savings.

Formally, these deviations are constructed as follows. Denote the co-state variables of the capital stocks in a maximin problem by μ_i, $i = 1, \ldots, n$. The maximin problem can be mathematically expressed as the maximization of the Hamiltonian $H(X,c) = \sum_{i=1}^{n} \mu_i \dot{X}_i$, subject to the constraint $U(X,c) \geq m(X)$ (Cairns and Long, 2006). It is equivalent to maximizing the Lagrangean:

$$
\begin{aligned}
L(c, X, \nu) &= H(X,c) + \nu\,(U(X,c) - m(X)) \\
&= \sum_{i=1}^{n} \mu_i \dot{X}_i + \nu\,(U(X,c) - m(X)),
\end{aligned}
$$

where ν is the dual variable of the equity constraint $U(X,c) \geq m(X)$. The term $\nu(U(X,c) - m(X))$ satisfies the complementarity slackness condition, and is always equal to zero. Cairns and Long (2006, proposition 1) show that the co-state variables of a maximin problem are equal to the derivatives of the maximin value function with respect to the state variables: $\mu_i = \partial m / \partial X_i$. Therefore, $M(X,c) = \sum_{i=1}^{n} \partial m(X)/\partial X_i \dot{X}_i = \sum_{i=1}^{n} \mu_i \dot{X}_i$, and the previous Lagrangean can be written as follows.

$$
L(c, X, \nu) = M\,(X,c) + \nu\,(U(X,c) - m(X)). \tag{6}
$$

Equation (6) yields a new interpretation of the maximin problem and its point-wise conditions: the maximin problem is tantamount to maximizing the net investment at maximin shadow values, subject to the constraint that consumption is no less than the maximin value.[4] We use this interpretation in characterizing the paths that deviate from maximin according to a parametric policy of consumption growth toward an eventual maximin path.

The maximin solution can be interpreted as follows. Along a regular maximin path, utility equals the highest level that is compatible with the satisfaction of the equity constraint (3). The consumption and investment decisions result in an egalitarian and efficient utility path. Along such a path, the Hamiltonian is nil (the Hartwick (1977) rule is satisfied), i.e., $H(X,c) = M(X,c) = \sum_{i=1}^{n} \mu_i \dot{X}_i = 0$. Net investment is maximized but takes a value of zero. There is no room for growth.

If utility is lower than the maximin level, the resources freed up can be invested to increase the maximin value, and hence the sustainability capacity of the economy. We consider 'deviations' from the maximin problem, in the sense that the sustainability constraint is relaxed and current utility is made lower than the maximin level. We explore the case in which the *ram* maximizes maximin investment subject to a postulated time

[4] In a context of efficiency and optimality (including the maximin case), Asheim and Buchholz (2004) interpret the maximization of the Hamiltonian as a strategy of 'no waste of welfare improvement,' which maximizes welfare improvement (i.e., net investment at some shadow values) subject to a constraint on current utility. In our non-efficient, non-optimal setting, when welfare is not specified to be the maximin value, the interpretation is modified, as we indicate below.

path of the utility levels $\bar{U}(t)$. This program is equivalent to maximizing the modified Lagrangean,

$$\tilde{L}(c, X, \tilde{\nu}) = M(X, c) + \tilde{\nu}\left(U(X, c) - \bar{U}(t)\right) .$$

That is to say, the program is to maximize $M(X, c)$ subject to a modified constraint, $U(X, c) \geq \bar{U}(t), t \geq 0$. The modified complementarity slackness condition is again equal to zero.

Definition 1. *Instantaneous maximization of sustainability improvement. The ram is said to maximize sustainability improvement at current time if decisions c maximize the increase of the maximin value subject to the given current utility:*

$$c \quad \text{maximizes} \quad M(X, c) = \sum_{i=1}^{n} F_i(X, c) \frac{\partial m(X)}{\partial X_i} \tag{7}$$
$$\text{s.t.} \quad U(X, c) \geq \bar{U}(t).$$

The interpretation of this particular *ram* is that the current generation consumes less than the sustainable, maximin level and relaxes the limit to growth as much as possible, given its own target utility. This mechanism has the advantage of generating trajectories along which the maximin value increases over time so long as the utility is lower than the maximin value. This program is in line with our focus on sustainable growth.[5]

3. Illustrative examples

To illustrate the links among initial utility, growth, and long-run utility, we study the development pattern generated by a *ram* in two canonical models. First, we use the simple fishery with a single decision to illustrate sustained development with a given growth pattern and the associated considerations on the trade-off between current sacrifice and long-run utility. In this model, the bliss level is the so-called maximum sustainable yield (MSY), which is not necessarily the long-run level achieved by the society. We then introduce the more sophisticated Dasgupta-Heal-Solow (DHS) model of production and consumption with a manufactured capital stock and a nonrenewable resource stock. Although this canonical model is still simple, it encompasses sufficient elements to allow us to discuss the cases of maximization of sustainability improvement. In the DHS model there is no exogenous bliss level and the long-run consumption is also endogenous.

3.1. The simple fishery

The natural rate of growth of the fish stock $S(t)$ at time t is $S(t)[1 - S(t)]$, fishing effort is denoted by $E(t)$ and the consumption (harvest) of the resource is $C(t) = S(t)E(t)$. We

[5]Other *ram* could result in trajectories along which the maximin value decreases at some time, in spite of the fact that current utility is lower than the maximin value (a case of 'sustainability decline due to investment choices' described in Cairns and Martinet, 2014).

study the following simple model of the evolution of the stock:[6]

$$\dot{S}(t) = S(t)\,[1 - S(t)] - S(t)E(t). \tag{8}$$

We assume that the effort E belongs to the interval $[0, 1]$. The open-access regime has $E(t) = E_0 = 1$.

In this model, the highest sustainable level of consumption is the MSY. Its value is

$$C_{MSY} = \max_S \,[S\,(1 - S)] = \tfrac{1}{4}.$$

The associated stock is $S_{MSY} = \frac{1}{2}$ and the equilibrium level of effort is $E_{MSY} = \frac{1}{2}$. The MSY stock is a benchmark for the study of both ecological and economic overexploitation.[7] If the initial state S_0 is lower than the stock producing the MSY, the maximin criterion (2) leads to a constant harvest in equilibrium, $C(t) = S_0(1 - S_0)$. Otherwise, the maximin value is equal to the MSY. The maximin value of a given state S is thus

$$m(S) = \begin{cases} S_{MSY}\,(1 - S_{MSY}) & \text{if } S > S_{MSY}, \\ S(1 - S) & \text{if } S \leq S_{MSY}. \end{cases} \tag{9}$$

If $S \leq S_{MSY}$, the level of effort, E^{mm}, on a maximin path is such that the harvest is equal to the natural growth, so that $E^{mm}S = S(1 - S)$, or $E^{mm} = 1 - S$.

Let the initial state S_0 be lower than the MSY biomass, i.e., $S_0 < S_{MSY}$, as may have occurred if the economy has been facing a 'tragedy of the commons' for some time because of an initial open access to the resource. The stock can be considered to be over-exploited, or vulnerable to over-exploitation, with a low sustainable (maximin) level of exploitation. A sacrifice is required for the stock to recover.

Effect of current sacrifice on instantaneous sustainability improvement We first examine the effect of current reduction of consumption with respect to the maximin level (steady-state consumption) on sustainability improvement.

In this simple fishery model, for a capital stock $S < S_{MSY}$, maximin improvement is given by

$$M(S, E) = \frac{dm(S)}{dt} = \frac{dm(S)}{dS}\dot{S} = (1 - 2S)(S(1 - S) - SE)\,. \tag{10}$$

Equation (10) gives the sustainability improvement as a function of the fishing effort E and current stock S. This equation can be interpreted as the product of two effects: a current stock effect $(1 - 2S)$, corresponding to the marginal productivity of the stock, and

[6] This model is often written using the parameters r, S_{sup} and q to represent the natural growth rate of the resource, its carrying capacity, and the catchability of the resource, respectively, so that

$$\dot{S}(t) = rS(t)\left(1 - \frac{S(t)}{S_{sup}}\right) - qS(t)E(t).$$

In our model, without loss of generality we define units of time, of effort and the resource such that $r = 1$, $S_{sup} = 1$, and $q = 1$. The expressions are less cumbersome, but one must be careful to keep track of the units in which the variables are measured.

[7] For sake of simplicity, we do not consider the cost of effort. As a result, the maximum economic yield (golden rule) coincides with the MSY. If there is a cost of effort, the golden rule stock is larger than the MSY stock.

a sacrifice effect corresponding to the net investment $(S(1-S)-SE)$ due to the foregone consumption with respect to the maximin level. Along a maximin path, one would have no sacrifice, with the full consumption of the produced resource, and a stationary resource stock, i.e., $SE = S(1-S)$ and $\dot{S} = 0$. The greater the sacrifice with respect to current maximin value, the larger the sustainability improvement. Also, the smaller the current resource stock, the larger the maximin improvement for a given sacrifice. At or above the MSY stock S_{MSY}, sacrifices have no effect.

Long-run recovery under constant effort We now consider the long-run effect of a reduction of the fishing effort, and thus of consumption, with respect to the sustainable, maximin level. For illustrative purposes, we consider a very simple *ram*, with constant fishing effort.[8]

Let a level of effort be chosen and remain constant at the level $E_0 \in [0, 1]$. Such a strategy could aim at increasing the available resource and sustainable consumption while maintaining an acceptable level of employment in the fishery. Consumption is given by $C(t) = E_0 S(t)$ and the dynamics of the exploited resource becomes

$$\dot{S}(t) = S(t)\left(1 - E_0 - S(t)\right). \tag{11}$$

Along this trajectory, the stock evolves as

$$S(t) = \frac{1 - E_0}{1 + (1 - E_0/S_0 - 1)\, e^{-(1-E_0)t}}. \tag{12}$$

The system tends toward a limit, $S_\infty = 1 - E_0$. For $E_0 = 1$, the stock is eventually exhausted.

The rule of constant effort completely determines the trajectory of the fishery. By equation (9), when $S < S_{MSY}$, the maximin level of effort is $E^{mm}(S) = 1 - S$. This level of effort maintains the stock at a stationary level that may correspond to a 'poverty trap.' In order to recover from a period of overfishing, society must harvest less than the maximin level $m(S) = S(1-S)$ so that the stock can grow and the maximin value function can increase along the trajectory. There is no 'free lunch' for the future. Current effort must be less than $E^{mm}(S_0)$, and consumption less than $C^{mm} = S_0(1 - S_0)$.

Under a strategy of constant effort, with $E(t) = E_0 < 1 - S_0 = E^{mm}(S_0)$, consumption increases with the stock size. Figure 2 depicts the following trajectories through time, beginning at the stock $S_0 = 0.1$:

- The natural growth of the stock (without harvesting).[9]
- The growth of the resource stock with constant fishing effort $E_0 = E^{MSY} = \frac{1}{2}$. The stock tends toward S_{MSY}. This trajectory is labeled 'stock recovery.'
- The trajectory of the maximin value along the trajectory for $E_0 = \frac{1}{2}$. The maximin value increases toward the MSY level.

[8]Many fisheries are managed under very simple rules, such as a constant effort or constant quotas. The Alaskan Pacific halibut stock (Singh *et al.*, 2006) and the Chilean jack-mackerel fishery (Martinet *et al.*, 2016) have been described as managed with a constant harvest rate. These rules have also been discussed in theoretical settings (see, e.g., Hannesson and Steinshamn, 1991; Quiggin, 1992).

[9]With no consumption ($C(t) = 0$, i.e., $E(t) = 0$), the dynamics of the resource stock is given by $S(t) = 1/1 + e^{-t}(1/S_0 - 1)$. The stock recovers faster, but the present generation does not consume at all.

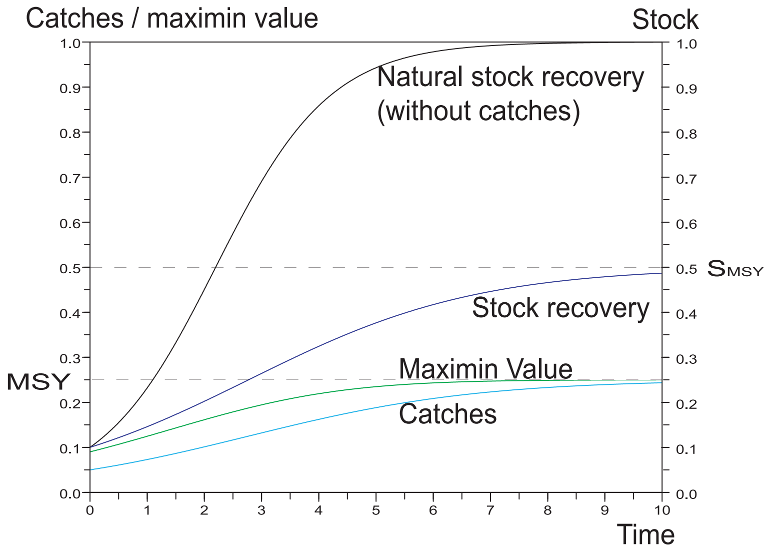

Figure 2. Evolution of the maximin value function along a constant effort trajectory $E(t) = E^{MSY} = \frac{1}{2}$ leading to maximum sustainable yield (MSY).

- The consumption pattern, which increases as the stock increases and catches up to the maximin value. Consumption tends toward the MSY.

We stress that the recovery of the fishery (and thus the increase in consumption) is possible only because consumption is lower than the maximin level at all times. The long-run consumption depends on the reduction of the current consumption, the constant fishing effort being between the maximin value $E^{mm}(S_0) = 1 - S_0$ and the MSY value $E^{MSY} = \frac{1}{2}$. A lower fishing effort, and hence current consumption, entails a higher long-run consumption.[10] Figure 3 presents the trajectories of maximin value and catches for three different recovery strategies (for three different effort levels) with, again, an initial fish stock $S_0 = 0.1$. For this stock, the initial maximin value is $m(S_0) = S_0(1 - S_0) = 0.1(1 - 0.1) = 0.09$. The evolutions of the stock for the different scenarios are not represented on the figure.

- The first strategy (trajectories denoted by $MV_{0.9}$ and $C_{0.9}$) corresponds to a constant fishing effort $E_0 = E^{mm} = 0.9$. At this effort level, the stock is in equilibrium at the initial value, i.e., $S_\infty = S_0 = 0.1$. The harvest is equal to the maximin value from the initial stock at all times, i.e., $C(t) = C^\infty = 0.09$. A policy maintaining effort or employment at this level entrenches poverty.
- The second strategy (trajectories denoted by $MV_{0.7}$ and $C_{0.7}$) corresponds to a constant fishing effort $E_0 = 0.7 < E^{mm}$. The fish stock increases asymptotically toward a limit, $S_\infty = 1 - E_0 = 0.3$. The harvest increases toward the maximin harvest for this limit, $C^\infty = S_\infty(1 - S_\infty) = 0.21$, which is lower than the MSY.

[10] Effort levels below $\frac{1}{2}$ are not considered as they would result in lower catches both for present and future generations, with a steady-state stock larger than the stock yielding the MSY.

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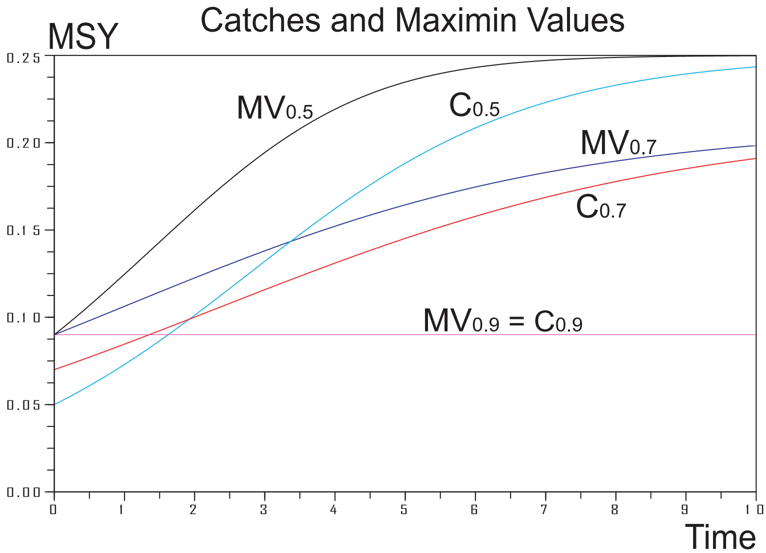

Figure 3. Sensitivity analysis with respect to the constant effort level, with values $E_0 = E^{MSY} = \frac{1}{2}$ (growth toward the MSY), $E_0 = 0.7$ (intermediate case), and $E_0 = E^{mm}(S_0) = 0.9$ (maximin path without growth).

- The third strategy (trajectories denoted by $MV_{0.5}$ and $C_{0.5}$) is that depicted in figure 2, with the fishing effort set constant at the MSY equilibrium effort, $E^{MSY} = 0.5$. The maximin value increases asymptotically toward the MSY value and the harvest increases toward the MSY, which is $C^{\infty} = 0.25$.

There is a non-linear relationship between C_0 and C_∞ which is determined by the chosen (constant) effort level. Recovery effort belongs to $[E^{MSY}, E^{mm}(S_0)]$. If the effort is small and equal to E^{MSY}, present consumption is low ($C_0 = E^{MSY} S_0$) and the limiting consumption is high, at the MSY. If the effort is equal to $E^{mm}(S_0)$, the stock remains at the initial level S_0, and the present and limiting consumption are equal. (There is no growth.) This is the maximin path. Intermediate cases are defined according to the relationship

$$C_\infty = \lim_{t \to \infty} E_0 S(t) = E_0 (1 - E_0) = \frac{C_0}{S_0} \left(1 - \frac{C_0}{S_0}\right), \tag{13}$$

for $C_0 \in [S_0/2, S_0]$, i.e., for $E_0 \in [1/2, 1]$. The possibility frontier between present and long-run consumption is described by figure 4.

Any pair (C_0, C_∞) that is achievable with constant effort $E_0 \geq 1/2$ belongs to this frontier. For this family of *ram*, different normative criteria would prescribe different initial consumption and result in different long-run consumption. Several particular solutions are represented in figure 4, including the green golden rule (Chichilnisky *et al.*, 1995), that maximizes C_∞, the myopic behavior from open access, maximizing C_0, and the maximin that accounts for the minimal consumption level and results in no growth, with $C_0 = C_\infty$.

The results in this section emphasize that there is a trade-off between the current consumption and long-run sustainability, given the postulated growth pattern. The model is, however, too simple to study investment patterns. We thus turn to a two-sector model.

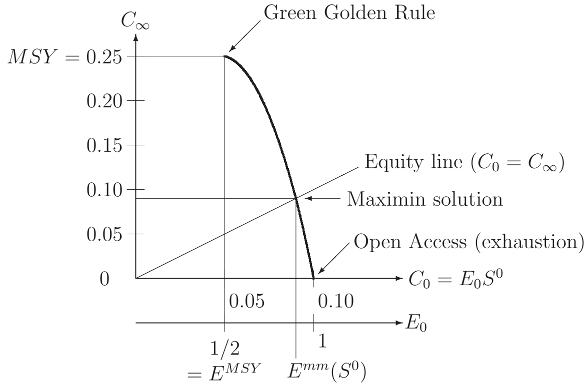

Figure 4. Trade-off between present consumption and long-run consumption in a fishery with constant effort and $S_0 = 0.1$.

3.2 The Dasgupta-Heal-Solow model

Consider a society that has stocks of a nonrenewable resource, S_0, and of a manufactured capital good, K_0, at its disposal at time $t = 0$. It produces output (consumption c and investment $\dot{K}$) by use of the capital stock and by depleting the resource stock at rate

$$r(t) = -\dot{S}(t), \tag{14}$$

according to a Cobb-Douglas production function:[11]

$$c + \dot{K} = F(K, r) = K^\alpha r^\beta, \text{ with } 0 < \beta < \alpha, \text{ and } \alpha + \beta \leq 1 . \tag{15}$$

This model has been used by many authors to study the implications of exhaustibility of an essential resource for sustainability. If the discounted-utility criterion is applied to this economy, consumption decreases asymptotically toward zero (Dasgupta and Heal, 1974, 1979). Analysis of how consumption can be sustained requires a different approach from discounted utilitarianism. For given levels of the capital and resource stocks, Solow (1974) and Dasgupta and Heal (1979) show that the maximal consumption that the economy can sustain, the *maximin* value, is given by

$$m(S, K) = (1 - \beta)(\alpha - \beta)^{(\beta/1-\beta)} S^{(\beta/1-\beta)} K^{(\alpha-\beta/1-\beta)} . \tag{16}$$

This aggregate of the two stocks is an increasing function of both stocks. It measures the capacity of the economy to sustain utility.

Effect of current sacrifice on instantaneous sustainability improvement Here again, we start by considering the case in which consumption is reduced with respect to the maximin value, and how it improves sustainability.

[11] We consider the Cobb-Douglas case for simplicity, because it guarantees the existence of a regular maximin path, and the maximin value is well-known for this case. Asheim *et al.* (2013) characterize the conditions for a maximin solution to exist in this model in general.

In the DHS model, maximin improvement is given by

$$M(K,S,c,r) = \frac{dm(K,S)}{dt} = \frac{\partial m(K,S)}{\partial K}\dot{K} + \frac{\partial m(K,S)}{\partial S}\dot{S}$$
$$= \frac{\partial m(K,S)}{\partial K}(K^{\alpha} r^{\beta} - c) - \frac{\partial m(K,S)}{\partial S} r. \quad (17)$$

Contrary to the single-stock problem of the fishery, a sacrifice does not necessarily entail sustainability improvement in the DHS model. For a level of consumption $c < m(S,K)$, the expression in equation (17) may be either positive or negative depending on the extraction level, and thus production and investment.

As explained previously, we consider a particular *ram* that is a deviation from the maximin path. This is for illustrative purposes only, and the same exercise could be done with another *ram*. The level of natural resource extraction that maximizes M conditional on the consumption level is given by the following extraction rule $\hat{r}(K,S)$:[12]

$$\hat{r}(K,S) = (\alpha - \beta)^{(1/1-\beta)} S^{(1/1-\beta)} K^{-(1-\alpha/1-\beta)} . \quad (18)$$

This feedback rule is the same as the one along the maximin path. At any given state, maximizing sustainability improvement entails producing the same as for the maximin path at the current state, and investing any amount of capital freed up by a sacrifice of current consumption.[13]

Under this strategy of maximizing sustainability improvement through the extraction rule $\hat{r}(K,S)$, net investment can be expressed as a function of the state variables and the consumption only:

$$M(K,S,c,\hat{r}(K,S)) = \frac{\partial m}{\partial K}(K^{\alpha}\hat{r}^{\beta} - c) - \frac{\partial m}{\partial S}\hat{r} .$$

This expression is linear in the current consumption (or symmetrically, current sacrifice), just as in the fishery model. It equals zero when the consumption equals the maximin level and net investment is nil, and is positive whenever the consumption is lower than the maximin level, corresponding to a positive net investment. The marginal effect of the sacrifice is proportional to the shadow value of the capital stock.

Trade-off between current consumption and long-run consumption for constant growth rate development paths Sustainable growth in the DHS model corresponds to growth from an initial level of consumption c_0 lower than the maximin value $m(S_0, K_0)$. In this model, sustainable growth without limit is possible, for example when considering hyperbolic discounting (Pezzey, 2004) or following a constant saving rate rule, which can correspond to some undiscounted utilitarian optima (Asheim and Buchholz, 2004; Asheim *et al.*, 2007; D'Autume and Schubert, 2008). As our purpose is to illustrate cases

[12] Mathematical details are provided in the appendix.

[13] Note that this path can be implemented by controlling the resource price. In a competitive economy, the resource is used up to the point at which its marginal product equals its price. The marginal product is $F'_r = \beta K^{\alpha} r^{\beta-1}$, which is equal to $(\beta K)/[(\alpha - \beta)S]$ for the feedback extraction rule (18). This provides a pricing rule for the resource. Note that this pricing rule is exactly the same as along the maximin path (but the actual price evolves differently from the maximin price over time as the stocks evolve differently). The program only deviates from maximin with respect to the consumption.

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in which growth induces a catching up of the maximin value in finite time, we consider that society initially pursues consumption growth at a constant rate $g > 0$, a growth pattern that is not sustainable in the long run without technological progress (Stiglitz, 1974; Llavador *et al.*, 2011).[14] We assume that the consumption side of the economy is determined by the constant growth rate pattern, with consumption

$$c(t) = c_0 e^{gt} . \tag{19}$$

To complete the *ram*, we assume that the current generation maximizes sustainability improvement, though the feedback extraction rule $\hat{r}(K, S)$.

The limit to growth. There is a limit to the time for which growth can be supported at rate g. If growth is pursued without considering the maximin value and its evolution, consumption overshoots the sustainable level at some time, and the economy ultimately collapses, as illustrated in figure 5. To avoid this unsustainable type of trajectory, the economy must switch at some time T from the exponential growth path to a maximin path characterized by constant consumption $c_\infty \equiv m(S(T), K(T))$. In fact, the long-run level of consumption is endogenous, and is defined at the time at which consumption catches up with the maximin level.[15] Figure 6 illustrates two sustained-development paths starting from the same initial state and thus the same maximin value, but with different initial consumption and growth rates. The long-run sustained consumption is different. The first path corresponds to a sustainable version of the overshooting path of figure 5, in which the consumption pattern switches to constant consumption once the maximin level is reached. The other path has a lower initial consumption (higher sacrifice), a higher growth rate, a longer growth period and a larger long-run consumption level.

Growth at rate $g > 0$ demands that $c_0 < m(S_0, K_0)$. The larger the initial sacrifice, the more room there is for growth. In the example in figure 6, the maximin value is increased by about 50 per cent with the path with the largest sacrifice (and by about 10 per cent for the other path), in spite of a larger growth rate. The growth rate and the duration of the growth period are linked to the initial consumption and the long-run consumption. If two of the four are given, the two others can be derived. For any initial pair of stocks (S_0, K_0) and the associated maximin value $m_0 = m(S_0, K_0)$, it is possible at time $t = 0$ to

[14] Any other growth pattern could have been used for illustrative purposes. A constant rate of growth seems a natural choice as it has been used by other authors, such as Lowenstein and Sicherman (1991) and Frank and Hutchens (1993). The World Bank (2011) assumes that consumption changes at a constant rate in its study of sustainable development and Llavador *et al.* (2011) provide that utility changes at a constant rate. Note that the opsustimal path also catches up to the maximin value in finite time in the DHS model (Asheim, 1988; Pezzey, 1994; Asheim and Buchholz, 2004). This efficient path completely sets the trade-off we want to study, by determining optimal initial and long-run consumption levels. As such, it is less illustrative than our parametrized growth pattern for our purpose. A possibility we do not explore would be to vary the utility discount rate of the opsustimal path to examine how it influences the trade-off between current utility and long-run utility. It would require having a full analytical expression of the optimal path, however, which is not obtainable except under strong assumptions on parameter values.

[15] Another possibility is to imagine a path for which the consumption smoothly approaches the maximin value. For example, the path followed could be a logistic growth curve. This path would be more difficult to solve than the path proposed in the text but would give no more insight into the problem. With quasi-arithmetic growth (Asheim *et al.*, 2007), there is no limit to growth: the dynamic limit represented by the maximin value increases forever along with consumption.

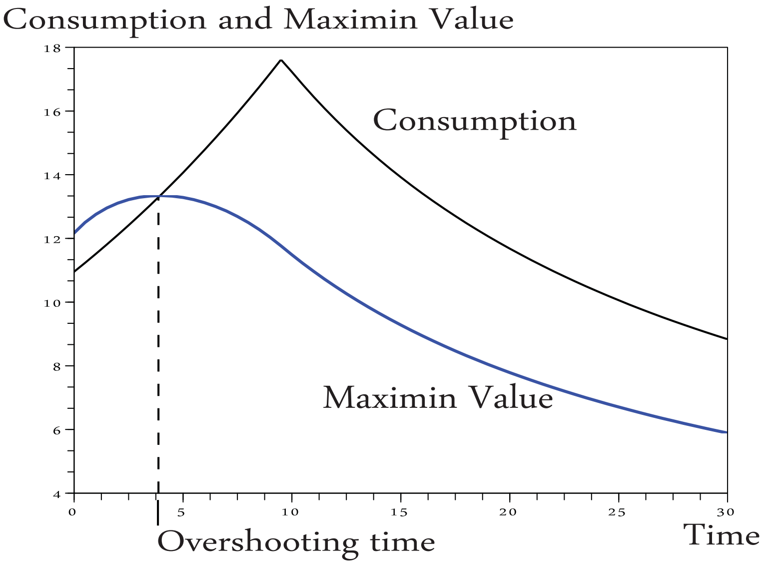

Figure 5. Exponential consumption and maximin value function, for $(K_0, S_0) = (10, 100)$, $\alpha = 2/3$ and $\beta = 1/3$, $m(K_0, S_0) \approx 12.17$, initial consumption $C_0 = 0.9m(K_0, S_0) \approx 10.95$ and growth rate $g = 0.05$.

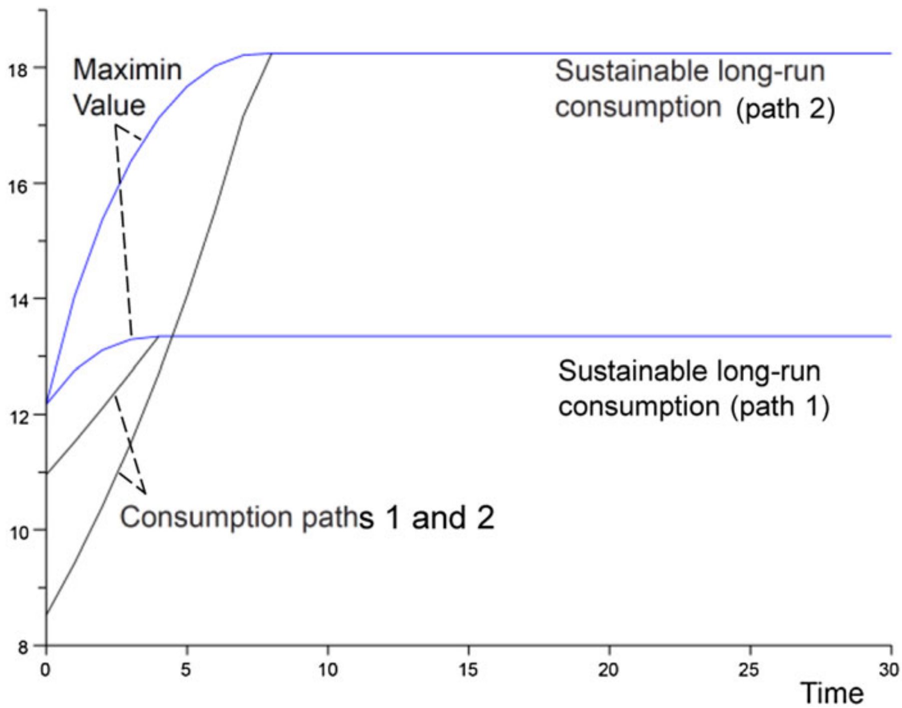

Figure 6. Exponential consumption and maximin value function, for $(K_0, S_0) = (10, 100)$, $\alpha = 2/3$ and $\beta = 1/3$, $m(K_0, S_0) \approx 12.17$. Path 1 is plotted with $C_0 = 0.9m(K_0, S_0) \approx 10.95$, a growth rate $g = 0.05$ and $C^\infty \approx 13.35$. Path 2 is plotted with $C_0 = 0.7m(K_0, S_0) \approx 8.52$, a growth rate $g = 0.1$ and $C^\infty \approx 18.25$.

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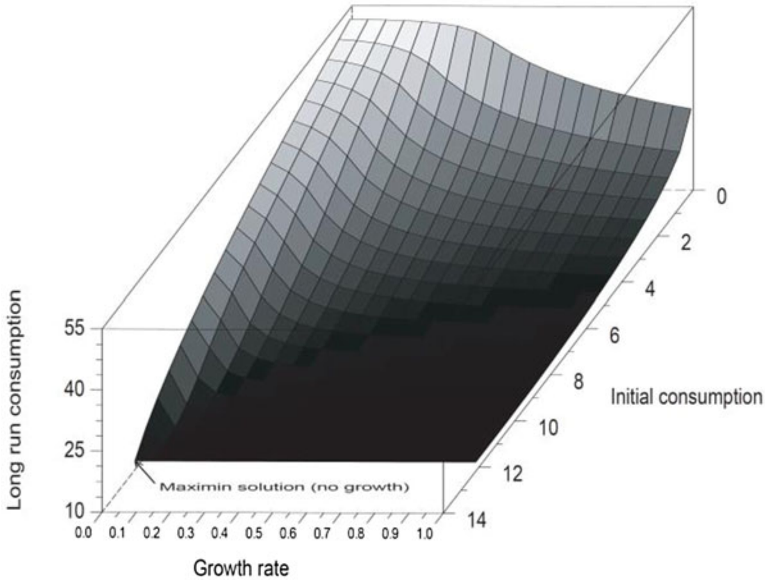

Figure 7. Links between initial consumption, growth rate, and long-run (sustained) consumption in the DHS model.

choose any pair

$$(c_0, g) \in \{]0, m_0[\times]0, \infty[\} \cup (m_0, 0) \ .$$

The path in which $(c_0, g) = (m_0, 0)$ is the maximin path starting from the initial state. It has no growth. A path in which $(c_0, g) \in \{]0, m_0[\times]0, \infty[\}$ (so that $c_0 < m_0$ and $g > 0$) has growth. However, growth at a constant rate cannot go on forever. There is an endogenous time $T(S_0, K_0, c_0, g)$ at which consumption catches up to the dynamic maximin value, i.e., $c(T) = c_0 e^{gT} = m[S(T), K(T)]$. From then on, growth is no longer sustainable, and the level of consumption must remain at the maximin level; i.e., for $t \geq T$, sustainability implies that $c(t) = m(S(T), K(T))$.[16]

Along any growth path, the *ram* that society implements imposes the long-run utility level. In our example of an exponential growth path, initial consumption, the rate of growth, and the very long-run consumption are interconnected. Figure 7 depicts the convex-concave correspondence from the initial pair (S_0, K_0) to the attainable frontier, $\left\{(c_0, g, c_\infty) \text{ feasible from } (S_0, K_0)\right\}$. Growth is possible only if $c_0 < m(S_0, K_0)$.[17] For a given growth rate, a lower level of initial consumption allows a higher long-run level. For a given initial consumption, a lower growth rate allows a higher long-run consumption (as the actual consumption catches the maximin level more slowly). Given the initial level of consumption c_0, there is a trade-off between the eventual maximin consumption that is sustained after some time T (endogenous to the path followed) and the rate of

[16] At the endogenous time T, only the part of the *ram* that drives consumption has to change, from allowing consumption to grow at rate g to keeping consumption constant at $c_\infty = m(S(T), K(T))$. Resource use is still determined by the maximization of sustainability improvement. Generations from time T enjoy the high, sustainable consumption level reached.

[17] 'Degrowth' ($g < 0$) is required if $c_0 > m(S_0, K_0)$.

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growth that is sustained up to that level. A level of present consumption that is closer to the maximin value $m(S_0, K_0)$ entails a lower prospect for growth.

4. Discussion

The previous examples illustrate the links between sacrifice and long-run sustainability that we stressed in the conceptual section. Under a given development pattern, the endogenous limit to growth, whenever there is one, is given by the co-evolution of current utility and maximin value. In a fishery driven by a constant-effort rule, there is a direct link between present consumption and long-run consumption. In the DHS model, even for a given consumption growth pattern, the evolution of the maximin value depends on the investment decisions, and thus on the *ram*. The more decision variables there are in the economy, the more ways there are to deviate from the maximin path. This calls for a careful examination of the long-run consequences of the current *ram* on sustainability. It is important to study the effect of the current *ram* on the maximin value, in the short and the long run, even when maximin is not a social objective, for two reasons.

The first reason is that, whenever the current development pattern is not consistent with infinite growth, it is of interest to assess the very-long-run consequences of current decisions on sustainability. Chichilnisky (1996) argued that the long-run utility should be accounted for in the definition of an optimal sustainable development path. We essentially carry the same message, but in a non-optimal framework. Few criteria make this analysis possible. Discounted utility neglects the very long run (Chichilnisky's dictatorship of the present). Both the green golden rule and the undiscounted utility criterion are oversensitive to the very long run, and exhibit Chichilnisky's dictatorship of the future. Comparing non-optimal trajectories with these criteria is thus unsatisfying. Using Chichlinisky's criterion would be a solution, but it requires knowing the full development path. Relying on the information available at the current time is less demanding in terms of assumptions. An important stream of literature adopts such a short-term, instantaneous perspective, studying the links between current net investment on the one hand, and welfare improvement and sustainability on the other hand. This literature focuses on (different measures of) net national product (NNP) and the significance of its growth through genuine savings, mainly in an optimality framework with intertemporal welfare defined as discounted utility (Asheim, 2000, 2007; Asheim and Weitzman, 2001).[18] The main objective of this literature is to determine how to account for changes in society's productive capacities, and how to interpret them. It emphasizes that wealth accounting is a proper way to account for both welfare improvement and sustainability improvement (Asheim, 2000; Dasgupta and Mäler, 2000). There are some connections between sustainable income (i.e., the maximin value) and NNP (Asheim, 2000). A necessary condition for the maximin value to be non-decreasing along an optimal discounted utility path is that green NNP increases (Onuma, 1999). But it is clear from the literature that a positive growth of NNP in the discounted utility framework does not indicate sustainability, either in the short or in the long run (Asheim, 1994, 2000; Pezzey, 1994). Asheim (2003) provides an interesting synthesis of the way different assumptions can

[18] Papers departing from the optimality or efficiency assumptions (e.g., Dasgupta and Mäler, 2000; Arrow *et al.*, 2003) use discounted utility as a welfare function. Papers departing from discounting utility assume efficiency, and relate their results to optimality with respect to the maximin criterion or undiscounted utility (Asheim and Buchholz, 2004; Asheim, 2007). No paper departs from both assumptions.

be combined to obtain different results. No results are obtained without assuming optimality or discounted utility as a measure of welfare, either regarding welfare accounting or sustainability accounting (see Asheim, 2003, tables 1 and 2). If one has no legitimate definition of welfare to work with, and if efficiency is dropped too, one is left with no tool to assess a current situation's sustainability.[19] We argue that, in this case, accounting for net maximin investment is of interest. Such an accounting does not measure welfare improvement, but sustainability improvement. This is consistent with the idea that 'sustainability is about making it possible for future generations to achieve some outcome, [...] sustainability defined in this way can be ascertained without making a precise prediction about the future generations' decisions, since only their possibility set matters' (Fleurbaey, 2015, p. 37). Sustainability improvement informs current generations about the 'room' left for growth to future generations, without making any assumption on what welfare is, or what future generations' decisions may be.

The second reason is that, when a growth pattern that is not sustainable in the long run is followed, at some point in time the maximin value is reached, imposing a necessary change of *ram* for sustainability. Along an optimal discounted utilitarian growth path, instantaneous utility increases if and only if the present value of genuine savings are decreasing (Hamilton and Withagen, 2007),[20] meaning that optimal growth comes with a decreasing net investment. Along the opsustimal paths defined by Pezzey (1994), the economy must follow the maximin path as soon as the growing consumption reaches the (possibly decreasing) maximin value. In their study of discounted utilitarian paths with a sustainability constraint, Asheim and Buchholz (2004, p. 378) show that 'real NNP growth approaching zero indicates that unconstrained development is no longer sustainable.' It is thus important to assess when net investment becomes inconsistent with sustainability, whether the development path is optimal or not, efficient or not. Unfortunately, genuine savings computed at discounted-utility prices are of no use. Pezzey (1994) emphasized that, along the discounted utilitarian path in the DHS model, there is a time after the overshooting of the maximin value and before the peak of the discounted-utility optimal path when aggregate wealth (accounted for at efficient prices) rises in spite of the unsustainable consumption, 'because wealth is being measured at 'unsustainable' prices' (p. 102). Computing genuine savings with discounted utility prices can yield 'false positive value.' Pezzey (1994) concludes in chapter 3 that theoretical research should analyze 'whether anything useful can be said about the 'sustainability prices' which would make a change in aggregate wealth an accurate measure of sustainability' (p. 143). Studying the evolution of the maximin value offers such a measure. Knowing how current utility gets close to it informs on the timing of a necessary change in *ram*. Examining how current *ram* differs from the maximin decisions also informs on how important the changes have to be to implement sustainability.

In this paper, we proposed a discussion of the links between current consumption sacrifice (with respect to the maximin value) and long-run sustainability improvement,

[19] In a single capital stock model *à la* Ramsey, if there is no technological change and future generations have the same technology as the current generation but differ in terms of preferences (and possibly preferences regarding growth), consumption-NNP may be used to compare the consumption opportunities offered by the capital stock transmitted over time (Asheim, 2011). Such a metric is, however, not available in economies with several capital stocks (see also Asheim, 1994).

[20] This result is due to conservation laws along optimal growth paths (Martinet and Rotillon, 2007), and more specifically to the conservation of the current value Hamiltonian, which is equal to the sum of instantaneous utility and net investment.

without optimality or efficiency assumptions. Even if our analysis focuses mainly on illustrative cases, the issues we raise are general. First, sustainable growth requires current utility to be lower than the maximin level *and* investment to be such that the maximin value is increasing so that current sacrifice makes room for growth. Second, there may be a limit to the increase of the maximin value, and hence of the utility reached in the long run, that is endogenous to the chosen growth pattern, and so to the current *ram*.

Our DHS example relies on a particular *ram* combining a given consumption growth pattern with the instantaneous maximization of sustainability improvement. In a sense, the investment strategy is such that the current generation deviates from the maximin path to improve sustainability, but without taking into account that the future generations may do the same. The *ram* maximizes an instantaneous criterion and not an intertemporal one. The development path it generates is surely not efficient. Studying the trade-off between current sacrifice and long-run utility could be done by maximizing the sustainable level $m(X(T))$ reached in the long run. Such an intertemporal optimization will likely yield investment strategies that differ from the maximization of sustainability improvement, and ensure intertemporal efficiency. It, however, requires assuming that the resulting path will be followed by future generations, given the risk that computing again the optimal path as time passes generates time-inconsistency. Our myopic approach eschews making such an assumption and considers only the potential room for growth bequeathed to future generations by the current one. This may be considered as a benefit, because it does not rely on assumptions on what future generations will do, nor bind them to a given intertemporal trajectory. The question of the time-consistency of long-run strategies to improve sustainability is, however, of interest.

An interesting question is to study how the recursive choice of successive generations for sustainable growth may shape the development path. In a discrete time DHS model, Asheim (1988) shows that combining maximin with nonpaternalistic altruistic preferences results in a time-inconsistent path. He defines a 'just programme' along which consumption grows as long as savings are 'permanently utility-productive' (i.e., 'savings increase [current generation] altruistic utility in spite of its children sharing the return with later generations') and follows an egalitarian path (maximin path) as soon as it is no longer the case. The path is not Pareto-efficient but is a subgame-perfect equilibrium, and is thus time-consistent. Fleurbaey (2015) discusses different ways to incorporate sustainability indicators within a welfare function, including an approach that evaluates the future only in terms of the ability of future generations to sustain utility. In a discrete time framework, the intergenerational welfare is then $W^d(U(X_0, c_0), m(X(1)))$, where $X(1)$ is the capital stock inherited by the next generation. This criterion depends on present utility and future maximin value. The closest criterion in a continuous time framework would combine current utility with the current change in the maximin value, i.e., $W^c(U(X_0, c_0), M(X_0, c_0))$. As emphasized by Fleurbaey (2015, p. 44) and by Geir Asheim (private discussions), if all generations act to maximize such a welfare measure, the resulting development path is likely to be inefficient. Here again, there seems to be a trade-off between efficiency and time-consistency.

Another possible extension is to consider risk. Gerlagh (2017) examined the effect of current decisions on the highest maximin value that could be reached. He defines 'generous sustainability' as the requirement that the growth pattern reduces neither the instantaneous maximin utility nor the attainable maximin utility (i.e., the golden rule). The simple models we used to illustrate our point do not include risk or irreversibility, except if one considers the complete exhaustion of one of the capital stocks. As such, the attainable utility is not modified by the particular *ram* we studied. In a model in which current *ram* modifies the highest maximin value, the recursive dimension mentioned above is even more important, because current decisions can jeopardize future options.

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Appendix A:

A.1. *Mathematical details for the DHS model*

Sustainability improvement maximizing extraction rule. By differentiating the maximin value function equation (16) logarithmically with respect to time, we express the rate of growth of the maximin value as

$$\frac{\dot{m}}{m} = \left[\frac{\alpha - \beta}{1-\beta}\frac{\dot{K}}{K} + \frac{\beta}{1-\beta}\frac{\dot{S}}{S}\right] = \left[\frac{\alpha - \beta}{1-\beta}\frac{(K^{\alpha}r^{\beta} - c)}{K} - \frac{\beta}{1-\beta}\frac{r}{S}\right].$$

Taking the derivative of this expression with respect to r and equalizing to zero gives us the extraction rule $\hat{r}(K, S)$ that maximizes the rate of growth of the maximin value (whatever is the consumption):

$$\hat{r}(K, S) = (\alpha - \beta)^{(1/1-\beta)} S^{(1/1-\beta)} K^{-(1-\alpha/1-\beta)} .$$

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