Forum

Interference to GPS

Walter Blanchard

I read Captain Gyldeń's account of failure of his GPS set (Vol. **50**, 328, May 1997) with some interest. A few days earlier we had had a few thunderstorms while I had my personal hand-held GPS running, using its own built-in antenna and a 12 volt battery. It was lying on a bench inside a wooden hut where I keep some of my amateur radio equipment. After one particularly close flash and bang, which produced a one-inch spark from my transceiver antenna, the transceiver locked up and stopped responding to keyboard commands. Then I noticed the GPS set had also stopped working, showing only random symbols on its readout. I feared the worst but, after switching them both off, leaving them for a few minutes, and then back on again, they worked perfectly.

Microprocessors locking up in strong local electrostatic fields, perhaps? Maybe if I had simply left them alone they would have started working again after the charge had leaked away. Next time I nearly get hit by lightning I'll try it.

Note from Editor

As indicated after Captain Gyldeń's paper in May 1997, reports of failures of GPS receivers, for whatever reason, would be welcomed by the Institute.

'Fast Craft and the COLREGS'

Hanno Weber *writes*

The May 1997 issue of the *Journal* contains an essay by R. D. Pike[1] on fast craft. I very much agree with his opinion on factual matters. However, I do not agree with the author's legal opinion on the existing COLREGS and I would like to suggest for consideration a much simpler solution in the future COLREGS.

Nowadays, fast vessels do take early actions. These early manoeuvres are never covered by Rule 17(b) and only sometimes by Rule 17(a)(ii). In the majority of encounters the fast craft does not wait until she becomes a stand-on vessel; she acts under Rule 2(a) at a long distance; that is, before there is 'risk of collision' and a consequent duty to give way/stand on.

Action at such an early stage is specifically required already today in all cases where a vessel is obliged NOT TO IMPEDE the passage or safe passage of another vessel. It

seems proper to classify fast craft in the same way so that they are required not to impede the safe passage of other vessels. There should be no need for special lights and shapes, and local needs could be covered by local rules.

REFERENCE

[1] Pike, R. D. (1977). Fast craft and the COLREGS. This *Journal*, **50**, 256.

KEY WORDS

1. Collision regulations.   2. Fast craft.

'Future of Radar Enhancers'

Richard M. Trim *writes*

It was with the greatest interest that I read the two important papers by Dr N. Ward, published in the May edition of the *Journal of Navigation*.[1,2]

In his paper on the 'Future of Radar Target Enhancers',[1] Dr Ward draws attention to the characteristic of a radar target enhancer (RTE) in that it has a maximum microwave output power (saturated output) which, as a result of radar signal processing, can result in a diminishing displayed RTE response as range decreases, possibly resulting in the suppression of the RTE response at close range.

A relative advantage of the passive radar reflector over the RTE is that it does not have such a saturation characteristic so that it is less susceptible to suppression due to radar signal processing.

As I suggested in my paper 'Radar Transponders and Radar Echo Enhancers',[3] published in the *Journal of Navigation* in September 1995, one approach could be to combine an RTE with a passive radar reflector. In this way, the benefits of RTE performance at medium and longer ranges would be combined with the absence of saturation effects in the case of the passive reflector.

A further advantage of the use of a passive reflector could be that it might give a useful echo enhancement at S-Band, especially at close range which, as Dr Ward points out, is not provided by an X-Band-only RTE.

A potential disadvantage of a passive radar reflector of useful performance, especially in the context of sailing yachts, is its size, weight and windage and the risk of it snagging rigging.

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[2] Ward, N. (1997). The future of radar target enhancers. This *Journal*, **50**, 248.
[3] Trim, R. M. (1995). Radar transponders and radar echo enhancers. This *Journal*, **48**, 396.

KEY WORDS

1. Radar.   2. Radar reflectors.   3. Radar target enhancers.

'Traditional Aids to Navigation: The Next 25 Years.'

C. Colchester

In the May 1997 issue of the *Journal*, a paper on this subject[1] was presented by Captain Turner on behalf of the General Lighthouse Authorities. I read the paper with interest, and in general concur with their assessment of the situation. However, I think that the situation regarding visual Aids to Navigation (AtoN) needs further consideration.

In paragraph 3 – 'Assumptions regarding AtoN provision' – it is stated that 'visual AtoN will continue to be required for the next 25 years and probably beyond for confirmation of position and for position fixing for some ships'. It seems to me that although this is probably correct, the requirement for visual AtoN, particularly fixed beacons and larger lights, mainly concerns the need to keep a visual lookout.

During the next decade, it seems fairly certain that the main, and quite likely secondary means, of navigating all ships, large and small, will be some form of radionavigation, very likely involving some form of electronic chart. At the same time, we may expect radar to develop and be carried on all ships, apart from yachts, and that this will be the main means of collision avoidance. However, it has still never been suggested that these navigation aids will substitute for the need to keep a visual lookout and, even in the future, it is envisaged that the bridge watchkeeper will spend most of his time on this activity. This means that, in spite of back-up systems and alarms (cf: for example, the grounding of the *Royal Majesty*), the first indication of trouble in the navigation system could very likely be the sighting of a visual AtoN. In other words, it will really only be possible to dispense with visual AtoN when there is no longer any need for visual watchkeeping.

My second point is that, if future AtoNs are no longer mainly required as 'bearing references', but rather as 'hazard warnings', then it would be necessary to re-examine the specifications, specifically the flash lengths of the lights. IALA makes no recommendations on this subject, but the US Coast Guard maintains that the human eye can perceive a flash length as short as 0·01 seconds, although it is not possible to take an accurate bearing unless the flash is repeated in a group.

If flash lengths could be made this short, possibly by using flash-tube equipment or the proposed laser, then the energy requirement, even on a 50,000 candela light, could theoretically be cut to a level equivalent to a buoy light. This should lead to much reduced maintenance and infrastructure costs, and incidentally, make the saving to be achieved by cancellation of the light, insignificant.

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[1] Turner, N. M. (1997). Traditional aids to navigation: the next 25 years. This *Journal*, **50**, 234.

KEY WORDS

1. Marine navigation.   2. Aids to navigation.   3. Lights

Middle Latitude Sailing Revisited

Roy Williams

1. INTRODUCTION. Many problems in navigation can be best viewed and solved as problems in analytical geometry. We only need to understand the geometry of two 'navigable' surfaces; the sphere and the ellipsoid of revolution. The ellipsoidal model is generated by revolving an ellipse about its minor axis and this model is used as a global model for the surface of the Earth. The eccentricity of the meridian ellipse is small (≈ 0.082) so we sometimes refer to this surface as a 'spheroid' since the surface is still 'sphere-like'. The physical Earth is, in fact, referred to as a 'geoid' whose surface is that which approximates global mean sea level. The mathematical representation of the geoid is not trivial and the ellipsoid of revolution is an extremely good approximation to it.

The first (and, in the past, most frequently used) approximation to the shape of the Earth in navigation is a sphere. This is not such a bad approximation if used consistently but bad practices crept in, among which was the habit of using methods of computation which contained elements from the spherical and ellipsoidal models in the same formula. For example, the following mistake was made in two different publications of nautical tables; a correction to apply to the mean latitude in order to obtain the 'middle latitude' for an observer travelling along the arc of a rhumb line was computed from a formula which determined the cosine of the middle latitude by the ratio of difference of latitude and difference of meridional parts. As is well known, the difference of latitude is the number of minutes of arc of the meridian on the surface of a sphere but the difference of meridional parts used to compute the cosine of the middle latitude was taken from ellipsoidal data. If the meridian distance along the arc of the meridian of the ellipsoid had been used instead of the difference of latitude, then the formula would have been correct. As it was, the table made no sense, yet it was published through several editions of the nautical tables and the error went apparently unnoticed. It seems that the theory behind the method of computation known as *Middle Latitude Sailing* was never widely understood by navigators.

Although the method is now mainly historical, it is the purpose here to give a rigorous derivation of the theory behind the method known as 'Middle Latitude Sailing'. This is a method of computation which is used by an observer travelling along the arc of a rhumb line mainly when the angle between the meridian and the rhumb line is large. W. M. Smart[1] presented a complete analysis of Middle Latitude Sailing for both the sphere and the ellipsoid. We will use a different approach to Smart and make one or two additional comments. In both cases the theory relies upon the computation of meridional parts provided by the method of Mercator Sailing so that the method of Middle Latitude Sailing does not exist independently. We will set the Earth in a spherical coordinate system whose origin coincides with the centre of the Earth but we use a system which is not quite standard for these coordinates; r is, as usual, the distance of a point from the origin, but θ is the longitude and ϕ is the geocentric latitude. Since arc length and geodetic (astronomical) latitude are intrinsic properties of the surface we use the standard notation for intrinsic coordinates and denote the arc length by s and the geodetic latitude by ψ.

For comparatively small gains in accuracy, the problem of computing course and distance or final position for an observer travelling along the arc of a rhumb line on the
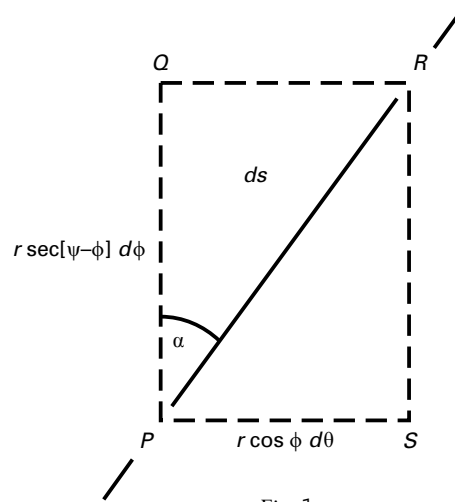
Fig. 1

surface of an ellipsoid requires some fairly high-powered numerical methods to aid its solution, but it does stimulate a lot of theoretical interest and the best way to find a numerical solution still promotes discussion in the pages of the *Journal of Navigation*. Williams[2] had a paper published in 1950 concerning the computation of distance along the arc of a rhumb line and, since then, Sadler[3] and others have written on the same theme. The latest contributions, which are well worth reading, are by Carlton Wippern[4] and Bennett.[5]

2. RHUMB LINES. A rhumb line is a curve on a surface of revolution which cuts all the meridians at the same angle α; $0 < \alpha < \frac{1}{2}\pi$. 'Rhumb' comes from the Greek ῥυμβος meaning spiralling. The curve is also known as a loxodrome which is also derived from greek words λοξος (oblique) δρομος (course). On the surface of a sphere or an ellipsoid of revolution a rhumb line is a curve of finite length which spirals endlessly to limit points at the poles. When $\alpha = \frac{1}{2}\pi$ then the curve is a parallel of latitude and when $\alpha = 0$ the curve is a meridian.

3. UNITS OF MEASUREMENT. The coordinate angles ϕ and θ and the geodetic angle ψ are measured in radians and, over the surface of the ellipsoid, lie in the ranges.

$$-\tfrac{1}{2}\pi \leqslant \phi, \psi \leqslant \tfrac{1}{2}\pi \quad \text{(North positive)}$$
$$0 \leqslant \theta < 2\pi \qquad \text{(East positive)}.$$

However, for an observer sailing along the arc of a rhumb line from a point $P_0(\phi_0, \theta_0)$ to the point $P_n(\phi_n, \theta_n)$ we must admit the circumstances where the difference of longitude $|\theta_n - \theta_0| > 2\pi$ since the rhumb line spirals endlessly.

All measurements of distance or length of arc given by the formulae which follow are expressed in the units of the length of one minute of arc of the equator which is the geographical mile. The length of the geographical mile will, of course, vary according to the dimensions of the ellipsoid which is used to approximate the shape of the Earth. Since all angles are expressed in radians the term $a(\theta_n - \theta_0)$ which represents the difference of longitude is then in minutes of arc a being the equatorial radius.

4. DEPARTURE AND MIDDLE LATITUDE ON THE SURFACE OF THE ELLIPSOID. Let us consider an observer on the surface of the terrestrial ellipsoid

travelling from a point P_0 to a point P_n along the arc of a rhumb line on course α. At a point P along the track let the differential element of the rhumb line be ds (see Fig. 1). $PQ(= r \sec [\psi - \phi] \, d\phi)$ lies along the tangent to the meridian through P and is the differential element of the meridian distance. PS ($= r \cos \phi \, d\theta$) lies along the tangent to the parallel of latitude through P and is the differential element of departure.

If we denote the departure by λ then the departure made good by an observer travelling along the rhumb line track from the point P_0 where the geocentric latitude is ϕ_0, the geodetic (astronomical) latitude is ψ_0 and the longitude is θ_0 to the point P_n where the geocentric latitude is ϕ_n, the geodetic latitude is ψ_n and the longitude is θ_n is given by

$$\lambda = \int_{\theta_0}^{\theta_n} r \cos \phi \, d\theta. \tag{1}$$

On the surface of the ellipsoid r is a function of ϕ as given by

$$r[\phi] = a \quad \left(\frac{1 - e^2}{1 - e^2 \cos^2 \phi} \right)$$

where a is the equatorial radius of the ellipsoid. ϕ can be expressed in terms of ψ by

$$\tan \phi = (1 - e^2) \tan \psi \tag{2}$$

and ψ is a function of θ given in implicit form by

$$\rho[\psi] \cot [\tfrac{1}{4}\pi + \tfrac{1}{2}\psi] = \rho[\psi_0] \cot [\tfrac{1}{4}\pi_0 + \tfrac{1}{2}\psi_0] e^{(\cot \alpha)(\theta - \theta_0)} \tag{3a}$$

if the observer is in the 'navigable' latitudes, or

$$\rho[\psi] \tan [\tfrac{1}{4}\pi - \tfrac{1}{2}\psi] = \rho[\psi_0] \tan [\tfrac{1}{4}\pi - \tfrac{1}{2}\psi_0] e^{-(\theta - \theta_0)(\cot \alpha)} \tag{3b}$$

if the observer is in high latitudes and where

$$\rho[\psi] = a \left(\frac{1 + e \sin \psi}{1 - e \sin \psi} \right)^{\frac{1}{2}e}.$$

Equations (1) and (2) are found from considerations of the geometry of the ellipse, equation (3a) is the image of the rhumb line in the Mercator projection and equation (3b) is the equation of the equiangular spiral which is the image of the rhumb line in the stereographic plane.

To express the integrand $r \cos \phi$ in equation (1) as a function of θ is therefore possible, but not very practical. We can find an exact expression for the departure without doing so. Equation (2) defines a mapping between the closed interval $[\psi_0, \psi_n]$ of the geodetic latitude and the closed interval $[\phi_0, \phi_n]$ of the geocentric latitude which is both one-to-one and onto. Similarly, each of the equations (3a) and (3b) defines a mapping between the closed interval $[\theta_0, \theta_n]$ of the longitude (where we must admit the possibility that $|\theta_n - \theta_0| > 2\pi$), and the closed interval $[\psi_0, \psi_n]$ of the geodetic latitude. These mappings are both one-to-one and onto, and so are their inverses. The compound mapping between the closed interval $[\theta_0, \theta_n]$ of the longitude and the closed interval $[\phi_0, \phi_n]$ of the geocentric latitude defined by the successive applications of equations (2) and either of equations (3a) or (3b) are therefore also one-to-one and onto.

These properties of the mappings show that we can therefore apply the mean value

theorem for integrals to equation (1) whereby we find a value χ of ϕ corresponding to a value θ_χ of θ where $\theta_0 \leqslant \theta_\chi \leqslant \theta_n$ and where $\phi_0 \leqslant \chi \leqslant \phi_n$ such that

$$\lambda = \frac{r[\chi]}{a} \cos \chi \left(a \int_{\theta_0}^{\theta_n} d\theta \right). \tag{4}$$

Since $0 \leqslant (r[\phi]/a) \cos \phi \leqslant 1$ and the expression $(r[\phi]/a) \cos \phi$ is monotonically decreasing in the closed interval $[0, \frac{1}{2}\pi]$ then the mapping $\phi \to \cos^{-1}(r[\phi]/a) \cos \phi$ is both one-to-one and onto and maps the interval $[0, \frac{1}{2}\pi]$ onto itself. We can therefore find an angle ξ such that

$$\cos \xi = \frac{r[\chi]}{a} \cos \chi. \tag{5}$$

The angle ξ is known as the *Middle Latitude* and equation (4) may be written

$$\lambda = a(\theta_n - \theta_0) \cos \xi$$

where θ is measured in radians and which is, for the navigator, the familiar formula

$$\text{Departure} = \text{DLong} \times \cos(\text{mid lat}).$$

We can also use the same argument for the latitude interval $[-\frac{1}{2}\pi, 0]$ except that the expression $(r[\phi]a) \cos \phi$ is then monotonically increasing so that, over the whole range $[-\frac{1}{2}\pi, \frac{1}{2}\pi]$ of latitude, we can find angle ξ whose sign is the same as χ and which satisfies equation (5). To calculate the value of the middle latitude and departure we recall the 'sailing triangles' shown here in Fig. 2 a, b. In the figure P_0 and P_n are points on the surface of the ellipsoid and the points P_0^* and P_n^* are their images, respectively, in the Mercator projection.

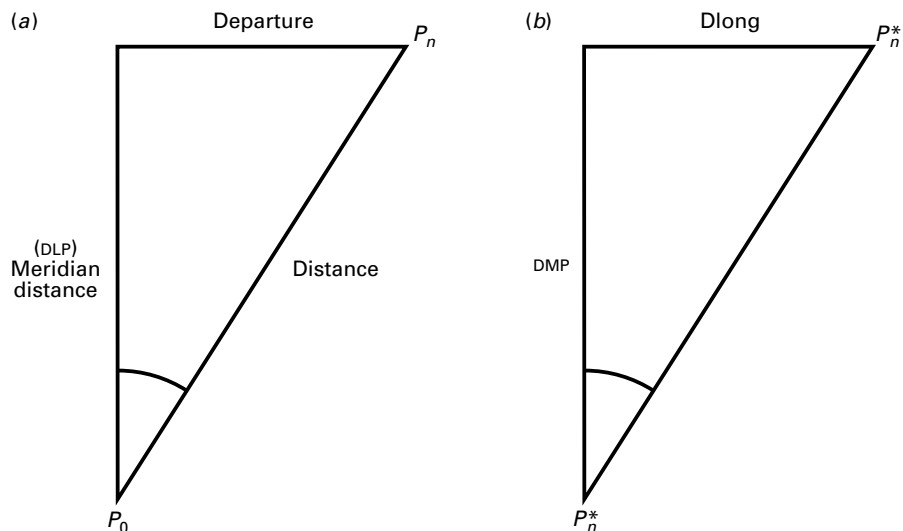
Fig. 2

From the triangles in Fig. 2 a, b (since the triangles are similar) we see that

$$\text{Departure} = \text{DLong}\left[\frac{\text{DLP}}{\text{DMP}}\right]$$

so that

$$\cos(\text{midlat}) = \frac{\text{DLP}}{\text{DMP}} \qquad (6)$$

and this is the most practical method of determining departure and middle latitude.

We use the term Difference of Latitude Parts (DLP) to mean the same thing as Meridian Distance. The term 'Latitude Parts' was coined to use alongside 'Meridional Parts' on the surface of the ellipsoid (Williams[6]). The number of Latitude Parts at a point P on a meridian at a given geodetic latitude, ψ, is the length of the meridian from the equator to P in geographical miles.

The ratio DLP/DMP is stable and, for navigational purposes, can be calculated accurately using only the arithmetic available on a hand calculator when the difference of latitude is as little as one minute of arc except in the small region of a few minutes of arc either side of the equator where we may assume anyway that the value of the limit of the ratio is unity.

5. SAILING ALONG A PARALLEL OF LATITUDE. As the difference of latitude, $\Delta\phi$, approaches zero and the geodetic latitude approaches the constant value ψ_0 we find that the cosine of the middle latitude also approaches a limit which is given by

$$\lim_{\Delta\phi \to 0}\left(\frac{\text{DLP}}{\text{DMP}}\right) = \frac{\cos\psi_0}{\sqrt{[1 - e^2 \sin^2\psi_0]}}.$$

For an observer travelling along the parallel to latitude $\psi = \psi_0$ from the point where the longitude is θ_0 to the point where the longitude is θ_n the departure is the distance travelled and this is

$$\text{distance} = \frac{a[\theta_n - \theta_0]\cos\psi_0}{\sqrt{[1 - e^2 \sin^2\psi_0]}}.$$

6. CONCLUSION. A correct table showing the correction (in minutes of arc) to apply to mean latitude in order to find the middle latitude on the surface of the WGS84 ellipsoid is shown in the appendix. This table was calculated using 10 digit arithmetic on a modern computer and only rounded on completion. The entries only differ in a few cases with the entries in the similar table published in *Inman's Nautical Tables*[7] which has always been considered correct but, considering the year that these tables were published, we believe we have the advantage with our modern computing aid. The table applies to the Northern hemisphere, where positive corrections are North and negative corrections are South. When applied in the Southern hemisphere then the situation is reversed. There would, however, be no need to keep such a table since the computation will find the cosine of the middle latitude directly from equation (6). Entries in the first line of the table where the mean latitude is 0°, such as the entry for DLAT 20° where the correction is given as 346 minutes of arc, may seem unreal and indicate an apparent lack of symmetry. The mathematics, however, has taken care of the fact that, in truth, the path along the rhumb line from 10° S to 10° N should be considered in two separate parts; from 10° S to 0° and from 0° to 10° N. The mean latitudes for the separate pieces would then be 5° S and 5° N, respectively. The correction to the mean latitude of 5°

to find middle latitude would then be 46′ in each hemisphere and this makes the middle latitude into 5°46′ (346′) which is the correction to apply to the mean latitude of 0° and gives the same middle latitude in either case.

APPENDIX. CORRECTION IN MINUTES TO MEAN LATITUDE TO FIND MIDDLE LATITUDE

Mean (°)	Difference of latitude									
	2°	4°	6°	8°	10°	12°	14°	16°	18°	20°
0	35	70	104	139	173	208	242	277	312	346
1	9	32	60	91	123	156	190	223	257	292
2	4	18	39	63	91	120	151	182	214	247
3	3	12	27	47	70	95	122	151	180	211
4	2	9	21	37	56	78	101	127	154	182
5	1	7	17	30	46	65	86	109	134	159
6	0	6	14	25	39	55	75	95	117	141
7	0	4	12	21	34	48	65	84	105	126
8	0	4	10	19	30	43	58	75	94	114
9	−1	3	9	16	26	38	52	68	85	104
10	−1	2	7	15	24	35	48	62	78	95
11	−1	2	6	13	22	32	44	57	72	88
12	−1	1	6	12	20	29	40	53	67	82
13	−2	1	5	11	18	27	37	49	63	77
14	−2	0	4	10	17	25	35	46	59	73
16	−2	0	3	8	15	22	31	41	53	66
18	−3	−1	2	7	13	20	28	38	48	60
20	−3	−1	2	6	11	18	26	35	45	56
24	−4	−2	1	4	9	15	22	31	41	51
28	−4	−3	0	3	8	14	20	28	37	47
36	−5	−3	−1	2	7	12	19	26	35	44
42	−5	−4	−1	2	7	13	19	27	36	46
48	−5	−4	−1	3	8	14	21	30	39	50
52	−5	−3	0	4	9	16	23	32	42	54
56	−5	−3	0	5	11	18	26	36	47	60
58	−5	−3	1	6	12	19	28	38	50	64
60	−4	−2	1	6	13	21	30	41	54	68

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KEY WORDS

1. Rhumb lines.　　　2. Loxodromes.　　　3. Geodesy.