

## ARTICLES

# SEASONALITY AND MONETARY POLICY

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Seasonal fluctuations are as large as cyclical fluctuations. Monetary policy in the United States has dealt with seasonality by smoothing nominal rates of interest. The original motivation for this was that seasonality in nominal interest rates put recurring strain on the banking system. We build a model of monetary policy in the presence of seasonality that puts financial market conditions in the foreground. Our findings are as follows. Insulating nominal rates of interest or rates of inflation from seasonal variation are the basis for a formula for generating indeterminacy of equilibria and excess volatility. Preventing seasonality in the rate of monetary growth does not suffer from this problem. Moreover, under any method for conducting monetary policy, the setting of the target value of the monetary instrument will affect the degree of seasonal variability in most endogenous variables. This is a new channel by which monetary policy can have real effects. The case for eliminating seasonality in nominal rates of interest is strongest when seasonal impulses derive from shifts in money demand. It is weakest when seasonal impulses derive from the real sector.

**Keywords:** Seasonal Fluctuations, Monetary Policy, Nominal Interest Rates, Inflation Rates

In a system in which the monetary authorities effectively control the money stock, they must decide explicitly how much seasonal change to introduce. . . . Should they determine the seasonal change so as to eliminate any seasonal movement in interest rates? . . . Or should they determine the seasonal change to introduce into money by an observed seasonal movement in velocity?

Friedman and Schwartz (1963, p. 295)

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## 1. INTRODUCTION

Prior to the advent of the Federal Reserve System, the U.S. economy was beset by recurring financial crises. As noted by Miron (1986) and Champ et al. (1996), bank panics were closely associated with seasonal financial stringencies. Indeed, bank panics tended to occur roughly at the same time as crops were harvested and moved to market. Crop moving was itself associated with a simultaneous high level of currency demand to make payments,<sup>1</sup> and a high level of credit demand because workers had to be paid at harvest and crops had to be purchased and shipped before they could be sold and revenue could be realized. As a result, there were recurring seasonal “strains” on the banking system. In the views of Andrew (1908), Sprague (1910), and Goodhart (1969), when these strains occurred, it took only small additional pressure on the banking system to create a crisis.

Accompanying these seasonal strains were large seasonal movements in nominal rates of interest. Contemporaries, such as Laurence Laughlin (1912), took the view that banking panics were but one—and perhaps a relatively minor—manifestation of the costs of large seasonal variation in nominal interest rates.

The Federal Reserve System was empowered to create an “elastic currency” that would eliminate recurrent seasonal tightness in financial markets. And, “the system was almost entirely successful in the stated objective of eliminating seasonal strain” Friedman and Schwartz (1963, p. 293). Nonetheless, the question posed by Friedman and Schwartz remains as pertinent today as it was when they wrote it, or as it was in 1914. Does it represent “good” monetary policy for any central bank to eliminate seasonal variability in nominal rates of interest? A central bank could, for example, be equally concerned about seasonal movements in the price level or the rate of inflation. Or, it could follow conventional monetarist prescriptions that it should maintain a stable rate of growth in some monetary aggregate, so that the rate of growth in the appropriate defined money stock would not vary seasonally. Would any of these alternatives constitute a better means of conducting monetary policy?

The appropriate seasonal conduct of monetary policy continues to be of great importance. Barsky and Miron (1989, p. 509) found that “deterministic seasonal fluctuations account for more than 85% of the fluctuations in the rate of growth of real output and more than 55% of the (percentage) deviations from trend.” In addition, although there is no seasonality in nominal rates of interest, “seasonal dummies account for approximately 50% of the variation in the log growth rate of money” (1989, p. 513). Interestingly, the policy of insulating nominal interest rates from seasonal variability also seems largely to insulate the rate of inflation from seasonal variability. Barsky and Miron (p. 509) found that “seasonal dummies explain only 3.0% of total variation” in the growth rate of the price level.

There is also reason to believe that the magnitude of seasonal fluctuations and the magnitude of cyclical fluctuations are related. Beaulieu et al. (1992) found a strong positive correlation across countries and industries between the standard

deviation of the seasonal component and the standard deviation of the cyclical component of aggregate variables. In the same spirit, Wen (2001) found that seasonal shocks explain 50% of the cyclical fluctuations in aggregate output. Thus, the economic propagation mechanism transmitting seasonal fluctuations from exogenous to endogenous variables may be systematically related to that transmitting business cycles.

To summarize, seasonal fluctuations have been, and continue to be, of major economic importance. They are, arguably, more important than business-cycle fluctuations. Yet, although the literature on the appropriate response of monetary policy to business-cycle fluctuations is enormous, there is, relatively speaking, little modern literature on the appropriate treatment of seasonality by a monetary authority.<sup>2</sup>

We propose to conduct an analysis of the appropriate seasonal stance of monetary policy. In contrast to much of the existing literature on seasonality—and in keeping with the views of Andrew (1908), Sprague (1910), Friedman and Schwartz (1963) and Goodhart (1969)—we will proceed on the basis of the notion that seasonal fluctuations exert much of their effect by affecting conditions in money and credit markets. Thus, we base our analysis on a model that brings banking and “credit market” conditions to the forefront.

More specifically, we consider a model in which spatial separation and limited communication create a transactions role for currency, and in which randomness associated with agents’ patterns of movement creates a role for banks to insure agents against idiosyncratic liquidity needs. We also introduce deterministic seasonal fluctuations. On the basis of the notion that the appropriate seasonal conduct of monetary policy could, at least in principle, depend on the source of seasonal impulses, we consider three possible sources of seasonality. One is seasonal variability in income (here, endowments). A second is seasonal variability in real rates of return. Such variability could arise, for example, as a result of seasonal fluctuations in credit demand [Sargent and Wallace (1982)]. Here it arises from seasonal fluctuations in a technological parameter that affects real rates of interest. And the third source of seasonality arises from fluctuations in a parameter that governs the demand for money.

In this setting, we consider four alternative methods of conducting monetary policy. Three of these are quite simple: we analyze policies that prevent seasonal fluctuations from being manifested in either (a) the rate of money creation, (b) the nominal interest rate, or (c) the rate of inflation. We then go on to consider a further policy in which the central bank targets an optimal seasonal time path for the nominal rate of interest. In the context of this last policy, we can analyze the optimal degree of seasonal variability in nominal rates of interest.

We analyze these policies along several dimensions. One is their implications for welfare. However, we also consider the implications of different methods of conducting monetary policy for the determinacy of equilibria, and for the potential for endogenously generated volatility to arise. Here, our thinking is guided by Friedman (1960, p. 23), who argued that “the central problem [of monetary policy]

is not to construct a highly sensitive instrument that can continuously offset instability introduced by other factors, but rather to prevent monetary arrangements from themselves becoming a primary source of instability.”

The results we obtain are as follows. First, preventing seasonal fluctuations from being manifested in nominal interest rates and/or the rate of inflation is a policy that can easily—and under empirically plausible assumptions about parameter values *will*—lead to an indeterminacy of perfect-foresight equilibria. Thus, a policy of insulating nominal rates of interest or the rate of inflation from seasonal fluctuations can easily create additional, market-generated, sources of variability over and above the exogenous sources of seasonal variation in an economy. The same problem does not arise under a policy that maintains a constant rate of money growth.

Second, in a steady-state or purely periodic equilibrium, we show that whether or not it is a “good idea” to eliminate seasonality in the nominal rate of interest depends heavily on the source of seasonal impulses. If seasonal fluctuations derive primarily from changes in the demand schedule for money, then smoothing nominal rates of interest (and the rate of inflation) seasonally is a relatively “good” policy from a welfare perspective. However, if seasonality arises from exogenous fluctuations in income (endowments), eliminating seasonality in the nominal rate of interest is a relatively “bad” policy. An optimal policy allows nominal rates of interest to display larger seasonal movements than the movements in income. Somewhere in between lie seasonal fluctuations that affect real rates of return. Here, the optimal policy allows the nominal rate of interest to vary seasonally. However, seasonal variations in it should be fairly small relative to seasonal movements in the rate of interest. Interestingly, the flavor of these results has much in common with Poole’s (1970) results regarding when it is and is not desirable to prevent the nominal rate of interest from responding to either random or deterministic exogenous fluctuations.

Furthermore, we show that in an economy with seasonal variation of a magnitude similar to that observed in the United States, the Friedman rule (a zero nominal rate of interest) need not be optimal. In particular, it may allow for too little (i.e., no) seasonal variation in nominal rates of interest.

Finally, within any given monetary regime, the setting of the appropriate target variable for monetary policy will affect the magnitude of seasonal fluctuations in most endogenous variables. Thus, for example, if the central bank targets the rate of growth of the money supply and insulates it from any seasonal variation, we are able to show that higher rates of money growth will induce greater seasonal variability in real balances, investment, and the rate of inflation. This finding, for which we provide some empirical support, has an important policy implication. Since seasonal fluctuations are large relative to business-cycle fluctuations, and since monetary policy can affect the magnitude of seasonal fluctuations in endogenous variables, an important channel by which monetary policy can affect the real economy has largely been overlooked.

The remainder of the paper proceeds as follows. Section 2 describes the economic environment and the behavior of agents. Sections 3 and 4 discuss a general

equilibrium under different methods of conducting monetary policy. Section 5 explores the welfare consequences of smoothing the nominal rate of interest. Finally, Section 6 offers some concluding remarks.

## 2. MODEL

### 2.1. Environment

We consider a discrete-time economy populated by an infinite sequence of two-period-lived, overlapping generations. Let  $t = 1, 2, \dots$  index time. In each period, economic activity takes place in two physically separate locations, denoted locations 1 and 2. These locations are symmetric, so that there is no need to index variables separately by location.

At each date  $t$ , a new young generation appears in each location. This generation consists of a continuum of agents with mass  $N_t = (1+n)^t$  in each location.

We consider a single-good, pure exchange economy (with storage). At date  $t$ , young agents have a (before-tax) endowment of  $w_t > 0$  units of the good. Old agents have no endowment.

For simplicity, we assume that agents value consumption only in the second period of life.<sup>3</sup> As a result, young agents save their entire after-tax endowment. And, if  $c$  denotes second-period consumption, young agents have the lifetime utility  $u(c) = c^{1-\rho}/(1-\rho)$ . Throughout, we assume that  $\rho \in (0, 1)$ .<sup>4</sup>

There are two (primary) assets in this economy: money and storage. All agents have access to a technology for storing the good whereby one unit stored at  $t$  yields  $x_t$  units of consumption at time  $t+1$ . We assume that  $x_t > 1+n$  holds, so that use of the storage technology is socially efficient.

### 2.2. Spatial Separation and Limited Communication

Following Townsend (1987), we use spatial separation and limited communication to motivate a transactions role for currency. And, following Champ et al. (1996) or Schreft and Smith (1997), we use stochastic relocation among disparate locations to generate a role for banks.

At the beginning of each period, each young agent is assigned to one of the locations. At this point in time, agents cannot move between or communicate across locations. Thus, goods and asset-market transactions occur within each location at the beginning of each period. After trade and savings decisions have occurred at time  $t$ , some randomly selected fraction  $\pi_t$  of young agents from location 1 is chosen to move to location 2 and vice versa, so that the two locations remain symmetric. We assume that stored goods are not transportable, either because transporting them is too costly or investment returns have not been realized at the time when agents are relocated. Moreover, as in Townsend (1987) or Champ et al. (1996), limited communication implies that relocated agents cannot transact with privately issued liabilities in their new location. As a result, relocated agents need currency to transact. However, within-location communication does permit

“credit” transactions. Thus, agents who are not relocated do *not* need currency to make purchases. Relocation constitutes a physical story about which goods are “cash goods” (those purchased by relocated agents) and which are “credit goods” (those purchased by nonrelocated agents).

The need to transact in cash, if one is relocated, implies that agents who move between locations must liquidate all other assets prior to moving, and convert them into currency. If currency is dominated in the rate of return—the situation on which we focus throughout the paper—then the necessity of converting higher-yielding assets into lower-yielding currency constitutes an adverse shock that agents will wish to be insured against. As in Diamond and Dybvig (1983), such insurance can be provided by banks. Their activities are described below.

### 2.3. The Government

The government in this economy has a variety of policies from which to choose. It could follow a policy of maintaining a constant rate of money creation, or it may choose to target either the nominal rate of interest or the rate of inflation. However, independent of the “operating procedure for monetary policy,” we assume that all required injections or withdrawals of money are accomplished through lump-sum tax/transfers to young agents.

Let  $M_t$  be the value of the time  $t$  nominal money stock, let  $p_t$  be the time  $t$  price level, and let  $\tau_t$  be the real value of lump-sum transfers made to young agents at  $t$ . Then, the government’s budget constraint is

$$N_{t+1}\tau_{t+1} = \frac{M_{t+1} - M_t}{p_{t+1}}. \quad (1)$$

### 2.4. Banks

As we have noted, the fact that agents are subject to stochastic relocation implies that there is a role for banks to provide liquidity. We now describe the behavior of these banks.

As in Diamond and Dybvig (1983), all savings will be intermediated. Hence, at the beginning of period  $t$ , each young agent deposits his/her after-tax endowment (savings),  $w_t + \tau_t$ , with a bank. The bank uses these deposits to acquire money and storage investments. Money is held by the bank as cash reserves to pay relocated agents.

We assume that the time  $t$  relocation probability,  $\pi_t$ , is known by all agents at the beginning of period  $t$ . Thus, the fraction of agents who will withdraw early—and who require payment in currency—is known with certainty by each bank.

Let  $z_t$  and  $k_t$  denote the per-depositor quantity of real balances and storage investment, respectively, acquired by a bank at  $t$ . The gross real rate of return on real balances between  $t$  and  $t + 1$  is  $p_t/p_{t+1}$ ; the gross real rate of return on storage is  $x_t$ . The bank is competitive in asset markets. Therefore, it takes these values as given. At the same time, banks offer two gross real rates of return to depositors:  $r_t^m$

per unit deposited to agents who are relocated, and  $r_t$  per unit deposited to agents who are not relocated. The bank faces the following constraints on its choices of  $z_t$ ,  $k_t$ ,  $r_t^m$ , and  $r_t$ . First, bank assets must not exceed bank liabilities, so that

$$z_t + k_t \leq w_t + \tau_t. \tag{2}$$

Second, payments to relocated agents at  $t$ ,  $\pi_t(w_t + \tau_t)r_t^m$ , cannot exceed the time  $t + 1$  value of the bank’s cash reserves because relocated agents require currency to make transactions. Hence,

$$\pi_t(w_t + \tau_t)r_t^m \leq z_t \frac{p_t}{p_{t+1}}. \tag{3}$$

The presence of the term  $p_t/p_{t+1}$  in this expression reflects any depreciation of currency values that occurs between the time when currency is acquired ( $t$ ) and when it is spent ( $t + 1$ ).

Finally, payments to nonrelocated agents,  $(1 - \pi_t)(w_t + \tau_t)r_t$ , cannot exceed the income from the bank’s capital investments and bond holdings. Thus,

$$(1 - \pi_t)(w_t + \tau_t)r_t \leq k_t x_t. \tag{4}$$

Implicit in these constraints is the notion that banks do not carry real balances between periods. Given the predictability of withdrawal demand, this will be the case if currency is dominated in the rate of return, so that  $x_t > p_t/p_{t+1}$  holds—as we henceforth assume.

As a coalition of ex ante identical depositors, the bank maximizes depositor expected utility,

$$\pi_t \frac{((w_t + \tau_t)r_t^m)^{1-\rho}}{1 - \rho} + (1 - \pi_t) \frac{((w_t + \tau_t)r_t)^{1-\rho}}{1 - \rho}$$

by choice of  $z_t$ ,  $k_t$ ,  $r_t^m$ , and  $r_t$ . The solution to this problem can be described as follows. First, the optimal ratio of reserves to deposits is given by

$$\frac{z_t}{w_t + \tau_t} \equiv \gamma(I_t, \pi_t) = \frac{1}{1 + \frac{1 - \pi_t}{\pi_t} I_t^{\frac{1-\rho}{\rho}}},$$

where  $I_t = x_t(p_{t+1}/p_t)$  denotes gross nominal rate of interest.<sup>5</sup> The ratio of storage to deposits is then given by  $k_t/(w_t + \tau_t) = 1 - \gamma(I_t, \pi_t)$ . In addition, the gross rates of return obtained by depositors can then be derived from (3) and (4) at equality. It is easy to verify that  $r_t = I_t^{1/\rho} r_t^m$ . Thus, when nominal rates of interest are positive, agents receive less than complete insurance against the event of being relocated. This is true because, in order to provide such insurance, the bank must hold cash reserves. However, the bank perceives an opportunity cost of doing so, which is related to the nominal rate of interest. Note that the higher the nominal rate of

interest (the greater the opportunity cost of holding reserves), the greater is the “wedge” between the consumption of agents who are and are not relocated.

The solution to the bank’s maximization problem is fully summarized by the optimal reserve deposit ratio  $\gamma(I_t, \pi_t)$ . For future reference, we summarize some properties of this function.

LEMMA 1.

- (a)  $\gamma(1, \pi_t) = \pi_t$ .
- (b)  $\frac{I_t \gamma(I_t, \pi_t)}{\gamma(I_t, \pi_t)} = -\frac{1-\rho}{\rho} (1 - \gamma(I_t, \pi_t)) < 0$ .

*With  $\rho < 1$ , increases in the nominal interest rate induce banks to economize on reserve holdings.*

### 2.5. Sources of Seasonality

Our intention is to consider how seasonal fluctuations affect this economy, and to consider how monetary policy should best respond to exogenous seasonal variability. Of particular interest is whether it is a good idea to insulate the nominal interest rate,  $I_t$ , from seasonal fluctuations. As we have just seen, if  $I_t$  fluctuates seasonally, the wedge between rates of return received by relocated and nonrelocated agents will also fluctuate seasonally, affecting the provision of insurance by banks. We investigate whether it is desirable for the government to prevent such seasonal fluctuations.

The answer to this question might, in principal, depend on the source of seasonality. Here, we consider three different potential sources of exogenously driven seasonality. One might derive from changes in production conditions that induce seasonal variability in real rates of return. We can capture seasonal variations that affect real rates of return by allowing the rate of return on storage to vary in the following deterministic manner<sup>6</sup>  $x_t = x_e(x_o)$  for  $t$  even (odd).

Second, there may be seasonal variations in the general availability of resources. We can capture this type of seasonality by allowing endowments to satisfy  $w_t = w_e(w_o)$  for  $t$  even (odd).

Finally, seasonal fluctuations may derive from deterministic variation in the need for currency in transactions. For instance, historically, cash use rose during harvest seasons as crops were moved to market. Alternatively, the need for cash may rise in seasons when the volume of other transactions is fairly large, say at Christmas. In any event, we can induce seasonality in cash use by letting the relocation probability,  $\pi_t$ , vary deterministically:  $\pi_t = \pi_e(\pi_o)$  for  $t$  even (odd).

In the analysis, we consider each possible source of seasonality, although often we assume that only one source of seasonality operates at a time.

### 3. GENERAL EQUILIBRIUM

The basic condition of equilibrium is that the supply of money and the demand for money be equal. Since all savings are intermediated, all beginning-of-period asset



demand derives from banks. It follows that the money market clears at each date if

$$z_t = \gamma(I_t, \pi_t)(w_t + \tau_t); \quad t \geq 1. \tag{5}$$

We now consider how an equilibrium is determined under different simple policies that the government might follow.

### 3.1. Constant Money Growth Rate Rule

One policy that the government might follow is to insulate the rate of money growth from seasonal fluctuations. Here we represent such a policy simply by letting the nominal stock of money grow at the constant gross rate  $\sigma$ , which the government selects exogenously—and once and for all—at date 1. Thus,  $M_{t+1} = \sigma M_t$ ;  $t \geq 0$  and where  $M_0 > 0$  is given as an initial condition. Equation (5), the government budget constraint (1), and the definition of  $z_t$  imply that

$$\tau_t = \frac{\sigma - 1}{\sigma} z_t, \quad \forall t \geq 1.$$

Then, it follows from the money-market-clearing condition (5) that

$$z_t = \left( \frac{\gamma(I_t, \pi_t)}{1 - \frac{\sigma - 1}{\sigma} \gamma(I_t, \pi_t)} \right) w_t. \tag{6}$$

Moreover,

$$I_t = x_t \frac{p_{t+1}}{p_t} \equiv \sigma \frac{x_t}{1 + n} \frac{z_t}{z_{t+1}}$$

holds. Using these relations, Gomis-Poqueras and Smith (2002) derive the following result.

LEMMA 2. *Under a constant money growth rate rule, the equilibrium law of motion for  $z_t$  satisfies*

$$z_{t+1} = \frac{z_t \sigma x_t}{1 + n} \left( \frac{1 - \pi_t}{\pi_t} \frac{z_t}{w_t - \frac{z_t}{\sigma}} \right)^{\frac{\rho}{1-\rho}}. \tag{7}$$

### 3.2. Nominal-Interest-Rate Targeting

An alternative method of conducting monetary policy, and one that is closer to current and historical practice in the Federal Reserve System, is to prevent the nominal rate of interest from fluctuating seasonally. A simple policy that would

accomplish this is for the central bank to target a constant value  $I$  for the rate of interest, so that  $I_t = I \forall t \geq 1$ .<sup>7</sup> Under this policy,

$$\frac{p_{t-1}}{p_t} = \frac{x_{t-1}}{I_{t-1}}, \quad t \geq 2. \tag{8}$$

It then follows from the government budget constraint that

$$\tau_t = z_t - z_{t-1} \frac{p_{t-1}}{p_t} \frac{1}{1+n} = z_t - z_{t-1} \left( \frac{x_{t-1}}{1+n} \right) \frac{1}{I_{t-1}}. \tag{9}$$

Using (9) in the money-market-clearing condition (5), and using the fact that  $I_t = I$ , we obtain the following equilibrium law of motion for  $z_t$ :

$$z_t = \frac{\gamma(I, \pi_t)}{1 - \gamma(I, \pi_t)} \left( w_t - \frac{z_{t-1}x_{t-1}}{I(1+n)} \right) = \frac{\pi_t}{1 - \pi_t} (I)^{\frac{\rho-1}{\rho}} \left( w_t - \frac{z_{t-1}x_{t-1}}{I(1+n)} \right), \quad t \geq 2. \tag{10}$$

### 3.3. Inflation Targeting

A third “operating procedure” for monetary policy might be to prevent seasonal fluctuations from being reflected in the time path of the inflation rate (or, possibly, from being reflected in the time path of the nominal price level). One policy that would accomplish this would be to have a central bank target a value for the initial price level,  $p_1^*$ , and thereafter to maintain a target inflation rate, so that

$$\frac{p_{t+1}}{p_t} = \eta \forall t \geq 1.$$

Under inflation (or price-level) targeting rules, the government budget constraint becomes

$$\tau_t = z_t - z_{t-1} \frac{p_{t-1}}{p_t} \frac{1}{1+n} = z_t - \frac{z_{t-1}}{\eta(1+n)}, \quad t \geq 2. \tag{11}$$

Using (11) in the money-market-clearing condition (5), we obtain the following equilibrium law of motion for  $z_t$ :

$$z_t = \frac{\gamma(I_t, \pi_t)}{1 - \gamma(I_t, \pi_t)} \left( w_t - \frac{z_{t-1}}{\eta(1+n)} \right) = \frac{\pi_t}{1 - \pi_t} I_t^{\frac{\rho-1}{\rho}} \left( w_t - \frac{z_{t-1}}{\eta(1+n)} \right), \quad t \geq 2, \tag{12}$$

where  $I_t = \eta x_t$ . Notice that, unless  $x_t$  exhibits seasonal fluctuations, the policies of targeting the nominal rate of interest and of targeting the time path of the price level lead to identical equilibrium laws of motion for real balances. Given the low level of observed seasonality in the real rate of interest, this is consistent with the observation made in the introduction: The policy of eliminating seasonality in the nominal rate of interest also largely eliminates seasonality in the rate of inflation.

4. SEASONALITY AND MONETARY POLICY

We now examine the consequences of different methods of conducting monetary policy. We begin by examining periodic equilibria under alternative operating procedures for the central bank.

4.1. Constant Rates of Money Growth

As before, we begin by considering a policy that maintains a constant gross rate of money creation,  $\sigma$ . In addition, as noted earlier, we seek periodic equilibria, that is, equilibria with  $z_t = z_e(z_o)$  for  $t$  even (odd), and  $I_t = I_e(I_o)$  for  $t$  even (odd). When  $z_t$  evolves according to (7), we in fact have

$$I_t = \left( \frac{\sigma x_t}{1+n} \right) \frac{z_t}{z_{t+1}}. \tag{13}$$

Then, equation (6) implies that, in order for the money market to clear,

$$z_t = \frac{\gamma \left( \frac{\sigma x_t}{1+n} \frac{z_t}{z_{t+1}}, \pi_t \right) w_t}{1 - \frac{\sigma-1}{\sigma} \gamma \left( \frac{\sigma x_t}{1+n} \frac{z_t}{z_{t+1}}, \pi_t \right)}. \tag{14}$$

Define the function  $Q(y, \sigma)$  as follows:

$$\begin{aligned} Q(y, \sigma) &\equiv \left( \frac{\gamma \left( \frac{\sigma x_e}{1+n} y, \pi_e \right) w_e}{\gamma \left( \frac{\sigma x_o}{1+n} \frac{1}{y}, \pi_o \right) w_o} \right) \frac{1 - \frac{\sigma-1}{\sigma} \gamma \left( \frac{\sigma x_o}{1+n} \frac{1}{y}, \pi_o \right)}{1 - \frac{\sigma-1}{\sigma} \gamma \left( \frac{\sigma x_e}{1+n} y, \pi_e \right)} \\ &\equiv \frac{1 + \frac{1-\pi_o}{\pi_o} \sigma^{1/\rho} \left( \frac{x_o}{1+n} \frac{1}{y} \right)^{\frac{1-\rho}{\rho}}}{1 + \frac{1-\pi_e}{\pi_e} \sigma^{1/\rho} \left( \frac{x_e}{1+n} y \right)^{\frac{1-\rho}{\rho}}}. \end{aligned}$$

Then, it follows from (13) and (14) that, in any nontrivial periodic equilibrium,

$$\frac{z_e}{z_o} = Q \left( \frac{z_e}{z_o}, \sigma \right). \tag{15}$$

Lemma 4 describes some properties of the function  $Q$ . Its proof appears in Gomis-Poqueras and Smith (2002).

LEMMA 4.  $Q(y, \sigma)$  satisfies

- (a)  $\lim_{y \rightarrow 0} Q(y, \sigma) = \infty$ ,
- (b)  $\lim_{y \rightarrow \infty} Q(y, \sigma) = 0$ ,

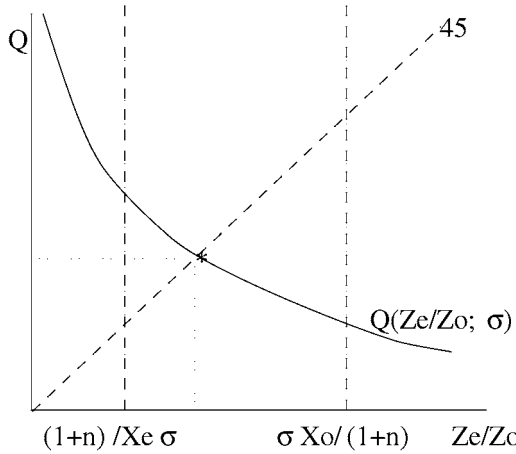


FIGURE 1. Determination of periodic equilibria under a constant rate of money growth.

- (c)  $Q_1(y, \sigma) < 0$ ,
- (d)  $Q_2(y, \sigma) \geq (<)0$  holds if  $\sqrt{\frac{x_o}{x_e} \left( \frac{\pi_e}{1-\pi_e} \frac{1-\pi_o}{\pi_o} \right)^{\frac{\rho}{2(1-\rho)}}} \geq (<)y$ ,
- (e)  $Q\left(\sqrt{\frac{x_o}{x_e} \left( \frac{\pi_e}{1-\pi_e} \frac{1-\pi_o}{\pi_o} \right)^{\frac{\rho}{2(1-\rho)}}}\right) = 1$ .

A periodic equilibrium with  $I_e > 1$  and  $I_o > 1$  is a solution to (15) that satisfies the additional condition

$$\frac{\sigma x_o}{1+n} > \frac{z_e}{z_o} > \frac{x_e}{(1+n)\sigma}. \tag{16}$$

The determination of such an equilibrium is depicted in Figure 1. The following result is immediate from an inspection of the figure.

PROPOSITION 1. A periodic equilibrium with  $I_e > 1$  and  $I_o > 1$  exists iff

$$Q\left(\frac{1+n}{\sigma x_e}, \sigma\right) > \frac{1+n}{\sigma x_e} \tag{17}$$

$$Q\left(\frac{\sigma x_o}{1+n}, \sigma\right) < \frac{\sigma x_o}{1+n}. \tag{18}$$

If (17) and (18) hold, then the desired periodic equilibrium exists, and we can ask questions about the consequences of varying the rate of money growth,  $\sigma$ . Part (d) of Lemma 4 implies that an increase in  $\sigma$  shifts  $Q(z_e/z_o, \sigma)$  upward (downward) in Figure 2 if

$$\frac{z_e}{z_o} \leq (\geq) \sqrt{\frac{x_o}{x_e} \left( \frac{\pi_e}{1-\pi_e} \frac{1-\pi_o}{\pi_o} \right)^{\frac{\rho}{2(1-\rho)}}}.$$

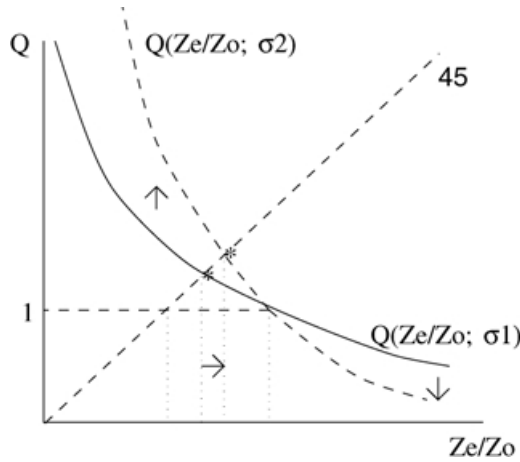


FIGURE 2. Effect of an increase in the money growth rate on periodic equilibria when  $z_e/z_o > 1$  and  $\sigma_2 > \sigma_1$ .

Thus,

$$\frac{\partial(z_e/z_o)}{\partial\sigma} > (<)0 \quad \text{if} \quad z_e/z_o > (<)1.$$

Figure 2 depicts the case  $z_e/z_o > 1$ . The opposite case is similar, and the following result is then immediate.

**PROPOSITION 2.** (a) An increase in the rate of money creation increases the magnitude of seasonal fluctuations in real balances. (b) If  $x_e = x_o$  (there are no seasonal fluctuations in real rates of return), then an increase in the rate of money creation increases the magnitude of seasonal fluctuations in the nominal rate of interest. (c) If  $x_e = x_o$ , then an increase in the rate of money creation increases the magnitude of seasonal fluctuations in the rate of inflation.

In particular, if  $z_e/z_o > (<)1$ , then real balances are highest in even (odd) periods. Also, an increase in  $\sigma$  raises (lowers)  $z_e$  relative to  $z_o$ . Moreover,  $I_e/I_o = (x_e/x_o)(z_e/z_o)^2$ . Thus, if  $x_e = x_o$  and if  $z_e/z_o > (<)1$ , nominal interest rates are relatively high in even (odd) periods. Furthermore, an increase in  $\sigma$  raises (lowers)  $z_e$  relative to  $z_o$ , and hence raises (lowers)  $I_e$  relative to  $I_o$ . Finally, when  $t$  is even,  $(p_{t+1}/p_t)(p_{t-1}/p_t) = (x_o/x_e)(I_e/I_o)$ , and then the same observations imply that increases in  $\sigma$  increase seasonal variation in the rate of inflation.

Proposition 2 has an important corollary. The setting of monetary policy (the choice of  $\sigma$ ) affects the size of seasonal fluctuations in all endogenous variables. Moreover, as we have noted, seasonal fluctuations are large relative to business-cycle fluctuations. Thus, monetary policy has considerable scope to influence variability in economic activity. Also, failure to attribute some seasonal fluctuations to the conduct of monetary policy underemphasizes the importance of monetary policy for real fluctuations.

**4.2. Constant Nominal Interest Rates**

We now examine how monetary policy affects the magnitude of seasonal fluctuations when the central bank follows the policy of maintaining a constant value for the nominal rate of interest.<sup>8</sup> Also, as before, in this section we restrict our attention to periodic equilibria, so that  $z_t = z_e(z_o)$  for  $t$  even (odd).

Finally, to reduce the number of possible cases that require consideration, in this section we assume that

$$1 > \frac{\pi_e}{1 - \pi_e} \frac{\pi_o}{1 - \pi_o} (I^2)^{-1/\rho} \frac{x_e x_o}{(1 + n)^2}, \tag{19}$$

where  $I$  is the target value for the nominal rate of interest.<sup>9</sup>

In a periodic equilibrium, equation (10) implies that the equilibrium values of real balances satisfy

$$z_t = \frac{\frac{\pi_t}{1 - \pi_t} w_t I^{\frac{\rho-1}{\rho}} - \frac{\pi_t}{1 - \pi_t} \frac{\pi_{t-1}}{1 - \pi_{t-1}} I^{\frac{\rho-2}{\rho}} \frac{x_{t-1}}{1 + n} w_{t-1}}{1 - \frac{\pi_t}{1 - \pi_t} \frac{\pi_{t-1}}{1 - \pi_{t-1}} I^{\frac{-2}{\rho}} \frac{x_{t-1} x_t}{(1 + n)^2}}. \tag{20}$$

Equation (19) implies that the target nominal rate of interest must be set so that the denominator of the right-hand side of equation (20) is positive.

Equation (20) implies that

$$\frac{z_e}{z_o} = \frac{\frac{\pi_e}{1 - \pi_e} w_e - \frac{\pi_e}{1 - \pi_e} \frac{\pi_o}{1 - \pi_o} I^{\frac{-1}{\rho}} \frac{x_o}{1 + n} w_o}{\frac{\pi_o}{1 - \pi_o} w_o - \frac{\pi_e}{1 - \pi_e} \frac{\pi_o}{1 - \pi_o} I^{\frac{-1}{\rho}} \frac{x_e}{1 + n} w_e}. \tag{21}$$

Using this relation, Gomis-Poqueras and Smith (2002) prove the following result.

**PROPOSITION 3.** *The equilibrium corresponding to a constant nominal interest rate satisfies*

(a)  $z_e \geq (<) z_o$  if

$$\frac{\pi_e}{1 - \pi_e} w_e - \frac{\pi_o}{1 - \pi_o} w_o \geq (<), \tag{22}$$

$$\frac{\pi_e}{1 - \pi_e} \frac{\pi_o}{1 - \pi_o} \left( \frac{x_o}{1 + n} w_o - \frac{x_e}{1 + n} w_e \right) I^{-1/\rho};$$

(b)  $[\partial(z_e/z_o)]/\partial I \geq (<) 0$  holds if

$$\frac{\pi_o}{1 - \pi_o} x_o w_o^2 \geq (<) \frac{\pi_e}{1 - \pi_e} x_e w_e^2. \tag{23}$$

Proposition 3 indicates that, when the central bank insulates the nominal rate of interest from seasonal fluctuations, changes in the interest rate may have complicated

consequences for the magnitude of seasonal fluctuations. This point is illustrated by the following example.

**Example 1.** Suppose that  $\pi_e w_e / (1 - \pi_e) > \pi_o w_o / (1 - \pi_o)$ ,  $\pi_e x_e w_e^2 / (1 - \pi_e) < \pi_o x_o w_o^2 / (1 - \pi_o)$ , and  $x_o w_o > x_e w_e$  hold. Suppose further that

$$\frac{\pi_e}{1 - \pi_e} w_e - \frac{\pi_o}{1 - \pi_o} w_o < \frac{\pi_e}{1 - \pi_e} \frac{\pi_o}{1 - \pi_o} \left( \frac{x_o}{1 + n} w_o - \frac{x_e}{1 + n} w_e \right)$$

is satisfied. Then, if the target value of  $I$  is set close to 1,  $z_e < z_o$  will hold. Moreover, increases in  $I$  will increase  $z_e / z_o$ . Thus, over some range of target values for the nominal rate of interest, higher settings of the target will reduce the magnitude of the seasonal fluctuations in real balances. However, if  $I$  is set sufficiently beyond this point, further increases in the nominal rate of interest will continue to raise  $z_e / z_o$ , so that the magnitude of seasonal variation in real balances will increase. In short, changes in monetary policy can easily have nonmonotonic effects on the degree of seasonal variability.

Moreover, the effects of changes in the target nominal interest rate will affect not only the magnitude of seasonal fluctuations in the level of real balances, but also the magnitude of seasonal fluctuations in investment (storage). To see this, note that, under a policy of maintaining a constant nominal rate of interest,

$$\frac{k_e}{k_o} = \frac{1 - \gamma(I, \pi_e)}{1 - \gamma(I, \pi_o)} \frac{\gamma(I, \pi_e)}{\gamma(I, \pi_o)} \frac{z_e}{z_o} = \frac{1 - \pi_e}{\pi_e} \frac{\pi_o}{1 - \pi_o} \frac{z_e}{z_o}$$

in a periodic equilibrium. Thus, anything that affects the degree of seasonal fluctuations in real balances will also affect the degree of such fluctuations in investment. Indeed, monetary policy changes must affect the degree of seasonality in investment because monetary policy works by injecting reserves into and withdrawing reserves from the banking system. Changes in the target nominal rate of interest modify the required seasonal pattern of reserve injections and withdrawals. As a consequence, bank portfolios must be affected. This has implications for the aggregate investment just noted.

### 4.3. Testable Predictions

We now briefly consider the extent to which our predictions about the conduct of monetary policy and the size of the seasonals are reflected in the data. However, in doing so, we are severely limited by the lack of comparable seasonally unadjusted data series across countries.

As we have observed, our analysis predicts that, within any given monetary regime, the setting of the appropriate target variable for monetary policy will affect the magnitude of seasonal fluctuations in most endogenous variables. For example, under a policy of maintaining a constant target rate of money creation, any increase in the (geometric) average rate of inflation leads to an increase in the magnitude of seasonal fluctuations in a periodic equilibrium. To find evidence related to this

proposition, we employ cross-country data on seasonal fluctuations, compiled by Beaulieu et al. (1992). Using their data set, the main empirical question we try to answer is whether there is a positive relation across countries between the amount of seasonal variation in macroeconomic variables and the average inflation rate.

As we have noted, if the degree of seasonal variation in real interest rates is small,<sup>10</sup> then an increase in the (geometric) average rate of inflation (the rate of money growth), under a policy of preventing seasonality in the rate of base money growth, will increase the seasonal standard deviation of the rate of inflation. In a sample of 14 countries,<sup>11</sup> the simple correlation between the average rate of inflation and the seasonal standard deviation of inflation is 0.843. This correlation is significant at the 10% level. Similarly, in a sample of 15 countries,<sup>12</sup> the simple correlation between the average rate of inflation and the seasonal standard deviation of industrial production is 0.491. This correlation is significantly different from zero at the 5% level.

On the basis of at least this casual evidence, we can conclude that there is some support for the notion that higher rates of money creation (inflation) are associated with a greater degree of seasonal variation in the data.

#### 4.4. Nonperiodic Equilibria

We now briefly examine the scope for equilibria that are not purely periodic to exist under different methods of conducting monetary policy. We begin with the case of a constant rate of money growth.

*4.4.1. Constant rate of money growth.* When monetary policy is conducted by holding the rate of money creation constant, equation (5) governs the evolution of real balances. In Gomis-Poqueras and Smith (2002), we prove the following claim.

**PROPOSITION 4.** *When there are nontrivial seasonal fluctuations, there is a unique equilibrium in which money retains value asymptotically. This is the periodic equilibrium derived above.*

Thus, from a monetary policy perspective, the periodic equilibrium analyzed previously is the only interesting equilibrium.

*4.4.2. Constant nominal rate of interest.* When monetary policy maintains the nominal rate of interest at a constant target value, equation (10) implies that, for  $t$  even,

$$\begin{aligned}
 z_t = & \frac{\pi_e}{1 - \pi_e} (I)^{\frac{\rho-1}{\rho}} w_e - \frac{\pi_e}{1 - \pi_e} \frac{\pi_o}{1 - \pi_o} \frac{x_o}{(1 + n)} (I)^{\frac{\rho-2}{\rho}} w_o \\
 & + \frac{\pi_e}{1 - \pi_e} \frac{\pi_o}{1 - \pi_o} \frac{x_o x_e}{(1 + n)^2} (I)^{\frac{-2}{\rho}} z_{t-2}.
 \end{aligned}
 \tag{24}$$

Evidently, there is a unique periodic equilibrium, and this equilibrium is asymptotically stable if (19) holds. Thus, in particular, when (19) holds, as we argue below is



the empirically most plausible case, there is a continuum of nonstationary equilibria, all of which converge to the periodic equilibrium. In short, a policy of insulating the nominal rate of interest from seasonal fluctuations leads to an indeterminacy. Moreover, all nonstationary equilibria will display seasonal fluctuations in real balances exceeding those that would be observed in a purely periodic equilibrium.

Why does targeting the nominal rate of interest create so much scope for multiplicity of equilibria exhibiting endogenously generated volatility? The answer is that, if agents demand high levels of real balances at  $t$ , the central bank, to prevent the nominal interest rate from rising must inject reserves into the banking system. This reserve injection, in turn, increases the volume of deposits and, consequently, the demand for reserves. As a result, actions that the central bank take to hold the nominal rate of interest constant validate the endogenously high level of demand for real balances at date  $t$ . Moreover, for this process not to be destabilizing, the central bank must withdraw reserves at  $t + 1$ , implying a low level of deposits and a low level of demand for real balances. Thus, any equilibria where real balances deviate from their steady-state level must display oscillation.

Notice that a policy of nominal interest-rate targeting can, then, induce deterministic fluctuations. These resemble seasonals, except that these oscillations dampen asymptotically. This observation suggests that a policy of nominal-interest-rate targeting might lead to equilibria where seasonal variability is apparently amplified. Such a conjecture is easily verified.

Finally, we observe that, along dynamical equilibrium paths where  $z_t$  fluctuates, economic volatility is not confined to fluctuations in real balances. The variability of central bank reserve injections (lump-sum transfers) will also translate into variability in investment activity (storage).

**5. SEASONALITY, MONETARY POLICY, AND WELFARE**

In this section, we confine our attention to periodic equilibria, and we ask whether current Federal Reserve policy, which is to insulate nominal interest rates from seasonal fluctuations, is or is not a “good policy.” In particular, we now allow the central bank to target a (periodic) time path for the nominal rate of interest; that is, we focus on policies where  $I_t = I_e(I_o)$  for  $t$  even (odd).

We also assume that  $I_e I_o = I^2 > 1$ , where  $I$  is the target value for the geometric average for the nominal rate of interest. We take  $I$  as given, and, for a given value of  $I$ , setting  $I_e = I_o = I$  (preventing seasonal fluctuations from being manifested in the nominal rate of interest) is a “good policy” if it maximizes “steady-state welfare.” We now describe what we mean by the term steady-state welfare.

When banks choose their portfolios optimally, the expected utility of an agent born at  $t$  is

$$\frac{(w_t + \tau_t)^{1-\rho}}{1 - \rho} \left( \pi_t \left( \frac{\gamma(I_t, \pi_t)x_t}{\pi_t I_t} \right)^{1-\rho} + (1 - \pi_t) \left( \frac{(1 - \gamma(I_t, \pi_t))x_t}{1 - \pi_t} \right)^{1-\rho} \right).$$

To write this more compactly, we define the functions  $V_t$  for  $t$  even (odd) by

$$V_t(I_t) = \pi_t^\rho \left( \frac{\gamma(I_t, \pi_t)x_t}{I_t} \right)^{1-\rho} + (1 - \pi_t)^\rho (x_t(1 - \gamma(I_t, \pi_t)))^{1-\rho}.$$

Then, the expected utility of an agent born in an even period is  $[(w_e + \tau_e)^{1-\rho} / (1 - \rho)] V_e(I_e)$ , and the expected utility of an agent born in an odd period is  $[(w_o + \tau_o)^{1-\rho} / (1 - \rho)] V_o(I_o)$ . In general, different choices of  $I_e$  and  $I_o$  satisfying  $I_e I_o = I^2$  (with  $I$  given) will result in welfare gains for one group (e.g., those born in even periods) at the expense of another group (e.g., those born in odd periods). We therefore define steady-state welfare as follows:

$$W(I_e, I_o) = \frac{1}{2} \frac{(w_e + \tau_e)^{1-\rho}}{1 - \rho} V_e(I_e) + \frac{1}{2} \frac{(w_o + \tau_o)^{1-\rho}}{1 - \rho} V_o(I_o). \tag{25}$$

This welfare measure can be interpreted as the ex ante expected utility of an arbitrary young agent who, prior to our knowing his date of birth, has an equal probability of being born in an even or an odd period. We now examine how policy choices affect this welfare measure.

To do so, it is necessary to determine the equilibrium values of the transfers  $\tau_e$  and  $\tau_o$ . The government budget constraint implies that

$$\tau_t = z_t - z_{t-1} \frac{x_{t-1}}{1 + n} \frac{1}{I_{t-1}} \tag{26}$$

for all  $t$ . Using (26) in equation (5) then allows us to determine that

$$z_t = \frac{\gamma(I_t, \pi_t)}{1 - \gamma(I_t, \pi_t)} \left( w_t - z_{t-1} \frac{x_{t-1}}{(1 + n)I_{t-1}} \right). \tag{27}$$

Using the fact that  $\gamma(I_t, \pi_t) / (1 - \gamma(I_t, \pi_t)) = (\pi_t / (1 - \pi_t)) I_t^{(\rho-1)/\rho}$  in (27), and solving the resulting system of equations for  $z_e$  and  $z_o$  yields

$$z_e = \frac{\frac{\pi_e}{1 - \pi_e} I_e^{\frac{\rho-1}{\rho}} w_e - \frac{\pi_e}{1 - \pi_e} \frac{\pi_o}{1 - \pi_o} (I^2)^{\frac{\rho-1}{\rho}} \frac{x_o w_o}{(1 + n)I_o}}{1 - \frac{\pi_e}{1 - \pi_e} \frac{\pi_o}{1 - \pi_o} I^{\frac{-2}{\rho}} \frac{x_o x_e}{(1 + n)^2}} \equiv H(I_e, I_o) \tag{28}$$

and

$$z_o = \frac{\frac{\pi_o}{1 - \pi_o} I_o^{\frac{\rho-1}{\rho}} w_o - \frac{\pi_e}{1 - \pi_e} \frac{\pi_o}{1 - \pi_o} (I^2)^{\frac{\rho-1}{\rho}} \frac{x_e w_e}{(1 + n)I_e}}{1 - \frac{\pi_e}{1 - \pi_e} \frac{\pi_o}{1 - \pi_o} I^{\frac{-2}{\rho}} \frac{x_o x_e}{(1 + n)^2}} \equiv G(I_e, I_o). \tag{29}$$

It then follows that

$$\begin{aligned}
 w_e + \tau_e &= w_e + H(I_e, I_o) - \frac{x_o}{1+n} \frac{G(I_e, I_o)}{I_o} \\
 &= \frac{\left(1 + \frac{\pi_e}{1-\pi_e} I_e^{\frac{\rho-1}{\rho}}\right) \left(w_e - \frac{\pi_o}{1-\pi_o} \frac{x_o}{(1+n)} (I_o)^{-1/\rho} w_o\right)}{1 - \frac{\pi_e}{1-\pi_e} \frac{\pi_o}{1-\pi_o} I^{\frac{-2}{\rho}} \frac{x_o x_e}{(1+n)^2}} \equiv Q(I_e, I_o)
 \end{aligned}
 \tag{30}$$

and

$$\begin{aligned}
 w_o + \tau_o &= w_o + G(I_e, I_o) - \frac{x_e}{1+n} \frac{H(I_e, I_o)}{I_e} \\
 &= \frac{\left(1 + \frac{\pi_o}{1-\pi_o} I_o^{\frac{\rho-1}{\rho}}\right) \left(w_o - \frac{\pi_e}{1-\pi_e} \frac{x_e}{(1+n)} (I_e)^{-1/\rho} w_e\right)}{1 - \frac{\pi_e}{1-\pi_e} \frac{\pi_o}{1-\pi_o} I^{\frac{-2}{\rho}} \frac{x_o x_e}{(1+n)^2}} \equiv R(I_e, I_o).
 \end{aligned}
 \tag{31}$$

Thus, steady-state welfare is given by the expression

$$W(I_e, I_o) = \frac{1}{2} \frac{(Q(I_e, I_o))^{1-\rho}}{1-\rho} V_e(I_e) + \frac{1}{2} \frac{(R(I_e, I_o))^{1-\rho}}{1-\rho} V_o(I_o). \tag{32}$$

We now consider the problem of a government that chooses  $I_e$  and  $I_o$  to maximize  $W(I_e, I_o)$  subject to the constraints that  $I_e I_o = I^2$  (with  $I^2 > 1$  given),  $I_e \geq 1$ , and  $I_o \geq 1$ . This problem is sufficiently complicated that it is necessary to use numerical methods to investigate the optimal choices of  $I_e$  and  $I_o$ . To do so, we choose parameter values that satisfy the following conditions. First, according to the 2000 edition of the *Economic Report of the President*, the average value of the ratio of reserves-to-demand deposits over the 1990’s in the United States was 0.14. Second, Rasche (1972) reports that the interest elasticity of excess reserves<sup>13</sup> (with respect to the nominal rate of interest on Treasury bills) is about  $-0.3$ . These figures suggest that, in the absence of seasonal fluctuations,  $\gamma(I, \pi) = 0.14$  and

$$\frac{I\gamma_1(I, \pi)}{\gamma(I, \pi)} = - \frac{(1-\rho)(1-\gamma(I, \pi))}{\rho} = -0.3$$

should hold.

For the gross (safe) real rate of return, in the absence of seasonality, we set  $x = 1.04$ . This corresponds to Prescott’s (1986) calibrated value for a safe annual real rate of return, in a steady state. In addition, according to the 2000 edition of *The Economic Report of the President*, the average gross rate of inflation over the

1990's was 1.03. Thus, in a steady state (if there were no seasonal fluctuations), we would like to set  $p_{t+1}/p_t = I/x = 1.03$ . Thus,  $I = 1.0712$ . We can then determine that  $\pi \approx 0.14$ , and  $\rho = 0.74$ . Finally, we set the rate of population growth to 1% annually ( $n = 0.01$ ), and—again absent seasonal variation—we set the endowment  $w$  equal to per capita GDP. Thus,  $w = 27,924$ .

To introduce seasonal fluctuations, we proceed as follows. Since it is quite possible that the nature of an optimal policy depends on the source of seasonal variability—or on the relative importance of different sources of seasonal variability—we examine different economies in which only one particular source of seasonality is operative at any one time. Thus, if  $w_e \neq w_o$ , for example, we set  $x_e = x_o$  and  $\pi_e = \pi_o$ . Of course, ultimately, we examine all three possible sources of seasonal impulses.

For whichever exogenous variable does vary seasonally, we assume that its geometric average is equal to the steady-state value given previously. Thus, if  $w_e \neq w_o$ , we impose that  $\sqrt{w_e w_o} = w = 27,924$ . Similarly, if  $x_e \neq x_o$  ( $\pi_e \neq \pi_o$ ), we require that  $\sqrt{x_e x_o} = x = 1.04$  ( $\sqrt{\pi_e \pi_o} = \pi = 0.14$ ).

It remains to describe the magnitude of the exogenous seasonal impulse. To do so, if  $w_e \neq w_o$ , we let  $w_e = \Omega w_o$ . Similarly, if  $x_e \neq x_o$  ( $\pi_e \neq \pi_o$ ), we set  $x_e = \Omega x_o$  ( $\pi_e = \Omega \pi_o$ ). With this parameterization, if  $\sigma_w$  is the standard deviation of the logarithm of  $w$  (and, similarly,  $\sigma_x$  and  $\sigma_\pi$  are the standard deviations of the logarithms of  $x$  and  $\pi$ ), we have  $\sigma_w = \ln\sqrt{\Omega}$ . To fix ideas, we will always let  $\Omega > 1$  hold.

Barsky and Miron (1989) find that the percentage standard deviation in per-capita income, due to *seasonal fluctuations*, is 2.51%. This observation, along with the calculations above, suggests that a reasonable value for  $\Omega$  (at least with respect to seasonal income) is 1.051. Thus, we set  $\Omega = 1.05$ . However, we experimented with other values for  $\Omega$  ( $\Omega = 1.01$  and  $\Omega = 1.1$ ) with no qualitative difference in results.

**Example 2.** Suppose we substitute  $I_o = I^2/I_e$  into the government's objective function. Figure 3 plots the function  $\bar{W}(I_e) = W(I_e, I^2/I_e)$  if  $x_e \neq x_o$  (implying  $w_e = w_o$  and  $\pi_e = \pi_o$ ), when  $I = 1.0712$ . Clearly, the function  $\bar{W}(I_e)$  is concave, and it attains a maximum at  $I_e = 1.086$  (implying  $I_o = 1.056$ ). Note that the implied magnitude of seasonal fluctuation in the nominal rate of interest is  $I_e/I_o = 1.028$ , so that an optimal policy involves the nominal rate of interest exhibiting smaller seasonal variation than the real rate of interest. Here, "peak-to-trough" seasonal movements in the rate of inflation are given by the ratio  $(I_e/I_o)(x_e/x_o)$ . It follows that the rate of inflation exhibits even smaller seasonal variability than the nominal rate of interest. In addition, the rate of inflation will be lowest in seasons when the real rate of interest is highest.

To summarize, the optimal policy smooths the seasonality in nominal interest rates (relative to the magnitude of the underlying seasonal impulse). However, it also smooths the seasonality in inflation (relative to the magnitude of the same underlying seasonal impulse). For this reason, it is not optimal to completely insulate the nominal rate of interest from seasonal factors.

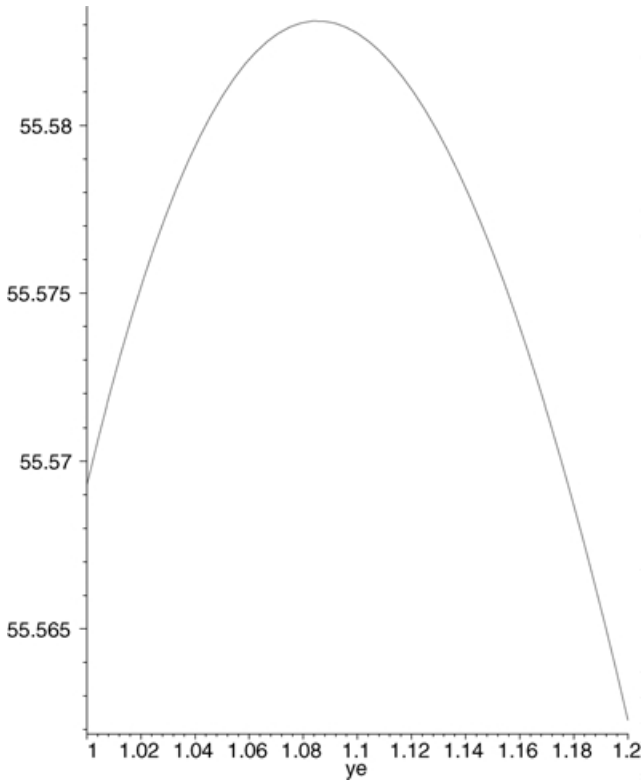


FIGURE 3. The function  $\bar{W}(I_e) = W(I_e, I^2/I_e)$ , seasonality in real interest rates.

TABLE 1. Seasonality in real returns: Optimal nominal rates of interest seasonally for alternative “average” nominal rates of interest

$I$	$I_e^*$	$I_e^*/I_o^*$	$I_e$	Transfer	Transfer F.R.
1.05	1.07	1.035	1.08	0.001	-1.5
1.07	1.09	1.029	1.10	0.9	0.2
1.09	1.11	1.025	1.12	0.9	3.86
1.11	1.14	1.054	1.15	1.2	10.2
1.13	1.15	1.031	1.16	0.95	15.7
1.15	1.17	1.03	1.18	1.28	23.1
1.17	1.19	1.028	1.2	1.0	32.0

Table 1 reports the optimal value of  $I_e$ , and the optimal ratio  $I_e/I_o$ , for the same economy but for different average values of the nominal rate of interest ( $I$ ). Evidently, as the average value of the nominal rate of interest increases, optimal monetary policy initially involves a reduction in the magnitude of seasonal variability in nominal rates. However, this is not true globally: Once  $I$  is sufficiently

large, further increases in it render it optimal to allow the magnitude of seasonal fluctuations in the nominal rate of interest to increase.

To describe the magnitude of welfare losses from suboptimal settings of policy, Table 1 also reports the compensation (additional, seasonally independent transfer) that each young agent would have to receive to make them indifferent between the optimal policy and a policy that sets  $I_e$  one percentage point higher than its optimal level (with  $I_e I_o = I^2$  continuing to hold). The necessary transfer is tiny—about 3/10,000th of 1% of per-capita income. This finding suggests the relative unimportance—from a welfare perspective—of smoothing or not smoothing the nominal rate of interest seasonally. However, two caveats to this remark should be offered. First, the method by which money is injected into the economy works to minimize welfare losses from suboptimal policy settings. In particular, young agents get high transfers when the nominal interest rate is high, in effect “insuring” agents against being born in unfortunate seasons. Thus, if money were injected into the economy by different means, the welfare losses from suboptimal policy choices would tend to be higher.

Second, the fact that deviations from the optimal setting of policy do not have large welfare effects does *not* imply that monetary policy choices necessarily have small effects on equilibrium quantities. For instance, it is widely thought that the welfare costs of business-cycle fluctuations are small [Lucas (1987)], and yet, this does not lead anyone to argue that how monetary policy affects business-cycle fluctuations is an unimportant issue.

Finally, Table 1 also reports the welfare “loss” associated with deviations from the Friedman rule (maintenance of a zero nominal rate of interest). Note that, for  $I = 1.05$ , this loss is negative—that is, the Friedman rule is not optimal here. It allows insufficient (specifically, no) seasonal variation in nominal rates of interest. However, the welfare loss from following the Friedman rule is also not large.

**Example 3** ( $w_e \neq w_o$ ). Again, the function  $\bar{W}(I_e)$  is concave, and it attains a maximum at  $I_e = 1.112$  (implying  $I_o = 1.0313$ ). At an optimum,  $I_e/I_o = 1.078$  holds. Thus, when seasonal impulses derive from endowments (income), the nominal interest rate should vary more, seasonally, than the exogenous driving variable. Moreover, with real returns constant, the same will be true for the rate of inflation.

Intuitively, the monetary authority can transfer utility from agents born in even periods (those with high endowments) to agents born in odd periods by making the nominal rate of interest relatively high (low) in even (odd) periods. In this case, optimal transfers involve relatively large seasonal variation in the nominal rate of interest. In other words, when the source of seasonality derives from exogenous income movements, it is completely inappropriate to engage in smoothing either nominal interest rates or the rate of inflation seasonally.

Table 2 reports the optimal values of  $I_e$ , and  $I_e/I_o$ , for alternative “average” values of the nominal rate of interest. Clearly,  $I_e/I_o$  does not vary monotonically with  $I$ . The table also reports the compensation (transfer) necessary to compensate

**TABLE 2.** Seasonality in endowments: Optimal nominal rates of interest seasonally for alternative “average” nominal rates of interest

$I$	$I_e^*$	$I_e^*/I_o^*$	$I_e$	Transfer	Transfer F.R.
1.05	1.09	1.073	1.10	0.2	-7.2
1.07	1.11	1.078	1.12	0.9	-4.74
1.09	1.13	1.072	1.14	0.1	-0.845
1.11	1.16	1.09	1.16	1.3	4.5
1.13	1.17	1.076	1.19	0.49	11.53
1.15	1.20	1.083	1.22	1.5	18.0
1.17	1.22	1.08	1.24	1.65	27.6

**TABLE 3.** Seasonality in liquidity demand: Optimal nominal rates of interest seasonally for alternative “average” nominal rates of interest

$I$	$I_e^*$	$I_e^*/I_o^*$	$I_e$	Transfer	Transfer F.R.
1.05	1.07	1.038	1.08	1.6	-1.78
1.07	1.08	1.019	1.09	0.4	0.7
1.09	1.10	1.018	1.11	0.8	4.5
1.11	1.12	1.018	1.14	0.9	10.22
1.13	1.14	1.013	1.16	1.35	16.7
1.15	1.17	1.012	1.19	1.99	24.9
1.17	1.18	1.010	1.2	0.4	33.4

agents if  $I_e$  is set one percentage point higher than its optimal level. Again, the implied welfare loss is quite small relative to per-capita income. However, the previous caveats still apply. Finally, note once again that the Friedman rule is not optimal.

**Example 4** ( $\pi_e \neq \pi_o$ ). As before, the function  $\bar{W}(I_e)$  is concave, and it attains a maximum at  $I_e = 1.081$  (implying  $I_o = 1.0609$ ). Here, an optimum has  $I_e/I_o = 1.019$ . Thus, the optimal policy in the presence of seasonal variation in liquidity demand does closely approximate eliminating seasonality from nominal interest rates.

Table 3 shows how the optimal seasonal pattern of nominal interest rates varies with the “average” nominal rate of interest. As is clear from the table, this pattern is essentially independent of the average of the nominal rate of interest. Also, as before, the welfare losses associated with minor deviations from the optimal policy are quite small. Finally, once again, a pursuit of the Friedman rule is suboptimal in this example.

To summarize, eliminating seasonality in the nominal rate of interest is close to an optimal policy when the source of seasonality is driven by the demand for liquidity needs. When seasonality derives from exogenous fluctuations in income, smoothing nominal rates of interest or inflation seasonally is a relatively “bad”

policy. In fact, nominal interest rates should display seasonal fluctuations that are large relative to seasonal fluctuations in income. Finally, seasonality driven by exogenous technological fluctuations that affect real rates of return should be met by allowing the nominal rate of interest to vary seasonally. However, seasonal movements in the nominal rate of interest should be small relative to exogenous seasonal impulses. Interestingly, these results on the optimal seasonal time path for the nominal rates of interest are nearly seasonal analogs of Poole's (1970) results on the optimal choice of interest rate versus money supply rules.

## 6. CONCLUSIONS

From a practical perspective, seasonal fluctuations are quite important. They equal or exceed cyclical fluctuations in magnitude. In addition, from the perspective of monetary policy, the presence of seasonality poses an important question. How should seasonal fluctuations be dealt with, from a policy perspective? Moreover, should monetary policy seek to insulate nominal rates of interest, the rate of inflation, or the rate of money growth from seasonal influences?

Historically, at least in the United States, seasonal fluctuations had their most dramatic manifestation in the form of recurring strains on the financial system. Therefore, we considered these issues in a framework that places the banking system in the forefront. In this context, we have demonstrated that policies intended to insulate nominal rates of interest, or the rate of inflation, from seasonal fluctuations are formulas for generating an indeterminacy of equilibrium. They thus fail Friedman's test that desirable policies should not inject additional sources of economic noise. In contrast, the policy of maintaining a constant rate of monetary growth does not suffer from this problem.

We have also shown that, within any particular monetary policy regime, settings of the monetary policy target in question will generally affect the magnitude of seasonal fluctuations in all endogenous variables (other than the one targeted). This is an important result because it suggests the existence of a previously unexplored channel by which monetary policy might have real effects.

Finally, we have shown that the case for smoothing nominal rates of interest depends on the source of seasonal impulses. If seasonality derives largely from shifts in the demand schedule for money, insulating nominal interest rates from seasonal fluctuations is not far from an optimal policy. However, if seasonal fluctuations are driven by real factors, such as changes in technology or endowments, the case for eliminating seasonals in nominal rates of interest is much weaker. This suggests that the current Federal Reserve policy, which does eliminate seasonality in nominal rates of interest, can only be justified if seasonal fluctuations derive largely from shifts in the demand for money. It would be interesting to see the case articulated that this is, in fact, the situation we confront.



## NOTES

1. In other words, the currency deposit ratio was high when crops were being sent to market.
2. Important exceptions to this statement include Sargent and Wallace (1982), Mankiw and Miron (1991), Chatterjee (1997), Braun and Evans (1998), and Liu (2000).
3. This assumption is relatively innocuous. The consequences of relaxing this assumption are described by Gomis-Poqueras and Smith (2002).
4. The assumption that  $\rho < 1$  will imply that banks' demands for reserves are a decreasing function of the nominal rate of interest.
5. As noted previously, our derivations are predicated on currency being dominated in the rate of return, so that  $I_t > 1$  holds for all  $t$ . Environments similar to ours, where the Friedman rule ( $I_t \equiv 1$ ) is shown to be suboptimal, are described by Paal and Smith (2000) and Smith (2002). In addition, as we show, the Friedman rule may be suboptimal in this environment as well.
6. Seasonality that affects real rates of return is very similar to that considered by Sargent and Wallace (1982) and Champ et al. (1996).
7. It continues to be the case, under this policy, that  $M_0$  is given as an initial condition.
8. The policy of maintaining a constant rate of inflation would work in a similar fashion—and, in fact, in an identical fashion—if  $x_e = x_o$ . Hence, we do not explicitly consider that policy here.
9. As we demonstrate later, this condition will be satisfied for empirically plausible parameter values.
10. Small seasonal variation in real interest rates is the empirically plausible case since, as observed in the Introduction, a policy of eliminating seasonals in the nominal rate of interest also seems to virtually eliminate seasonality in the rate of inflation.
11. The countries are Austria, Belgium, Canada, Finland, France, Germany, Greece, Italy, Japan, Luxembourg, Portugal, Spain, Sweden, and the United States from 1961 to 1987.
12. The countries are Australia, Austria, Belgium, Canada, Finland, France, Germany, Ireland, Italy, Japan, Luxembourg, Netherlands, Norway, Spain, Sweden, and the United States from 1961 to 1987.
13. See Schreft and Smith (2001) for a justification of this elasticity in a more modern context.

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