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Multi-legged walking machine body design

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SUMMARY

The effects of body geometry on walking machine performance have been investigated, and a body design procedure proposed. The relationships between static workspace, body-geometry and installed joint torques have been derived. A body design procedure that uses this data is then described, and two design examples discussed. The procedure results in a body geometry which minimises the installed joint torques, and hence the machine weight, for the desired workspace area.

KEYWORDS: Walking machine; Multi-legged; Body design; Static workplace.

1 INTRODUCTION

There have been many publications on the study of walking machines. But much of the work has concentrated on navigation, gait generation and control.[1–3] When prototypes have been developed it has been assumed that the enabling technologies are in place and the problem is one of applying them. In practice this is far from the truth, as the abysmal performance of existing prototypes testifies. There has been little work on the fundamentals of walking machine design and the emphasis has been on the leg mechanisms without considering the effect of the body geometry on machine performance.[4–7]

This paper focuses on the mechanical design of multi-legged walking machines, and in particular on the design of body geometry. Mechanical design includes body geometry design, leg mechanism design, and joint strength distribution (actuator sizing). The design of body geometry is closely linked to joint strength distribution, and leads to a leg specification, which can be used as the starting point for leg mechanism design. Important performance measures for multi-legged walking machines include power to weight ratio; efficiency; body and foot workspaces (reachable areas) and stability. Two parameters that have a very direct effect on these measures are machine weight and workspace. In other words, for a given total load (machine weight plus payload), a design that is light and has a large workspace is likely to perform well in all respects. A machine's weight is directly related to the strength of the installed joint actuators; therefore in this paper the performance measures adopted will be the installed joint torques and workspace area. In this context, the workspace is the range over which a reference point on the machine's body (usually the cg) can be moved without violating a kinematic or static constraint. The workspace is generally limited to two dimensions by assuming that the machine remains at constant height and parallel to the ground.

In previous work the focus has been on leg mechanism design without considering the relationships between leg design and body design. Nothing has been published on the effect of body geometry on the installed joint torques, and hence on the performance of machines. If the design procedure is to include both leg and body design, two approaches can be envisaged. Either body and leg design are treated as a single problem, or they are treated as separate problems and tackled sequentially. In the former case, all of the body and leg design parameters are considered together. The advantage of this approach is that a truly optimal design could be sought. However, the sheer number of parameters and the difficulty of defining a sensible objective function make this approach rather impractical.

In this paper, the second approach has been adopted, that is the body design problem has been decoupled from the leg design problem. The proposed procedure considers body design first by treating the legs as "black boxes". In other words, the capability of the legs to support the body is defined without specifying the actual design. When treated as a "black box", a leg can be specified by defining:—

a) the compressive force it can support along the axis joining the leg-body attachment point (hip) and the foot
b) the hip torque it can support, that is, the installed joint torques at the hip.
c) the nominal position of the foot relative to the hip, i.e., the centre of the foot's kinematic workspace.
d) the kinematic workspace of the foot

The installed torque at the knee joint or its equivalent is not defined as it depends upon the particular leg design.

So the body design problem becomes one of finding the body geometry and "black box" leg specification that give the best machine performance. As explained earlier, in this paper the performance measures will be static workspace area and installed hip-joint torques (a surrogate for machine weight). To simplify the design problem, the following assumptions and definitions are made:

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a) The "installed joint torque" is defined to be the maximum resultant torque available at the hip joints; no limit is imposed on leg compression. Although the machine weight will also be a function of the leg's compressive strength, this is highly dependent on the particular leg design. Fortunately, if hip strength is reduced whilst neglecting knee strength, this reduces the size and weight of two actuators at the expense of one. Also, the knee strength and hence leg weight can be considered at the leg design stage, as are the kinematic constraints.
b) There are no kinematic constraints on the body's workspace. The implication of this is that the required leg kinematic workspace will be determined by the body design and is achievable.
c) The body geometry is defined by a single parameter, the foot-body ratio, denoted by η_k. The foot–body ratio is defined to be the ratio of the hip location to the foot location when the machine is in its centre position, that is when the hip and foot location polygons are concentric. This definition assumes that the hip and foot location polygons are similar in shape, one being a scaled version of the other, and that the hip and foot locations are measured from the centre of both polygons.

In the following sections: the relationships between workspace, installed joint torque, and the foot–body ratio are established; and a body design procedure is proposed based on those relationships. The workspaces are determined using existing procedures developed by the authors.[8] What is new in this paper is the body design study itself and in particular the effect of foot–body ratio on machine performance.

2. WORKSPACE DETERMINATION

This section describes the procedure used for static workspace determination. The static workspace of the body is constrained by both environmental and mechanical design constraints. The environmental constraints limit the ground reactions at the feet, the normal ground reaction must be compressive and the tangential ground reaction must not cause foot slip. The mechanical-design constraints are, for statics purposes, the installed joint torques. Little has been published on the static workspaces of limbed vehicles. The authors have studied the static workspace of a walker designed based on the insect leg and the stewart platform.[9] But, the static workspace investigation method proposed in reference 9 is limited to a statically determinate walker. Because most multi-legged walking machines are statically indeterminate, different ground force decomposition methods lead to different joint torques and hence various sizes and shapes of static workspace.

So, to establish whether a given walker position is truly feasible it is necessary to identify whether any equilibrium solution exists that does not break the constraints. In this way, the true (largest) workspace could be identified by searching the potential workspace, and at each point searching the equilibrium solution space for a feasible solution. This would have to be repeated for different values of installed joint torque to produce a family of workspaces. It is apparent that the computation involved would be excessive. Based on the method in reference 9, the authros have proposed an alternative approach,[8] which is summarised below. For the convenience of analysis, the following definitions are given:[8]

The tipping polygon is the static workspace of the body obtained when only the tipping constraint is applied, that is, the normal ground reactions are constrained to be compressive. The tipping polygon is the largest workspace possible for a machine that cannot grip the surface it walks over. When the external wrench is the gravity force only, and the machine is moving over a horizontal surface, the tipping polygon coincides with the foot support polygon. For a general external wrench, the tipping polygon is the same size and shape as the support polygon but is offset from it. In this paper, the tipping polygon boundary is simply called **the tipping boundary.**

The maximum joint torque, M_{kcf}, is defined as the largest absolute value of the joint torques, $m_1, \ldots, m_N$, of the N legs on the ground, for one of the equilibrium solutions for one particular geometrical configuration of the machine. $M_{kcf} = \max\{\text{abs}(m_1), \ldots, \text{abs}(m_N)\}$, where: $k = 1 \cdots N_k$ corresponds to one set of the design parameters, and N_k is the number of all possible sets; $c = 1 \cdots N_c$ corresponds to one geometrical configuration, where N_c is the number of all possible configurations; $f = 1 \cdots N_f$ corresponds to one equilibrium solution, where N_f is the number of possible solutions for a given geometrical configuration.

The minimised joint torque, M_{kc}, is defined as the minimum value of the maximum joint torque for a given geometrical configuration of the machine, $M_{kc} = \min\{M_{kc1}, \ldots, M_{kcN_f}\}$. In other words, it corresponds to the optimum equilibrium solution.

The installed joint torque, M_k, is defined as the maximum of the minimised joint torques for all required geometrical configurations for a given set of design parameters, $M_k = \max\{M_{k1}, \ldots, M_{kN_c}\}$. It is the joint torque required to achieve the desired workspace.

The optimum installed joint torque, $M_{\min}$, is defined as the minimum value of the installed joint torques for all possible sets of the design parameters, $M_{\min} = \min\{M_1, \ldots, M_{N_k}\}$. In other words, it corresponds to the optimum design.

A grid of points is selected that covers the potential workspace and at each point the installed joint torque is found that would make that point a limiting position, i.e., on a workspace boundary. This is the minimised joint torque, M_{kc}, as defined above. So a grid of data points is produced, each one having a minimised joint torque associated with it. Intermediate points for other minimised joint torques can be obtained by interpolation, and the resulting data used to construct a set of workspaces. This procedure reduces the amount of computation required substantially because the potential workspace is covered once only and the grid of points need not be closely spaced.

The grid of points should include the boundary of the machine tipping polygon. This workspace is important because it is the maximum possible for a machine that cannot grip the surface it walks over. In addition, a reasonable spread of points is required inside the tipping polygon so that the much smaller workspaces, corresponding to low joint strengths, can be constructed.

In this paper, a walking machine supported by three legs is used as the example because many multi-legged walkers have either four or six legs and the popular gaits are the wave gait for the quadrupeds and the alternating tripod gait for the hexapods. The body is supported by three legs during the key phases of these two gaits. In this case, for a general wrench, a 3 dimensional equilibrium solution space must be searched to find the minimised joint torque at each point.

To provide an illustrative example of the methodology with less computational effort, the following assumptions were made.

(i) The machine is supported by three legs, as is often the case for quadrupeds and hexapods.
(ii) The foot support polygon is an equilateral triangle. This considerably eases the burden of computation whilst still providing a realistic example.
(iii) The machine is standing on a horizontal surface and the external wrench is the gravity force, $\boldsymbol{W}_b = [0, 0, -100, 0, 0, 0]^T$. In other words, the total load (payload plus machine weight) is 100 N.
(iv) The frictional coefficient between the foot and the ground is $\mu = 0.7$;
(v) The height of the machine is 180 (mm), the position vectors of feet 1, 2 and 3 in the earth frame are $r_{s1} = [115.45, -200, 0]^T$ (mm), $r_{s2} = [115.45, 200, 0]^T$ (mm) and $r_{s3} = [-230.9, 0, 0]^T$ (mm) as shown in Figure 1.

The resulting symmetry means that by finding solutions along just one tipping boundary and one centre line, sufficient grid points are generated to construct approximate workspaces, as shown in Figure 1.

Figures 2 and 3 show the minimised joint torque along the centre line and tipping boundary respectively, for eleven different foot-body ratios. For given foot positions, when η_k is zero the body size is zero; when η_k is one, the body is identical to the foot support polygon, that means that, on a horizontal surface, the legs are vertical when the body is in its centre position.

As Figures 2 and 3 apply to all three centre lines and all three tipping boundaries, they provide sufficient data to construct the approximate static workspace as follows:

(i) Select an installed joint torque, M_k, and a foot-body ratio η_k;
(ii) Find the displacements on both the $M_{kc} \sim y_{eb}$ and $M_{kc} \sim x_{eb}$ curves, which correspond to the installed joint torque;
(iii) Determine the co-ordinates of the vertices of the static workspace using the displacements found in step 2;
(iv) Draw the static workspace using the co-ordinates determined in step 3.

Unfortunately, this is not adequate for smaller

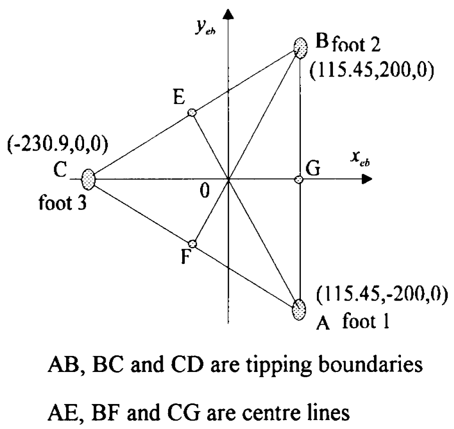

Fig. 1. Equilateral foot support triangle of the numerical example.

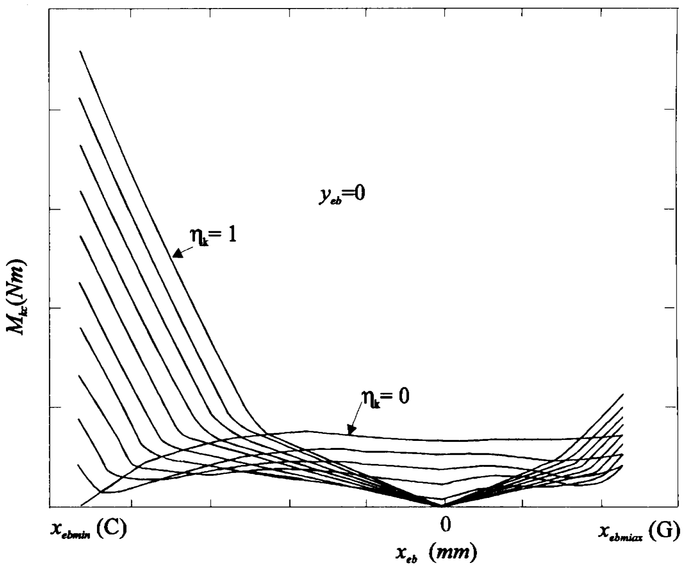

Fig. 2. Minimised joint torque along centre line ($C \rightarrow G$).

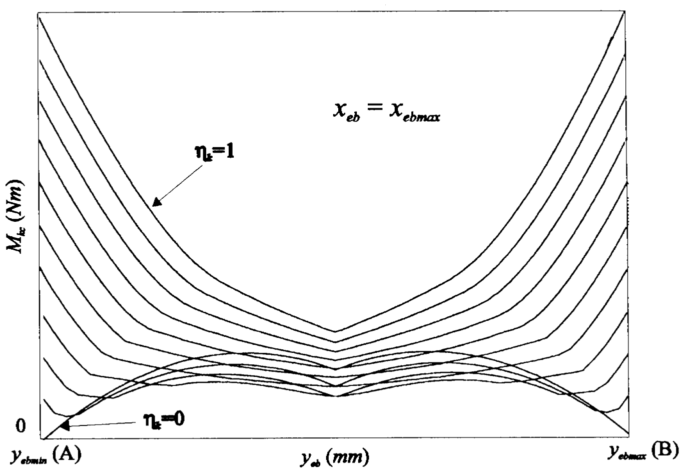

Fig. 3. Minimised joint torque along the tipping boundary ($A \rightarrow B$).

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foot-body ratios because of the complex workspace shape. Therefore, in these cases additional grid points, not covered by Figures 2 and 3, are required.

3. THE RELATIONSHIPS BETWEEN STATIC WORKSPACE, FOOT-BODY RATIO AND INSTALLED JOINT TORQUE

The static workspaces for the example machine, as defined in section 2, are shown in Figures 4–7 for four different foot-body ratios, $\eta_k = 1$ (full body geometry), 0.6 (medium body geometry), 0.15 (small body geometry) and 0 (zero body geometry). In the figures, the full triangle represents the maximum possible workspace, that is the static workspace due to the tipping constraint (compressive normal ground reactions); the other polygons represent the workspaces due to the foot slip constraints and the installed joint torque constraints.

Referring to Figures 2 and 3, it can be seen that there are local maxima in one or both of the M_{kc} curves when $\eta_k < 0.45$. This means that if the installed joint torque is reduced below a critical level, the workspace may consist of several separate parts, as shown in Figure 7(B). In this paper, we assume that only the central workspace is of practical value, that is, the workspace that includes the geometric centre of the tipping polygon. For example, in Figure 7(B) the quoted workspace area would not include the three secondary workspaces at the corners. In Figures 4–6 and 7(A) only the useful (central) workspaces are shown as this makes the figures easier to interpret.

For convenience we define the following torques: the maximum installed joint torque, $M_{k\,\max}$, is the value when the workspace just coincides with the tipping polygon; the minimum installed joint torque, $M_{k\,\min}$, is the critical value below which there is no central workspace that includes the centre of the tipping polygon. Thus, referring to Figure 6, $M_{k\,\min} = 2.35$ Nm and $M_{k\,\max} = 3.87$ Nm for $\eta_k = 0.15$.

From Figures 2–6, it can be seen that

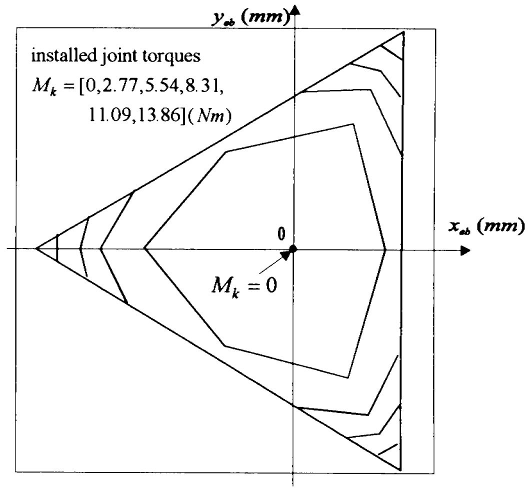

Fig. 5. Static workspaces for different installed joint torques and $\eta_k = 0.6$.

(i) The difference between $M_{k\,\max}$ and $M_{k\,\max}$ reduces as the foot body ratio is reduced (Figures 4–6).

(ii) The maximum installed joint torque, $M_{k\,\max}$, is greatest when $\eta_k = 1$ ($M_{k\,\max} = 23$ Nm); reduces to $M_{k\,\max} = 3.87$ Nm when $\eta_k \approx 0.15$; and increases again to $M_{k\,\max} = 4.81$ Nm when $\eta_k = 0$. It is apparent from this that, if a static workspace equal to the tipping polygon is desired, then a small foot body ratio ($\eta_k \approx 0.15$) should be adopted.

(iii) For the large foot-body ratios, as the installed joint torque is reduced from $M_{k\,\max}$ to $M_{k\,\min}$, workspace is lost at the three corners of the tipping polygon.

Using Figures 4–7 the relationships between installed joint torque, M_k, workspace area, A_p, and foot-body ratio, η_k, can be obtained (Figure 8). Workspace area is given as a percentage of the tipping polygon area. The dotted lines are obtained if the total workspace area including secondary workspaces is plotted, while the solid lines correspond to the central workspace only, and are the basis for what follows. Figure 9 shows the projection of Figure 8 onto the $M_k \sim A_p$ plane. Figure 10 is a contour plot of M_k, with A_p and η_k as the axes.

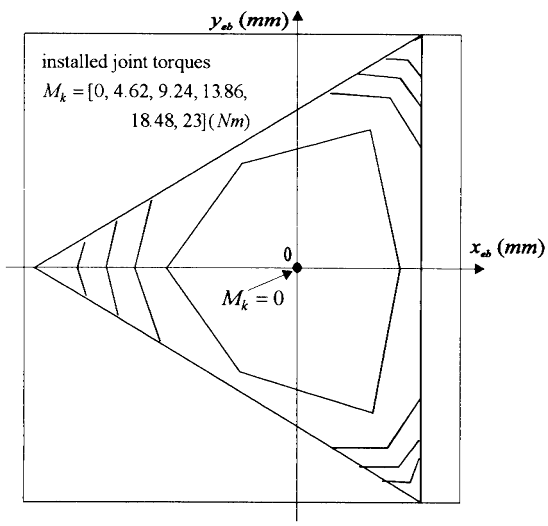

Fig. 4. Static workspaces for different installed joint torques and $\eta_k = 1$.

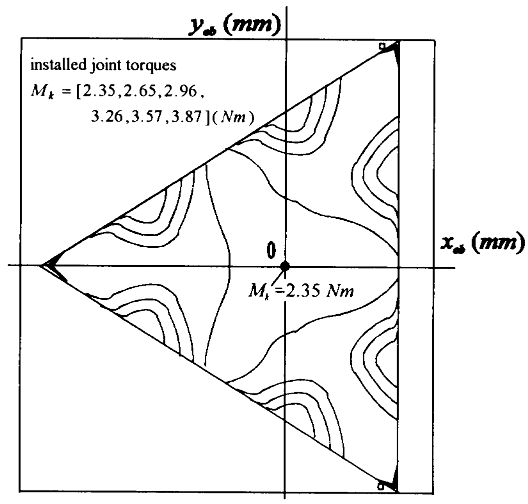

Fig. 6. Static workspaces for different installed joint torques and $\eta_k = 0.15$.

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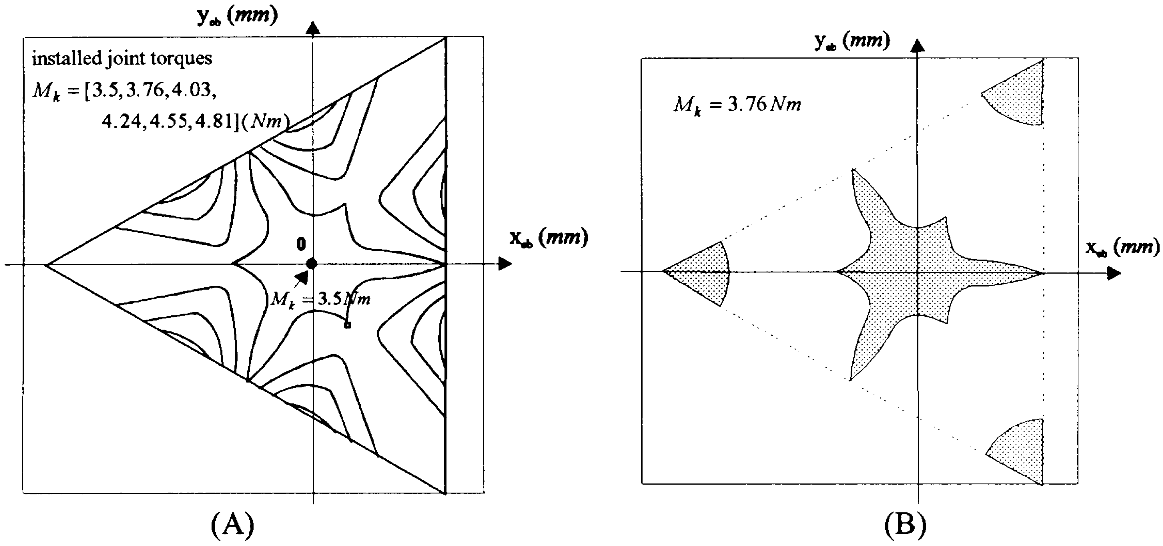

Fig. 7. Static workspaces for different installed joint torques and zero body size, $\eta_k = 0$.

4. BODY DESIGN

In this section, the use of Figures 8 to 10 for body design is discussed and a design procedure proposed. As explained earlier, in this context, body design is taken to mean the selection of an appropriate foot-body ratio and the corresponding leg specification. The leg specification depends on the installed hip-joint torque and body static workspace. Given these, the required compressive strength of the leg and the required kinematic workspace of the foot can then be determined.

To demonstrate the use of Figures 8 to 10, two extreme design requirements are considered. Firstly, consider a machine that need not move quickly or be particularly agile, but must have a good payload capability. In this case, reducing installed joint torque and hence machine weight for a given total load is more important than achieving a large workspace. By accepting a workspace are that is only 20% of the tipping polygon ($A_p = 20\%$), it is possible to reduce the installed hip-joint torque to approximately 2.38 Nm (Point A on Figure 10) by selecting a foot-body ratio of $\eta_k \approx 0.4$.

Secondly, consider a machine that must be agile and fast. For this reason it is decided that the full tipping polygon must be available ($A_p = 100\%$). In this case, the optimum foot-body ratio, that minimises installed joint torque, is $\eta_k \approx 0.15$ (point B on Figure 10); the corresponding installed joint torque is 3.87 Nm.

Referring to Figure 9, it is apparent that substantial reductions in installed joint torque (machine weight) can be obtained by relatively small reductions in workspace area for some foot-body ratios. This means that even when agility is required, it may be better to accept less than $A_p = 100\%$. For example, when $\eta_k \approx 0.4$, by accepting $A_p = 90\%$, the installed joint torque can be reduced by 33% from 8.66 to 5.77 Nm (Points C and D on Figure 10). The larger the foot-body ratio, the more the reduction of the installed joint torque. When $\eta_k \approx 1$, by accepting $A_p = 90\%$, the installed joint torque can be reduced by 45% from 23 Nm to 12.5 Nm (Points E and F on Figure 10).

From the above discussion it is possible to envisage a

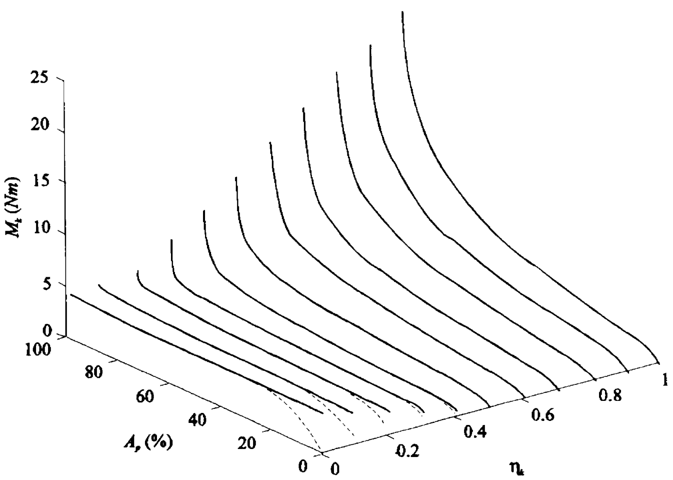

Fig. 8. $M_k \sim A_p \sim \eta_k$ plot, $z_{ab} = 180$ mm, $W_b = [0, 0, -100, 0, 0, 0]^T$.

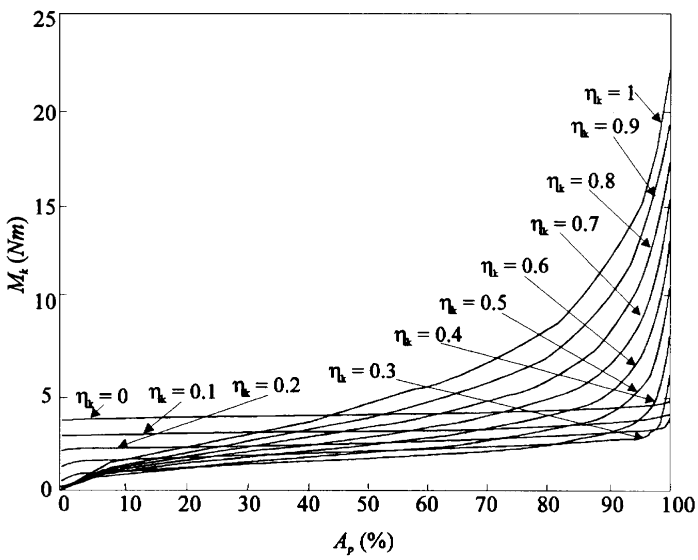

Fig. 9. $M_k \sim A_p$ for different η_k plot, $z_{eb} = 180$ mm, $W_b = [0, 0, -100, 0, 0, 0]^T$.

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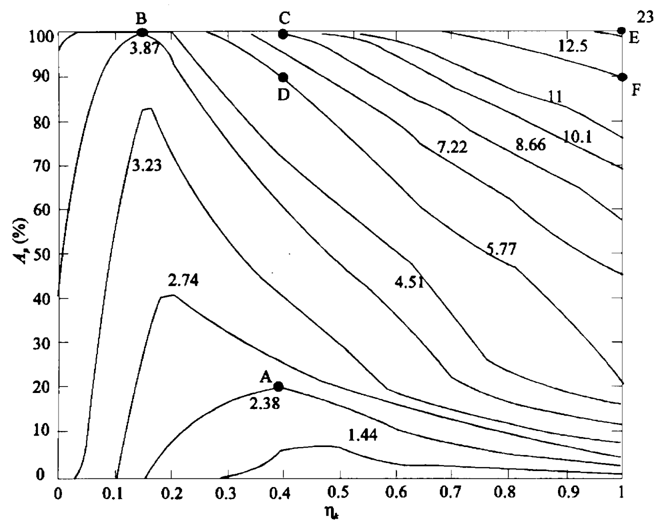

Fig. 10. M_k contour plot with A_p and η_k as axes, $z_{eb} = 180$ mm, $W_b = [0, 0, -100, 0, 0]^T$.

design procedure based on Figures 8 to 10; in broad terms this would include the following steps;

(i) From consideration of the basic design requirements (payload capability, agility, speed etc.), make an initial choice of workspace area, A_p.
(ii) From the figures establish the optimum foot-body ratio that minimises the installed joint torque (machine weight).
(iii) Study the sensitivity of the installed joint torque to changes in A_p. If significant reductions are possible without losing too much workspace, then repeats steps 1 and 2.
(iv) Having estabished a desired workspace area, A_p, and the corresponding foot–body ratio and installed hip-joint torque, derive the remaining leg specification parameters.

5. CONCLUSIONS

This paper has discussed the effects of body geometry on the performance of walking machines, and proposed a body design procedure. Firstly, the relationships between static workspace, foot–body ratio and installed joint torque were derived; this was done for a particular example, however, it is believed that the results are representative of the general case. Secondly, a design procedure is proposed that uses these relationships to select an optimal foot–body ratio for the chosen application. This also provides enough information to completely define the required leg specification, that can then be used as the basis for the leg mechanism design.

Two design examples are discussed: a machine that need not move quickly or be particularly agile, but must have a good payload capability; and a machine that must be agile and fast. In both cases possible design points are identified (points A and B on Figure 10). It is apparent that a significant reduction in installed joint torque, and hence machine weight, can be obtained by the proper selection of foot–body ratio, η_k and static workspace area.

The data presented here is based on the example defined at the end of section 2. Although this is sufficient to demonstrate the proposed design procedure, it is also necessary to investigate the effects of changes in machine geometry other than foot–body ratio; changes in the external wrench; and changes in the foot–ground friction coefficient. The results of current work in these areas are shortly to be published. Preliminary results indicate that increasing the height of the machine allows the installed joint torques to be reduced (for the vertical gravity wrench). This is to be expected, because the increased moment arm means that varying the tangential ground reactions has a greater effect on the installed joint torques. In practice the height will be limited by other constraints, probably related to the leg mechanism design. The authors have also shown that although changing the external wrench, or the foot-ground friction coefficient, does change the results, the best machine geometry is likely to be similar to that indicated by the data presented in this paper.

5. ACKNOWLEDGEMENT

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