Laser and Particle Beams (2001), *19*, 597–604. Printed in the USA.

DOI: 10.1017.S0263034601194061

Plasma wake-field excitation by relativistic electron bunches and charged particle acceleration in the presence of external magnetic field

V.A. BALAKIREV,[1] V.I. KARAS',[1] I.V. KARAS',[1] AND V.D. LEVCHENKO[2]
[1]National Science Center "Kharkov Institute of Physics and Technology," Kharkov, 61108, Ukraine
[2]Keldysh Institute of Applied Mathematics of Russian Academy of Sciences, Moscow A-47, 125047, Russia

(Received 21 December 2000; Accepted 11 May 2001)

Abstract

High-amplitude plasma wake waves are excited by high-density relativistic electron bunches (REB) moving in a plasma. The wake-fields can be used to accelerate charged particles, to serve as electrostatic wigglers in plasma free-electron lasers (FEL), and also can find many other applications. The electromagnetic fields in the region occupied by the bunch control the dynamics of the bunch itself. This paper presents the results of 2.5-dimensional numerical simulation of the modulation of a long REB in a plasma, the excitation of wake-fields by bunches in a plasma, in particular, in magnetoactive plasma. The previous one-dimensional study has shown that the density-profile modulation of a long bunch moving in plasma results in the growth of the coherent wake-wave amplitude. The bunch modulation occurs at the plasma frequency. The present study is concerned with the REB motion, taking into account the plasma and REB nonlinearities. It is demonstrated that the nonlinear REB/plasma dynamics exerts primary effect on both the REB self-modulation and the wake-field excitation by the bunches formed. We have demonstrated that a multiple excess of the accelerated bunch energy $\varepsilon_{\max}$ over the energy of the exciting REB is possible in a magnetoactive plasma for a certain relationship between the parameters of the "plasma–bunch–magnetic field" system (owing to a hybrid volume–surface character of REB-excited wake-fields).

Keywords: Charged particles acceleration; Nonlinear phenomena numerical simulation; Plasma accelerators; Relativistic electron bunches; Wake-fields excitation

1. INTRODUCTION

The ideas of using collective fields for acceleration in the plasma and noncompensated charged beams were stated as early by Budker (1956), Fainberg (1956), and Veksler (1956). The appearance and development of new powerful energy sources such as lasers, high-current relativistic electron beams, super-high-power microwave generators gave another impetus to the development of the collective methods of charged particle acceleration. As a result (Tajima & Dawson, 1979; Chen *et al.*, 1985), there appeared new modifications of the method of charged particle acceleration in a plasma by charge density waves (see reviews by Fainberg, 1987, 1994, 1997, 2000; Dawson, 1999; Power *et al.*, 1999; Suk *et al.*, 2001), where it was proposed that the accelerating fields should be excited by laser pules and relativistic electron bunches. The charged particle acceleration by charge density waves in a plasma and in uncompensated charged beams (Fainberg, 1956) appears to be a most promising trend in the collective methods of acceleration. The variable part of the charge density can be made to be very high (up to n_0, where n_0 is the unperturbed plasma density); therefore, the accelerating fields can reach 10^7 and 10^9 V/cm. Chen *et al.* (1985) have proposed a modification of the Fainberg (1956) acceleration method, consisting of using a train of bunches. Katsouleas (1986) has considered electron bunches with different profiles, namely, a bunch with a slow build-up in the density and its very quick fall-off, and also the bunch with the Gaussian-type distribution for different rise and fall-off times. It was established (Katsouleas, 1986) that the use of these nonsymmetric bunches instead of symmetric ones can provide the accelerating field E_{ac} value to be many times (10 to 20) higher than the retarding field E_{st} value. The so-called transformation coefficient $R_E = E_{ac}/E_{st} =$

Address correspondence and reprint requests to: Vladimir Artemovich Balakirev, National Science Center "Kharkov Institute of Physics and Technology," 1 Akademicheskaya Street, Kharkov, 61108, Ukraine. E-mail: karas@kipt.kharkov.ua

$\Delta\gamma_{ac}\gamma_b^{-1}$ is equal to $2\pi N$, where N corresponds to the number of wavelengths along the bunch length, γ_b is relativistic factor of a bunch. The excitation of nonlinear stationary waves in the plasma by a periodic train of electron bunches has been studied by Amatuni *et al.* (1979) and Rosenzweig (1990), where it was shown that the electric field of the wave in the plasma increases with γ_b at commensurable plasma/beam densities. The experiments undertaken in Rosenzweig (1990) on wake-field acceleration have demonstrated the importance of three-dimensional effects.

Usually, we consider two different regimes with high amplitudes of plasma wake-fields that are employed in the accelerator physics. The first regime makes use of an extended short beam, then the high-amplitude waves excited by this beam and having high-gradient longitudinal electric fields can be used to accelerate other bunches. In the second case, a strong focusing can be attained with a long narrow beam, making use of its intrinsic magnetic field which is unbalanced because of space charge compensation by the plasma.

2. 2.5-D NUMERICAL MODELING OF THE FORMATION OF A PLASMA CHANNEL AND OF THE LONG REB SELF-MODULATION

The excitation of wake-fields is investigated with an aid of the 2D3V axially symmetric version of the SUR code being, in turn, a further development of the COMPASS code (Batishchev *et al.*, 1994a, 1994b). Earlier, this code had been used to simulate the induction accelerator (Karas' *et al.*, 1992), the modulated relativistic electron beam (Batishchev *et al.*, 1993), and a single REB or a train of these bunches in a plasma (Batishchev *et al.*, 1994a, 1994b; Karas' *et al.*, 1996, 1997, 1998, 2000a).

Note that, in those studies (similar to the experiments described in Rosenzweig, 1990) the initial bunch radius $r_{b0} = R_0$ and the length L_0 of the bunch were smaller than skin depth $lc = c/\omega_{pe}$ (ω_{pe} is the Langmuir frequency) and the densities of the relativistic electron bunch and plasma were bounded varying. The numerical simulation carried out in Karas' *et al.* (1992, 1996) showed that the radius of the bunch propagating in a plasma varies considerably, which substantially changes the bunch density (by more than an order of magnitude) and the field excited by this bunch. It was also shown that the amplitudes of the axial and radial electric fields increase as each subsequent bunch is injected into the plasma. However unlike in the rigid bunch approximation (Keinigs & Jones, 1987), the increase in the field amplitudes is not proportional to the number of injected bunches.

2.1. Mathematical model and methods

The REB dynamics is described by the relativistic Belyaev–Budker equations for the distribution functions $f_a(\vec{r},\vec{p})$ of plasma particles of each species and by the Maxwell equations for the self-consistent electric E and magnetic B fields. We assume that, initially, a cold two-component background plasma ($m_i/m_e = 1840$, where m_i and m_e are the ion and electron masses) fills the entire region $[0,L]\times[0,R]$, where $L = 100$ cm and $R = 10$ cm. To analyze the dependence of the amplitude of the excited fields on the number of bunches injected into plasma, we carried out a series of calculations. A finite sequence of REB, which are specified by the expression

$$n_n(r,z,t) = n_b\theta(R_0 - r)\theta(v_b t - z + (n-1)\lambda_p)$$
$$\times\,\theta(z - v_p t + Z + (n-1)\lambda_p).$$

Here n is the number of the injected bunch; $V_b = c\sqrt{1 - 1/\gamma_b^2}$ is the bunch velocity; c is the speed of light; the initial bunch sizes are equal to 0.4 cm $\times$ (0.1–0.5) cm; n_b is the mean density of a relativistic electron bunch; $\lambda_p = 2\pi c/\omega_{pe}$. The scale on which the electric and magnetic fields vary is $m_e c\omega_{pe}/e$. We assume that the plasma/bunch particles escape from the calculated region through the $z = 0$ and $z = Z$ boundary surfaces and are elastically reflected from the $r = R$ surface. We also assume that cold background electrons and ions can return to the region under consideration from buffer zones $z < 0$ and $z > Z$. The boundary conditions for the fields correspond to the metal wall at the cylindrical surface $r = R$ and free emission of electromagnetic waves from the right and left plasma boundaries. The weight of the model particles was a function of the radial coordinate, and the total number of these particles was about 10^6. All the calculations were carried out on a Pentium-166 personal computer using the modified particle-in-cell simulation algorithm.

2.2.1. Numerical simulation of the formation of a plasma channel due to ion redistribution

Barov and Rosenzweig (1994) pointed out that, in the immobile-ion approximation, a channel with a neutralized charge can arise in the plasma when the background electrons escape from the region through which the REBs propagate. It has already been noted that, to analyze the formation of an ion channel in a realistic situation, we must take into account, along with the electron motion, the ion motion in self-consistent electromagnetic fields. We will show that the ion dynamics plays a key role in the formation of an ion channel.

Figures 1 and 2 illustrate the formation of an ion channel. The parameters of the plasma channel are governed by the ratio between the bunch and plasma densities and by the ratio between the bunch radius and the collisionless skin depth. Also, Figure 1 shows that the effective sizes of the plasma channel and its depth increase monotonically with time and in the direction against the z-axis. From Figure 2, we can see that substantial variations of electron density are associated only with the wake-field wave and have no permanent component contrary to ion density distribution.

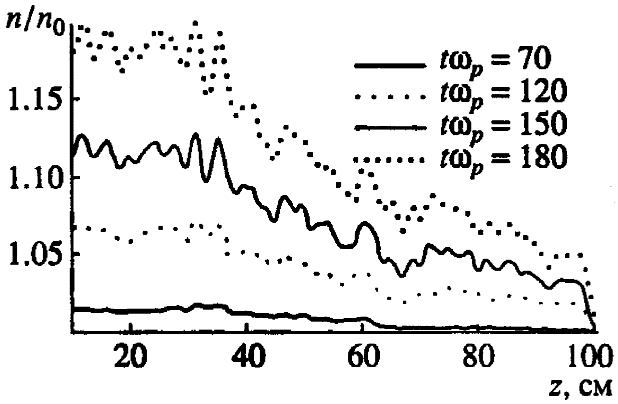

Fig. 1. Ion density versus the axial z-coordinate for $r = 0.5$ cm.

2.2.2. Self-modulation of a long relativistic electron bunch

Apart from the transverse forces, the bunch particles are also influenced by powerful longitudinal forces on the side of electric wake fields. The longitudinal fields will give rise to a longitudinal modulation of the electron bunch, that is, to a splitting of an originally uniform bunch into microbunches with a modulation period $\lambda_p = 2\pi c/\omega_{pe} = 3.36 \times 10^6 n_0^{-1/2}$ cm. In particular, in the plasma with a particle density of 10^{16} cm^{-3}, the modulation period is 0.3 mm. The effect of longitudinal REB modulation by wake fields can be used for developing plasma modulators of dense electron beams. It is pertinent to note one more feature of this phenomenon. Since the modulation frequency is coincident with the plasma frequency, the wake-fields of microbunches are then combined coherently. Therefore, the electron bunch modulation will involve an increase in the amplitude of the wake-field behind the bunch. This effect opens up a possibility of using long-pulse electron bunches to excite intense wake-fields in a plasma. It is particularly remarkable that the effect of longitudinal modulation at a plasma frequency takes place for a long laser pulse, too (Joshi *et al.*, 1987).

Previously in Balakirev *et al.* (1996), a theoretical study was made into the process of modulation of long electron bunches in a plasma by longitudinal wake-fields. Results were reported there for one-dimensional numerical simulation of nonlinear dynamics of bunch modulation. It was demonstrated in Balakirev *et al.* (1996) that the particle modulation of a long bunch moving in the plasma causes an increase in the wake-field amplitude.

This effect is accounted for by coherent combining of fields excited by microbunches, into which the bunch is split in the course of modulation. The bunch is modulated at a plasma frequency. The investigation of the one-dimensional approximation is justified in the case of great transverse dimensions ($2\pi r_b/\lambda_p \gg 1$). This paper, in part, deals with the 2.5-dimensional numerical simulation of wake-fields excited by a long relativistic electron bunch. Figures 3 to 5 show spatial distributions of the electric charge density of plasma electrons *el.Q*, longitudinal electric field dfld E_z, longitudinal current density of electrons *el.Jhz*, respectively, for the instants of time $t = 60\omega_{pe}^{-1}$ (a) and $t = 100\omega_{pe}^{-1}$ (b).

It is seen from Figure 3 that the longitudinal electric field rapidly grows, reaching $0.8 m_e c\omega_p/e$. Note that the original beam particle density was only 6% of the plasma electron density. The radial electric field E_r also grows, but it reaches a somewhat lower value of $0.4 m_e c\omega_p/e$. It is significant that: (1) the finite length of the initial bunch is responsible

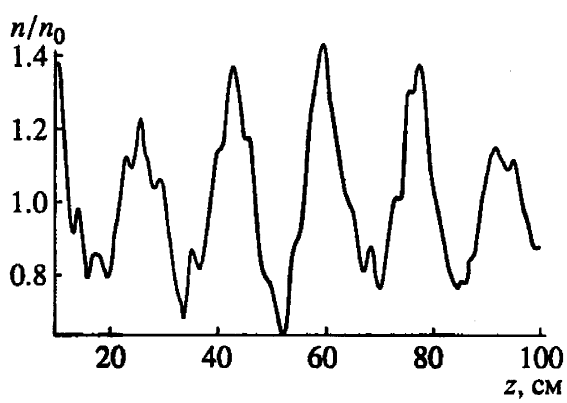

Fig. 2. Electron density versus the axial z-coordinate for $r = 0.5$ cm, $t\omega_{pe} = 180$.

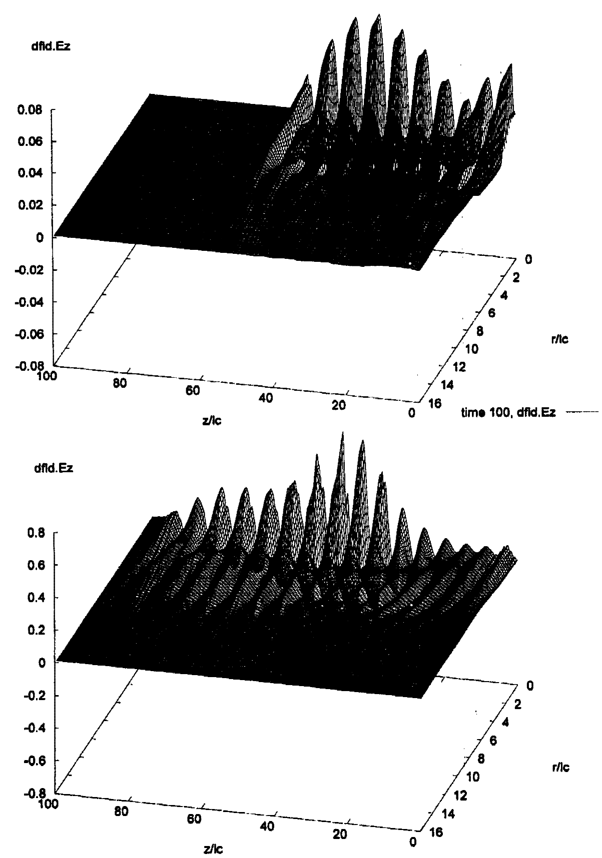

Fig. 3. 2D distribution of the longitudinal electric field dfld E_z for the instances of time $t = 60\omega_{pe}^{-1}$ and $t = 100\omega_{pe}^{-1}$.

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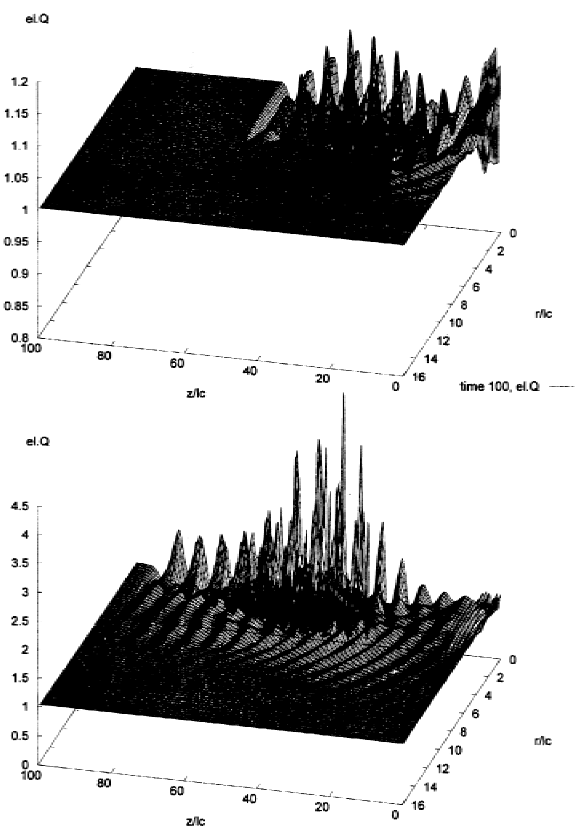

Fig. 4. 2D distribution of the plasma electron density *el.Q* for the instances of time $t = 60\omega_{pe}^{-1}$ and $t = 100\omega_{pe}^{-1}$.

for the formation of the growing electric field; (2) the electric field has a rather high amplitude near the axis, this being due to microbunch pinching; (3) the evolution of the instability, giving rise to microbunches, leads to some decrease in the phase velocity of the perturbed wake wave.

From Figure 4 it is seen that the electric charge density distributions of plasma electrons *el.Q* are similar to the spatial distributions of the longitudinal electric field E_z. The highest density value is attained for the eighth microbunch and is $4.5n_0$. It is of importance to note that the maximum of the beam particle charge density corresponds to the fifth microbunch rather than to the eighth microbunch and is equal to $1.6n_0$, this being two orders of magnitude higher than the initial beam particle density value in the long bunch.

The spatial distribution of the longitudinal current density of plasma electrons *el.Jhz* (Fig. 5) also correlates rigidly with the longitudinal electric field E_z distribution. Here attention must be given to the peak current value for the eighth microbunch, which is two orders of magnitude higher than the initial longitudinal current value or REB particles.

2.3. Results and discussion

The present results show that the nonlinear picture in the plasma–REB system drastically differs from both the initial picture corresponding to the rigid REB and the one by the scenario following from the one-dimensional numerical modulation (cf. Balakirev *et al.*, 1996). This supports in full measure the conclusion given in Rosenzweig (1990) about the necessity of taking into complete account the three-dimensional effects and the nonlinear behavior of both the plasma and the bunch.

The spatial density distributions of REB and plasma electrons obtained for the instances of time $t = 60\omega_{pe}^{-1}$ and $t = 100\omega_{pe}^{-1}$ show that the density ratio n_b/n_0 (the initial value being 0.018) reaches 0.04 as early as at $t = 60\omega_{pe}^{-1}$. At $t = 100\omega_{pe}^{-1}$, the highest beam particle density becomes commensurable with the plasma density, that is, a very strong modulation of beam particle density is observed. The spatial distributions of the longitudinal E_z and transverse E_r electric fields show that the E_z and E_r amplitudes grow owing to the enhancement in the density modulation. At $t = 100\omega_{pe}^{-1}$ the highest longitudinal-field amplitude reaches $0.8m_e c\omega_{pe}/e$, and the highest transverse-field amplitude is equal to $0.4m_e c\omega_{pe}/e$. It is essential that the amplitude growth occurs only within a moderate REB length. Therefore, there is little point in using the REB of the length greater than that corresponding to the highest longitudinal-field amplitude; otherwise no increase in the excited wake-field will be attained.

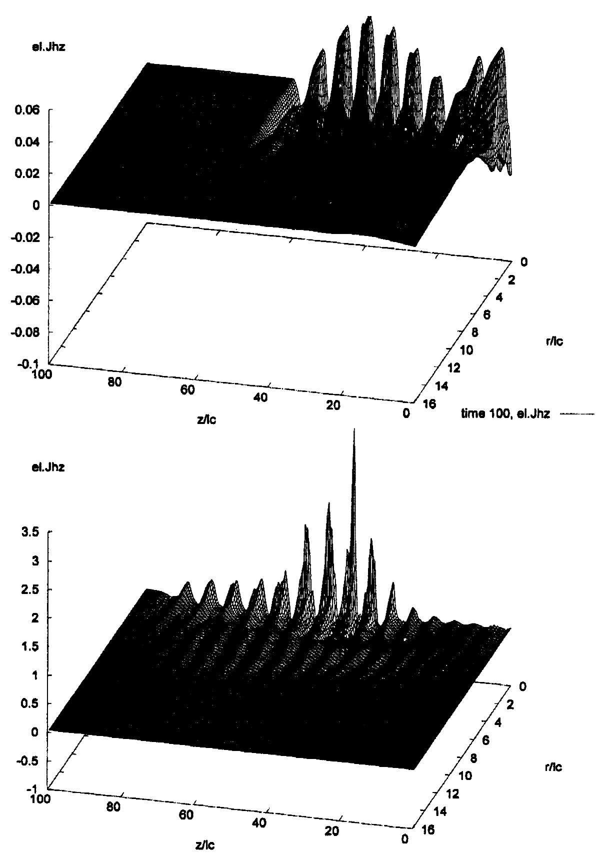

Fig. 5. 2D distribution of the longitudinal electric current density *el.Jhz* for the instances of time $t = 60\omega_{pe}^{-1}$ and $t = 100\omega_{pe}^{-1}$.

The undertaken numerical experiments have demonstrated that the nonlinear dynamics of the particles of plasma components and bunches results in the following effects: (1) the transverse dimension of bunches varies within a very wide range; (2) close to the axis of the system, an ion channel is formed, which is a contributory factor for the stabilization of bunch propagation and the growth of bunch-generated fields; (3) an essential increase in the amplitudes of excited electric fields takes place in the case of a long bunch as a result of its self-modulation. However, bunches of optimum length should be used, since any excess of the optimum length of the bunch fails to provide, even at self-modulation, the growth in the amplitudes of excited electric fields.

3. WAKE-FIELDS IN MAGNETOACTIVE PLASMAS

In our opinion, the excitation of accelerating fields in a plasma by an individual relativistic electron bunch appears most preferable, because it is nonresonant in character, and therefore, is a little sensitive to the longitudinal plasma density inhomogeneity observed in the experiment. Besides, to preclude the electromagnetic filamentation and slipping of instabilities, it is reasonable to use the stabilizing external longitudinal magnetic field (Keinigs & Jones, 1987). Aside from stabilization, the magnetic field also gives rise to a multitude of new wave branches, and this, as is shown below and in Amatuni *et al.* (1995), essentially extends the potentialities of the wake-field method of charged particle acceleration.

This section presents the results from theoretical studies into the processes of REB excitation of wake-fields in a magnetoactive plasma, both in cases of an unbounded plasma and the waveguide with a partial plasma living. Let us define the wake-field by an axially symmetric REB moving in the magnetoactive plasma along the z axis. We neglect the thermal motion of electrons, and assume ions to be immobile.

3.1. REB excitation of wake-fields in unbounded magnetoactive plasma

The solution to the nonuniform set of Maxwell equations has the form

$$H_{\omega z} = i\pi \frac{I_0}{c} T(\omega) \frac{k\varepsilon_2}{\varepsilon_1} \times \int [A_1 G(\lambda_1 r_0, \lambda_1 r) - A_2 G(\lambda_2 r_0, \lambda_2 r)] r_0\, dr_0 \psi(r_0),$$

$$E_{\omega z} = -\pi \frac{I_0}{c} T(\omega) \frac{1}{k_0 \varepsilon_3} \times \int [B_1 G(\lambda_1 r_0, \lambda_1 r) - B_2 G(\lambda_2 r_0, \lambda_2 r)] r_0\, dr_0 \psi(r_0),$$

where

$$A_i = \frac{\lambda_i^2}{\lambda_1^2 - \lambda_2^2}, \qquad B_i = \frac{\lambda_i^2 (p_H^2 - \lambda_i^2)}{\lambda_1^2 - \lambda_2^2}, \qquad k = \omega/V_b,$$

$$k_0 = \omega/c, \qquad \varepsilon_1 = 1 - \frac{\omega_{pe}^2(\omega + i\nu)}{\omega[(\omega + i\nu)^2 - \omega_{He}^2]},$$

$$\varepsilon_2 = \frac{\omega_{pe}^2 \omega_{He}}{\omega[(\omega + i\nu)^2 - \omega_{He}^2]}, \qquad \varepsilon_3 = 1 - \frac{\omega_{pe}^2}{\omega(\omega + i\nu)},$$

ω_{He} is the Larmor frequency of plasma electrons, ν is the effective collision frequency, I_0 is the peak value of the total current of bunch, $\psi(r)$ is the function describing the current distribution in the bunch cross section, the function T describes the longitudinal profile of the bunch (max $T = 1$), t is the time, z and r are, respectively, the longitudinal and radial coordinates, V_b is the bunch velocity, $\mathbf{e_z}$ is the unit vector in the direction of bunch motion and an external magnetic field $\mathbf{H}_0 = H_0 \mathbf{e_z}$, φ is the azimuthal coordinate, $\lambda_{1,2}$ are the transverse wave numbers of ordinary and extraordinary waves, respectively, and are the roots of the biquadratic equation. They can be written as:

$$\lambda_{1,2} = \frac{p_E^2 + p_H^2}{2} \pm \sqrt{\frac{(p_E^2 - p_H^2)^2}{4} + k^2 k_0^2 \varepsilon_3 \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2},$$

$$G(\lambda_i r_0, \lambda_i r) = \begin{cases} H_0^{(1)}(\lambda_1 r) J_0(\lambda_i r_0), & r > r_0 \\ H_0^{(1)}(\lambda_i r_0) J_0(\lambda_i r), & r < r_0, \end{cases}$$

where $J_0(\lambda_i r), H_0^{(1)}(\lambda_i r)$ are the Bessel functions and Hankel functions, respectively.

In the ultrarelativistic case, we can put $V_b = c$. Then, instead of earlier equation, we have

$$\lambda_{1,2}^2 = \frac{k_0^2}{\varepsilon_1} [\varepsilon_3(\varepsilon_1 - 1) \pm \varepsilon_2 \sqrt{\varepsilon_3}].$$

In the frequency range $\omega^2 < \omega_{pe}^2$ the transverse wave numbers are complex.

In the limiting case of a strong magnetic field, $\omega_{He}^2 \gg \omega_{pe}^2$, the expression for transverse wave numbers takes the form

$$\lambda_1^2 = k_0^2 - k^2, \qquad \lambda_2^2 = (k_0^2 - k^2)\varepsilon_3.$$

For simplicity's sake, we consider an infinitely thin annular bunch of radius r_b:

$$T(\omega) = 1 \quad \psi(r) = \frac{\delta(r - r_b)}{r_b}.$$

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Then, for the electric field on the axis of the system $r = 0$, we obtain the following integral representation:

$$E_z = -\frac{1}{2}\frac{Q_0}{V_b^2\gamma_b^2}\int_{-\infty}^{\infty}\exp(-i\omega t)H_0^{(1)}(k_\perp r_b)\omega\, d\omega,$$

where Q_0 is the total charge of bunch, $k_\perp^2 = (k_0^2 - k^2)\varepsilon_3$. Note that in the frequency range $\omega < \omega_{pe}$ the bunch emits an electromagnetic field in the radial direction, because $k_\perp^2 > 0$. We obtain the following expression for the longitudinal component of the electric wake-field:

$$E_z = \frac{2Q_0\omega_{pe}^2}{V_b^2\gamma_b^2}\frac{\tau}{\tau^2+\mu^2}\left(\sin\sqrt{\tau^2+\mu^2} + \frac{\cos\sqrt{\tau^2+\mu^2}}{\sqrt{\tau^2+\mu^2}}\right),$$

where

$$\tau = \omega_{pe}\left(t - \frac{z}{V_b}\right), \qquad \mu = \frac{\omega_{pe} r_b}{V_b\gamma_b}.$$

At large distances behind the bunch, the wake-wave field decreases as $1/\tau$. This is due to the fact that the oscillations in the plasma placed in a strong magnetic field have the finite group velocity. The radiation of plasma waves from the near-axis region causes the wake-field to decrease in the longitudinal direction (Balakirev *et al.*, 2000).

3.2. REB excitation of wake-fields in waveguide with a partial plasma filling

Let us consider the plasma waveguide with a partial plasma filling in the external magnetic field, that is, the waveguide, where, between the plasma boundary $r = a$ and the conducting housing $r = b$, there is a vacuum gap.

To derive a dispersion equation of eigenwaves of this plasma wave guide, it is necessary to find the electromagnetic fields in the vacuum gap $b < r < a$ and join them with the plasma fields through the use of boundary conditions on the plasma surface. The boundary conditions are standard, that is, the continuity of electromagnetic field tangential components. As a result, we obtain the dispersion equation, which can be conveniently written as a determinant Det $A = 0$, where the matrix A had the following components:

$$A_{11} = 1,\quad A_{12} = 1,\quad A_{13} = -1,\quad A_{14} = 0,$$

$$A_{21} = \Gamma_1\frac{\omega}{c\lambda_1}\frac{J_1(\lambda_1 a)}{J_0(\lambda_1 a)},\quad A_{22} = \Gamma_2\frac{\omega}{c\lambda_2}\frac{J_1(\lambda_2 a)}{J_0(\lambda_2 a)},$$

$$A_{23} = 0,\quad A_{24} = \frac{\omega}{cw},\quad A_{31} = \Gamma_1,\quad A_{32} = \Gamma_2,\quad A_{33} = 0,$$

$$A_{34} = Q(wa),\quad A_{41} = \varepsilon_3\frac{\omega}{c\lambda_1}\frac{J_1(\lambda_1 a)}{J_0(\lambda_1 a)},\quad A_{42} = \varepsilon_3\frac{\omega}{c\lambda_2}\frac{J_1(\lambda_2 a)}{J_0(\lambda_2 a)},$$

$$A_{43} = -\frac{\omega}{cw}F_1(wa),\quad A_{44} = 0,$$

where

$$Q(wa) = \frac{I_0(wa)K_1(wb) + K_0(wa)I_1(wb)}{I_1(wa)K_1(wb) - I_1(wb)K_1(wa)},$$

$$\Gamma_{1,2} = \frac{1}{2\varepsilon_1}\left\{(\varepsilon_1-\varepsilon_3)\left(\varepsilon_1 - \frac{k_2}{k_0^2} - \varepsilon_2^2\right.\right.$$
$$\left.\pm\left[\left((\varepsilon_1-\varepsilon_3)\left(\varepsilon_1-\frac{k^2}{k_0^2}\right)-\varepsilon_2^2\right)^2 + 4\varepsilon_2^2\varepsilon_3\frac{k^2}{k_0^2}\right]^{1/2}\right)\Bigg\},$$

$$\lambda_{1,2} = \frac{k_0}{2\varepsilon_1}\left\{(\varepsilon_1+\varepsilon_3)\left(\varepsilon_1-\frac{k^2}{k_0^2}\right)-\varepsilon_2^2\right.$$
$$\left.\pm\left[\left((\varepsilon_1-\varepsilon_3)\left(\varepsilon_1-\frac{k^2}{k_0^2}\right)-\varepsilon_2^2\right)^2 + 4\varepsilon_2^2\varepsilon_3\frac{k^2}{k_0^2}\right]^{1/2}\right\};$$

$$F_1 = \frac{I_0(w_p r)\Delta_0(wr_b, wb)}{I_0(w_p a)\Delta_0(wa, wb)};\qquad w_p = (k^2 - k_0^2\varepsilon_3)^{1/2};$$

$$w^2 = k^2 - k_0^2;\qquad \Delta_0 = I_0(wr)K_0(wb) - I_0(wb)K_0(wr).$$

The character of field distribution in the cross section for the waveguide is determined by the transverse wave numbers. If $\lambda_{1,2}^2 > 0$, then the wave is three-dimensional (volumetric). However, if $\lambda_{1,2}^2 < 0$, then the wave is pertaining to the surface. And finally, if $\lambda_{1,2}^2$ are the complex variables, then the wave is hybrid. The boundaries of the region, where $\lambda_{1,2}^2$ become complex, are defined by the inequalities $\omega_1 > \omega > \omega_2$, where

$$\omega_{1,2} = kc\,\frac{2\omega_{pe}^2 + \omega_{He}^2 \pm (w_{pe}^4 + \omega_{He}\omega_{pe}^2 - \omega_{He}^2 k^2c^2)^{1/2}}{\omega_{He}^2 + 4k^2c^2}.$$

To excite the REB of the hybrid wave, the relativistic factor must satisfy the following condition: $\gamma_b > \omega_{He}/2\omega_{pe}$.

The field pattern and the frequency of the hybrid wave being in synchronism with the bunch were found by numerical methods.

We attained the characteristic radial distribution of the longitudinal electric-field component at the following plasma and waveguide parameters:

$$\frac{\omega_{He}}{\omega_{pe}} = 6.3,\qquad \frac{\omega_{pe}a}{c} = 23.3,\qquad \frac{b}{a} = 2.4,\qquad \gamma_b = 4.6.$$

The wake hybrid wave frequency is here equal to $0.35\,\omega_{pe}$. It is shown that for the radius $r/a = 0.8$, the magnitude of the longitudinal electric-field component has a deep maximum corresponding to the energy transformation coefficient

$$R_E = \left| \frac{E_{z\,\max}}{E_{z(r=0)}} \right| = 37.$$

Note that a great value of the transformation coefficient corresponds to a significant (R_E times) excess of the maximum energy obtained by the accelerated bunch as compared to the energy of the bunch exciting the wake-field, because the energy transformation coefficient R_E is equal to the ratio of the amplitude of the electric field accelerating the guided bunch to the amplitude of the electric field decelerating the bunch that excites the accelerating wake-field. So, it has been demonstrated that a multiple excess of the accelerated bunch energy $\varepsilon_{\max}$ over the energy of the exciting REB is possible in a magnetoactive plasma at a certain relationship between the parameters of the "plasma–bunch–magnetic field" system (owing to a hybrid volume–surface character or REB-excited wake-fields), even without using the REBs contoured in the longitudinal direction, namely:

$$\varepsilon_{\max} = mc^2(R_E\,\gamma_b - 1).$$

4. CONCLUSION

This article is devoted to the theoretical study and numerical simulation of the excitation of wake-fields in plasma and their application for charged particle acceleration. It is shown that at a given relationship between parameters of the "plasma–bunch–magnetic field" system in the magnetoactive plasma owing to the hybrid space–surface character of wake-fields waves excited by a relativistic electron bunch, the accelerated bunch energy $\varepsilon_{\max}$ can appear essentially higher than the exciting bunch energy even if the longitudinal REB charge density profiling is not used. With the help of 2.5-D numerical simulation of wake-fields excited by a single REB or a REB sequence, we have established the following: the ion channel is formed owing to transverse ion motion in self-consistent electromagnetic fields and this channel stabilizes REB propagation and thus serves to increase the REB excited fields; the self-modulation of a long-pulsed REB is a very promising way both to obtain high rates of charged particle acceleration and to modulate the bunch and plasma densities (this gives evidence that the linear approach cannot be used to describe the plasma even in the low beam density case). These results make it possible to clarify the prospects and to evaluate the possibility of creating new types of charged particle accelerators which will have acceleration rates much higher than the conventional resonant accelerator has.

ACKNOWLEDGMENT

We acknowledge the financial support of the Basic Research State Foundation of the Ministry of Education and Science of the Ukraine, project no. 02.07/213.

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