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PROPERTY INVESTMENT APPRAISAL

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ABSTRACT

This paper considers the application of discounted cash flow (DCF) techniques to the analysis of the property investment market. The traditional method of property valuation is briefly outlined and its shortcomings highlighted. An alternative DCF procedure is derived to calculate the present value of a property investment. This method will be familiar to actuaries, but is not always used in property disciplines. The sensitivities of this formulation to changes in the force of real interest, force of real rental growth and force of inflation are derived. It is suggested how these formulae may be used for property investment appraisal and risk analysis. We conclude that DCF offers a more flexible and accurate means of estimating the value of a property, and that property valuers, financial economists and actuaries should work jointly to develop practical DCF methods. However, so long as traditional methods of valuation prevail, a rational investor must use both methods to identify mispriced property assets. There have been few property contributions to the actuarial literature in the United Kingdom; this paper is intended to build on the few previous papers and suggests directions for future work.

KEYWORDS

Property Valuation; Discounted Cash Flow Analysis; Sensitivity

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1. Introduction

1.1 Despite property having long been a major component of the portfolios of many institutional investors, until recently little attention has been paid to integrating its investment performance with that of other assets. One of the main reasons for this has been the investment characteristics of property.

1.2 All properties are unique and are traded infrequently in local markets, rather than centrally. This, combined with the high cost of any individual property, means that it is an illiquid asset for which price information is hard to obtain. As a result, property portfolio performance measurement is based on valuations of expected selling prices rather than actual transaction prices. Moreover, as property is a physical asset, it requires a substantial management input for rent collection, rent review, lease negotiation, maintenance and refurbishment.

1.3 The management of property has been dominated by the surveying profession, and a number of surveying specialisms have developed, focusing on the valuation, management and construction of the physical asset. Property valuation has developed within the surveying profession, separately from other asset markets, and it uses a language unfamiliar to other investment professionals. The techniques have been developed from discounted cash flow (DCF) analysis to accommodate property's particular cash flow patterns, but this link is not obvious. Fundamentally, a traditional valuation is an estimate of likely selling price derived from comparable market evidence. Accordingly, it is not, and can never be, an investment appraisal which assesses the worth of a property. Such an appraisal would require explicit DCF techniques linking property to the other capital markets. Already we can find a confusion of terminology between actuaries and surveyors. Investment analysts and actuaries will generally use the term 'price' to denote the market price of a unit of an investment asset; this would be known as 'value' in property professions. Investment analysts and actuaries refer to the present value put on an asset by a particular investor (this present value will then be compared with price in coming to a buy/sell/hold decision); property professionals would use the term 'worth' rather than 'value' in that context.

1.4 Despite property's special characteristics, it also has many of the characteristics of other investment assets. It generates cash flows; there is uncertainty attached to those cash flows; and there are embedded options in property (see Adams & Booth, 1996). There is a clear need for a multi-disciplinary approach to property investment and for other professions, such as actuaries and financial economists, to contribute to research in this area. This could then be absorbed into the standard methods of surveyors. However, not only is there a need for ideas to cross the divide from actuaries and financial theorists into the property professions, there is also a need for ideas to run in the other direction. Surveyors' knowledge of the micro-workings of the property market is typically better than that of actuaries or financial theorists. Furthermore, there are aspects of the traditional valuation process about which actuaries and financial theorists should know more. This requirement for the cross-fertilisation of ideas led to the creation of the Property Investment Sub-Committee of the Institute and the Faculty of Actuaries, and the publication of Ashurst *et al.* (1998) in the property literature and Cumberworth *et al.* (1997) in the actuarial literature.

1.5 Apart from these publications, actuaries have rarely made contributions to the literature on property appraisal and valuation, Hager & Lord (1985) being the most notable exception. However, actuaries and property economists have jointly published papers which concentrate on the role of property within an investment portfolio in the property literature (for example, Booth & Matysiak, 1996); and actuaries and surveyors have jointly published work on risk analysis in the property literature (for example, Adams, Booth & Venmore-Rowland, 1993). This paper continues the cross-fertilisation process. It presents ideas and research which offer new insights into the fundamental financial nature of property. These could lead to further research and the development of better techniques for valuation and investment appraisal.

1.6 In Section 2 traditional valuation techniques currently used in the property field are discussed and an alternative DCF approach is derived, both in nominal terms and in real terms. Although a 'real terms' model is rarely used by surveyors, the approach is familiar to actuaries and is not new. In Sections 3 and 4 sensitivity measures are discussed. These were introduced in Adams, Booth & Venmore-Rowland (1993), but this paper adds further insights, relates the measures to existing literature in the property finance field and discusses their practical application. Tables are provided to aid the practical application of sensitivity measures for use by surveyors in property investment appraisal and risk analysis. Section 5 is the conclusion.

2. Traditional Valuation and Explicit DCF Investment Appraisal

2.1 *The Traditional Method of Determining Open Market Value*

2.1.1 To simplify the discussion in this section, the analysis is restricted throughout to freehold property (feuhold in Scotland). Initially, discussion is restricted to property with no vacant space which has just been let at current open market rental value. Thereafter, there is a brief discussion of reversionary properties, that is, for which the contract rent is below the market rent. A fuller discussion and critique of traditional freehold and leasehold valuation methods can be found in Baum & Crosby (1995).

2.1.2 The open market value (OMV) of a property is the most likely selling price of that property under a set of conditions laid down in professional guidelines. These include: a willing seller; a reasonable period to negotiate a sale; a reasonable period for marketing; values remaining static during this period; and the absence of a special purchaser. In 1995 new guidance was issued by the Royal Institution of Chartered Surveyors (RICS) to professional valuers on additional and different bases for valuation, depending on the purpose of the valuation. For fully let property, surveyors capitalise rental income at a rate usually termed the initial or all risks yield (ARY) which includes implicit assumptions about income growth, risk and depreciation. It is determined from the yields on similar properties which have been sold recently — the comparables. Market information on the comparables is collected and adjusted, subjectively, to take into account factors such as differences in location, lease terms, rent review patterns, tenant security, size, condition and date of sale (to take account of market movements).

2.1.3 The income is treated as if it were fixed in perpetuity with the ARY (or k, the capitalisation rate) set at a level to allow for income growth at rent reviews (typically every five years in the United Kingdom). Its inverse ($1/k$) is known as the 'years purchase in perpetuity'. Thus, the simplest version of a valuation is:

$$\text{OMV} = R/k \tag{2.1}$$

where: OMV is the open market value; R is the rental income (which should be net of management costs); and k is the capitalisation rate. Strictly, costs of acquisition should also be taken into account, but, for simplicity, are ignored here.

2.1.4 This method was developed when property was let on long leases with substantial periods between rent reviews or no reviews. The capitalisation rate was typically taken as the conventional gilt yield plus a 2% premium for risk. As rent reviews became more frequent, income growth was treated implicitly through the all risks yield, derived from market transactions, and the explicit link to the capital markets was lost. Where a property is between rent reviews, and market rents have risen above the contract rent, the investment is said to have reversionary potential. (In the context of upwards only rent reviews, the possibility of a rent reduction does not occur at a rent review, but only at the termination of a lease.) Methods were developed to deal with the cash flow patterns of such properties, but here the link to the capital markets is even less obvious. There are two distinct methods for valuing reversionary properties: the *term and reversion* method; and the *layer* method.

2.1.5 In a term and reversion valuation, the income is divided into a fixed income to review (the term) and an income from review to perpetuity (the reversion). The latter income is taken to be the current open market rent, and is capitalised as if it were from a fully let property. It is then discounted to the valuation date. Three rates are required: a capitalisation rate for the reversion; a discount rate for the capitalised reversionary rent; and a discount rate for the term. The first is typically taken as the yield on comparable fully let properties, as both have income growth every five years on review. The second is conventionally taken as the same as the first, despite the fact that the market rent may grow between valuation and review. The third is taken as the first less 1%, on the grounds that the term income is more secure, despite it not involving growth. Baum & Crosby (1995, p90) suggest that the method is "logically incorrect and practically difficult to understand", but note that the low yield applied to the term tends to cancel the high discount rate applied to the reversion.

2.1.6 The layer method divides the income into a constant income from the present in perpetuity (the bottom slice) and an additional income from the rent review (the top slice). A low capitalisation rate, typically that of a fully let property, is applied to the bottom slice, as it is regarded as secure because of upward-only rent reviews. However, this income stream has no growth. The top slice, despite the fact that it contains growth, is capitalised at a higher yield because it is perceived to be more risky.

2.1.7 A development of both methods is known as the *equivalent yield* approach. In this method an internal rate of return is calculated from a comparable property, and this rate is applied to both parts of the income. The weaknesses of these techniques were exposed during the property market slump of the early 1990s. As market rents fell, many properties became 'over-rented', that is, the contract rent was above the market rent. With expectations of low market rental growth, and market rents well below contract rents, many properties had no prospect of rent rises at review during the period of the lease. Such properties had fixed income for the term of the lease, and so had all the income features of a corporate bond with a risk premium dependent on the tenant. Yield choice, therefore, required explicit comparison with the capital markets, and conventional valuation methods could not cope.

2.1.8 The methods outlined above dominate contemporary practice. Adair *et al.* (1996), in a survey of 203 valuations across all the main property sectors and across a wide range of towns and cities, found that all but one used these methods. Fundamentally, as it involves the use of market comparables, traditional valuation practice is an assessment of likely selling price, and not an appraisal of investment worth to an individual investor. The same applies, of course, to the use of the price/earnings ratio and dividend yield in comparing shares on a stockmarket.

2.1.9 Having briefly explained the traditional valuation method, the basic explicit DCF approach is now considered. This can provide a justification for traditional valuation methods under certain assumptions.

2.2 *Explicit DCF Investment Appraisal of a Fully Let Freehold Property with Regular Rent Reviews*

2.2.1 An explicit DCF valuation puts a value on the expected cash flows from a property, using techniques which are familiar to actuaries. For ease of exposition, in this section, it is additionally assumed that there are no tenant defaults, leases at expiry are immediately renewed on similar terms, and there is constant annual growth in full rental value with no allowance for ageing or obsolescence. (When considering historic rental growth for a particular location, it is necessary to distinguish between data based on 'always new' properties and those based on a property or a group of properties which age and become obsolete through time.) Future cash flows are assumed to extend to infinity, whereas no property will survive that long; however, the error should be small in practice.

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2.2.2 Assume, for the moment, that there are no refurbishment/redevelopment costs or other non-annual outgoings. The present value at a rent review of a fully let freehold property with regular rent reviews is then given by:

$$V_0 = R_1 \ddot{a}^{(4)}_{\overline{n}|} + \frac{R_1(1+g)^n}{(1+r)^n} \ddot{a}^{(4)}_{\overline{n}|} + \frac{R_1(1+g)^{2n}}{(1+r)^{2n}} \ddot{a}^{(4)}_{\overline{n}|} + \ldots \qquad (2.2)$$

where:

V_0 is the value of the property;
R_1 is the initial annual rental income, net of tax and other annual outgoings;
n is the rent review period (years);
g is the expected growth in open market rental value (OMRV) p.a. (so that rents increase by a factor $(1+g)^n$ at each rent review);
r is the investor's required annual rate of return, net of tax; and
$\ddot{a}^{(4)}_{\overline{n}|}$ is the present value of an annuity of 1 p.a. payable quarterly in advance for n years, using a rate of interest r.

2.2.3 Assuming $r > 0$ and $r > g > -1$, so that infinite or negative values are excluded, we obtain:

$$V_0 = \frac{R_1 \ddot{a}^{(4)}_{\overline{n}|}}{1 - \left(\dfrac{1+g}{1+r}\right)^n}. \qquad (2.3)$$

Thus, the required initial (rental) yield is:

$$\frac{R_1}{V_0} = \frac{1 - \left(\dfrac{1+g}{1+r}\right)^n}{\ddot{a}^{(4)}_{\overline{n}|}}. \qquad (2.4)$$

If the market initial yield is lower than this, the price payable is too high to provide the investor with the required rate of return.

2.2.4 In assessing the appropriate growth rate for the OMRV, due consideration should be given to the effect of the development cycle on rents, particularly in the City of London. Thus, high rents make it attractive to develop new space. However, the decisions to develop are taken separately by a number of players. Hence, excess development can take place, and 'high' rents can become 'low' rents; this can create a development cycle in property values.

2.2.5 Equation (2.3) can be adjusted to allow for non-annual outgoings. Suppose that such outgoings, net of tax, are estimated to be C_{t_1}, C_{t_2} ... at

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times t_1, t_2, ... , after the time of valuation, and expressed in prices at the time of valuation, and that the rate of increase in the building costs index is b. Then equation (2.3) becomes:

$$V_0 = \frac{R_1\, \ddot{a}_{\overline{n}|}^{(4)}}{1 - \left(\frac{1+g}{1+r}\right)^n} - \sum_{i=1}^{\infty} \frac{C_{t_i}(1+b)^{t_i}}{(1+r)^{t_i}}. \tag{2.5}$$

Equation (2.4), for the required initial rental yield, becomes:

$$\frac{R_1}{V_0} = \frac{1 - \left(\frac{1+g}{1+r}\right)^n}{\ddot{a}_{\overline{n}|}^{(4)}} \left(1 + \frac{1}{V_0}\sum_{i=1}^{\infty} \frac{C_{t_i}(1+b)^{t_i}}{(1+r)^{t_i}}\right). \tag{2.6}$$

2.3 *A Comparison of Traditional Open Market Valuation and Explicit DCF Investment Appraisal Methods*

2.3.1 Traditional open market valuation methods and DCF investment appraisal methods can give the same results. Ignoring the purchaser's costs, the price an investor is willing to pay for a property, using the DCF model, is given by equation (2.5). Using traditional valuation methods, the open market value is given by equation (2.1). If the ARY is the same as the required initial yield given by equation (2.6), which incorporates the investor's expectation of rental growth and required rate of return, the traditional OMV formula will give the same result as the DCF formula. For example, an ARY of 3.2% is the same as the required initial yield for an investor who expects rental growth of 4.5% p.a., and requires a rate of return of 7.5% p.a., ignoring non-annual outgoings. Looking at it a different way, if an investor requires a rate of return of 7.5% p.a. and the ARY is 3.2%, then a rental growth rate of 4.5% p.a. would be necessary to achieve this return.

2.3.2 The traditional open market valuation method is opaque, and does not bring out explicitly the factors which should affect the investor's required initial yield (the capitalisation rate). The appropriate ARY is subjectively adjusted to reflect a number of implicit factors relating to the risk of the property and the prospects for rental growth. In contrast, the DCF investment appraisal approach incorporates assumptions about future rental growth explicitly, and allows the investor's required rate of return to be adjusted for risk. It should be noted that DCF techniques are also flexible, and can be adapted to deal with problems such as rental voids and 'over-rented' properties. (For a discussion of the problems involved in valuing over-rented property, see Adams & Booth, 1996).

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2.3.3 In practice, explicit DCF methods are sometimes used as a check on traditional open market valuations. Thus, an implicit valuation method is used to determine the likely selling price in a market dominated by such methods, and where case law requires attention to be paid to comparables; and the explicit DCF analysis is used as an investment appraisal technique to assess investor's worth (see Baum & MacGregor, 1992; Baum *et al.*, 1996). This paper proposes that there should be a greater role for explicit DCF methods in the determination of open market value. This would lead to more rational valuations and ultimately to a convergence of the two approaches. Thus, techniques used in investment worth appraisal would, ultimately, drive those used in the assessment of open market value.

2.3.4 With the globalisation of capital markets, including greater integration of European markets, more investors will develop a facility for property investment on an international scale. Explicit DCF techniques will be essential in making comparisons across property markets of different countries, particularly as different lease structures make the implicit assumptions underlying the opaque traditional U.K. methods inappropriate.

2.3.5 Using the DCF investment appraisal formula, it is possible to carry out further analysis of a property investment. Of particular interest is the way in which the present value of a property might respond to changes in the financial variables used in the DCF investment appraisal process. A real terms DCF model is preferred for this analysis, because property is essentially a real asset. An exception is 'over-rented' property, which has bond characteristics until the OMRV grows to exceed the contract rent.

2.4 *DCF Investment Appraisal Modelling: a Real Terms Analysis*

2.4.1 The explicit DCF investment appraisal process can be carried out by discounting payments denominated in constant purchasing power terms, at a real rate of interest. A general advantage of this approach is that real variables can be estimated more easily than the equivalent nominal variables. There are, however, problems for property appraisal, because rental income is fixed in *nominal* terms between reviews, so real property income has an uncertain short-term relationship with inflation. Nonetheless, it is still held to offer long-term inflation protection (see Hamelink *et al.* 1997; Hoesli *et al.*, 1997).

2.4.2 In a real terms analysis, there is an appropriate benchmark (index-linked gilts) against which the required real rate of return from property can be measured. It could be argued, however, that the real redemption yield on long-dated index-linked gilts are artificially depressed at present, as a result of the minimum funding requirement (MFR) valuation rules for defined benefit pension funds. The increased demand which may be arising as a result of the application of these rules is happening at a time of reduced supply due to a reduction in government budget deficits. For a full discussion of this issue, see Debt Management Office (1999).

2.4.3 The nominal equivalent (conventional gilt) is more problematic for property analysis, as its risk characteristics are generally quite different from those of property, with the possible exception of over-rented property. Thus, as property is predominantly a real investment, it should be analysed within an appropriate conceptual framework.

2.4.4 Define j as the investor's required annual real rate of return, g_r as the expected annual real growth rate of rental values and b' as the estimated real building cost inflation rate. Then, by standard methods of discounting real payments in conditions of inflation (see Adams, Booth & Venmore-Rowland, 1993):

$$V_0 = \frac{R_1 \ddot{a}^{(4)}_{\overline{n}|}}{1 - \left(\frac{1+g_r}{1+j}\right)^n} - \sum_{i=1}^{\infty} C_{t_i} \frac{(1+b')^{t_i}}{(1+j)^{t_i}} \quad (j > 0, j > g_r > -1) \qquad (2.7)$$

where:

$\ddot{a}^{(4)}_{\overline{n}|}$ is calculated using the nominal rate of return $r = (1+j)(1+f) - 1$; and

f is the annual rate of increase in the general level of prices.

Again, non-annual outgoings expressed in prices at the time of valuation are estimated to be $C_{t_1}, C_{t_2}, \ldots$ at times $t_1, t_2, \ldots$ after the time of valuation.

2.4.5 The expression for the required rental yield is:

$$\frac{R_1}{V_0} = \frac{1 - \left(\frac{1+g_r}{1+j}\right)^n}{\ddot{a}^{(4)}_{\overline{n}|}} \left(1 + \frac{1}{V_0} \sum_{i=1}^{\infty} C_{t_i} \frac{(1+b')^{t_i}}{(1+j)^{t_i}}\right). \qquad (2.8)$$

This required initial yield can be compared with actual initial yields available in the market. If non-annual outgoings are ignored, the required initial yield will be determined by the following factors: the investor's required real rate of return; the real rental growth rate expected from the property; the rent review period; and the rate of inflation (which affects the value of $\ddot{a}^{(4)}_{\overline{n}|}$). The last two variables are normally much less significant than the former two variables.

2.4.6 Three stages may be identified in the estimation of the investor's required real rate of return j. First, the real yield on long-term index-linked gilts must be considered. Second, a premium must be added to reflect the higher risk, poorer liquidity and valuation difficulties of property compared with index-linked gilts. Finally, the risk characteristics of the particular property must be considered. These would include the probability of not re-letting if the tenant were to default, the probability of physical, environmental or economic obsolescence, the probability of changes in property or planning law and variability of returns in that market segment (location, lot size, age, construction, and so on). There is no standard or rigorous procedure for this, and an adjustment, usually of 0.25% - 1.0%, would often be made on the basis of the surveyor's perception of two or three of the most important factors.

2.4.7 Modern portfolio theory (MPT) suggests that it is the risk that a property contributes to a diversified portfolio that should be considered, but there are problems in the application of MPT to property. Firstly, the use of data derived from smoothed valuations understates risk. Secondly, diversification is difficult, especially with larger properties, and the analysis depends on whether or not we are considering property within a mixed asset portfolio.

2.4.8 For property in a mixed asset portfolio, the analysis normally assumes a properly diversified portfolio with negligible specific risk. If this is not so, any analysis using a diversified index is misleading, as it ignores the specific risk in the property portfolio. This is a problem for investors with less than about 20 properties (about £80m). The value of a High Street retail property in a medium-sized provincial town might be £700,000, and the value of a small office in a similar position might be £6m. Small portfolios also have problems gaining access to high value markets (such as City offices and shopping centres). This creates positions relative to an index (such as the IPD Index), so indexing is not possible.

2.4.9 There are also problems in using MPT to construct a property portfolio. Data are not available for individual properties, so asset classes (e.g. London offices, Scottish shops, etc.) are used. Again, we might assume that the portfolio contains diversified selections from these asset classes, but this is invalid for small portfolios. It is now fairly standard to amend the MPT analysis to accommodate specific risk. We assume that the asset classes capture the 'market' risk, and that there is a building specific component (uncorrelated with the market/asset components; and uncorrelated from individual property to property). If it is assumed that the variances of the specific components of individual properties are identical, then the contribution to portfolio risk is a function of that variance and the relative values of the properties. The variances of the market and specific components can then be added (as they are uncorrelated). This approach overcomes the problem, although it assumes that the asset classes capture all systematic factors.

2.4.10 The estimate of the real rental growth rate g_r, depends, first, on the supply and demand position in the property market as a whole, and, second, on the characteristics of the particular property under consideration. It should be stressed that both the risk adjustment to the real yield and the real rental growth rate are determined subjectively. As suggested in ¶2.4.6, this is not done according to any standard or rigorous procedure.

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Table 1. Rental yield %

Real rental growth rate %	Real interest rate %														
	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.50	5.00	5.50	6.00	7.00	8.00	9.00
0.00	2.35	2.61	2.86	3.12	3.37	3.62	3.88	4.13	4.63	5.13	5.62	6.11	7.09	8.05	9.00
0.25	2.10	2.36	2.61	2.87	3.13	3.38	3.64	3.89	4.39	4.89	5.39	5.88	6.86	7.83	8.79
0.50	1.84	2.10	2.36	2.62	2.88	3.14	3.39	3.65	4.15	4.66	5.16	5.66	6.64	7.61	8.57
0.75	1.59	1.85	2.11	2.37	2.63	2.89	3.15	3.40	3.91	4.42	4.92	5.42	6.42	7.39	8.36
1.00	1.33	1.59	1.86	2.12	2.38	2.64	2.90	3.16	3.67	4.18	4.69	5.19	6.19	7.17	8.14
1.25	1.07	1.33	1.60	1.86	2.12	2.39	2.65	2.91	3.42	3.94	4.45	4.95	5.96	6.95	7.92
1.50	0.81	1.07	1.34	1.60	1.87	2.13	2.39	2.66	3.18	3.69	4.21	4.72	5.72	6.72	7.70
1.75	0.54	0.81	1.08	1.34	1.61	1.87	2.14	2.40	2.92	3.44	3.96	4.47	5.49	6.49	7.48
2.00	0.27	0.54	0.81	1.08	1.35	1.61	1.88	2.14	2.67	3.20	3.71	4.23	5.25	6.26	7.26
2.25	-	0.27	0.54	0.81	1.08	1.35	1.62	1.89	2.42	2.94	3.47	3.99	5.01	6.03	7.03
2.50	-	-	0.27	0.55	0.82	1.09	1.36	1.62	2.16	2.69	3.21	3.74	4.77	5.79	6.80
2.75	-	-	-	0.27	0.55	0.82	1.09	1.36	1.90	2.43	2.96	3.49	4.53	5.56	6.57
3.00	-	-	-	-	0.27	0.55	0.82	1.09	1.63	2.17	2.70	3.23	4.28	5.32	6.34
3.25	-	-	-	-	-	0.28	0.55	0.82	1.37	1.91	2.45	2.98	4.03	5.07	6.10
3.50	-	-	-	-	-	-	0.28	0.55	1.10	1.64	2.18	2.72	3.78	4.83	5.86
3.75	-	-	-	-	-	-	-	0.28	0.83	1.38	1.92	2.46	3.53	4.58	5.62
4.00	-	-	-	-	-	-	-	-	0.56	1.11	1.65	2.20	3.27	4.33	5.38
4.25	-	-	-	-	-	-	-	-	0.28	0.83	1.38	1.93	3.01	4.08	5.13
4.50	-	-	-	-	-	-	-	-	-	0.56	1.11	1.66	2.75	3.83	4.88
4.75	-	-	-	-	-	-	-	-	-	0.28	0.84	1.39	2.49	3.57	4.63
5.00	-	-	-	-	-	-	-	-	-	-	0.56	1.12	2.22	3.31	4.38

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2.4.11 Table 1 gives the required initial yield for different required real rates of return and real rental growth rates, and ignoring non-annual outgoings. A rate of inflation of 2.5% and a rent review period of five years have been assumed, although the required initial yield is relatively insensitive to these assumptions. The table could be employed when considering the purchase of a property on a given initial rental yield.

3. Sensitivity Measures

3.1 *Introduction*

3.1.1 The concepts of duration and volatility have long been applied to bond investment and, in recent years, the concept of duration has also been applied to equities (for example Leibowitz *et al.*, 1989) and to property (for example Hartzell *et al.*, 1988; Ward, 1988). The duration of the expected payments from a property investment is calculated using a DCF model. Duration, so derived, gives insights into the interest rate risk of the investment. In this paper the concept of volatility (which measures the sensitivity of the present value to changes in financial variables), rather than duration, is preferred, although, mathematically, they amount to the same thing.

3.1.2 The possibility of changes in the determinants of present value other than changes in interest rates affecting the value of the property lends support to this approach. In addition, there may be some inter-relationships between the factors causing property present values to change, which can be investigated using the volatility (or sensitivity) measures derived here.

3.2 *Sensitivity Measures for Present Values*

3.2.1 Sensitivity measures give the investor an insight into the effect of changes in financial variables on property values. They also provide an insight into the relative risk of properties with different characteristics. Although the use of sensitivity measures as a risk management tool is no substitute for more sophisticated asset/liability management techniques such as deterministic (scenario-based) and stochastic cash flow modelling, an analysis of the sensitivity of asset values and liability values to changes in various financial variables can be helpful when taking portfolio selection decisions and analysing risk.

3.2.2 It should be emphasised that a fluctuation in present value does not necessarily imply a fluctuation in open market value, as open market values do not necessarily reflect the true worth of a property. The sensitivity measures are helpful, however, in giving an indication to investors of how changes in key variables *should* affect property values. They are also useful to those institutions which value property holdings on an explicit DCF basis for internal purposes. Furthermore, sensitivity measures would grow in importance and open market values would move closer to 'worth' if

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investors were increasingly to use explicit DCF methods in open market valuations.

3.2.3 Adams, Booth & Venmore-Rowland (1993) derive sensitivity measures for the nominal DCF model of property valuation. These measures indicate the sensitivity of the property value to changes in certain financial variables. The corresponding sensitivity measures for the real terms DCF model are also stated, without proof. This paper adopts, instead, a real terms approach, for the reasons discussed in Section 2.4.

3.2.4 A further advantage of analysing property in a real terms model is revealed when consideration is given to the factors which are most likely to affect a property's present value in the nominal DCF model developed in Section 2.2. An increase in nominal interest rates would reduce the present value of a property, and an increase in nominal rental growth rates would increase the present value. The most likely reason for an increase in nominal interest rates is an increase in inflationary expectations; this is likely to give rise to a simultaneous increase in expected long-term nominal rental growth rates. The effect of any increase in nominal interest rates is, therefore, very often difficult to ascertain.

3.2.5 Using a real terms DCF valuation model, it is possible to consider the effects on the present value of a property of changes in the force of real interest, the force of real rental growth, and then, explicitly, the effect of changes in long-term inflationary expectations. Sensitivity measures are now derived which indicate the effect of changes in these variables. The following basic formula for the property value, which ignores the non-annual outgoings in equation (2.7), will be used:

$$V_0 = \frac{R_1 \ddot{a}_{\overline{n}|}^{(4)}}{1 - \left(\frac{1+g_r}{1+j}\right)^n} \quad (j > 0, j > g_r > -1) \tag{3.1}$$

where $\ddot{a}_{\overline{n}|}^{(4)}$ is calculated using the nominal rate of return $r = (1+j)(1+f) - 1$.

3.2.6 Again, it should be stressed that the required real rate of return j and the real rental growth rate g_r in equation (3.1) are determined subjectively.

3.3 *Sensitivity of Present Values to Changes in the Force of Real Interest*

3.3.1 The *force of real interest sensitivity* is defined as:

$$S_{\delta_j} = -\frac{\partial V_0}{\partial \delta_j} \cdot \frac{1}{V_0}. \tag{3.2}$$

This shows the proportionate change in the present value of the property, per unit change in the force of real interest δ_j, for small changes in the force

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of real interest. For example, if S_{δ_j} is equal to 9, this means that, broadly speaking, the present value will rise (or fall) by 9% for each percentage point fall (or rise) in the force of real interest (ignoring the convexity effect).

3.3.2 A formula for S_{δ_j} is now derived analytically. To do so, it is first necessary to differentiate $\ddot{a}^{(4)}_{\overline{n}|}$ with respect to j.

$$\ddot{a}^{(4)}_{\overline{n}|} = \frac{1}{4}\{1-(1+j)^{-n}(1+f)^{-n}\}\{1-(1+j)^{-\frac{1}{4}}(1+f)^{-\frac{1}{4}}\}^{-1}. \tag{3.3}$$

Then:

$$\frac{\partial \ddot{a}^{(4)}_{\overline{n}|}}{\partial j} = \frac{n(1+j)^{-n+\frac{1}{4}}(1+f)^{-n+\frac{1}{4}} - \ddot{a}^{(4)}_{\overline{n}|}}{4(1+j)\{(1+j)^{\frac{1}{4}}(1+f)^{\frac{1}{4}}-1\}} \tag{3.4}$$

$$\Rightarrow \frac{\partial \ddot{a}^{(4)}_{\overline{n}|}}{\partial j} = \frac{nv^{n-\frac{1}{4}} - \ddot{a}^{(4)}_{\overline{n}|}}{(1+j)r^{(4)}} \tag{3.5}$$

where:

$$r^{(4)} = 4\{(1+j)^{\frac{1}{4}}(1+f)^{\frac{1}{4}}-1\} \text{ and } v = (1+j)^{-1}(1+f)^{-1}.$$

Differentiating equation (3.1) partially with respect to j gives:

$$\frac{\partial V_0}{\partial j} = R_1\left[\frac{\dfrac{\partial \ddot{a}^{(4)}_{\overline{n}|}}{\partial j}}{\left(1-\left(\dfrac{1+g_r}{1+j}\right)^n\right)} - \frac{n\ddot{a}^{(4)}_{\overline{n}|}(1+g_r)^n(1+j)^{-(n+1)}}{\left(1-\left(\dfrac{1+g_r}{1+j}\right)^n\right)^2}\right]. \tag{3.6}$$

Substituting for $\dfrac{\partial \ddot{a}^{(4)}_{\overline{n}|}}{\partial j}$ from equation (3.5), gives:

$$\frac{\partial V_0}{\partial j} = R_1\left[\frac{nv^{n-\frac{1}{4}} - \ddot{a}^{(4)}_{\overline{n}|}}{(1+j)r^{(4)}\left(1-\left(\dfrac{1+g_r}{1+j}\right)^n\right)} - \frac{n\ddot{a}^{(4)}_{\overline{n}|}(1+g_r)^n(1+j)^{-n}}{(1+j)\left(1-\left(\dfrac{1+g_r}{1+j}\right)^n\right)^2}\right]. \tag{3.7}$$

Multiplying throughout by $\dfrac{-(1+j)}{V_0}$ and using equation (3.1), produces:

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$$\frac{-\partial V_0}{\partial j}\cdot\frac{(1+j)}{V_0}=\frac{\ddot{a}^{(4)}_{\overline{n}|}-nv^{n-\frac{1}{4}}}{r^{(4)}\,\ddot{a}^{(4)}_{\overline{n}|}}+\frac{n}{\left(\frac{1+j}{1+g_r}\right)^n-1}. \tag{3.8}$$

Thus:

$$S_{\delta_j}=\frac{\ddot{a}^{(4)}_{\overline{n}|}-nv^{n-\frac{1}{4}}}{r^{(4)}\,\ddot{a}^{(4)}_{\overline{n}|}}+\frac{n}{\left(\frac{1+j}{1+g_r}\right)^n-1} \tag{3.9}$$

because:

$$\frac{\partial V_0}{\partial \delta_j}=\frac{\partial V_0}{\partial j}\cdot\frac{dj}{d\delta_j}=\frac{\partial V_0}{\partial j}.(1+j).$$

3.3.3 Note that the force of real interest sensitivity increases as the real growth rate of rents increases. This is only to be expected, as a higher real rental growth rate will weight the receipts from a property investment to a later point in time. The real interest rate, rate of inflation and rent review period also affect the force of real interest sensitivity. Table 2 enables analysts to read off the force of real interest sensitivity for properties with a five-year rent review period and an assumed future annual rate of inflation of 2.5%.

3.3.4 The sensitivity measures in Table 2 apply to small changes in the force of real interest (say 0.1% rather than 1%). Only for small changes do upward and downward movements in real interest rates produce symmetrical changes in capital values. Thus, assuming zero growth, 3.5% real return and 2.5% inflation, a fall in the real interest rate from 6% to 5.9% would increase capital value by about 2.9% and a rise in the real interest rate from 6% to 6.1% would decrease capital value by a similar percentage.

3.4 *Sensitivity of Present Values to Changes in the Force of Real Rental Growth*

3.4.1 A change in real rental growth will change the present value of all future receipts after the next rent review. Define the force of real rental growth as $\delta_{g_r}=\ln(1+g_r)$, the continuous real growth rate of rents equivalent to the annual effective real growth rate g_r. The *force of real rental growth sensitivity* is then defined as:

$$S_{\delta_{g_r}}=\frac{\partial V_0}{\partial \delta_{g_r}}\cdot\frac{1}{V_0} \tag{3.10}$$

that is, the proportionate change in the present value, per unit change in the force of real rental growth, for small changes in the force of real rental

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Table 2. Force of real interest sensitivity %

Real rental growth rate %	Real interest rate %														
	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.50	5.00	5.50	6.00	7.00	8.00	9.00
0.00	44.77	40.32	36.69	33.65	31.09	28.89	26.99	25.32	22.54	20.32	18.50	16.99	14.60	12.82	11.43
0.25	50.44	44.87	40.42	36.77	33.73	31.16	28.96	27.05	23.90	21.42	19.41	17.75	15.17	13.25	11.77
0.50	57.74	50.56	44.98	40.51	36.86	33.81	31.23	29.03	25.44	22.64	20.41	18.58	15.77	13.71	12.13
0.75	67.47	57.88	50.68	45.08	40.61	36.94	33.89	31.31	27.17	24.01	21.52	19.50	16.42	14.20	12.51
1.00	81.10	67.64	58.02	50.80	45.19	40.70	37.03	33.97	29.16	25.55	22.74	20.50	17.13	14.73	12.92
1.25	101.55	81.30	67.80	58.15	50.92	45.30	40.80	37.11	31.45	27.30	24.12	21.61	17.90	15.29	13.36
1.50	135.63	101.79	81.49	67.96	58.29	51.04	45.40	40.89	34.12	29.29	25.67	22.85	18.74	15.90	13.82
1.75	203.79	135.95	102.04	81.69	68.12	58.43	51.16	45.51	37.29	31.59	27.42	24.23	19.67	16.56	14.32
2.00	408.28	204.28	136.28	102.28	81.88	68.28	58.57	51.28	41.08	34.28	29.42	25.78	20.68	17.28	14.85
2.25	-	409.28	204.78	136.61	102.53	82.08	68.44	58.70	45.72	37.46	31.74	27.54	21.80	18.06	15.42
2.50	-	-	410.27	205.27	136.94	102.77	82.27	68.60	51.52	41.27	34.44	29.56	23.05	18.91	16.04
2.75	-	-	-	411.27	205.77	137.27	103.02	82.47	58.98	45.93	37.63	31.88	24.44	19.84	16.70
3.00	-	-	-	-	412.26	206.26	137.60	103.26	68.93	51.76	41.46	34.59	26.01	20.86	17.43
3.25	-	-	-	-	-	413.26	206.76	137.92	82.86	59.26	46.14	37.80	27.79	21.99	18.21
3.50	-	-	-	-	-	-	414.25	207.25	103.75	69.25	52.00	41.65	29.82	23.25	19.07
3.75	-	-	-	-	-	-	-	415.25	138.58	83.25	59.53	46.36	32.17	24.66	20.01
4.50	-	-	-	-	-	-	-	-	208.24	104.24	69.57	52.24	34.91	26.24	21.04
4.25	-	-	-	-	-	-	-	-	417.24	139.24	83.64	59.81	38.14	28.04	22.18
4.50	-	-	-	-	-	-	-	-	-	209.23	104.73	69.90	42.03	30.09	23.45
4.75	-	-	-	-	-	-	-	-	-	419.23	139.89	84.03	46.78	32.46	24.87
5.00	-	-	-	-	-	-	-	-	-	-	210.22	105.22	52.72	35.22	26.47

growth. $S_{\delta_{g_r}}$ is always positive, as an increase in real rental growth will always lead to an increase in the present value of the property. Differentiating equation (3.1) with respect to g_r gives:

$$\frac{\partial V_0}{\partial g_r} = \frac{nR_1 \ddot{a}^{(4)}_{\overline{n}|} \dfrac{(1+g_r)^{n-1}}{(1+j)^n}}{\left[1 - \dfrac{(1+g_r)^n}{(1+j)^n}\right]^2}. \tag{3.11}$$

As $\delta_{g_r} = \ln(1+g_r)$, therefore $\dfrac{dg_r}{d\delta_{g_r}} = (1+g_r)$

and

$$\frac{\partial V_0}{\partial \delta_{g_r}} = \frac{nR_1 \ddot{a}^{(4)}_{\overline{n}|} \dfrac{(1+g_r)^{n}}{(1+j)^n}}{\left[1 - \dfrac{(1+g_r)^n}{(1+j)^n}\right]^2} \tag{3.12}$$

because:

$$\frac{\partial V_0}{\partial \delta_{g_r}} = \frac{\partial V_0}{\partial g_r} \cdot \frac{dg_r}{d\delta_{g_r}} = \frac{\partial V_0}{\partial g_r} . (1+g_r).$$

Dividing throughout by V_0, produces:

$$S_{\delta_{g_r}} = \frac{n \dfrac{(1+g_r)^n}{(1+j)^n}}{\left[1 - \dfrac{(1+g_r)^n}{(1+j)^n}\right]} = \frac{n}{\dfrac{(1+j)^n}{(1+g_r)^n} - 1}. \tag{3.13}$$

3.4.2 $S_{\delta_{g_r}}$ increases as the rent review period decreases. This is because an increase in real rental growth does not take as long to take effect if the rent review period is shorter. $S_{\delta_{g_r}}$ also increases as j decreases and as g_r increases. This would imply that properties with low initial yields (implying low j and/or high g_r) would have higher real rental growth sensitivity as well as higher force of real interest sensitivity.

3.4.3 Table 3 gives the force of real rental growth sensitivity for properties valued with different values for g_r and with different required real rates of return. An inflation rate of 2.5% and a rent review period of five years are assumed.

3.4.4 Again, the sensitivity measures apply to small changes in real rental growth. Only for small changes do upward and downward movements produce symmetrical changes in values.

Table 3. Force of real growth sensitivity %

Real rental growth rate %	Real interest rate %														
	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.50	5.00	5.50	6.00	7.00	8.00	9.00
0.00	42.49	38.05	34.42	31.39	28.83	26.64	24.74	23.08	20.31	18.10	16.29	14.78	12.42	10.65	9.28
0.25	48.16	42.60	38.15	34.51	31.48	28.91	26.71	24.81	21.67	19.20	17.20	15.55	12.98	11.08	9.62
0.50	55.46	48.29	42.71	38.25	34.60	31.56	28.99	26.78	23.20	20.42	18.20	16.38	13.59	11.54	9.99
0.75	65.20	55.61	48.41	42.82	38.35	34.69	31.64	29.06	24.94	21.79	19.30	17.29	14.24	12.04	10.37
1.00	78.82	65.36	55.75	48.54	42.93	38.45	34.78	31.73	26.93	23.33	20.53	18.30	14.95	12.56	10.78
1.25	99.27	79.02	65.53	55.89	48.66	43.04	38.55	34.87	29.22	25.07	21.91	19.41	15.72	13.13	11.21
1.50	133.35	99.52	79.22	65.70	56.03	48.79	43.15	38.65	31.89	27.07	23.45	20.64	16.56	13.74	11.68
1.75	201.51	133.68	99.77	79.42	65.86	56.18	48.91	43.27	35.05	29.37	25.21	22.02	17.48	14.40	12.17
2.00	406.00	202.01	134.01	100.02	79.62	66.03	56.32	49.04	38.85	32.06	27.21	23.58	18.50	15.11	12.70
2.25	-	407.00	202.51	134.35	100.27	79.82	66.20	56.46	43.49	35.23	29.52	25.34	19.62	15.89	13.28
2.50	-	-	408.00	203.01	134.68	100.52	80.02	66.36	49.29	39.05	32.22	27.35	20.86	16.74	13.89
2.75	-	-	-	409.00	203.51	135.01	100.77	80.22	56.75	43.71	35.42	29.68	22.26	17.67	14.56
3.00	-	-	-	-	410.00	204.01	135.35	101.02	66.70	49.54	39.25	32.39	23.83	18.69	15.28
3.25	-	-	-	-	-	411.00	204.51	135.68	80.62	57.03	43.93	35.60	25.60	19.83	16.06
3.50	-	-	-	-	-	-	412.00	205.01	101.52	67.03	49.79	39.45	27.64	21.09	16.92
3.75	-	-	-	-	-	-	-	413.00	136.35	81.02	57.32	44.15	29.98	22.49	17.86
4.50	-	-	-	-	-	-	-	-	206.01	102.02	67.36	50.04	32.72	24.08	18.89
4.25	-	-	-	-	-	-	-	-	415.00	137.01	81.42	57.60	35.96	25.87	20.04
4.50	-	-	-	-	-	-	-	-	-	207.01	102.52	67.70	39.85	27.92	21.31
4.75	-	-	-	-	-	-	-	-	-	417.00	137.68	81.82	44.60	30.29	22.73
5.00	-	-	-	-	-	-	-	-	-	-	208.01	103.02	50.54	33.06	24.32

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3.5 *Sensitivity of Present Values to Changes in the Force of Inflation*

3.5.1 Having expressed all the variables in the DCF model as real variables, it is now possible to analyse explicitly the effect of a change in an investor's assumed inflation rate on the present value of a property investment. Consideration is given only to the effect of a change in the assumed constant future force of inflation, immediately after a rent review. The results are slightly different if the changes occur at other times, and depend on the proximity to a rent review. Define δ_f to be the force of inflation equivalent to the annual rate of inflation f, so that $\delta_f = \ln(1+f)$. The *force of inflation sensitivity* is then defined as:

$$S_{\delta_f} = -\frac{\partial V_0}{\partial \delta_f} \cdot \frac{1}{V_0}. \tag{3.14}$$

3.5.2 The negative sign ensures a positive value for S_{δ_f}. The force of inflation sensitivity measures the proportionate change in present value, per unit change in the force of inflation, for small changes in the force of inflation.

3.5.3 Adams, Booth & Venmore-Rowland (1993) derive the following formula for force of inflation sensitivity:

$$S_{\delta_f} = \frac{1}{r^{(4)}} - \frac{n}{(1+r)^n - 1} \tag{3.15}$$

where $r = (1+f)(1+j) - 1$.

3.5.4 The force of inflation sensitivity varies with the nominal rate of interest (and, hence, with the real rate of interest and assumed future rate of inflation) and with the rent review period. The rent review period is the most important variable. Table 4 gives the inflation sensitivity of properties with different rent review periods and real rates of interest, at an annual rate of inflation of 2.5%.

3.5.5 Property can be regarded as a 'quasi-real' investment. The present value of a property can fluctuate significantly with changes in anticipated inflation. This technical aspect of the quasi-real nature of property should be separated out from other likely causes of fluctuation in the value of real property caused by inflation. These other causes of fluctuation include: a re-rating of property as compared with other investments; distortions, caused by the way in which inflation is transmitted through the economy, to the level of rental growth (see Hoesli *et al.*, 1997); and the severe effect that the operation of tax and monetary policy can have on the property market in the short to medium term.

3.5.6 Inflation sensitivity can be interpreted broadly as the percentage change in the present value per one percentage point change in the rate of

Table 4. Force of inflation sensitivity %

Rent review period (years)	Real interest rate %								
	2.25	2.75	3.25	4.00	5.00	6.00	7.00	8.00	9.00
1	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37
2	0.86	0.86	0.86	0.85	0.85	0.85	0.84	0.84	0.84
3	1.34	1.34	1.33	1.33	1.32	1.31	1.31	1.30	1.29
4	1.81	1.81	1.80	1.79	1.78	1.77	1.75	1.74	1.73
5	2.28	2.27	2.26	2.24	2.22	2.20	2.18	2.16	2.15
6	2.73	2.72	2.71	2.68	2.66	2.63	2.60	2.57	2.55
7	3.18	3.16	3.14	3.12	3.08	3.04	3.00	2.96	2.93
8	3.63	3.60	3.57	3.54	3.49	3.44	3.39	3.34	3.29
9	4.06	4.03	3.99	3.95	3.88	3.82	3.76	3.70	3.64
10	4.49	4.45	4.41	4.35	4.27	4.19	4.12	4.04	3.97
11	4.90	4.86	4.81	4.74	4.64	4.55	4.46	4.37	4.28
12	5.31	5.26	5.20	5.12	5.00	4.90	4.79	4.68	4.58
13	5.72	5.65	5.58	5.49	5.36	5.23	5.11	4.98	4.87
14	6.11	6.04	5.96	5.85	5.70	5.55	5.41	5.27	5.13
15	6.50	6.41	6.33	6.19	6.02	5.86	5.70	5.54	5.39
16	6.88	6.78	6.68	6.54	6.34	6.16	5.97	5.80	5.63
17	7.26	7.14	7.03	6.87	6.65	6.44	6.24	6.04	5.85
18	7.62	7.50	7.37	7.19	6.95	6.71	6.49	6.27	6.06
19	7.98	7.84	7.70	7.50	7.23	6.98	6.73	6.49	6.26
20	8.33	8.18	8.03	7.80	7.51	7.23	6.96	6.70	6.45
21	8.68	8.51	8.34	8.09	7.78	7.47	7.18	6.90	6.63
50	15.89	15.12	14.40	13.39	12.18	11.13	10.20	9.40	8.70
100	20.25	18.61	17.17	15.35	13.42	11.90	10.69	9.71	8.89
Infinity	21.18	19.17	17.52	15.52	13.48	11.93	10.70	9.71	8.90

inflation. If there are no rent reviews, a property will have the same inflation sensitivity as the interest rate sensitivity of a perpetuity paid quarterly in advance. For a one-year rent review period, there would only be a very small percentage change in the present value of the property for a one percentage point change in anticipated inflation. A property with seven-yearly rent reviews would have a force of inflation sensitivity of around 3 (see Table 4) which is only slightly less than the volatility of a conventional bond with four years to maturity and paying a 10% coupon half-yearly.

3.5.7 The inflation sensitivity arises as a result of the periodic rent review in the traditional institutional lease. Different contract terms would give rise to different sensitivities to changes in the underlying fundamental variables. For example, the use of rents linked to price indices, as in Germany and France, with long review periods, would give rise to virtually zero inflation sensitivity, but greater real rental growth sensitivity. On the other hand, rents linked to retail turnover, as is common in the United States retail property market, provide the freeholder with more of an equity-type interest.

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3.5.8 It is possible for the property market to adapt so that unanticipated inflation is less disadvantageous to the investor. The shortening of rent review periods in the 1970s is one example of this. Other possible ways in which contracts could evolve to deal with unanticipated inflation include annual price indexation of rents (if this were more common in the U.K., a longer period between rent reviews would be possible) or the linking of rents to other real variables. Turnover rents have been used in the case of shops, and are common in the U.S. This means fewer voids in times of recession but still changes considerably the income stream aspects of the risk profile of such property.

4. Sensitivity Measures in Property Market Analysis

4.1 *Sensitivity Measures in Property Market Appraisal*

4.1.1 The neglect of analytical valuation techniques in the determination of the open market value of property, together with the inherent lack of liquidity, means that property markets are sluggish in response to changes in other investment markets. It is widely accepted that, as a result, property returns calculated from traditional open market valuations appear to vary far less over time than return series calculated from other asset classes (see, for example, Barkham & Geltner, 1994; MacGregor & Nanthakumaran, 1992). The yield gap, yield ratio, real yield gap and real yield ratio are used extensively to link valuations of gilts, index-linked gilts and equities. Property analysts could make greater use of such statistics to link movements in securities markets to possible future movements in the property market. Nevertheless, it can be argued that the current sluggish response of property to changes in other markets is an advantage within the portfolio theory context.

4.1.2 The existence of both conventional and index-linked gilts in the U.K. allows the derivation of a real yield curve and an inflation expectations curve (see Deacon & Derry, 1994). The sensitivity formulae derived in Section 3 can be used to assess the possible effects on the property market of: changes in real yields from index-linked gilts; changes in risk premiums required for property investment; changes in inflationary expectations; and changes in expected real rental growth rates. A real yield curve and an inflation expectations curve can be useful in assessing the effects of changes in risk-free real interest rates and inflationary expectations.

4.1.3 Booth (1993) gives an example of how objectively observed movements in other investment markets might affect property markets. At that time index-linked gilt yields had fallen significantly, suggesting increases in the investment values of both equities and property. There was, indeed, a significant increase in equity values over the period, but there was a lagged effect on the property market.

Table 5. Predicted and actual changes in property values

Year	Index-linked gilt yield change %	Sensitivity measure	Present value change required by changes in yields %	Property capital value index change %	Property capital value index change following year %
1990	0.52	20.5	−10.7	−14.2	−10.5
1991	0.23	20.5	−4.7	−10.5	−10.0
1992	−0.39	20.5	8.0	−10.0	10.8
1993	−0.95	20.5	19.5	10.8	4.0
1994	0.86	20.5	−17.6	4.0	−4.1
1995	−0.33	20.5	6.8	−4.1	2.0
1996	0.03	20.5	− 0.6	2.0	9.0
1997	−0.57	20.5	11.7	9.0	4.6
1998	−0.95	20.5	19.5	4.6	–

4.1.4 In the following paragraphs consideration is given to the possible effect on property DCF values of changes in real yields from index-linked stocks between 1 January 1990 and 1 January 1999. Three approximations will be made when applying the sensitivity measures. First, the sensitivity measures are applied to small finite changes in investment yields rather than to infinitesimal changes. Secondly, sensitivity measures derived with respect to the force of real interest are applied to changes in annual real rates of interest convertible half-yearly. Thirdly, the assumed real yield at which the discounting is carried out will not be changed as index-linked gilt yields change. All these three approximations have a negligible effect.

4.1.5 Columns in Table 5 show the change in real yield from index-linked gilt yields over each year, as measured by the benchmark 2.5% Index-linked Treasury 2016; the property sensitivity measure; the change in property value which would result from the change in real yields; and the actual change in property capital values over the year, as measured by the change in the IPD capital growth index. Property indices are based on valuations rather than on market prices, and so are likely to respond only slowly to changes in the economy or other capital markets. The last column, therefore, shows the change in property capital values over the following year, as measured by the change in the IPD capital growth index. The property sensitivity measure is used for a rent review period of five years, a required real rate of return of 6% and a real growth rate of rents of 1%. Table 2 gives a value for S_{δ_j} of 20.5% for this sensitivity measure.

4.1.6 Consideration of the DCF models and sensitivity measures would suggest that the other main explanation as to why property values may not have changed in the way indicated by the real yield sensitivity measure alone is that expectations of real rental growth changed. Objective information is not available on this issue. Property analysts should use real yield changes in assessing property values, and greater joint consideration of investment

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markets (including property) should lead to better investment decisions. If we assume that the effects of real interest rate changes and real growth rate changes on property values are approximately additive, an analyst can assess the relative value of property by considering whether the change in expected rental growth has been sufficient to move the property market to make up the difference between the move predicted when one applies interest rate sensitivity factors and the actual move in a particular period. For example, changes in real yields would require a movement in capital values of +8.0% in 1992; actual property values changed by −10.0%. If changes in anticipated real rental growth did not explain the −18% discrepancy between these two figures, and the change in the index-linked gilt yield is appropriate in the economic circumstances, property must, at some point, have been mis-priced. This provides opportunities for arbitrage, provided the protracted period involved in property transactions does not mean that the mis-pricing is reversed within the transaction period.

4.1.7 A change in present value does not necessarily feed through to a change in open market value, and the additional factors to consider when estimating how open market values will move are discussed in Booth (1993). Open market values may not fully reflect all available information either before or after changes in financial variables. Anticipated inflation and the risk premium required by investors may also change; only changes in the risk-free real rate of return and real growth rates have been considered. However, changes in real interest rates should certainly affect an investor's estimate of the true worth of property. Given the clear linkages, there should be more emphasis on research which connects property markets with the real and money economies and with other investment markets. Sensitivity measures can assist in this joint analysis.

4.1.8 As well as being useful in analysing the connections between the property market and the economy and other investment markets, a DCF valuation approach, in conjunction with sensitivity measures, can be used in property market forecasting. In particular, the sensitivity measures can be used, together with surveyors' forecasts of real rental growth and economists' forecasts of real interest rates and inflation, in the analysis of possible future capital value changes in the property market. Traditional methods are much less amenable to this kind of analysis.

4.2 *Sensitivity Measures in Property Risk Analysis*

4.2.1 Sensitivity analysis is commonly used in property investment as a method of assessing risk. Key parameters are identified and varied, one at a time, to assess the sensitivity of the investment's present value (or internal rate of return) to variations in each parameter. A basic example of sensitivity analysis is given in Baum & Crosby (1995). There is no attempt to attach probabilities to possible changes in relevant financial variables, the sensitivity analysis is more a deterministic 'what if?' analysis along the lines of life office scenario testing. Sykes (1983), however, attempts to show how probabilities of financial variables taking different values could be applied to the analysis.

4.2.2 Hargity & Yu (1993) also describe a more sophisticated approach. They define the expected present value of the property to be the present value of each of the possible future period cash flows multiplied by the probability of receipt; there are, in fact, conceptual problems in defining expected present values in this way, as it ignores Jensen's inequality. (In general, the expected value of a function is not equal to the function of the expected value. In this case, the expected present value is not necessarily the same as the present values of the expected cash flows. There is considerable theoretical debate as to whether this kind of approach is valid.) The variance of the cash flows is calculated by the standard procedure of assuming that it was a linear combination of variances of, and covariances between, the various cash flows. A simulation method of obtaining a probability distribution for the present value of future cash flows may also be proposed to overcome the difficulties of finding such a distribution analytically. The approach of Hargity & Yu (1993) is helpful, but, as explained below, sensitivity measures can provide a useful additional tool. In particular, Hargity & Yu do not consider variability in market interest rates as a possible factor affecting property values.

4.2.3 Morely (1988) looks at the variability of the sale value of a property by including the variance of the projected income stream growth and the variance of the capitalisation rate at sale (although he suggests that the capitalisation rate and income stream growth rate are statistically independent). He also considers a simulation approach, allowing for variability in both the rental growth rate and the capitalisation rate. If simulation is used, it is relatively easy to include any correlations between these variables.

4.2.4 The advantage of using simulation and sensitivity analysis is that a range of 'what if?' scenarios can be assessed. Simulation appears to provide an overall assessment of a property's risk. However, it hides inadequacies in the model used to estimate variances and covariances. Ward (1988) argues for a duration-based approach to assessing risk. The sensitivity measures estimated here develop this argument further.

4.2.5 In conclusion, if the present value can be written in a neat mathematical form, as in equation (3.1) for freehold property, it may be possible to calculate sensitivity measures. These provide an economical way of summarising the effect of changes in key financial variables, and can be an important complement to traditional sensitivity analysis. When combined with estimates of the probabilities of movements of different magnitudes in the financial variables, a better indication of risk can be obtained for property investments with different characteristics.

4.2.6 It is worth distinguishing between the different risks which can be

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analysed using sensitivity measures. An institution, such as a pension fund, holding a portfolio of assets which is well matched to the fund's liabilities, will be less concerned about a change in the general level of interest rates. Such a change would affect the liability values of the fund in the same way as it would affect the asset values. Indeed, the force of real interest sensitivity measure may be used as a measure of duration in asset-liability management. A pension fund would, however, be concerned about a fall in a property value caused by a fall in its long-term income earning power. The force of real rental growth sensitivity would, therefore, be of considerable interest. The formulation of the Minimum Funding Requirement (MFR) in the 1995 Pensions Act means that changes in property yields cannot be taken into account when determining the value of the liabilities of the scheme. Therefore, interest rate sensitivity measures may be important in this context, as property is effectively not a 'matching asset' for MFR purposes.

4.2.7 Many property investment organisations are more concerned about short-term capital values than long-term income earning power. Such organisations might include property companies, banks which have made loans to property developers, or general insurance companies. They will be interested in both force of real interest sensitivity and force of real rental growth sensitivity of property values.

5. Discussion and Conclusion

5.1 This paper has considered the application of DCF techniques to property investment analysis. The traditional method of property valuation was briefly outlined and its shortcomings highlighted. The traditional method was originally developed for an asset with very different characteristics from those of modern property investment. Prior to the 1960s, leases were long, rent reviews infrequent or non-existent, and property was infrequently traded. Property had many of the features of a conventional bond, and illiquidity was relatively unimportant. It was valued as a constant income discounted at the gilt yield plus a 2% premium. As rent reviews became more frequent and trading became more important, income growth and illiquidity emerged as important factors in a valuation. Rather than going back to the first principles of financial mathematics, the traditional method was developed in a manner which is illogical and unintuitive. Moreover, the explicit link with the capital markets was lost, and links to the real and money economies were not forged.

5.2 The paper derives an alternative DCF procedure to calculate the present value of a property investment and the sensitivities of this formulation to changes in the force of real interest, force of real rental growth and force of inflation. These formulae may be used for the appraisal of property and for risk analysis. The general conclusion is that DCF offers a more flexible and

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accurate method of property investment analysis than does the traditional method of property valuation. In particular, it enables explicit cash flow analysis and assessment of the sensitivity to key inputs. However, criticism of traditional valuation methods must be qualified in two ways. First, their dominance of the market means they are important, and, second, practical implementation of alternative methods is difficult.

5.3 The conventional method of valuation, being based on direct capital comparison of 'similar' properties sold close to the valuation date, produces an estimate of likely selling price, and not an assessment of investment worth. This purpose is backed by professional codes of practice and by case law. This is a fundamental consideration which must not be overlooked in a desire to extend explicit valuation techniques. It is reasonable to suppose that professional valuations influence market prices, and thus, if traditional methods dominate, there will be scope for mispricing which can be exploited by the rational investor. In these circumstances, it is essential to understand the traditional method to enable an estimate of likely market price to be made, against which an explicit analysis of worth may be compared. Short leases and over-rented property are two examples of systematic mis-pricing which has occurred, and which can be attributed, at least in part, to the prevalence of irrational methods.

5.4 It is also important to distinguish the availability of a technique from the practicability of its application. Lest it be supposed that the surveying profession has been wholly irrational in not adopting full DCF procedures, it should be stressed that these are particularly difficult to apply in a market where each property is unique and where relevant data are lacking. The links between the economy and capital markets have, until recently at least, not been well-researched. Forecasting models linking a national index of rents to measures of economic activity and to the supply of property have been in widespread use at the property portfolio level for no more than 10 years. The extension of such models to the local level, let alone the individual property level, is fraught with difficulties, as modelling frameworks become more difficult to design and data problems become severe. Forecasts are also required of depreciation and risk; no easy matter at the individual property level. Furthermore, professional guidance and case law stresses the importance of direct capital comparison. In these circumstances, it is hardly surprising that traditional methods dominate the market, and that property valuations have lost the explicit link to capital markets and the economy.

5.5 On a more positive note, contemporary practice, particularly in larger institutions and in international firms of surveyors, is developing to embrace fully DCF techniques as an adjunct to traditional methods. The current education of the future generations of surveyors, at least in the better surveying schools, focuses on setting valuation in its proper economic and capital market context. There has also been a mushrooming of quality

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research, both in academia and in practice, over the last 10-15 years. Researchers with non-surveying backgrounds, such as from economics, business and finance, have brought new approaches and methods to the analysis of property investment. Future development would be enhanced if investment advisers, financial economists and actuaries were to engage more fully with the property market. In this way, practical methods of valuation and appraisal could be developed and mutual understanding improved.

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