Macroeconomic Dynamics, **25**, 2021, 970–997. Printed in the United States of America.
doi:10.1017/S136510051900049X

FISCAL MULTIPLIERS AT THE ZERO LOWER BOUND: THE ROLE OF GOVERNMENT SPENDING PERSISTENCE

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In this paper, I examine the role of government spending persistence on fiscal multipliers at the zero lower bound (ZLB) in a more realistic environment while keeping the model simple enough to identify mechanisms driving the result. In particular, I build on a standard dynamic New Keynesian (DNK) model with an occasionally binding ZLB and Rotemberg pricing with rebates, where the probability of hitting the ZLB and the government purchase shock are in line with US data. Moreover, I compute the multiplier in a state that mimics the Great Recession. The main findings of the paper are as follows: (1) the multiplier is non-monotonic in the persistence of government spending while the economy is at the ZLB; (2) given the persistence estimated from US data, the multiplier is 1.25; and (3) in the framework with perfect foresight or with aggregate resource cost for adjusting prices, the multiplier is around 1 or less.

Keywords: Government Spending Multiplier, Zero Lower Bound, ZLB, Rotemberg Price Adjustment, Rebate, Nonlinear Method

1. INTRODUCTION

The effectiveness of fiscal policy has received much attention from economists and policymakers since the target federal funds rate hit the zero lower bound (ZLB) in December 2007 and the conventional monetary policy became ineffective in stimulating economic activities. Most research based on standard dynamic New Keynesian (DNK) models focuses on the effectiveness of government spending when the spending is perfectly correlated with the ZLB shock, which follows a two-state Markov process with an absorbing non-ZLB state, for example, Woodford (2011) and Boneva et al. (2016). The main purpose of doing so is to derive a closed-form solution for the policy function and the spending multiplier. The trade-off is that one cannot study the roles of government spending

I would like to thank the editor and anonymous referees for useful comments/suggestions. This paper is a substantial revision of a part of my previously circulated working paper entitled "Does Calvo Meet Rotemberg at the Zero Lower Bound?," coauthored with Jianjun Miao. Address correspondence to: Phuong V. Ngo, Department of Economics, Cleveland State University, 2121 Euclid Avenue, Cleveland, OH 44115, USA. e-mail: p.ngo@csuohio.edu. Phone: (617) 347 2706. Fax: (216) 687 9206.

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persistence and an occasionally binding ZLB on the multiplier while the economy is at the ZLB.

In this paper, I examine the role of government spending persistence on fiscal multipliers at the ZLB in a more realistic environment than previous research while keeping the model simple enough to identify mechanisms driving the result. In particular, I build on a standard New Keynesian model with an occasionally binding ZLB and Rotemberg pricing with rebates, where the probability of hitting the ZLB and the government purchase shock are in line with US data. Moreover, I compute the multiplier in a state that mimics the Great Recession. I also provide empirical evidence about the persistence of the government spending for the USA and the corresponding multiplier.

The main findings of this paper are as follows: (1) the multiplier is non-monotonic in the persistence of government spending while the economy is at the ZLB; (2) at the persistence that is in line with US data, the multiplier is 1.25; (3) in the perfect foresight framework or the conventional Rotemberg price setting without rebates, the multiplier is around 1 or less.

The first finding of the paper is an important complement to Woodford (2011), where he finds that the government spending multiplier is monotonically decreasing in the persistence of government spending after financial disturbance ends and the ZLB is no longer binding. As seen in Figure 3 of Woodford (2011), when the ZLB is expected to bind for 10 periods, the multiplier is around 2.3 if the government spending ends right after the financial disturbance that makes the ZLB binding dies out. However, the multiplier decreases monotonically if the persistence of government spending increases.

In this paper, I focus on the relationship between the government persistence and the spending multiplier while the economy is still at the ZLB.[1] I find that the relationship is non-monotonic. The multiplier first increases with the persistence. It then declines if the persistence is greater than a certain value. The intuition for the non-monotonicity is that when the persistence of government spending increases, future inflation is expected to be higher, leading to a smaller expected real interest rate as long as the ZLB binds. A smaller real interest rate would raise private consumption. This substitution effect would cause output and, as a result, the multiplier to increase. However, government spending also generates a negative wealth effect because of higher lump-sum taxes, which are levied to finance this government spending. This negative wealth effect lowers private consumption. When the persistence is moderate, the positive substitution effect dominates the negative wealth effect, causing private consumption, output, and the multiplier to increase. However, if government spending is too persistent, the negative wealth effect dominates the positive substitution effect so that consumption falls. Therefore, the multiplier starts decreasing when the persistence is sufficiently high.

The first finding is also an important complement to Coenen et al. (2012), where the authors find that fiscal policy is most effective if it has moderate persistence and if monetary policy is accommodative. However, in their experiment, the monetary policy accommodation is not due to a binding ZLB. Instead, they calibrate shocks such that the nominal interest rate remains 100 basis points above its steady state. Moreover, they did not compute their spending multiplier in a state that mimics the Great Recession. Rather, they assume that the economy is initially in a steady state. It is well known that in a fully nonlinear model, the government spending multiplier is state dependent. In addition, the length of government spending is imposed arbitrarily instead of following an autoregressive (AR) process.

The second finding of this paper that the multiplier is 1.25 at the spending persistence of 0.86 is greater than what is found by Boneva et al. (2016), even though the parameters used in this paper are very similar to those in their paper. They show that the government spending multiplier is less than 1 even when the expected duration of the ZLB is about 10 quarters. The discrepancy comes from three sources: (1) I compute the multiplier based on the government spending persistence of 0.86 that is consistent with US data; (2) I allow for an occasionally binding ZLB; and (3) in my model, the aggregate resource adjustment cost is paid to workers, making the Rotemberg pricing almost equivalent to the Calvo pricing.[2] All of these features are important in producing a multiplier greater than 1. Without these characteristics, the multiplier is around 1 or less. This is also the third finding of this paper.

My paper contributes to the burgeoning body of literature that investigates the effectiveness of fiscal policy at the ZLB. A non-exhaustive list of research in this area includes Christiano et al. (2011), Woodford (2011), Boneva et al. (2016), Eggertsson (2009), Eggertsson (2011), Eggertsson and Krugman (2012), Coenen et al. (2012), Ramey (2017), Hall (2009), Nakata (2016), Hills and Nakata (2017), and Ercolani and Azevedo (2018). These papers are different from my paper because either they do not take into account the unconditional probability of hitting the ZLB that conforms to US data or they do not investigate the role of government spending persistence in an occasionally binding ZLB framework.[3]

In terms of solution methods, my paper is most closely related to papers by Fernandez-Villaverde et al. (2015), Ngo (2014), Gust et al. (2012), and Aruoba and Schorfheide (2013).[4] All these papers use global projection methods to approximate agents' decision rules in a DNK model with a ZLB constraint.[5] Fernandez-Villaverde et al. (2015) also study the government spending multiplier at the ZLB. However, they do not focus on the role of government spending persistence on the magnitude of spending multiplier. In addition, they do not compute the multiplier at a state that mimics the Great Recession. It is well known that economic responses and government spending multipliers are state dependent in a fully nonlinear framework.

2. MODEL

The model I use in this paper is a standard DNK model that features Rotemberg price setting and an occasionally binding ZLB.[6] I intentionally consider a standard DNK model so that I can clearly understand the difference between my results and notable results of previous research. Note that the comparisons of estimated nested models is obscure.

My model consists of a continuum of identical households, a continuum of identical competitive final goods producers, a continuum of monopolistically competitive intermediate goods producers, and a government (i.e., monetary and fiscal authorities).

2.1. Households

The representative household maximizes his expected discounted utility

$$E_t\left\{\left(\frac{C_t^{1-\gamma}}{1-\gamma}-\chi\frac{N_t^{1+\eta}}{1+\eta}\right)+\sum_{\tau=1}^{\infty}\left\{\left(\Pi_{j=0}^{\tau-1}\beta_{t+j}\right)\left(\frac{C_{t+\tau}^{1-\gamma}}{1-\gamma}-\chi\frac{N_{t+\tau}^{1+\eta}}{1+\eta}\right)\right\}\right\} \quad (1)$$

subject to the budget constraint

$$P_{t+\tau}C_{t+\tau}+(1+i_{t+\tau})^{-1}B_{t+\tau}=W_{t+\tau}N_{t+\tau}+B_{t+\tau-1}+\Pi_{t+\tau}+T_{t+\tau} + AC_{t+\tau} \text{ for } \tau=0,1,2,\ldots, \quad (2)$$

where C_t is consumption of final goods, i_t is the nominal interest rate, B_t denotes one-period bond holdings, N_t is labor, W_t is the nominal wage rate, Π_t is the profit income, T_t is the lump-sum tax, AC_t is the price adjustment cost paid to the representative household in terms of lump-sum payment, and β_t denotes the preference shock. With Rotemberg pricing, the price adjustment cost is a cost for intermediate goods-producing firms, and therefore it lowers the firms' profits. However, this cost would make the dead-weight loss unrealistically high if it were not paid to the household as in the standard DNK model.[7] In addition, Miao and Ngo (2018) show that the Calvo pricing and the Rotemberg pricing with rebates generate very similar results at the ZLB when they are equivalent at a first-order approximation.[8] Therefore, in this paper we assume that the cost of adjusting prices is paid to the representative household, so it enters the household's budget constraint, increasing his revenues.[9]

The preference shock β_t follows an AR(1) process

$$\ln(\beta_{t+\tau})=(1-\rho_\beta)\ln\beta+\rho_\beta\ln(\beta_{t+\tau-1})+\varepsilon_{\beta,\,t+\tau}, \quad (3)$$

where $\rho_\beta\in(0,1)$ is the persistence of the preference shock and $\varepsilon_{\beta t}$ is the innovation of the preference shock with mean 0 and variance σ_β^2. The preference shock is a reduced form of more realistic forces that can drive the nominal interest rate to the ZLB. It is common in the literature to model an occasionally binding ZLB in this way.[10]

The first-order conditions for the household optimization problem are given by

$$\chi N_t^\eta C_t^\gamma=w_t, \quad (4)$$

and

$$E_t\left[M_{t,t+1}\left(\frac{1+i_t}{1+\pi_{t+1}}\right)\right]=1, \quad (5)$$

where $w_t = W_t/P_t$ is the real wage, $\pi_t = P_t/P_{t-1} - 1$ is the inflation rate, and the stochastic discount factor is given by

$$M_{t,t+1} = \beta_t \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}. \tag{6}$$

2.2. Final Goods Producers

To produce the final goods, final goods producers buy and aggregate a variety of intermediate goods indexed by $i \in [0, 1]$ using a constant elasticity of substitution (CES) technology

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}},$$

where ε is the elasticity of substitution among intermediate goods. The profit maximization problem is given by

$$\max \; P_t Y_t - \int_0^1 P_t(i) Y_t(i) \, di,$$

where $P_t(i)$ and $Y_t(i)$ are the price and quantity of intermediate good i. Profit maximization and the zero-profit condition give the demand for intermediate good i

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t, \tag{7}$$

and the aggregate price level

$$P_t = \left(\int P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}. \tag{8}$$

2.3. Intermediate Goods Producers

There is a unit mass of intermediate goods producers on [0, 1] that are monopolistic competitors. Suppose that each intermediate good $i \in [0, 1]$ is produced by one producer using the linear technology

$$Y_t(i) = N_t(i), \tag{9}$$

where $N_t(i)$ is labor input. Cost minimization implies that each firm faces the same real marginal cost

$$mc_t = mc_t(i) = w_t. \tag{10}$$

2.4. Price Setting

Following Rotemberg (1982), I assume that each intermediate goods firm i faces costs of adjusting prices in terms of final goods. In this paper, I use a quadratic adjustment cost function, which was proposed by Ireland (1997) and which is one of the most common functions used in the ZLB literature:

$$\frac{\varphi}{2}\left(\frac{P_t(i)}{P_{t-1}(i)}-1\right)^2 Y_t,$$

where φ is the adjustment cost parameter which determines the degree of nominal price rigidity.[11] The problem of firm i is given by

$$\max_{\{P_t(i)\}} E_t \sum_{j=0}^{\infty}\left\{M_{t,t+j}\left[\left(\frac{P_{t+j}(i)}{P_{t+j}}-mc_t\right)Y_{t+j}(i)-\frac{\varphi}{2}\left(\frac{P_{t+j}(i)}{P_{t+j-1}(i)}-1\right)^2 Y_{t+j}\right]\right\} \quad (11)$$

subject to its demand (7). In a symmetric equilibrium, all firms will choose the same price and produce the same quantity (i.e., $P_t(i)=P_t$ and $Y_t(i)=Y_t$). The optimal pricing rule then implies that

$$(1-\varepsilon+\varepsilon w_t-\varphi\pi_t(1+\pi_t))\,Y_t+\varphi E_t\left[M_{t,t+1}\pi_{t+1}(1+\pi_{t+1})\,Y_{t+1}\right]=0. \quad (12)$$

2.5. Monetary and Fiscal Policies

The central bank conducts monetary policy by setting the interest rate using a simple Taylor rule, subject to the ZLB condition

$$\frac{1+i_t}{1+i}=\max\left\{\left(\frac{GDP_t}{GDP}\right)^{\phi_y}\left(\frac{1+\pi_t}{1+\pi}\right)^{\phi_\pi},\ \frac{1}{1+i}\right\}, \quad (13)$$

where $GDP_t\equiv C_t+G_t$ denotes the gross domestic product (GDP) and GDP, π, and i denote the steady-state GDP level, the targeted inflation rate, and the steady-state nominal interest rate, respectively.[12]

Following Fernandez-Villaverde et al. (2015), Gust et al. (2017), and Aruoba et al. (2018), I assume that the government consumes a stochastic fraction of GDP

$$\frac{G_t}{GDP_t}=S_g g_t,$$

where S_g denotes the steady-state share of government spending and g_t denotes the government spending shock that follows an AR(1) process

$$\ln g_t=\rho_g\ln g_{t-1}+\varepsilon_{gt},$$

where $\rho_g\in(0,1)$ is the persistence parameter and ε_{gt} is the innovation with mean 0 and variance σ_g^2.[13,14]

Some researchers such as Woodford (2011) and Boneva et al. (2016) model the ZLB following a two-state Markov process with one absorbing state, which is the non-ZLB state. They also model government spending as perfectly correlated with the ZLB state. The main purpose for doing so is to derive a closed-form solution for the policy function and the spending multiplier. The trade-off is that the roles of government spending persistence and an occasionally binding ZLB on the multiplier while the economy is at the ZLB cannot be studied. In this paper,

https://doi.org/10.1017/S136510051900049X Published online by Cambridge University Press

I solve the model using a fully nonlinear method. Thus, I can keep the ZLB and government spending processes flexible, and I can study the roles of government spending persistence and ZLB uncertainty on the multiplier.

2.6. Equilibrium Systems

With the Rotemberg price setting, the aggregate output satisfies

$$Y_t = N_t, \tag{14}$$

and the price adjustment cost is

$$AC_t = \frac{\varphi}{2}\pi^2 Y_t.$$

It is important to note that in this paper I assume the price adjustment cost is paid (rebated) to the household so that it is not an aggregate waste and does not show up in the resource constraint, which is given by

$$C_t + G_t = Y_t. \tag{15}$$

As explained above, Miao and Ngo (2018) recommend that ZLB research should use either Calvo pricing or Rotemberg pricing with rebates as in this paper. The reason for doing so is that the model using Rotemberg pricing with rebates produces very similar results to Calvo pricing and can avoid an astronomical price adjustment cost at the ZLB, which is also the dead-weight loss if the cost is not paid to the household, as noted in Eggertsson and Singh (2016).[15]

The equilibrium system for the Rotemberg model consists of a system of six nonlinear difference equations (4), (5), and (12)–(15) for six variables w_t, C_t, i_t, π_t, N_t, and Y_t.

3. CALIBRATION AND SOLUTION METHOD

The values of the parameters used in this paper are listed in Table 1. The quarterly subjective discount factor β is set at 0.997 such that the annual real interest rate is 1.2%, as in Christiano et al. (2011) and Boneva et al. (2016). The constant relative risk aversion parameter γ is 1, corresponding to a log utility function with respect to consumption. This utility function is commonly used in the business cycles literature. The labor supply elasticity with respect to wages is set at 1, or $\eta = 1$, as in Christiano et al. (2011). The value of χ is calibrated to obtain the steady-state fraction of working hours of 1/3. The elasticity of substitution among differentiated intermediate goods ϵ is 7.66, corresponding to a 15% net markup that is in the range found by Diewert and Fox (2008). This value is also popular in the literature [e.g., Boneva et al. (2016)].

I set the price adjustment cost parameter in the Rotemberg model $\varphi = 495.8$, as is done in Boneva et al. (2016). This value, together with the other parameters, implies that the slope of the Phillips curve is 0.0269, which is within the range estimated by Ball and Mazumder (2011) for the US using 1985:q1–2007:q4 data.

TABLE 1. Calibration

Symbol	Description	Values
β	Quarterly discount factor	0.997
γ	Constant relative risk aversion (CRRA) parameter	1
η	Inverse labor supply elasticity	1
ε	Monopoly power	7.66
φ	Price adjustment cost parameter in the Rotemberg model	495.8
π	Inflation target	0
ϕ_π	Weight of inflation target in the Taylor rule	1.75
ϕ_y	Weight of output target in the Taylor rule	$\frac{0.5}{4}$
S_g	Share of the government spending at the steady state	0.2
σ_β	Standard deviation of preference innovations	$\frac{0.1}{100}$
ρ_β	AR-coefficient of preference shocks	0.85
σ_g	Standard deviation of government spending innovations spending shocks	$\frac{0.3}{100}$
ρ_g	AR-coefficient of government spending shocks	[0, 0.94]

I set the parameters in the Taylor rule at $\phi_\pi = 1.75$ and $\phi_y = 0.25$, which are close to the estimates by Gust et al. (2017). The conventional average share of the government spending in output $S_g = 0.20$, as in Christiano et al. (2011).

Based on my empirical assessment using US data in a subsection next, I set the persistence of government spending shock to $\rho_g = 0.86$ and the standard deviation for the shock innovations to $\sigma_g = \frac{0.3}{100}$. This persistence value is considered to be the benchmark of this paper. To study the role of the government spending process at the ZLB, I also report the results using different persistence values.

The most important parameters left to calibrate are those regarding the preference shock. Following Gust et al. (2017), I set the persistence of preference shock at 0.85. I set the standard deviation for preference innovations to $\sigma_\beta = \frac{0.1}{100}$ so that the unconditional probability of hitting the ZLB is 17%, which is consistent with recent US data. In particular, using the method from Ball (2013) and Ngo (2018), I find that the probability of the nominal interest rate reaching the ZLB would be between 16.1% and 19.7% if the Fed were to keep the inflation target as low as 2%.[16]

In terms of a solution, I use projection methods, which is a similar approach to that in Miao and Ngo (2018). In particular, I approximate the expectations as a function of state variables using a finite element method called the linear spline interpolation [Judd (1998) and Miranda and Fackler (2002)]. The nominal interest rate is always determined by equation (13) at every state, within or outside of the set of collocation nodes. The main advantage of this approach is that I do not have to worry about the kink when the ZLB starts binding. Furthermore, expectations can smooth out the kink. The detailed algorithm and computation errors can be found in Appendix C.

4. RESULTS

To see the role of the persistence of government spending shocks, I first solve the model using different values for the persistence, ranging from 0 to 0.94. I then compute the government spending multipliers under 1% of the government spending shock for the state that mimics the Great Recession. At the state that partially mimics the Great Recession, based on the model result, the quarterly output gap is about −6.5%, the inflation rate is about −0.7% per quarter (or about −3% per year), and the median of ZLB duration is about 10 quarters.[17]

Although this expected ZLB duration of 10 periods is debatable, the ZLB literature tends to use this number.[18] I use this number so that my result is more comparable to those found in the literature. In addition, although the output gap is in line with US data (about −6.5%), the inflation rate is debatable. In particular, based on the US Consumer Price Index (CPI), the inflation rate was −3.5% per quarter at the trough of the Great Recession. However, the inflation rate is only −1% per quarter based on the GDP deflator, and −0.04% based on the core Personal Consumption Expenditure (PCE) and CPI.[19] Our number of about −0.7% per quarter still complies with the US data.

I compute the spending multiplier based on conventional impulse responses of GDP and government spending.[20] In particular, I first compute the responses of GDP and government spending, $(GDP_t^1, G_t^1)_{t=1}^T$, under only an adverse preference shock that puts the economy at a state similar to the Great Recession. While the preference shock dies out according to its motion equation, the other shocks (for both the present and future) are imposed at the median values. I then compute the responses of GDP and government spending, $(GDP_t^2, G_t^2)_{t=1}^T$, under both the preference shock and additional 1% government spending shock. Afterward, I compute the conventional IRFs as $IRF_t^X = X_t^2 - X_t^1$, where $X = (GDP, G)$. The (impact) multiplier is computed using the following formula.[21,22]

$$m_{Impact}^{ZLB} = \frac{IRF_1^{GDP}}{IRF_1^G}. \qquad \textbf{(16)}$$

For comparison, I also compute the multiplier when the ZLB is not binding. In particular, the initial state is at the steady state, which is also the median state. The results are presented in Figure 1.

It is well known in the literature that outside the ZLB, larger government spending would cause private consumption to decrease. This decrease occurs because an increase in government spending will cause higher prices and inflation. Under the Taylor principle, the central bank would raise the nominal interest rate by an amount greater than the increase in inflation, resulting in an increase in the real interest rate that lowers private consumption. The more persistent the government spending is, the more inflation is created, and the higher the nominal interest rate is raised by the central bank under the Taylor rule. This higher nominal interest rate results in a greater increase in the real interest rate and a larger crowding-out effect. This is why the multiplier is less than one and monotonically decreasing

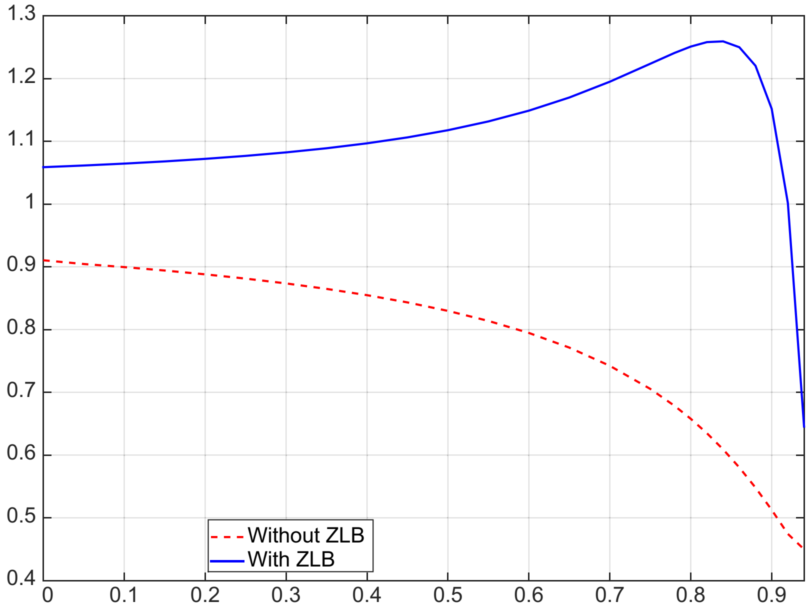

FIGURE 1. Government spending multipliers. In the case at the ZLB, the ZLB binds for 10 periods on average.

with the persistence of government spending when the economy is not at the ZLB, as presented by the dashed red line of Figure 1.

With the ZLB imposed, the multiplier can be larger than one. More importantly, the multiplier is non-monotonic in the persistence of government spending, as shown by the solid blue line in Figure 1. This finding is an important complement to Woodford (2011), where he finds that the government spending multiplier is monotonically decreasing in the persistence of government spending after the financial disturbance ends and the ZLB is no longer binding. As seen in Figure 3 of Woodford (2011), when the ZLB is expected to bind for 10 periods, the multiplier is around 2.3 if the government spending ends right after the financial disturbance that makes the ZLB bind ends. However, the multiplier decreases monotonically if the persistence of government spending increases. In this paper, I focus on the relationship between the government persistence and the spending multiplier while the economy is still at the ZLB. I find a non-monotonic relationship between government spending and the multiplier while the economy is at the ZLB.

The intuition for the non-monotonicity is as follows: when the persistence of government spending increases, future inflation is expected to be higher, leading to a smaller expected real interest rate as long as the ZLB binds. A smaller real interest rate would raise private consumption. This substitution effect causes output and, as a result, the multiplier to increase. However, government spending also generates a negative wealth effect because of higher lump-sum taxes, which are levied to finance the government spending. This negative wealth effect lowers private consumption and output. When the persistence is moderate, the positive substitution effect dominates the negative wealth effect, causing the multiplier to increase. However, if government spending is too persistent, the negative wealth effect dominates the positive substitution effect so that consumption falls. Therefore, the multiplier starts decreasing when the persistence is sufficiently high; it can be smaller than one if the persistence is greater than 0.9.

This finding is also an important complement to Coenen et al. (2012), where the authors find that fiscal policy is most effective if it has moderate persistence and if monetary policy is accommodative. In their experiments, the monetary policy accommodation is not due to a binding ZLB. Instead, they calibrate shocks such that the nominal interest rate remains 100 basis points above its steady state. Moreover, they did not compute their spending multiplier in a state that mimics the Great Recession. Instead, they assume that the economy is initially in a steady state. It is well known that in a fully nonlinear model, the government spending multiplier is state dependent. In addition, the length of government spending is imposed arbitrarily instead of following an AR(1) process.

4.1. What Persistence Fits the US Data?

The answer to this question turns out to be very important because the persistence determines the magnitude of the government spending multiplier at the state that mimics the Great Recession. To answer this question, I first collect real GDP and real government purchases on final consumption and investment from the Federal Reserve Economic Data (FRED) website.[23] I then compute the government spending shock as fraction of the government purchases to GDP, as described in Section 2. However, the government purchases-to-GDP series is nonstationary. The null hypothesis of stationary is rejected at the 1% level based on the augmented Dicky–Fuller test with four lags, which is chosen by the Baysian Information Criteria (BIC). Given the model is stylized and does not deal with very slowly moving components, such as technology growth shocks, it is reasonable to filter these very slowly-moving components out and keep cyclical components only. Figure 2 shows the filtered series based on the Hodrick–Prescott filtering method for the period 1960q1–2017q2.

It is noticeable that the US government purchase as share of GDP is countercyclical. It increases sharply in most recessions and decreases during expansions. In particular, it increased about 2% during the Great Recession from 2007q4 to 2009q2. This is the largest increase since 1975. However, it is important to note that the government purchase is not perfectly correlated with recessions. Therefore, the assumption that the government purchase is perfectly correlated with recessions or with binding ZLB should be relaxed.

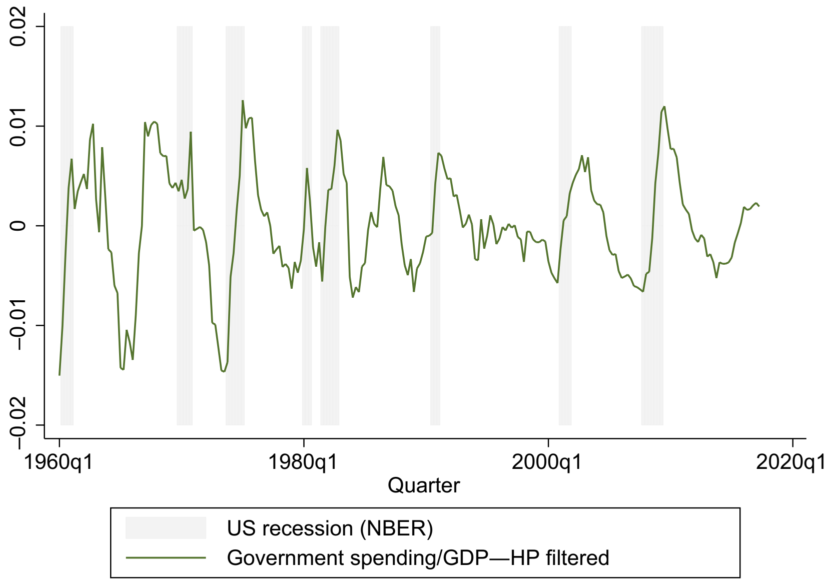

FIGURE 2. Government purchase as fraction of GDP: 1960q1–2017q2.

To determine the persistence of government purchases, I use the data from 1960q1 to 2017q2 to fit an AR(1) model. The regression result is presented in Table 2.

The estimate of the government purchase persistence is 0.86 for the sample from 1960q1 to 2017q3. The standard deviation of the government purchase innovations is approximately $\frac{0.3}{100}$. These estimation results are robust when I use different subsamples as shown in Table 2. Given the persistence estimate of 0.86, the government spending multiplier is approximately 1.25. However, if I take into account the estimation uncertainty by using the 95% confidence interval for the spending persistence, the multiplier would range from around 1 to 1.26.[24]

4.2. Why is My Result Different from Other Recent Research?

Boneva et al. (2016) find that the government spending multiplier is less than 1 under the parameterization which is very similar to ours except for the persistence of government spending.[25] Several reasons explain the difference. The main reason is that the government spending persistence in my paper is in line with US data. As seen in Figure 1, when the spending is very transient or very persistent ($\rho_g = 0$ or $\rho_g > 0.9$), the multiplier is small (around 1 or less).

Another important reason is that the ZLB is occasionally binding in my model. The dot-dashed black line in Figure 3 shows the multiplier for the case of perfect

https://doi.org/10.1017/S136510051900049X Published online by Cambridge University Press

TABLE 2. Government purchase shock process

Dependent variable Government purchase/GDP	(1) 1960q1–2007q3	(2) 1980q1–2007q3	(3) 1960q1–2017q2
L.government purchase/GDP	0.85***	0.84***	0.86***
	(0.04)	(0.05)	(0.03)
Constant	0.00	−0.00	0.00
	(0.00)	(0.00)	(0.00)
Observations	190	111	229
RMSE (%)	0.28	0.22	0.27
Adjusted R^2	0.74	0.70	0.76

Notes: Standard errors in parentheses.
*** denotes p-value $< 1\%$.

foresight, which is very different from the benchmark results with an occasionally binding ZLB. Specifically, when the persistence is 0.86, the occasionally binding ZLB method generates the multiplier of 1.25, while the perfect foresight method produces the multiplier of approximately 1. This occurs because the recession is worse under the occasionally binding ZLB due to the possibility of the ZLB coming back. Therefore, the substitution effect caused by persistent spending shock is larger, and the multiplier is bigger under an occasionally binding ZLB.

The last reason is that in my model there is not any aggregate resource price adjustment cost and, as a result, dead-weight losses to the whole economy. The dashed red line of Figure 3 shows the multiplier for the case with both aggregate resource cost and perfect foresight. As seen from this figure, allowing an aggregate resource cost to price adjustments (the case with perfect foresight and without rebate) causes the multiplier to decline further to 0.9 when the government spending persistence is 0.86. This less-than-one multiplier is consistent with the result in Boneva et al. (2016).

Woodford (2011) and Christiano et al. (2011) find that the multiplier is around 2. Specifically, Woodford (2011) finds that the multiplier is around 2.3 while the economy is at the ZLB. Christiano et al. (2011) finds that the multiplier is in the range from 1.6 to 2.3. The main reason for the difference is that the ZLB state is more persistent in their models than in my model.[26] In addition, they calibrate price rigidity such that the slope of the Phillips curve is greater in their models than in mine. In this paper, I calibrate the price adjustment parameter such that the slope of the Phillips curve is in line with US data.[27] It is well known that the greater the slope, the greater the increase in inflation under an increase in government spending (and output gap), leading to a greater decline in real interest rate if the nominal interest is stuck at the ZLB. The greater decline in the real interest rate leads to a more substantial increase in consumption (and output), increasing the government spending multiplier.

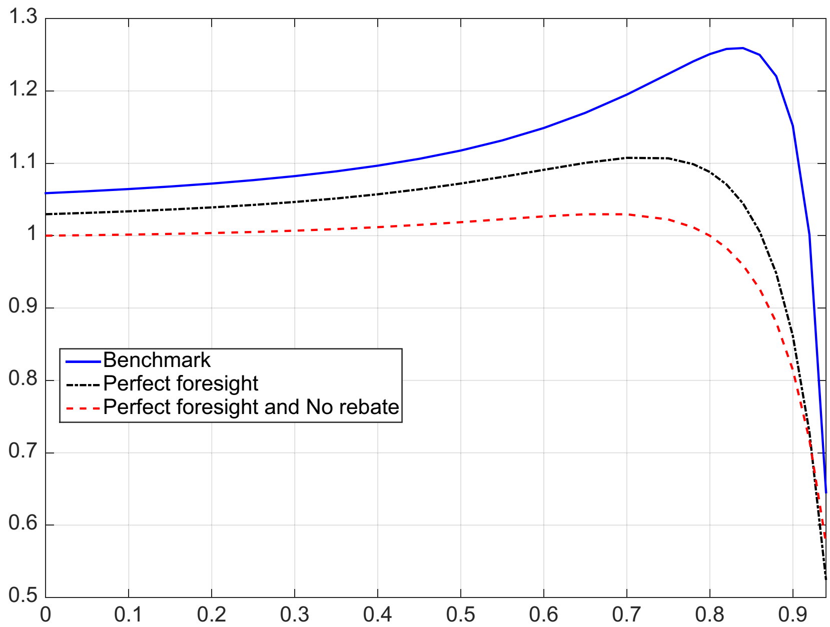

FIGURE 3. Government spending multipliers at the ZLB. The ZLB binds for 10 periods on average.

5. SENSITIVITY ANALYSIS

5.1. Generalized IRF

Due to the ZLB constraint, the policy functions, especially the one for the nominal interest rate, are highly nonlinear. Therefore, the impulse responses are both shock and state dependent, as in Koop et al. (1996). In Section 4, I use the conventional IRF to compute the spending multiplier.[28] In this subsection, I implement a robustness check to see if using generalized IRFs (GIRFs), as described in Koop et al. (1996), would change the main results. Intuitively, a GIRF for a state is the average of many IRFs starting from that state. Due to computational expensiveness resulting from a Monte Carlo simulation related to GIRFs, I only compute GIRFs for the case when the government spending persistence is 0.86, which is the benchmark value. I also plot the conventional IRFs, which I use to compute the spending multiplier in Section 4. The results are presented in Figures 4 and 5 for two different states: the steady state and the state that mimics the Great Recession. The GIRFs are computed using 9999 draws of shocks, with each having 20 periods.[29]

From Figure 4, we are able to see that the conventional IRFs and GIRFs are very similar, especially for GDP and government spending, as seen in panels E and F. This means that the government spending multiplier based on the IRFs and its counterpart based on GIRFs are the same.

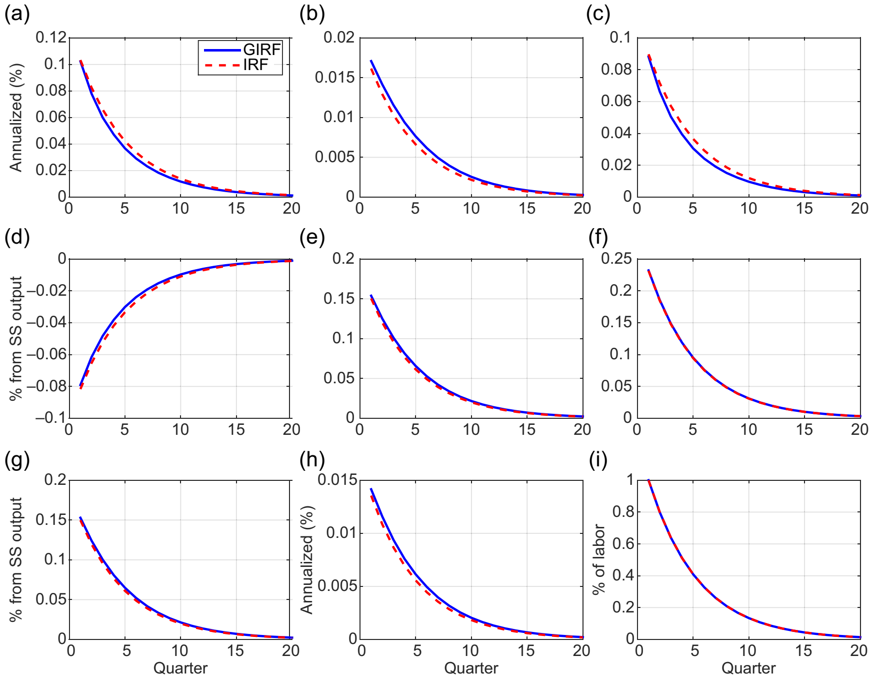

FIGURE 4. Conventional IRFs and GIRFs at the steady state. The GIRFs are computed as the average of 9999 IRFs starting from the steady state. See Koop et al. (1996) for more detail. (a) Nominal interest rate. (b) Inflation. (c) Real interest rate. (d) Consumption. (e) GDP (C+G). (f) Gov spending. (g) Labor. (h) Expected inflation (%). (i) Government spending shock.

At the ZLB state, the IRFs and GIRFs are also the same for GDP and government spending, as seen in panels E and F of Figure 5. As a result, the government spending multiplier is the same regardless of using IRFs or GIRFs.

However, it is interesting to note that the IRF and GIRF for nominal interest rate are very different. Again, the GIRF for the nominal interest rate is the average of 9999 IRFs starting from the same ZLB state. If I compute the GIRF using the median IRF from the set of 9999 ones, the IRF and GIRF are very similar. The reason for the difference is that the distribution of the nominal interest rate is skewed to the right, so the mean is greater than the median.

5.2. Magnitude of Government Spending Shock

Because of the nonlinear policy functions, it is likely that the marginal effect of a government spending shock and, as a result, the spending multiplier also depend on the magnitude of the initial shock. In this subsection, I compute and plot the spending multiplier under 0.5%, 1%, 2%, and 5% shocks to government spending. The results are presented in Figure 6.

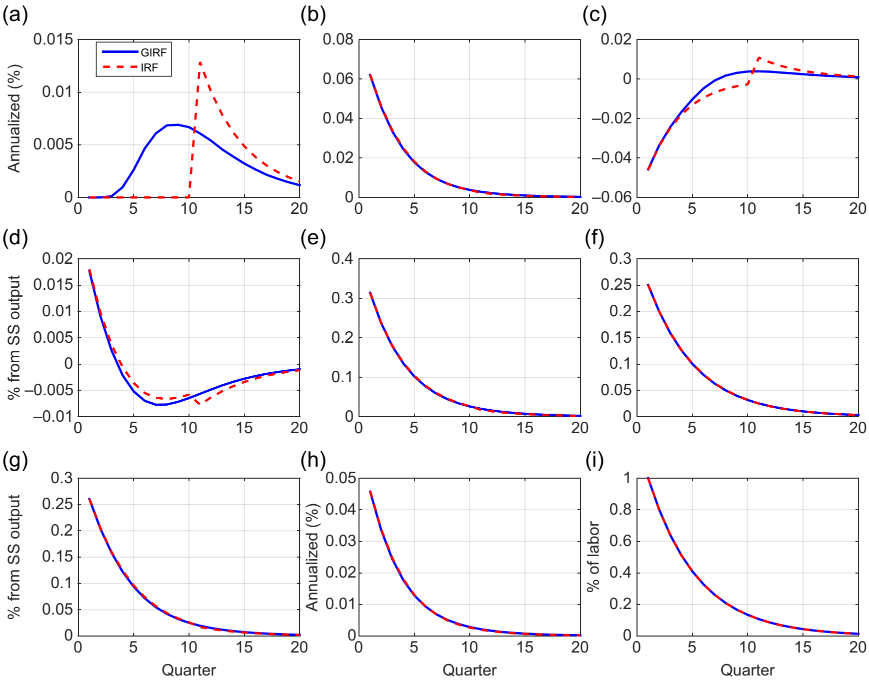

FIGURE 5. Conventional IRFs and GIRFs at the ZLB state. The GIRFs are computed as average of 9999 IRFs starting from the ZLB state. See Koop et al. (1996) for more detail. (a) Nominal interest rate. (b) Inflation. (c) Real interest rate. (d) Consumption. (e) GDP (C+G). (f) Gov spending. (g) Labor. (h) Expected inflation (%). (i) Government spending shock.

As seen from Figure 6, under the benchmark spending persistence of 0.86, the greater the initial government spending shock, the smaller the multiplier at the ZLB. However, the results are quite robust when the magnitude of the spending shock is around 1%. The decline of the multiplier in the magnitude of the initial spending shock is consistent with the findings found in Fernandez-Villaverde et al. (2015) and Christiano et al. (2011).

5.3. Cumulative Spending Multiplier

Although the impact multiplier is commonly used in the literature for the ZLB and fiscal policy, I would like to see if the results of the paper are robust if I use the present value multiplier. The present value multiplier is computed using the following formula:

$$m^{ZLB}_{PresentValue} = \left(\frac{\sum_{t=1}^{T} \left(\Pi_{j=0}^{t-1}\beta_j\right) \left(GDP_t^2 - GDP_t^1\right)}{\sum_{t=1}^{T} \left(\Pi_{j=0}^{t-1}\beta_j\right) \left(G_t^2 - G_t^1\right)} \right), \tag{17}$$

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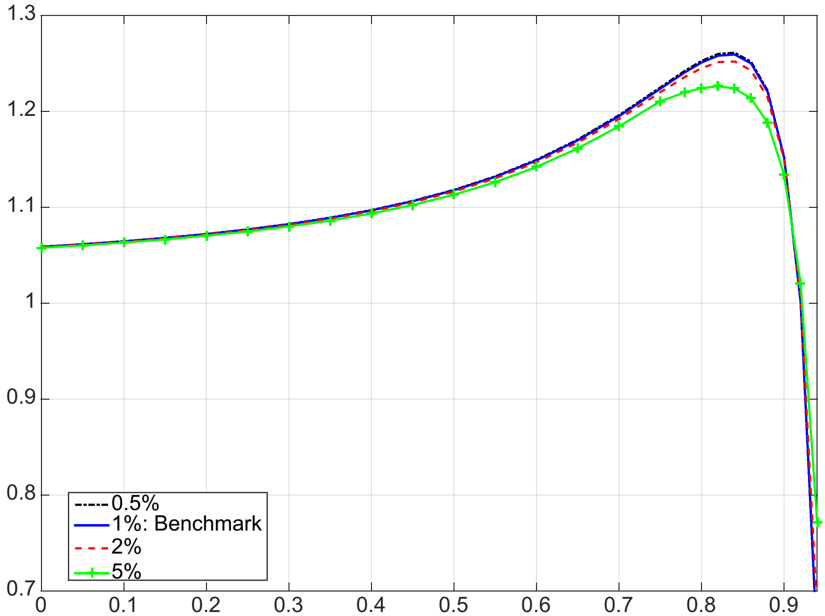

FIGURE 6. Government spending multipliers at the ZLB under various values of initial government spending shock. The ZLB binds for 10 periods on average.

where $(GDP_t^1, G_t^1)_{t=1}^T$ denotes the responses of GDP and spending under only preference shock; $(GDP_t^2, G_t^2)_{t=1}^T$ denotes the responses of GDP and spending under both preference and government spending shocks.[30]

The present value multiplier is presented together with the impact multiplier in Figure 7 for the benchmark case and for the case with perfect foresight and no rebate. We can see that the non-monotonic relationship between the present value multiplier and spending persistence still holds. At the persistence of 0.86, the present value multiplier is still above 1. However, for the case with perfect foresight and no rebate, both the impact multiplier and the present value multiplier are less than 1. In particular, the present value multiplier in this case is only around 0.8.

5.4. Multiplier and ZLB Duration

To see the impact of an expected ZLB duration on the effectiveness of government spending, I compute the impact multiplier at different ZLB states where the expected ZLB duration varies. The results are presented in Figure 8. From this figure, when the expected ZLB duration increases, the multiplier increases. In particular, at the benchmark government spending persistence of 0.86, the multiplier is 1.45 if the ZLB is expected to bind for 12 periods. This value is greater than 1.25 under the benchmark case with an average 10-period binding ZLB. The

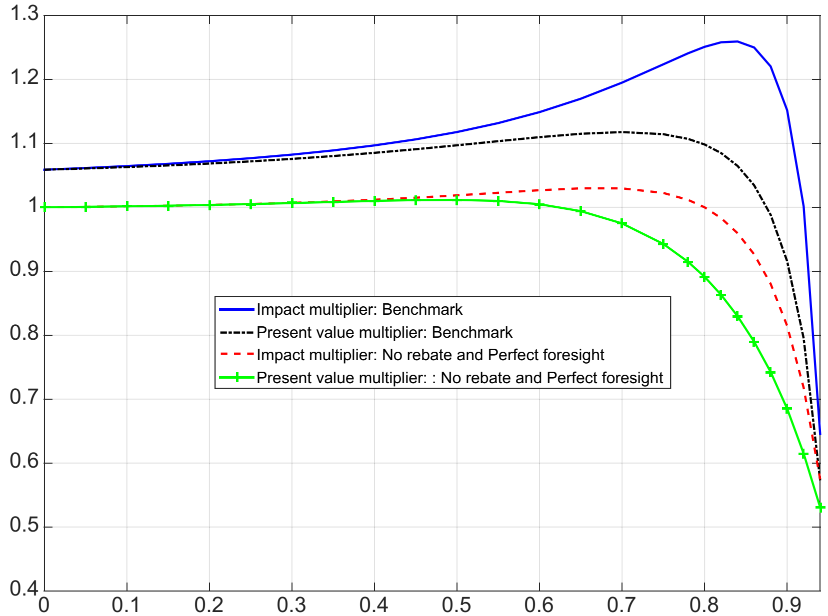

FIGURE 7. Government spending multipliers at the ZLB. The ZLB binds for 10 periods on average.

results are consistent with the ZLB literature, such as Fernandez-Villaverde et al. (2015) and Woodford (2011).

It is interesting to note that I do not see the discontinuity in the impact multiplier when the expected ZLB duration is greater than 10 as found in some papers, including Boneva et al. (2016). The difference comes from the settings of the two models. In their model, the expected ZLB duration is determined by the exogenous probability of staying at the ZLB regime, while in this paper the expected ZLB duration is determined by both the magnitude of the preference shock and the persistence of the shock.

5.5. Multiplier and Price Adjustment Costs

Figure 9 shows that the government spending multiplier and the price adjustment cost parameter ϕ are negatively correlated. When the price adjustment cost parameter decreases, the multiplier increases. The intuition is straightforward. A smaller adjustment cost parameter leads to more price flexibility. As a result, inflation increases more under an increase in government spending, leading to a greater decline in the real interest rate if the nominal interest rate remains at the ZLB. This greater decrease in the real interest rate causes consumption and output to further increase, raising the government spending multiplier.

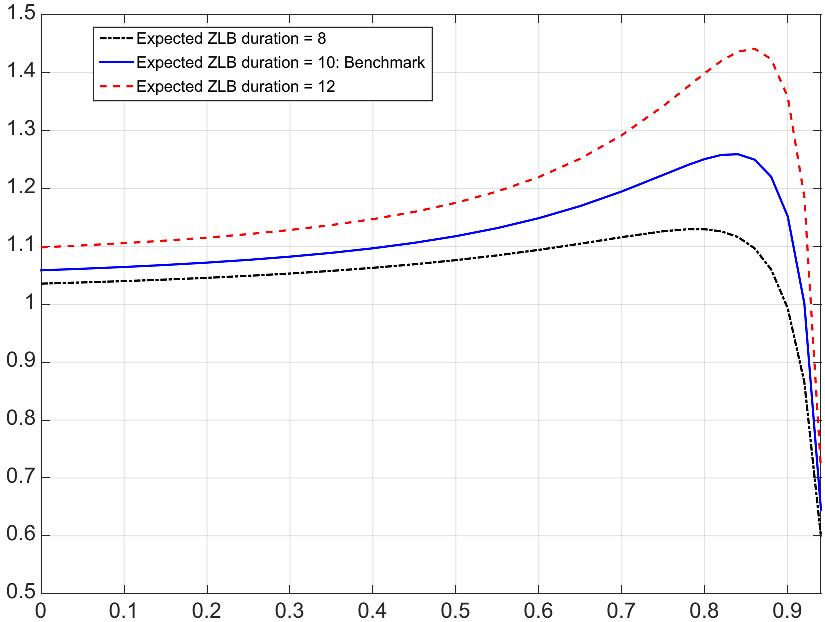

FIGURE 8. Government spending multipliers at the ZLB. The ZLB binds for 8, 10, and 12 periods on average.

6. CONCLUSION

This paper contributes to the literature regarding the ZLB and the role of fiscal policy by investigating the magnitude of government spending multipliers in a standard DNK model that allows for an occasionally binding ZLB. My approach is novel because it takes into account the unconditional probability of hitting the ZLB that is in line with US data. Moreover, I compute the multiplier in a state that mimics the Great Recession. I also study the role of government spending persistence, not just the magnitude of spending, on the government spending multiplier while the economy is at the ZLB.

The main findings of the paper include (1) the magnitude of the government spending multiplier is a non-monotonic function of the persistence of government spending shocks while the economy is at the ZLB; (2) at the estimated persistence of 0.86, the multiplier is around 1.25; (3) under the perfect foresight condition or conventional Rotemberg price setting without rebates, the multiplier is quite small, around 1, or less.

In addition, I show that conventional IRFs and GIRFs generate very similar results, except for the nominal interest rate. Future research might study further the difference between these two methods in the framework of New Keynesian DSGE models. If the difference is actually small, then using GIRFs might not be a good choice given its computational cost due to Monte Carlo simulations.

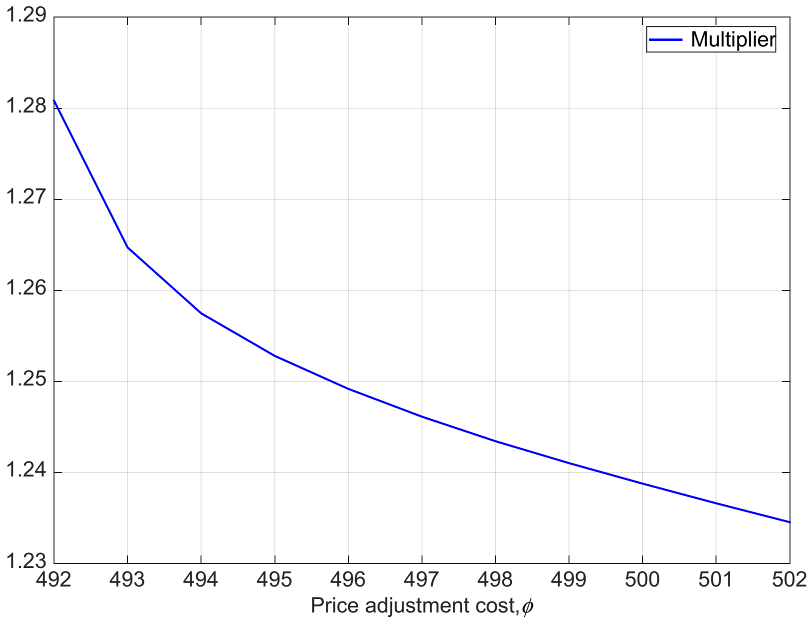

FIGURE 9. Government spending multipliers at the ZLB and price adjustment costs. The ZLB binds for 10 periods on average.

NOTES

1. The expected duration of the ZLB remains unchanged when I change the persistence of government spending because the magnitude of the government spending shock is small enough.
2. Eggertsson and Singh (2016) show that the price adjustment cost as a fraction of aggregate demand could be highly unrealistic at the ZLB in a model without rebates. As a result, the deadweight loss could be unrealistically high if the cost is not paid to the household. In addition, Miao and Ngo (2018) recommend using Rotemberg pricing with rebates in order to generate results equivalent to the Calvo pricing. However, the Rotemberg pricing has a significant computational advantage over the Calvo pricing with an economy-wide labor market, which is commonly used in the literature, because it has a one less state variable.
3. To save space, I do not discuss all these papers in detail. I only discuss the difference between this paper and the papers that are most closely related, including Boneva et al. (2016) and Woodford (2011).
4. In addition to the papers cited in the main text, an incomplete list of papers using nonlinear models with a ZLB constraint includes Wolman (2005), Nakata (2016), Ngo (2014), Richter and Throckmorton (2015), and Miao and Ngo (2018).
5. Except for Aruoba et al. (2018), these papers solve the targeted inflation equilibrium only.
6. Mitra et al. (2019) investigate fiscal policy multipliers in a real business cycle (RBC) model with learning. They show that learning helps raising the multipliers to be in line with empirical values.
7. Eggertsson and Singh (2016) show that the price adjustment cost as a fraction of aggregate demand could be highly unrealistic at the ZLB in a model without debates. This means that the deadweight loss caused by adjusting prices could be highly unrealistic.
8. The number of states in a model with Calvo pricing and with an economy-wide labor market is greater than that of a model with Rotemberg pricing due to the appearance of relative price dispersion.

Given the number of times I have to solve the model in this paper, having one more state variable would be very computational expensive. Therefore, I use the Rotemberg pricing with rebates to avoid the computational expense.

9. See Ascari and Rossi (2012), Note 12, for more detail.

10. For an example, see Aruoba et al. (2018) and Ngo (2014), among others.

11. For an example, see Nakata (2016) and Aruoba and Schorfheide (2013), among others. It would also be interesting to compare the time-dependent Calvo price setting to another state-dependent price setting as in Dotsey et al. (1999) at the ZLB.

12. Some researchers use the flexible price equilibrium output as the output target in the Taylor rule, and some researchers also include the lagged interest rate. These alternative specifications will not change my key insights.

13. Please note that the fiscal policy in the paper is a kind of rules that stabilizes the government spending-to-GDP ratio. I have also modeled government spending G_t, instead g_t, following an AR(1) process. The main results and insights are robust. To save space, I do not report them here; however, the additional results are available upon request.

14. In the conventional New Keynesian model with a representative household used in this paper, equilibrium debts (bonds) are zero. All government purchases are financed via lump-sum taxes. In order to model a fiscal rule that stabilizes the debt-to-GDP ratio, debts must be nonzero. So, we must modify the model significantly by either (i) using heterogenous agents as in Eggertsson and Krugman (2012) or (ii) allowing heterogenous asset markets, that is, short-term bonds and long term-bonds as in Chen et al. (2012) or both (i) and (ii) as in Leeper et al. (2017). These modifications though interesting will make nonlinear computation very burdensome and will not likely change the key insight of this paper. Modifying the model to allow a fiscal rule that stabilizes the debt-to-income ratio and to study the role of government purchase persistence would be an interesting extension of this paper. It requires a substantial amount of work and deserves a separate paper.

15. The model with Rotemberg pricing has a significant computational advantage over the model with Calvo pricing and an economy-wide labor market, which is commonly used in the literature, because it has one less state variable—the relative price dispersion.

16. See Appendix A for my calculation of the unconditional probability of hitting the ZLB.

17. Based on my simulation, there are many simulated series where the ZLB binds more than 30 periods consecutively.

18. See Woodford (2011), Boneva et al. (2016), and Christiano et al. (2011)

19. The data come from the Federal Reserve Economic Data website. The output gap is computed as the percent difference between the real GDP and real potential GDP. The actual real GDP series is GDPC1. The real potential GDP series is GDPPOT. The GDP deflator series is A712RD3Q086SBEA. The CPI series is CPIAUCSL. The core CPI is CPILFESL. The core PCE series is PCEPILFE.

20. Note that I am aware of the fact that the policy functions are nonlinear, so the impulse response functions are both shock and state dependent. Therefore, in Section 5, I also compute the multiplier based on GIRFs, as described in Koop et al. (1996) and in Miao and Ngo (2018). The results based on GIRFs are quite similar to those based on the conventional IRF explained in this section.

21. I also compute the results under 0:5%, 1%, 2%, and 5% government spending shocks. The additional results are presented in Section 5 and are consistent with the findings in Fernandez-Villaverde et al. (2015) and Christiano et al. (2011) that the larger the government spending, the smaller the spending multiplier. However, the difference is very small for shocks in the range of 1%–3%.

22. There are other multipliers including cumulative multipliers and present value multipliers; however, most of the literature compute and report impact multipliers. Therefore, in this section I use impact multipliers for meaningful comparison. In Section 5, I compute and report both cumulative and present multipliers, and find that the main results of this paper hold.

23. The series for real GDP and real government purchases on final consumption and investment are GDPC1 and GCEC1, respectively.

24. Intuitively, if we include more of the lower-frequency components in the filtering, the persistence would be larger than the estimated value of 0.86. In particular, if we do not filter the government

purchases-to-GDP ratio at all, the estimated persistence is 0.9988 and the Augmented Dicky–Fuller test with four lags is not able to reject the null that the ratio process is nonstationary. As shown in Figure 1, if the persistence is greater than 0.92, the multiplier is less than 1.

25. They also assume the ZLB is expected to bind for 10 periods as is done in my paper.

26. Ercolani and Azevedo (2018) show that allowing substitutability between private and government consumption would significantly reduce the size of government spending multiplier. This feature is not included in the standard NK literature with the ZLB, including my paper.

27. See the calibration subsection for more detail.

28. As explained in Section 4, the conventional impulse responses are computed as the difference between the responses under both a preference shock and a government spending shock and the ones under only the preference shock.

29. Computing results based on GIRFs for the whole paper (and for different values of persistence) is computationally expensive because it is based on Monte Carlo simulations. My main purpose is to show that GIRF and IRF results are not much different in this paper. So, it is safe to use IRF results and avoid expensive computation of GIRFs.

30. I also compute the cumulative multiplier using the formula

$$m_{Cumulative}^{ZLB} = \left(\frac{\sum_{t=1}^{T} \left(GDP_t^2 - GDP_t^1 \right)}{\sum_{t=1}^{T} \left(G_t^2 - G_t^1 \right)} \right). \quad \mathbf{(18)}$$

However, the cumulative multiplier is very similar to the present value multiplier. To save space I do not report the cumulative multiplier in this paper. The additional results are available upon request.

31. For a robust check, I also use the PCE index, instead of the CPI of All Items Less Food and Energy. The result is quite robust.

32. Ball (2013) argues that in the three out of seven recent recessions excluding the 2007-2009 recession, the nominal interest rate would have hit the ZLB if the inflation rate had been around 2% at the start of the recessions. These three recessions are the 1969-1970 recession, the 1973-1975 recession, and the 1980 recession. Hence, the probability of hitting the ZLB conditional on a recession would be around 50%, or four out of eight recessions, if the Fed targeted a 2% inflation rate post World World II.

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A. APPENDIX: PROBABILITY OF HITTING THE ZLB

The probability of hitting the ZLB plays a key role in determining the level of the government spending multiplier. Many economists, including Mishkin (2011), believe that the 2007–2009 recession with a binding ZLB is a rare disaster that occurs once every 70 years. However, other economists, for example, Ball (2013), disagree. In this appendix, I will use different methods to estimate the unconditional probability of hitting the ZLB using the actual US data.

The first method to compute unconditional probability of hitting the ZLB is

$$\Pr(\text{Policy rate} = 0) = \frac{\text{No. of periods the target FFR} = 0}{\textit{No}\text{. of all periods where the target FFR is available}}. \quad \textbf{(A1)}$$

Using the US data, the target Federal Funds Rate (FFR) data span from 1982:IV to 2017:II. There are 139 observations in total and 28 observations with zero (from 2008:IV to 2015:III). Thus, the unconditional probability of hitting the ZLB is

$$\Pr(\text{interest rate} = 0) = \frac{28}{139} = 0.2014 \text{ or } 20.14\%. \quad \textbf{(A2)}$$

The second method is to answer the question raised in Ball (2013): what the unconditional probability of hitting the ZLB would have been if the Fed had targeted the inflation rate of 2%? To this end, I follow Ball (2013) and use the real interest rate to answer the question. Specifically, the nominal interest rate equals the real interest rate plus the expected inflation rate. Therefore, we can interpret the ZLB on the nominal interest rate as a lower bound of minus expected inflation for the real interest rate. If the target inflation rate is 2%, the expected inflation rate would be 2% and the lower bound on the real interest rate would be -2%. However, Ball (2013) argues that a recession is likely to push expected inflation down somewhat and that history suggests that the inflation rate fell about 1% during the past recessions that started with 2%–3% inflation rates. Therefore, he finds that the bound on the real interest rate is -1%.

Figure A.1 shows (i) the effective federal funds rate; (ii) the real interest rate computed as the effective federal funds rate minus the inflation rate, where the inflation rate is calculated as a percentage change of the CPI of All Items Less Food and Energy from a year ago; and (iii) the lower bound of the real interest rate. The data span from 1957:IV, when the data for the CPI of All Items Less Food and Energy was first available, to 2017:II. So, we have 239 observations in all.

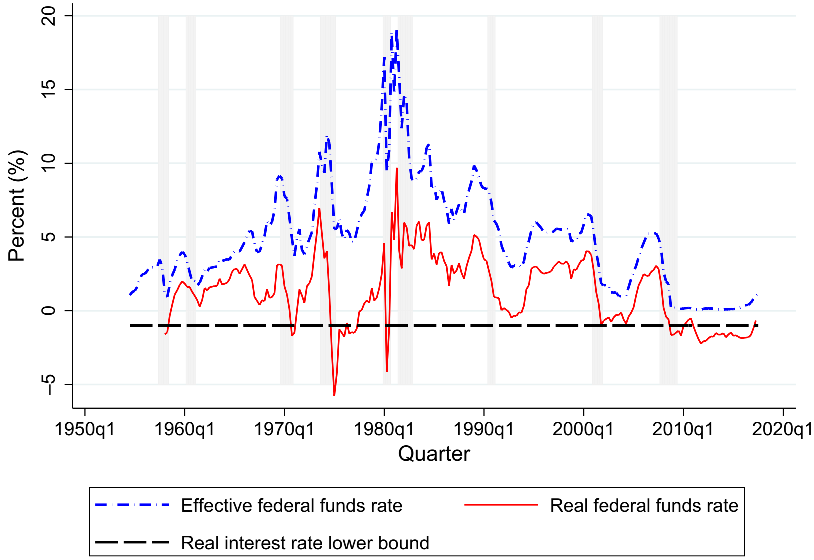

FIGURE A.1. Real federal funds rate is the effective federal funds rate minus the inflation rate computed as a percentage change in the CPI of All Items Less Food and Energy a year ago. The shaded areas indicate the US recessions. Source: the FRED.

From the figure, we are able to see that the real interest rate was smaller than the bound, and, as a result, the nominal interest rate might have hit the ZLB, in the five recessions: 1957:III–1958:II, 1969:IV–1970:IV, 1973:IV–1975:I, 1980:I–1980:IV, and 2007:IV–2009:II.[31] Especially using the real interest rate, we can very well infer that the nominal interest rate reached the ZLB during the 2007–2009 recession. In addition, the nominal interest rate almost hit the ZLB in the 2001 recession.

Examining the real interest rate since 1957:IV, when the CPI data were first available, I find that the ZLB was binding in 47 quarters. Given that the sample has 239 quarters, the unconditional probability of hitting the ZLB is 19.7%. When I compute the real interest rate using the CPI of All Items, the probability of hitting the ZLB is slightly smaller, around 16.1%.[32]

In this paper, I calibrate the preference shock to match the unconditional probability of hitting the ZLB 17%, which is in the lower range of [16.1%, 20.14%].

B. APPENDIX: THE GREAT RECESSION

It is well known that the magnitude of the government spending multiplier depends on the state of the economy. In this subsection, I will discuss different ways to measure the adversity of the Great Recession based on that I calibrate the shocks and compute the government spending multiplier.

https://doi.org/10.1017/S136510051900049X Published online by Cambridge University Press

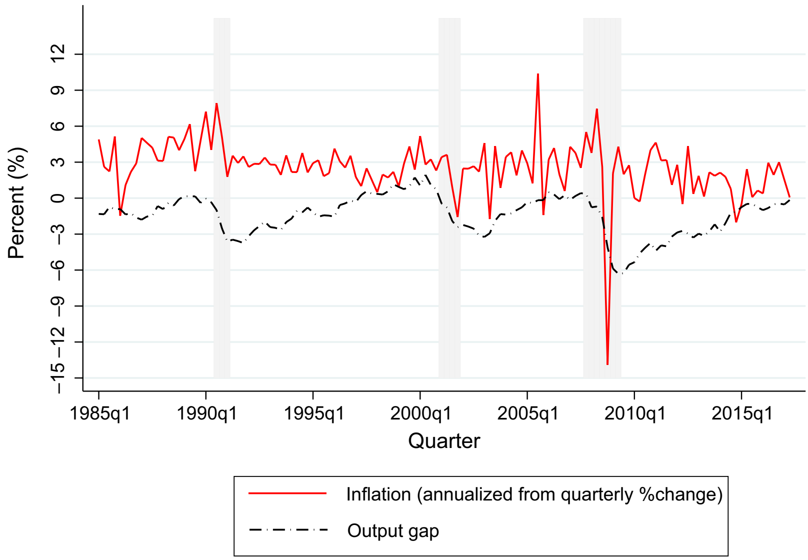

FIGURE B.1. Output gap and inflation in the US Source: the FRED.

Figure B.1 shows the output gap and inflation series for the USA. To compute the output gap, I take the percentage difference between real GDP and potential real GDP. To compute inflation, I first compute the quarterly percentage change in the CPI, then annualize it by multiplying it by 4. The quarterly data on CPI (of All Items), real GDP, and potential real GDP are collected from the FRED website hosted by the Federal Reserve Bank of St. Louis.

As seen from this figure, at the trough of the Great Recession, the output gap was as large as around −6.5% (the dash-dotted black line), and the annualized inflation rate (the solid red line) was approximately −14% or −3.5% per quarter. If I use core CPI that excludes food and energy prices, the inflation rate was much smaller. To be conservative, I target an inflation rate of about −0.7 per quarter (or −3% per year). In conclusion, I compute the multiplier at the state that partially mimics the Great Recession: the expected ZLB duration is 10 quarters, the output gap is around −6.5%, and the inflation rate is about −0.7% per quarter.

C. APPENDIX: SOLUTION METHOD

Our solution method is close to, but slightly different from, the one used in Fernandez-Villaverde et al. (2015). Similar to their method, we do not approximate the policy function for the nominal interest rate. Instead, the nominal interest rate is always determined by equation (13) at every state, in or out of the set of collocation nodes. However, different from them, we approximate the expectations as function of state using a finite element method called the cubic spline interpolation; see Judd (1998) and Miranda and Fackler (2002) for more details.

The main advantage of this approach is we do not have to worry about the kink when the ZLB starts binding.

Following Miranda and Fackler (2002), we rewrite the functional equations governing the equilibrium in a more compact form:

$$f\left(s, X\left(s\right), E\left[Z\left(X\left(s'\right)\right)\right]\right) = 0, \tag{C1}$$

where

- $f: R^{2+6+2} \to R^6$ is the equilibrium relationship;
- $s = (\beta, g)$ is the current state of the economy;
- $X(s) = (R(s), C(s), N(s), w(s), \Pi(s), Y(s))'$, and $X: R^2 \to R^6$ is the policy function, where $R = 1 + i$ is the gross interest rate and $\Pi = 1 + \pi$ is the gross inflation rate.
- s' is the next period's state that evolves according to the following motion equation:
  $s' = g(s, \varepsilon) = \begin{bmatrix} \beta' = \beta^{\rho_\beta} \exp(\varepsilon_\beta) \\ g' = g^{\rho_g} \exp(\varepsilon_g) \end{bmatrix},$
  where ε_β and ε_g are the innovations of the preference and the government spending shocks;
- $Z\left(X\left(s'\right)\right) = \begin{pmatrix} Z_1\left(X\left(s'\right)\right) = \frac{C(s')^{-\gamma}}{\Pi(s')} \\ Z_2\left(X\left(s'\right)\right) = \frac{1}{C(s')^{-\gamma}}\left(\Pi\left(s'\right) - 1\right)\Pi\left(s'\right) Y\left(s'\right) \end{pmatrix}.$

Instead of solving policy function, we actually solve the expectations as functions of state using a finite element method called the cubic spline interpolation. Define $h(s) = E\left[Z\left(X\left(s'\right)\right) | s\right]$; the following is the simplified algorithm:

- *Step 1: Define the space of the approximating functions and collocation nodes* $S = (S_1, ..., S_N)$, where $N = N_\beta \times N_g$, and N_β and N_g are the numbers of grid points along each dimension of the state space. In this paper, we approximate the expectations:
  $h(s) = \left(\phi(s)\theta_{h_1}, \phi(s)\theta_{h_2}\right)'$ or
  $h(s) = \phi(s)\Theta,$
  where
  - $\phi(s)$ is a $1 \times N$ matrix of cubic spline basis functions evaluated at state $s \in S = (S_1, ..., S_N)$.
  - $\Theta = (\theta_{h_1}; \theta_{h_2})$ is a $N \times 2$ coefficient matrix that we want to approximate.
- *Step 2: Initialize the coefficient matrix* Θ^0 *and set up stopping rules.*
- *Step 3: At each iteration* j *given the corresponding* Θ^j, *we implement the following substeps:*

  1. At each collocation node s_i, $s_i \in \{S_1..S_N\}$, compute $h(s_i)$ using the approximating functions for the expectations.
  2. Solve for $X(s_i)$ such that $f\left(s_i, X(s_i), h(s_i)\right) = 0$. We solve this complementarity problem using the Newton method.

https://doi.org/10.1017/S136510051900049X Published online by Cambridge University Press

- *Step 4: Update h using the following substeps:*

  1. Approximate policy functions for C, Π, Y using a cubic spline interpolation.
  2. At each collocation node s_i, $s_i \in \{S_1..S_N\}$, update $h(s_i) = (h_1(s_i), h_2(s_i))$ using

$$h_1(s_i) = \sum_j^{25} w_j \left[\frac{C\,(s')^{-\gamma}}{\Pi\,(s')} \right], \tag{C2}$$

$$h_2(s_i) = \sum_j^{25} w_j \left[\frac{(\Pi\,(s') - 1)\,\Pi\,(s')\,Y\,(s')}{C\,(s')^{-\gamma}} \right], \tag{C3}$$

  where the innovations for the preference and government spending shocks are discretized using the Tauchen and Hussey (1991) method with 25 nodes.

- *Step 5: Update* $\Theta^{j+1} = \Phi^{-1}\Theta^j$, *where* $\Phi = (\phi(s_1), ..., \phi(s_N))'$ (see Note 10).
- *Step 6: Check the stopping rules. If not satisfied, go to Step 3, otherwise go to Step 7.*
- *Step 7: Report results.* We use the approximated expectation functions to solve for the equilibrium value at any state. So, we are able to find almost exactly the kink for the nominal interest rate.

In addition, we write our code using a parallel computing method that allows us to split up a large number of collocation nodes into smaller groups assigned to different processors to be solved simultaneously. This procedure reduces computation time significantly. We obtain the maximal absolute residual across the equilibrium conditions of the order of 10^{-8} for almost all states off the collocation nodes. For a few states when the ZLB becomes binding, the maximal absolute residual is of the order of 10^{-5}. This is quite standard given the kink in the interest rate policy function; see Miranda and Fackler (2002) and Judd et al. (2011) for more information.