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ORIGINAL ARTICLE

A duration estimator for a continuous time war of attrition game

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Abstract
We developed a maximum likelihood estimator corresponding to the predicted hazard rate that emerges from a continuous time game of incomplete information with a fixed time horizon (i.e., Kreps and Wilson, 1982, *Journal of Economic Theory* **27**, 253–279). Such games have been widely applied in economics and political science and involve two players engaged in a war of attrition contest over some prize that they both value. Each player can be either a strong or weak competitor. In the equilibrium of interest, strong players do not quit whereas weak players play a mixed strategy characterized by a hazard rate that increases up to an endogenous point in time, after which only strong players remain. The observed length of the contest can therefore be modeled as a mixture between two unobserved underlying durations: one that increases until it abruptly ends at an endogenous point in time and a second involving two strong players that continues indefinitely. We illustrate this estimator by studying the durations of Senate filibusters and international crises.

Keywords: Duration and survival analysis; maximum likelihood estimation (MLE)

1. Introduction

Politics is replete with situations in which two players, or groups of players, decide whether to compete for a prize. As initially conceived in evolutionary biology (e.g., Maynard Smith 1974; Riley 1980) and then further developed by Kreps and Wilson (1982) in the context of explaining the value of a firm's reputation, these games can be characterized as *wars of attrition*, in that once both sides decide to fight for the prize, the contest continues until one player concedes or time runs out. In particular, the equilibrium of interest involves strong players contesting the issue and fighting until time runs out, whereas weak players play a mixed strategy in which they probabilistically quit the contest over time.

This equilibrium produces a number of distinct empirical predictions. First, weak players exit at an increasing rate, resulting in a hazard rate for contests involving at least one weak player that exhibits positive duration dependence. Second, weak players will never contest the issue until time expires. Instead, there is an endogenously determined point in the game by which all weak players will have conceded. Since strong players never quit, any contest lasting beyond this point has a hazard of zero until time expires, producing a discontinuity as the overall hazard drops precipitously from the large, increasing hazard down to zero.

Although these features of the equilibrium provide for rich theoretical insights into the contest being modeled, they also make it difficult to test the predictions of the theoretical model with existing duration estimators. We therefore develop an estimator that captures the salient features of the equilibrium outlined above. As such, the estimator we derive provides a way for scholars to test the predictions of continuous time games of incomplete information with a fixed end point.

Our estimator uses a mixture model approach to account for the fact that the observed data from the war of attrition game we describe represent a combination of two distinct processes. The first—in our model, those involving at least one weak player—has a distribution that permits an increasing or decreasing hazard but with a support ending with the endogenous time horizon. The second—those involving two strong players—allows any non-negative duration and also features a monotonic hazard. By estimating a mixture between these distributions we are able to assess whether the resulting empirical hazards fit those predicted by the war of attrition theory, including estimating the endogenous time point and testing whether contests involving at least one weak player exhibit an increasing hazard. In addition to evaluating the presence and shape of the two hazards, our estimator, which we release as a Stata package, allows researchers to include covariates explaining the hazard of contests involving at least one weak player and the mixture probability. Thus researchers can separately assess how shifts in players' types affect the number of contests involving weak players and the failure rate of such contests contribute to changes in the observed durations.

We demonstrate the value of our estimator with two applications. First, we examine the duration of filibusters in the United States Senate to evaluate claims that the modern filibuster no longer serves an informational role by sorting out weak and strong sides to a given issue. Our second application examines the duration of international crises, which has long been viewed through the war of attrition framework. Both applications exhibit the features of a war of attrition, with a small hazard for contests involving two strong players and an increasing hazard with an endogenous end point that occurs well below the largest observed durations. We also find evidence that ignoring the mixture structure of the data or factors that influence the mixture leads to incorrect inferences about covariates' influence on the hazard for contests involving at least one weak player.

Our analysis thus provides new insights into the significant topics of international crises and Senate filibusters. But our contributions extend well beyond these specific applications. To begin with, our approach follows the logic of previous studies that espouse the importance of deriving empirical estimators directly from theoretical models to improve our ability to understand empirical phenomena (see, e.g., Morton 1999; Signorino 1999). In this respect, our study is closely tied in spirit to the Empirical Implications of Theoretical Models program (see, e.g., Granato *et al.* 2010).

This tighter link between theory and empirical work is especially relevant in the context of war of attrition models, given the strikingly widespread use of such models to examine political phenomena. For international crises, our new estimator provides a more nuanced evaluation of the crises bargaining process, one that outperforms standard duration models and provides evidence consistent with the presence of two types of states (weak and strong) that use the bargaining process as a way to signal their willingness to resolve disputes via conflict. For filibusters, we illustrate our estimator's value by addressing an important question that has plagued scholars and commentators alike: does the modern filibuster (i.e., since cloture reform in 1975) still serve an informational role by sorting out weak and strong sides to a given issue? Our results suggest it does.

Finally, as Powell (2017: 22) recently noted, "[w]ars of attrition provide workhorse models for analyzing many different kinds of conflict." Within political science, the war of attrition has been used to understand a wide variety of phenomena, including deterrence and bargaining (e.g., Fearon 1994, 1995), cabinets and coalitions in parliamentary democracies (e.g., Carmignani 2001) and budgetary negotiations (e.g., Klarner *et al.* 2012). Our Supplementary appendix references over two dozen relevant examples in Political Science and other fields. Clearly, this theory has been applied to a strikingly wide range of noteworthy political topics. Yet, empirical work has lagged behind. Our approach solves some of the problems associated with testing these models, and thus allows for more accurate and appropriate interrogation of the data across a broad set of political (and non-political) phenomena.

In this section, we outline the basic assumptions and results of the Kreps and Wilson (1982) continuous time war of attrition model with incomplete information. Our presentation is a bit brief, so we refer interested readers to the discussion in Dion *et al.* (2016) and its very detailed Supplementary technical appendix.

1.1 Preliminaries

The original application of the model was to competition between a monopolist and a potential entrant (Kreps and Wilson 1982), which was treated as a continuous time war of attrition. The model involves competition between two players, $i = 1, 2$ over an issue, worth $a > 0$ to player 1 and $b > 0$ to player 2. The game proceeds in two stages: first, players must decide whether to contest the issue, then, if both players do contest, they engage in a co$T$ntinuous time war of attrition that ends either when one player concedes or when time runs out. The maximum length of time, T, is normalized to 1. The player who does not concede receives the prize, the value of which decreases over the course of a contest, whereas the other side receives no value for the prize. Discounting occurs as the contest continues so that the prize's value is reduced to $(1 - t)a$ if player 1 wins and $(1 - t)b$ if player 2 wins.[1]

Both players have one of two types: weak or strong. Weak players pay a cost for prolonging the contest whereas strong ones gain. Payoffs are based on whether a player concedes or wins the contest and also on when the contest ends. A strong player 1 will receive $(1 - t)a + t$ if she wins at time t compared to t if she concedes at t. For a weak player 1 the payoff of winning a contest at time t is $(1 - t)a - t$ and $-t$ for losing one. Player 2 has similar payoffs, with b replacing a in the preceding. Each player knows their type, but is uncertain of their opponent's type. In particular, it is common knowledge that there are prior probabilities p that player 1 is strong and q that player 2 is strong, with $0 < p < 1$ and $0 < q < 1$.

We focus on the sequential equilibrium identified by Kreps and Wilson (1982). Given their positive value for fighting, strong players always contest the issue and continue fighting as long as necessary. Weak players do not always contest the issue but, when they do, they quit fighting at some positive rate. This rate is determined by the need to keep weak players indifferent between continuing to fight and conceding. For this to occur, a precise balancing condition must be met at each point in time during the contest:

$$q_t = p_t^{b/a}, \tag{1}$$

where p_t and q_t give the probability that each player is strong at time t.

When the two players have the same value for the prize (i.e., when $a = b$), then we have a symmetric situation in which the probability of facing a strong type is equal for both players. When $a > b$, player 1 has a greater reason, *ceteris paribus*, to continue fighting. To maintain indifference, then, the probability of facing a strong player 2 must be greater, and in fact $q_t > p_t^{a/b}$ if $a > b$. The situation is reversed, of course, when $a > b$.

Since the balancing condition must be satisfied at all time periods, it must be satisfied at the outset (i.e., where $t = 0$). This is unlikely to be the case given exogenously specified prior beliefs (p, q): either there will be an excess probability that player 2 is strong (C) or that player 1 is strong $(q < p^{a/b})$. For simplicity, we assume the side with the excess probability will play a mixed strategy in the first stage, bringing beliefs at $t = 0$ to the required balance.[2] This assumption affects only entry rates and does not affect the general shape of the hazard once a contest begins.

[1]Dion *et al.* (2016, Supplemental appendix) solve a version of the game in which the value of the prize does not decrease. That version predicts two flat hazards and therefore constitutes a special case of the estimator we derive here.

[2]See Figure 1 in the technical appendix of Dion *et al.* (2016) for an illustration of the entry process and the constraint of Equation 1 in equilibrium.

Depending on which of these two cases the game starts in, the equilibrium strategies are represented differently in terms of the parameters of the model, but are still analogous to each other. We focus on the first case (an excess probability that player 1 one is weak) for ease of presentation.

1.2 Predicted hazard rate

In the equilibrium just outlined, the rates of exit for each type (subscript S or W) of player (superscript 1 or 2) at time t are as follows:

$$h_S^1(t) = 0, \tag{2}$$

$$h_S^2(t) = 0, \tag{3}$$

$$h_W^1(t) = \frac{1}{a(1-t) - aq(1-t)^{(a-1)/a}}, \tag{4}$$

$$h_W^2(t) = \frac{1}{b(1-t) - bq^{a/b}(1-t)^{(b-1)/b}}. \tag{5}$$

Furthermore, since at $t = 0$, Equation 1 implies that $q^{a/b} = p$, the initial distribution over pairs of types at $t = 0$ is:

$$\psi_{SS}(0) = q^{(a+b)/b}, \tag{6}$$

$$\psi_{SW}(0) = q^{a/b}(1-q), \tag{7}$$

$$\psi_{WS}(0) = (1 - q^{a/b})q, \tag{8}$$

$$\psi_{WW}(0) = (1 - q^{a/b})(1-q), \tag{9}$$

where $\psi_{lk}(t)$ is the proportion of the dyads with player 1 of type $l \in \Theta = \{S, W\}$ and player 2 of type $k \in \Theta$.

These hazard rates imply a survival function equal to one for strong players and the following for weak players:

$$S_W^1(t) = \frac{(1-t)^{1/b} - q^{a/b}}{1 - q^{a/b}}, \tag{10}$$

$$S_W^2(t) = \frac{(1-t)^{1/a} - q}{1-q}. \tag{11}$$

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The survivor function for any given contest is the product of the survivor function for the two players involved in it, since a contest ends when just one player decides to quit. Note that for these two survivor functions to be positive, it must be the case that $t \leq 1 - q^a$. This condition implies that all contests involving at least one weak player must end by $1 - q^a$. We refer to this endogenous stopping time as t^*.

In practice, however, we will never know for sure what types of players are involved in a contest since information about each player's type is private information (though we could employ some measures to capture variation in the prior probability of players). We therefore calculate the expected hazard rate by taking the weighted average of the unobserved hazards. In doing so we distinguish between those involving at least one weak player, which have an increasing hazard through t^*, and those involving two strong players, which have a flat hazard through $t = 1$. This allows us to develop an estimator that more closely approximates the theoretically-derived data-generating process and which offers stark predictions about the overall hazard.

We start by averaging the survival function for the three different pairings including at least one weak player (i.e., $\Theta_W^2 = \{SW, WS, WW\}$):

$$S_W(t) = \sum_{ij \in \Theta_W^2} \psi_{ij}(t) S_i(t) S_j(t), \tag{12}$$

$$= \frac{(1-t)^{(a+b)/ab} - q^{(a+b)/b}}{1 - q^{(a+b)/b}}. \tag{13}$$

We then derive the expected hazard rate for contests involving one of these three pairings:

$$h_W(t) = -\frac{\partial \ln S_W(t)}{\partial t}, \tag{14}$$

$$= \left[\frac{a+b}{ab(1-t)}\right]\left[\frac{(1-t)^{(a+b)/ab}}{(1-t)^{(a+b)/ab} - q^{(a+b)/b}}\right]. \tag{15}$$

Importantly, since all weak players have exited by some point $t^* = 1 - q^a$ the hazard function for contests involving at least one weak player is only defined up through $t^* = 1 - q^a$. Importantly, this hazard increases with t until weak types have exited by t^*.

The second part of the hazard describes contests with two strong players and, since they never quit, is zero until the game ends: $h_{SS}(t) = 0$. The mixture probability between those two hazards is given by the proportion of contests involving two strong players, $\psi_{SS} = q^{(a+b)/b}$, which means that the probability of at least one weak player is $1 - q^{(a+b)/b}$. Figure 1 plots examples of the hazards for contests with two strong players and those involving at least one weak player.

2. A war of attrition hazard estimator

In this section we propose an estimator that captures the main features of the model just described: a mixture over two types of contests, one of which has an endogenous time point by which all such contests have ended. Since the outcome of interest constitutes a duration we draw from methods developed to model time to failure from survival or duration analysis. Since strong–strong contests all fail at the same time and after all the other contests have ended, the predicted distribution of failure involves a positive density up through t^*, followed by a density of exactly zero, and ending with a point mass for the end of the strong–strong

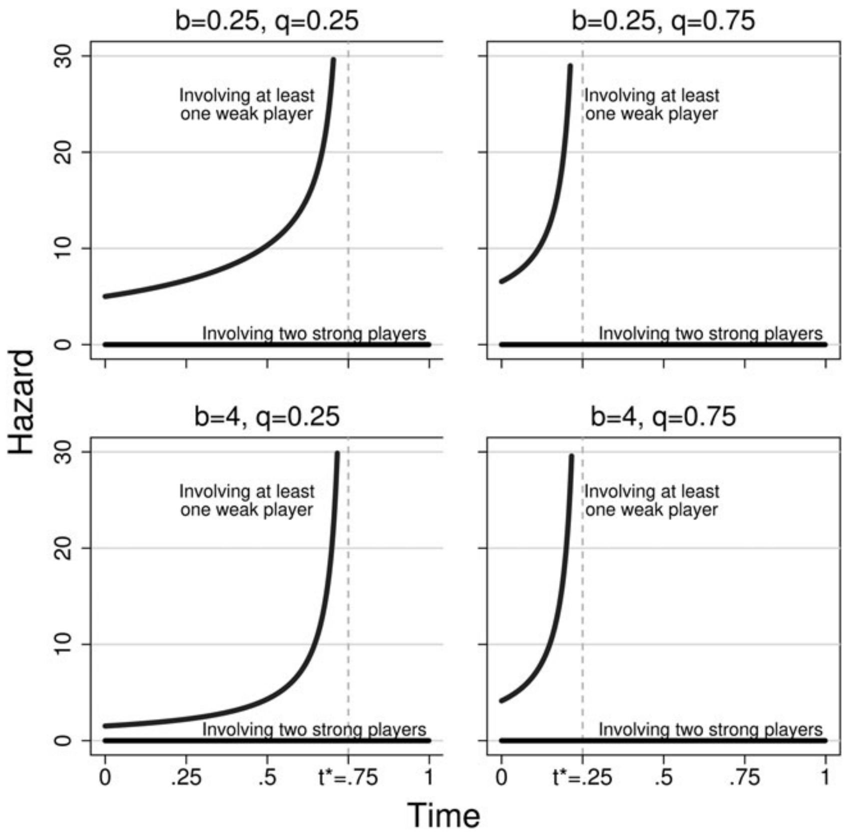

Figure 1. Predicted hazard rates for contests between two strong players and contests involving at least one weak player, varying *b* and *q*.
Note: Value of *a* set to 1.

contests. In empirical applications, however, we wouldn't know the values of t^* or the end of time, T. Yet, maximum likelihood estimation (MLE) typically assumes that the estimated parameters do not determine the support of the distribution (see, e.g., King (1998: 75) for a discussion of this). If the support for the distribution of strong–strong contests did not include values of t immediately greater than t^* (which it would not if it were a point mass strictly greater than t^*), then our estimate of t^* would in fact help set the support of the mixture distribution.

We therefore propose an estimator that deviates slightly from the theoretical prediction by allowing any non-negative duration value. We maintain the estimation of t^* since that constitutes a critical feature of the war of attrition: the informational battle leads to an endogenous time horizon after which only one type of contest remains. We therefore allow failures for contests involving two strong players to fail at any point in time. We expect that in practice the hazard of such failures will be small relative to the hazard for contests involving weak players. To test comparative statics predictions from the theory, the estimator allows users to parameterize the duration process and the mixing probabilities.[3]

[3]Our estimator bears some similarities to so-called cure duration models (Schmidt and Witte, 1989). Although both have two populations, two crucial distinctions arise: the presence of the endogenous time horizon that to estimate and the fact that in a war of attrition all observations will fail whereas in a cure model "cured" observations never fail.

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2.1 The likelihood function

To build the likelihood for the theoretically-derived data-generating process, we start with the hazard for contests involving strong–strong pairs. Although the incomplete information model predicts a hazard of zero for these contests (until the end of the period, when they all end), we relax this for the reasons described above by assuming a non-zero hazard rate, which we model as a Weibull duration. If the structure of the theoretical model is correct, we expect this hazard to be small relative to the hazard for contests involving weak types. Although we have no particular sense of "smallness" here, to be consistent with the hazards depicted in Figure 1, the hazard for strong–strong contests should be near zero or at least almost entirely below the other hazard. Such a hazard could arise by averaging over the differing end points from each of multiple instances of the war of attrition, leading to a hazard when aggregated over those contests. Depending on how they are spread out, that distribution could be increasing, e.g., if they are uniform; flat, e.g., if they are exponential; or decreasing, e.g., if they are Weibull with negative duration dependence.[4]

For strong–strong pairs the observed duration, t_i^{SS}, which we superscript by players' types for purposes of exposition, is generated according to a standard Weibull distribution:

$$t_i^{SS} = \exp(-\alpha) \times \epsilon_i.$$

When ϵ_i^p follows a standard exponential distribution, ϵ_i follows a Weibull distribution with shape parameter $p_{SS} > 0$. When the shape parameter is greater than one the data exhibit an increasing hazard over time and when it is less than one they exhibit a decreasing hazard. The Weibull has the following cumulative density function:

$$F_{\text{wbl}}(\epsilon_i) = 1 - \exp(-\epsilon_i^{p_{SS}}). \tag{16}$$

Since the rate of failure for pairs of strong players does not depend on any parameters in the theoretical model, we merely parameterize it with a constant hazard. Define $\lambda^{SS} = \exp(\alpha)$, where α captures the constant baseline hazard rate, which allows us to write $\epsilon_i = \lambda^{SS} t_i^{SS}$.[5] Inserting this into our c.d.f. and then taking the derivative with respect to t^{SS} gives us the density of an observation as

$$f_{\text{wbl}}(t_i^{SS}|\lambda_i^{SS}) = p_{SS}(\lambda_i^{SS})^{p_{SS}}(t_i^{SS})^{p_{SS}-1}\exp(-(\lambda_i^{SS}t_i^{SS})^{p_{SS}}). \tag{17}$$

For contests involving at least one weak player, the war of attrition model predicts an increasing hazard and an endogenous end point, $t^\star$. Since a Weibull distribution allows an increasing hazard as a special case, we use it as the starting point for the distribution of their durations, which we denote t_i^W. To capture the end point we modify the associated Weibull distribution by truncating it at $t^\star$. This produces a distribution with support from zero to $t^\star$ and whose cumulative distribution reaches one at $t^\star$. Thus, as in the theoretical model, all contests in this population will end by $t^\star$.

The truncated Weibull arises by dividing the cumulative distribution function of a standard Weibull, as already presented above, by the probability that $t_i^W \leq t*$. Since we wish to parameterize the hazard of contests including a weak player we write $t_i^W = \exp(-X_i\beta) \times u_i$ and define $\lambda_i^W = \exp(X_i\beta)$, where the superscript W indicates that this parameterizes the hazard for contests that involve at least one weak player. From this we derive the density and the associated hazard to

[4]Note that in our motivation analogy of averaging over contests, one still obtains an increasing hazard for the contests involving at least one weak player.

[5]Note that this corresponds to a hazard interpretation to match the exposition of our theoretical results.

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use for interpretation after estimation:

$$f_{\text{twbl}}(t_i^W|\lambda_i^W, t_i^W \leq t^*) = \frac{p_W(\lambda_i^W)^{p_W}(t_i^W)^{p_W-1}\exp\left(-(\lambda_i^W t_i^W)^{p_W}\right)}{1-\exp\left(-(\lambda_i^W t^*)^{p_W}\right)}, \tag{18}$$

$$f_{\text{twbl}}(t_i^W|\lambda_i^W, t_i^W > t^*) = 0; \tag{19}$$

$$h_{\text{twbl}}(t_i^W|\lambda_i^W, t_i^W < t^*) = \frac{p_W(\lambda_i^W)^{p_W}(t_i^W)^{p_W-1}\exp\left(-(\lambda_i^W t_i^W)^{p_W}\right)}{\exp\left(-(\lambda_i^W t_i^W)^{p_W}\right)-\exp\left(-(\lambda_i^W t^*)^{p_W}\right)}. \tag{20}$$

The observed data represent a convex combination of these two duration processes depending on the types of players involved, which we do not observe. Let π denote the initial probability (i.e., after players decide whether or not to contest the issue but before the contest begins) that a contest involves two strong players and therefore corresponds to the exponential duration process. Combining these terms leads to the following likelihood function, in which we drop all superscripts on the outcome variable, t_i, since we cannot distinguish players' types for outcomes that end before t^*:

$$L(\beta, \alpha, \pi, p_{SS}, p_W, t^*|t, X) = \prod_{i=1}^{n} \pi f_{\text{wbl}}(t_i|\lambda^{SS}) + (1-\pi) f_{\text{twbl}}(t_i|\lambda_i^W), \tag{21}$$

$$= \prod_{i=1}^{n} \pi p_{SS}(\lambda_i^{SS})^{p_{SS}} t_i^{p_{SS}-1}\exp\left(-(\lambda_i^{SS} t_i)^{p_{SS}}\right) \tag{22}$$

$$+(1-\pi)c_i\left[\frac{p_W(\lambda_i^W)^{p_W} t_i^{p_W-1}\exp\left(-(\lambda_i^W t_i)^{p_W}\right)}{1-\exp\left(-(\lambda_i^W t^*)^{p_W}\right)}\right],$$

where $c_i \in \{0, 1\}$ indicates that an observations fails after t^* to ensure that the density of the truncated Weibull contributes zero to the likelihood in such cases. Importantly, note that although t^* determines the largest value of the truncated Weibull distribution, the possible values of t include all positive duration outcomes via the mixture with the standard Weibull for strong–strong pairs.

Lastly, we incorporate heterogeneity in the probability that a given contest involves two strong players by parameterizing π. This allows us to estimate variation across observations in the mixture probabilities for the possible types of contests, which improves prediction of the parameters of the duration for each type of contest. We model π with a logit equation, replacing it in the likelihood above with the observation-specific probability:

$$\pi_i = \frac{\exp(W_i\gamma)}{1+\exp(W_i\gamma)} \tag{23}$$

One can then take the log and proceed to maximization. The likelihood is easily adapted to account for data involving right censoring, as we show in our Supplementary appendix.

2.2 Estimation

We here discuss some practical issues that emerge when estimating this likelihood and discuss how we address them. The first lies in directly estimating t^*. As noted earlier, t^* is the endogenous point in time by which all contests involving at least one weak player will have ended. At one extreme, estimating it simultaneously with the other parameters proves difficult. At the other extreme, assuming a specific value works in practice, but makes the results sensitive to that choice. Our middle approach involves searching over many values to identify the best one. We classify observations by whether they exceed or fall below some candidate value of t^*, estimate the other parameters of the model given t^*, repeat for multiple candidate values, and choose the one with the largest final log-likelihood.[6] With a finite number of values in a given data set we can search over all observed t^*.[7] Note that the precision of the estimates will therefore depends on the realized duration outcomes in a given data set and, in particular, on whether observations exist near the true value of t^*. Of course, any estimator would suffer from a lack of richness in observed outcomes, so this problem is not specific to ours alone.

The coarseness of the observed outcomes leads to a second estimation issue for the endogenous time horizon. Specifically, the likelihood becomes flat when t^* achieves sufficiently large values. This makes sense since once t^* becomes very large, the Weibull cumulative density function equals 1 at or before t^*. The truncated Weibull distribution used for estimation always reaches one at t^*, but when the underlying, untruncated Weibull distribution reaches one increases in t^* do not affect the likelihood since all observations from the Weibull for weak contests will have failed with probability one even without the truncation adjustment. Such observations therefore contribute only through the weighted density for strong–strong contests, $\pi_i f_{\exp}(t_i|\lambda^{SS})$, for values above and just below t^*. Under some conditions, it may be the case that this flat region of the likelihood represents the maximum.[8] When this happens, we have multiple values of t^* that maximize the likelihood. We address this by leveraging the theoretical definition of t^*: since it represents the point by which all contests involving at least one weak player have ended, we select the smallest value of t^* that maximizes the likelihood. Since the c.d.f. evaluated at this point is one, then all such contests have, in fact, ended. Note that this choice does not affect the values of the other parameter estimates since it only affects the truncation adjustment, which remains equal to one among the set of t^* that maximize the likelihood.[9]

In the end our estimator produces a likelihood that corresponds to the theoretical data-generating process for a continuous time war of attrition as depicted in Figure 1. In fact, it is even more general than this, since it allows decreasing hazards for contests involving at least one weak player and does not assume that this hazard lies above the hazard for contest involving two strong players. Thus, the theoretical prediction represents a special case of our empirical estimator rather than being assumed by it. Furthermore, our estimator parameterizes the duration of weak contests and the mixture probability to test whether they vary as expected with measures of the parameters of the theoretical model. Finally, note that our estimator also includes both the Weibull and exponential models as special cases, allowing researchers to test whether the data merely represent the outcome of a standard duration process.[10] We have released a custom

[6]Note that estimating the parameters in this way may tend to understate the standard errors since each individual maximization treats t^* as fixed.

[7]In practice, estimation for values of t^* near the maximum and minimum sometimes proves difficult given that almost all observations would be classified into just one of the two underlying durations.

[8]For example, this may occur if the true maximum is a little below the point at which the c.d.f. equals one, but no observation with the relevant duration value exists in the data set.

[9]To assess the viability of these two strategies, we conducted a series of Monte Carlo experiments to evaluate our estimator. These are available from the authors upon request. In short, the estimator typically converged fairly quickly and produced estimates close to the true values.

[10]The Weibull case occurs either when t^* is infinitely large and $\pi_i = 0, \forall i$, or when $\pi_i = 1, \forall i$; the exponential case emerges when either the preceding two conditions hold and $p^W = 0$ or when $\pi_i = 1, \forall i$, and $p^{SS} = 0$. Note that since all of these tests

Stata package that users can install to easily run the models, view the final log-likelihoods profiled over the values of the outcome variable, and generate plots of the estimated hazards.[11] Both of our applications are performed in Stata 14.2 using this package.

3. Application 1: filibustering in the US Senate

In this section, we employ our new estimator to study the duration of filibusters in the United States Senate. This application is motivated by a long series of claims in the press (e.g., Will 1982) and academic literature (e.g., Binder and Smith 1997) regarding a supposed trivialization of the filibuster over the last few decades starting with a change to the cloture rule in 1975. In particular, we assess the validity of claims by multiple experts consistent with Wawro and Schickler's (2006) conclusion that that "(t)he contemporary context of lawmaking in the Senate has essentially eliminated the informational benefits that used to accrue from these kind of battles."[12] Dion *et al.* (2016) argue that the filibuster fits the war of attrition framework well with two sides—either weak or strong—clashing over some issue of value.

To assess these claims we utilize replication data from Dion *et al.* (2016), which employs Beth's (1994) measure of filibuster length.[13] We estimate the hazard of filibusters with variables accounting for the 1975 rule change and the importance of issues being filibustered. The former captures the shift in cloture from two-thirds to three-fifths of those present and voting (and revised to 60 votes in 1980). The latter codes the issues underlying filibusters as important, not important, and "unclear" using Binder's (1997) list of important issues. The war of attrition model predicts that issue importance decreases the hazard of weak filibusters and increases the probability of a strong–strong filibuster (Dion *et al.* 2016). We compare our results to those from a standard Weibull estimator.

To estimate the best overall value of the endogenous time horizon, t^*, we fix it at a particular value and then estimate the other parameters. We repeat this procedure for all observed filibuster lengths. Figure 2 plots the final log-likelihood for every case that converged (those that did not are indicated by ticks inside the horizontal axis; for presentation we did not plot results for values greater than 50). The maximum final likelihood occurs when $t^* = 9$. In line with our discussion of the estimation process the likelihood remains flat after $t^* = 18$ when the Weibull c.d.f. underlying the truncated Weibull equals one; after this point the parameter estimates do not change.

Table 1 reports the results of this estimation along with a second model that include covariates along with estimates from the corresponding Weibull models. Consider first the results with no covariates. The first two equations report the truncated Weibull results for contests involving at least one weak player, the second two equations report the Weibull results for contests involving two strong players, and the final equation reports the logit model capturing the mixture of these two types of contests. From these results we observe that contests with a weak player exhibit positive duration dependence, as expected in a war of attrition. Contests involving two strong players do as well, but with a smaller value indicating a much flatter hazard. The logit model has a negative constant parameter, which indicates slightly less than half of all contests involve two strong players. The Weibull results reported in the first column attempt to capture these two type of contests with one equation and indicate even smaller positive duration dependence that either of the two equations in the war of attrition results. A likelihood ratio test for the Weibull against our estimator can be performed since the Weibull corresponds to the case in which $t^* \to \infty$ and the probability of a strong–strong contest is zero, though we make an adjustment since the test involves two boundary values. Shapiro (1988) shows that the appropriate statistic comes from the $\bar{\chi}^2$ which here represents

involve boundary values in the null hypotheses adjustments must be made to the standard likelihood ratio test (Shapiro, 1988), which we detail later.

[11]Version 1.0 of the package is included with our replication data, but for the most current version run `net search warofatt` in Stata.

[12]For additional work on the filibuster, see Bawn and Koger (2008), Bell (2011), Binder *et al.* (2002); or Koger (2010).

[13]These data are available at https://www.journals.uchicago.edu/doi/suppl/10.1086/690223.

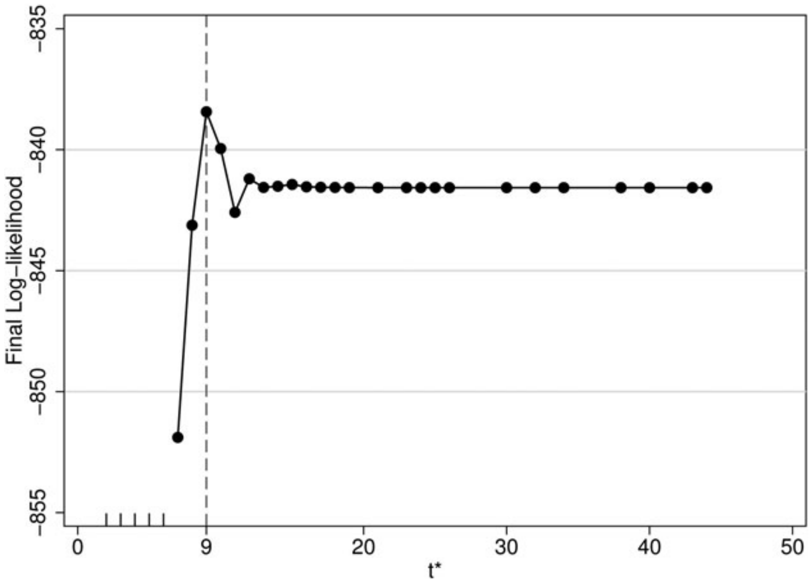

Figure 2. Value of final log-likelihood for different values of t^* (filibuster duration, no covariates).
Note: Plot represents the final value of the log-likelihood for the war of attrition duration estimator using all observed values of the outcome variable (filibuster length) in the data set. For presentation purposes, results for two values greater than fifty not included, but they continue the flat trend up through 50. The four upward ticks on the horizontal axis indicate values for which the estimator did not report regular convergence or exceeded 50 iterations.

the convex combination of three distributions, $\bar{\chi}^2 = (1/4)\chi_0^2 + (1/2)\chi_1^2 + (1/4)\chi_2^2$, and which has a critical value of 4.231.[14] The resulting test statistic exceeds this threshold.

More helpfully, Figure 3 plots the predicted hazards from the war of attrition estimator with no covariates along with their 95 percent confidence intervals.[15] The resulting hazards correspond quite closely to the plots depicted in Figure 1, with the hazard for filibusters involving at least one weak player increasing from the beginning of a filibuster up through nine days, by which point all such contests will have all ended. The hazard for filibusters with two strong players stays quite low and increases slightly, as expected from the results. We have no expectation regarding the form of duration dependence for strong–strong contests, but do expect it to be small relative to the one for weak contests. Note that for presentational purposes we only plot the hazards up through 40 days, but the strong–strong hazard increases very slowly to the longest observed filibuster, which lasts 97 days. Finally, the logit equation estimates that 42 percent of the observed filibusters involve two strong players.

The other pair of models adds in covariates. As predicted, high importance and unclear importance issues have lower hazards than low importance issues, with both significant in the war of attrition estimator. This is what we expect since weak players contesting important issues are willing to fight longer based on their greater value. In contrast, the results indicate no difference in the hazard for weak contests after the 1975 cloture reform. These results reverse themselves in the equation for the mixture probability, with the post-1975 period containing fewer strong players but no evidence of variation by issue importance.

[14]Stata's likelihood for the standard Weibull model drops the (constant) $-\sum_i \ln(t_i)$ term. We wrote our own Weibull MLE that includes it to ensure comparability; it also reports $\hat{\beta}$ directly whereas Stata's streg procedure reports $\hat{\beta}\hat{p}$.

[15]We generate the confidence intervals by sampling 10,000 draws of the parameters from their estimated distribution for each value of t, calculating the hazards, and then capturing the 2.5 and 97.5th percentiles. Because the distribution of the shape parameters is skewed, the upper bounds of the confidence intervals are further from the predicted value than the lower bounds.

Table 1. Weibull and partial cure hazard model results for duration of filibusters, 1919–1993 (hazard form)

	Weibull		War of attrition	
Weak Contests Hazard				
High Importance Policy		−0.486**		−0.463**
		(0.188)		(0.159)
Unclear Importance Policy		−0.305		−0.368**
		(0.206)		(0.164)
1975 and After		0.766**		0.013
		(0.131)		(0.132)
constant	−2.276**	−2.352**	−1.943**	−1.484**
	(0.057)	(0.207)	(0.144)	(0.179)
Weak Contests Duration Dep.				
constant ($\ln p_W$)	0.131**	0.277**	0.728**	0.732**
	(0.061)	(0.054)	(0.076)	(0.077)
Strong–Strong Contests Hazard				
constant			−2.772**	−3.042**
			(0.089)	(0.106)
Strong–Strong Contests Dur. Dep.				
constant ($\ln p_{SS}$)			0.193**	0.309**
			(0.083)	(0.101)
Probability both strong				
High Importance Policy				1.359
				(1.286)
Unclear Importance Policy				0.182
				(1.255)
1975 and After				−2.515**
				(0.594)
constant			−0.317*	−0.504
			(0.191)	(1.209)
Estimated value of t^*			9	15
Final log-likelihood	−877.62	−841.96	−838.43	−813.13
AIC	1759.25	1693.91	1688.87	1650.26
$\bar{\chi}^2$ test for model comparison			78.38	57.66

Note: $N = 274$. Robust standard errors reported. See the text and Shapiro (1988) for information on the $\bar{\chi}^2$ test for model comparison between the Weibull and our estimator given the presence of two boundary values. The critical value with two such parameters is 4.23.

These results contrast with those from the standard Weibull model. Although both models indicate that high importance issues have lower hazards, the Weibull alone indicates a positive and significant effect of the 1975 cloture reforms, corresponding to a greater hazard and shorter filibusters. This result appears to occur in the standard Weibull due to its inability to account for the reduction in the probability of strong participants since 1975, which it instead misattributes to an increase in the hazard. Not surprisingly given these differences, the statistical comparison of the two models rejects the standard Weibull in favor of the war of attrition estimator. Taken together, these results run contrary to the common wisdom that the 1975 reforms "trivialized" the filibuster. The hazard for filibusters did not move much after 1975, rather our results indicate a sizeable drop in the proportion of strong types willing to fight for their issues. The structure of the filibuster game itself did not change—the commitment of the players playing it did.

Figure 4 summarizes these results by plotting the estimated hazard rates for strong–strong filibusters and for the different types of filibusters involving at least one weak player. We do not report confidence intervals to more clearly interpret the effects of the covariates on the hazards.[16] Again, the overall shape closely resembles that of our theoretical model. But we can

[16]In our experience plotting confidence intervals suggests misleading conclusions about statistical significance. Because the shape parameter is exponentiated to keep it positive, the upper bound of the confidence interval of the hazard is much further away from the expected value than the lower bound and increasingly so as the value of time increases. This leads to substantial overlap in plots when comparing two hazards, thereby making easy to incorrectly infer that the two hazards do not differ in a statistical sense even if they are substantively different. To show the difference and assess statistical significance more

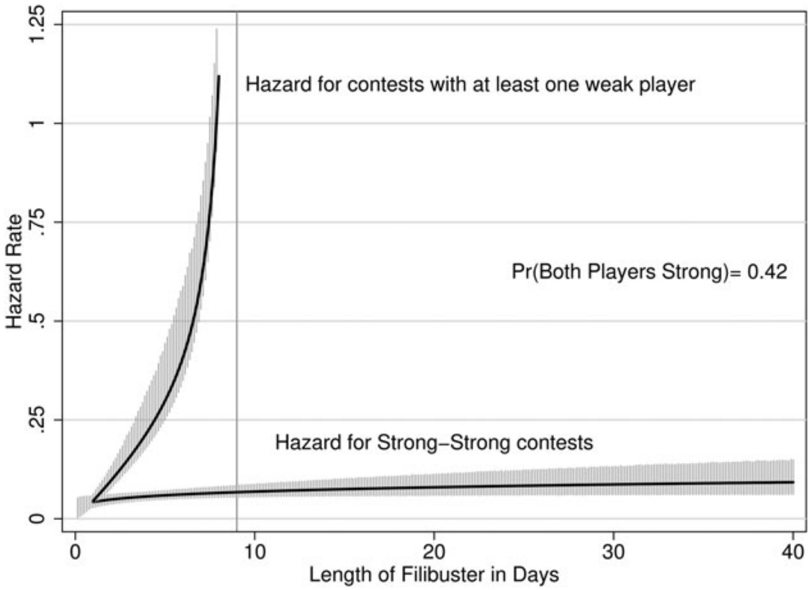

Figure 3. Estimated filibuster hazard rates from partial cure estimator.
Note: Estimates from Table 1 with no covariates ($t^* = 9$). Black lines indicate the predicted hazard rate for each type of contest. Gray bars give a nonparametric 95 percent confidence interval estimated by sampling 10,000 draws of the parameters from their estimated distribution and calculating the corresponding value of the hazard at many values of filibuster length.

also see how changes in various conditions affect the hazard for weak contests. The black line represents the baseline case, which consists of unimportant issues before the 1975 cloture reform. Increasing issue importance decreases the hazard by about one-half both for issues of unclear importance and important issues. The 1975 cloture reform has basically no effect, increasing the hazard ever so slightly relative to the baseline, which is consistent with the small estimated coefficient.

4. Application 2: the duration of international crises

We next apply our estimator to the duration of international crises. A significant part of the literature on crisis onset and escalation casts crises precisely as a "war of attrition" (Fearon 1994: 577). As Fearon (1995) notes elsewhere, imperfect information is one of a handful of ways that we might observe war since parties could otherwise perfectly negotiate away their differences. This imperfect information is often cast in terms of types for the two states involved (e.g., Banks 1990), with type capturing the level of cost states are willing to incur to win the prize (e.g., contested territory). Crisis escalation serves as a precursor to war, with both sides attempting to signal their types in the hopes that their opponent will back down.

We can therefore think of the crisis onset and bargaining stage in terms similar to the war of attrition model with two sides competing over a prize. Each state can be either weak or strong, which could correspond to their ability to suffer audience costs (Fearon 1994), the strength of the military forces (Morrow 1989), or their resolve to fight for the prize. They each choose whether to engage in the crisis and how long to escalate before backing down. The side that backs down loses the prize and bears the cost of the contest whereas the winner secures the prize. Strong states may gain from the crisis by enhancing their reputation or letting their leaders

clearly we therefore recommend plotting the difference between the hazards. In this case (not shown), this shows that the confidence interval for the difference only includes zero for very small values of filibuster length.

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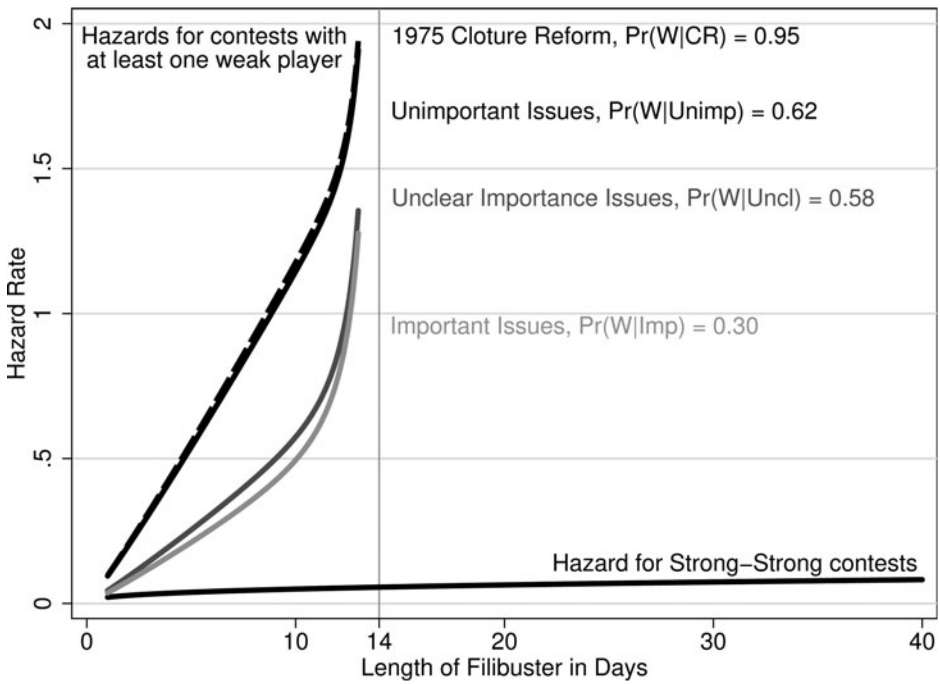

Figure 4. Estimated filibuster hazard rates from partial cure estimator with covariates.
Note: Estimates from Table 1 ($t^* = 15$). The reported hazards for contests involving at least one weak player each vary just one of the indicator variables at a time from the baseline (which represents unimportant issues pre-1975). We omit confidence intervals on this figure since it would be too cluttered and hard to interpret. Adding them, however, confirms what we learn from the table, i.e., the hazards for important and unclear issues differ from the hazard for unimportant issues.

display their strength whereas weak states incur costs for fighting, such as audience costs. If both sides are sufficiently strong and neither state backs down by some endogenous time horizon, then they settle the contest through a war. The crisis therefore serves as an explicit mechanism for learning about the strength of ones' opponent (Morrow 1989; Fearon 1994).

Although many similarities abound, key differences in Fearon's (1994) implementation lead to deviations from the predictions of the classic war of attrition model. He highlights two such differences: the option to escalate from a crisis to war rather than just continuing or backing down and the incurrence of audience costs only by the side that loses. In fact, his version of the war of attrition model for international crises predicts a negative expected hazard rate for crises up through the endogenous time horizon.

Despite these deviations from the classic war of attrition model, our estimator remains flexible enough to capture the salient features of these models of the duration of crisis bargaining in ways that previous research has not. The theoretical literature makes clear that crisis escalation serves as a signaling process through which weaker types attempt to appear strong, but often end up backing down. Stronger types, on the other hand, will remain committed and not back down. Finally, models of the process typically identify an endogenous time horizon after which neither party will exit (Fearon 1994). Our estimator captures these dynamics. Helpfully, it allows for both increasing and decreasing hazards among crises involving at least one weak player. Furthermore, as noted previously, it includes both the exponential and Weibull models as special cases. If the data do not mirror the implied process then we expect to obtain one of these special cases.

To guide our models, we draw on DeRouen and Goldfinch's (2005) analysis of the duration of the length of all international crises from 1918 to 1994 using the International Crisis Behavior

Table 2. Weibull and partial cure hazard model results for duration of international crises, 1918–1994 (hazard form)

	Weibull		War of Attrition		
Weak Contests Hazard					
Violence		−0.449**		−1.854**	−2.112**
		(0.102)		(0.330)	(0.394)
Log Relative Cap.		−0.042		−0.172	−0.085
		(0.038)		(0.109)	(0.071)
Joint Democracy		−0.099		−0.493	−0.780**
		(0.181)		(0.364)	(0.320)
Contiguity		0.174		0.394	0.336
		(0.109)		(0.262)	(0.258)
Rivals		0.035		−0.199	−0.183
		(0.105)		(0.244)	(0.172)
Ethnic		−0.584**		−0.459*	−0.570**
		(0.110)		(0.253)	(0.153)
Territory		−0.075		−0.080	0.079
		(0.108)		(0.324)	(0.288)
Social Unrest		−0.090		−0.356	−0.145
		(0.106)		(0.234)	(0.250)
constant	−4.687**	−4.462**	−2.937**	−3.273**	−3.026**
	(0.052)	(0.122)	(0.199)	(0.407)	(0.264)
Weak Contests Duration Dep.					
constant (ln p_W)	−0.249**	−0.188**	0.491**	0.504**	0.647**
	(0.025)	(0.025)	(0.116)	(0.156)	(0.256)
Strong–Strong Contests Hazard					
constant			−5.011**	−5.048**	−4.994**
			(0.065)	(0.099)	(0.110)
Strong–Strong Contests Dur. Dep.					
constant (ln p_{SS})			−0.108**	−0.159**	−0.164**
			(0.034)	(0.037)	(0.040)
Probability both Strong					
Log Relative Cap.					0.192**
					(0.092)
Joint Democracy					−0.333
					(0.863)
constant			1.260**	0.628**	1.173**
			(0.216)	(0.277)	(0.376)
Observations	699	674	699	674	674
Estimated value of t^*			44	309	308
Final log-likelihood	−4042.70	−3876.04	−4019.82	−3856.61	−3854.12
AIC	8089.39	7772.07	8051.65	7741.22	7740.23
$\bar{\chi}^2$ test for model comparison			45.74	38.85	43.84

Note: Robust standard errors reported. See the text and Shapiro (1988) for information on the $\bar{\chi}^2$ test for model comparison between the Weibull and our estimator given the presence of two boundary values. The critical value with two such parameters is 4.23.

data (Brecher and Wilkenfeld 2000).[17] The outcome variable indicates the number of days a state is in a crisis. DeRouen and Goldfinch's data include information on 710 actors from 220 distinct crises and capture information about both the target and the opponent. The remaining crises last anywhere from 1 to 1359 days. We replicate their analysis with the exception of employing a Weibull rather than a two-parameter gamma model, though they note little difference between the two. We employ the same eight independent variables.

Table 2 reports the results for the Weibull and war of attrition duration estimators. As with the filibuster analysis, the best way to interpret the results is by plotting the estimated hazard. Figure 5 reveals that such crises produce an increasing hazard rate through 44 days whereas crises involving two stronger states have a relatively small and decreasing hazard. Again, the plotted confidence intervals show clear separation between the two. The logit mixture equation estimates indicate that 78 percent of crises involve two strong players. Taken together, these results comport

[17]These data were downloaded from https://www.prio.org/JPR/Datasets/.

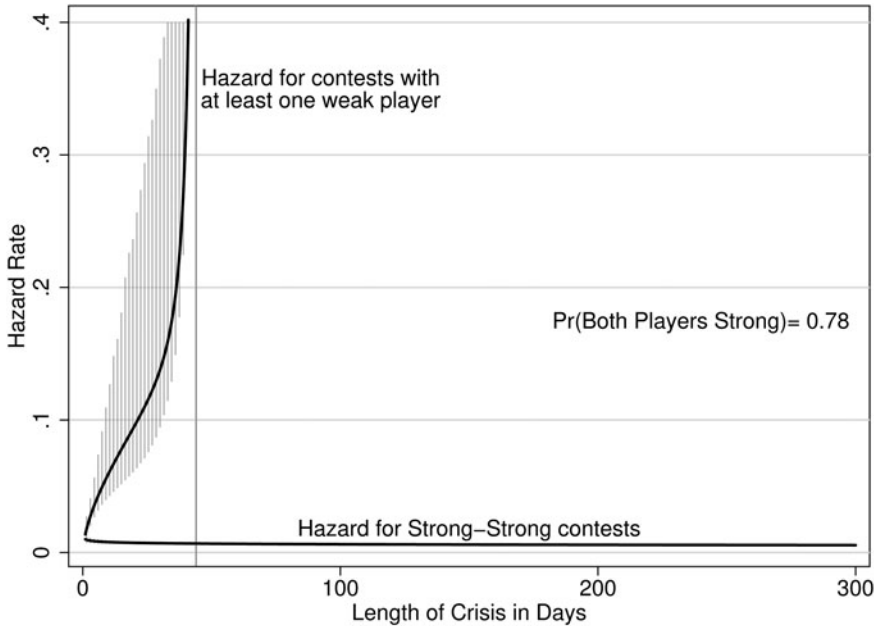

Figure 5. Estimated crisis hazard rates from partial cure estimator.
Note: Estimates from Table 2 with no covariates ($t^* = 44$). Black lines indicate the predicted hazard rate for each type of contest. Light gray bars give a nonparametric 95 percent confidence interval estimated by sampling the parameters from their estimated distribution and calculating the corresponding value of the hazard at many values of crisis duration. The upper bound of the confidence interval for the Weibull hazard is not shown past 0.4 since it approaches 30 and including it renders the other results imperceptible.

with those of the war of attrition model. A subtle difference from the filibuster hazards emerges with the truncated Weibull hazard for crises exhibiting an inflection point around 20 days. Its presence here reflects the smaller (though still positive) value of the duration dependence parameter interacting with the impending truncation point in the denominator of the hazard.

Contrast these results with those from the standard Weibull estimator, which indicate negative duration dependence, corresponding to a decreasing hazard rate. This finding emerges due to the downward pressure exerted by the high proportion of crises involving two strong players. Since their durations extend well beyond the endogenous truncation point and form a high proportion of all cases, the Weibull model for all durations accounts for this with an overall decreasing hazard. The decreasing rate comports with Fearon's (1994) prediction. Although not inconsistent with this prediction, our war of attrition estimator adds some nuance to that finding by capturing the presence of two qualitatively different kinds of contests, one of which has an increasing hazard rate as predicted by the war of attrition model.

Figure 6 offers another way to compare the relative performance of the two estimators. Here we follow Maller and Zhou (1995) by constructing a probability–probability plot that compares the predicted cumulative distributions from the Weibull and war of attrition estimators to the observed cumulative distribution. The closer the curves lie to the 45° line the better the performance of the estimator relative to the truth. Here we see that both underpredict failures in roughly the middle quintile. After about 0.35 the war of attrition estimator lies closer to 45° line, especially between 0.4 and 0.85. The correlations between the estimators and the observed data show the at Weibull at 0.91 and the war of attrition at 0.93.

The second and fourth models add covariates and further demonstrate the value of the war of attrition estimator. These produce similar patterns of significance for the parameterized hazard components, with the level of violence and the presence of an ethnic dispute decreasing the hazard. The war of attrition estimator continues to indicate a sizeable proportion of crises involve

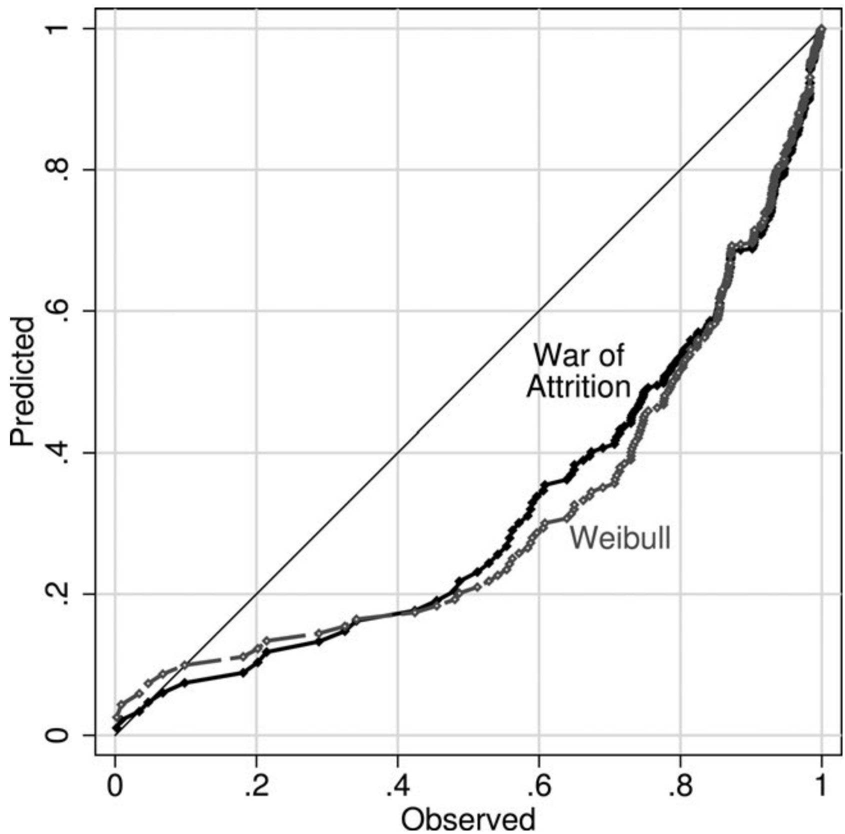

Figure 6. Probability–probability plot for comparison of model performance.
Note: Estimates from Table 2 with no covariates ($t^* = 44$). A probability–probability plot compares the observed cumulative distribution of the outcome variable with the cumulative distribution predicted by an estimator at the observed values. Perfect correspondence occurs when the plotted curve falls on the 45° line.

two strong players, however, and the $\bar{\chi}^2$ test continues to reject the basic Weibull model. Adding covariates increases the estimate for t^* from 44 to 309 days, though the overall shape of the hazards for both types of crises remains remarkably similar.[18]

Our final model investigates whether the probability that both states are strong depends on some of the observed covariates. Since the literature argues that democracies are better able to generate audience costs (Fearon 1994) and that states with greater military resources ought to be more resolved (Morrow 1989), we might expect both variables to increase the chance of both players being strong. Although our results produce coefficients with signs as expected, only capabilities emerge as significant. Furthermore, the inclusion of these variables in the mixture equation alters the results in the duration equation for weak contests: joint democracy produces a larger effect and its coefficient becomes significant. These results indicate that although capabilities predict states' type and willingness to enter a crisis, they do not affect their willingness to escalate a crisis once it starts. In contrast, once we account for the role of capabilities and joint

[18]We note here that the inclusion of the violence variable produces the large value of t^*. As DeRouen and Goldfinch (2005) note, this variable likely suffers from endogeneity since the occurrence of war ends the crises on that day; theory and our empirics indicate that the rate of occurrence of war depends specifically on the duration of the crises. We opted for exact replication, but our general results hold without the violence variable, though the significance of some covariates changes a bit.

democracy in predicting players' types, we learn that crises involving two democracies have a smaller hazard and therefore tend to last longer.

5. Discussion and conclusion

In this paper we present a novel full information maximum likelihood estimator inspired by the predictions from models of incomplete information such as the classic war of attrition game. We illustrate the value of this war of attrition estimator with applications to the duration of filibustering in the US Senate and international crises, finding evidence in both consistent with information revelation over the course of time and adding significant nuance to prior knowledge of these two processes. Application to other settings, such as those outlined in our Supplementary appendix, may prove fruitful as well.

Although the continuous-time war of attrition that inspired our estimator predicts an increasing hazard through the endogenous end point, the estimator as proposed can already capture increasing, flat, or decreasing hazards among the population involving contests with at least one weak player. Thus it can be applied to approximate other theoretical models with similar structures. There are a number of further developments that could be of value. For example, one could allow for alternate forms of duration dependence for the population involving at least one weak player, such as the log-normal or the two-parameter gamma distributions.

Furthermore, war of attrition games can be thought of as a particular type of game from a more general category of political economy games where mixture distributions should be helpful—in particular, games with multiple equilibria, which have been applied to topics in international relations (e.g., Morrow, 2014 on how the laws of war can lead to restraint by states and soldiers), political economy (e.g., Obstfeld, 1996 on currency crises), economic geography (Krugman, 1991 on economic geography), repression and protest (Tyson and Smith, 2018), and more. This broader set of games, of which wars of attrition can be viewed as a subset, are contests in which the mixture of potential outcomes and actions might combine distinct processes and which are therefore not appropriately modeled by standard approaches. Rather than limiting our theoretical focus to only games with pure strategy equilibria, or abandoning models that produce multiple equilibria because we cannot accurately test them empirically, our approach suggests how models with mixed strategy equilibria can be properly tested.

Supplementary material. The supplementary material for this article can be found at https://doi.org/10.1017/psrm.2020.29.

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