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# COUNTING THE POOR: AN ELEMENTARY DIFFICULTY IN THE MEASUREMENT OF POVERTY

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**Abstract**
This note suggests that the exercise of measuring poverty in a society is greatly aided by clarity on precisely what one means by "the extent of poverty". The latter concept may refer to the extent of poverty normalized for population size, or to the extent of poverty not so normalized. Absence of clarity on this distinction – which is both simple and non-trivial – could lead to rather straightforward problems of coherence and consistency in the measurement of poverty.

## 1. INTRODUCTION

The poverty of nations constitutes a fundamental subject of enquiry by economists. An important, even if by no means exhaustive, component of the subject of poverty is its measurement. Any comprehensive account of poverty would, presumably, depend upon the ability to measure it in a "satisfactory" way. In turn, a "satisfactory" measure of poverty must, at the very least, be expected to comply with a set of properties that are (a) ethically appealing, and (b) collectively coherent. These requirements of moral and logical acceptability fall naturally within the province of concern of the philosopher. The subject of poverty measurement, therefore, occupies an important place in the analytical area of intersection between economics and philosophy.

This note suggests that there exists a really rather basic set of conditions such that, if a poverty index is constrained to obey such a combination of axioms, then the very possibility of having a real-valued

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measure of poverty is called into question. More constructively, the impossibility result – simple and direct though it is – suggests the need for some clarity on precisely what the poverty analyst has in mind when seeking to measure the ''extent of poverty'' in a society. Specifically, one can have two conceptions of the ''extent of poverty'' – *when it is normalized for population size* (which seems to be the most favoured interpretation), and when it is *not* so normalized. Given this context, one could refer to the first of these conceptions as a ''*compassed*'' conception, and to the second, as an ''*uncompassed*'' conception. Failure to distinguish clearly between the two conceptions can lead to problems of ambiguity and inconsistency in the measurement of poverty. These issues are elaborated on in this note.

## 2. BASIC CONCEPTS

Let $x_i$ be the income of person i ($i = 1,...,n$), and let z stand for the *poverty line*, which is a positive level of income such that any i will be certified to be *poor* if and only if x is less than z. By a *poverty index* is meant a function P which, for every non-negative n-vector of incomes $x = (x_1,...,x_i,...,x_n)$, and every poverty line z, specifies a unique real number which is intended to signify the extent of poverty associated with the regime (x;z). It will throughout be assumed that the poverty index is *anonymous*, which is the requirement that, for all income vectors x,y and any poverty line z, if x is a permutation of y, then $P(x;z) = P(y;z)$. Given x and z, n(x) will stand for the dimensionality of x; $x^P{}_z$ will stand for the sub-vector of *poor* incomes in x; $x^N{}_z$ will stand for the sub-vector of *nonpoor* incomes in x; $x^N{}_z = \phi$ will signify that the vector x consists *only* of poor incomes; and q(x;z) will stand for the dimensionality of $x^P{}_z$.

Possibly the most elementary way of measuring poverty is to simply count the poor in any population, and to express the poverty index either as a ratio of the poor population to the total population (the socalled *headcount ratio* H) or as the size of the poor population (the socalled *aggregate headcount* A), viz., for any permissible combination of x and z: $H(x;z) = q(x;z)/n(x)$; and $A(x;z) = q(x;z)$. As is well-known (see, for example, Sen, 1976), the indices H and A violate the *monotonicity* property, which requires that, other things being equal, a dimunition in the income of any poor person should increase the value of the poverty index. An elementary measure of poverty which satisfies the monotonicity property is the *income-gap ratio* I which measures the proportionate shortfall of the average income of the poor, $\mu^P$, from the poverty line z, viz., for any permissible combination of x and z: $I(x;z) = [z-\mu^P(x;z)]/z$. The *per capita income-gap ratio*, R, is obtained as a product of the headcount and the income-gap ratios, viz.: $R(x;z) = [q(x;z)/n(x)].[(z-\mu^P(x;z))/z]$.

## 3. SOME ELEMENTARY AXIOMS FOR POVERTY MEASUREMENT

Stated below are three rudimentary properties of poverty indices arranged in the form of a set of axioms. The axioms are first presented together and subsequently discussed.

**Axiom SF (Strong Focus)**. For any pair of income vectors x and y, and any poverty line z, if $x^P{}_z = y^P{}_z$, then $P(x;z) = P(y;z)$.

**Axiom WPG (Weak Poverty Growth)**. For any pair of income vectors x and y, and any poverty line z, if $x^P{}_z$ is the q-vector $(x^o,...,x^o)$ for some non-negative real number $x^o$ and positive integer q; $y^P{}_z$ is the (q+1)-vector $(x^o,...,x^o)$; and $x^N{}_z = y^N{}_z \neq \phi$; then $P(y;z) > P(x;z)$.

**Axiom RI (Replication Invariance)**. For any pair of income vectors x and y, any poverty line z, and any positive integer k, if $y = (x,...,x)$ [k times], then $P(x;z) = P(y;z)$.

The Strong Focus Axiom says that, given any poverty line z, if any two income vectors are identical in respect of their respective sub-vectors of poor incomes, then the amount of poverty associated with the two vectors must be judged to be the same. This requirement is based on the reasonable notion, as Sen (1981; p. 186) puts it, ''the poverty measure is a characteristic of the poor, and not of the general poverty of the nation''. Sen was rationalizing a weaker version of Axiom SF, called the *Focus Axiom* (Axiom F), which requires that any two vectors sharing the same sub-vector of poor incomes should be judged to have the same amount of poverty, *provided* also that the dimensionality of the two vectors is the same. What Axiom F asserts is the ''independence of the income levels of those who are above the poverty line'' (Sen, 1981, p. 186). Given this motivational thrust, one can argue that Axiom F only partially captures the scope of the notion that ''the poverty measure is a characteristic of the poor, and not of the general poverty of the nation''; for, one should expect that the poverty measure should be invariant with respect to not only the incomes of the nonpoor but also the population size of the nonpoor. In other words, Axiom F is, properly speaking, an *income-focus* axiom; and there would appear to be a natural case for extending its coverage to incorporate a *population-focus* as well, as Axiom SF does.

Next, the Weak Poverty Growth Axiom is a weakened version of a certain kind of ''population monotonicity'' requirement advanced by Kundu and Smith (1983), and called the ''Poverty Growth Axiom [Axiom PG]''. The latter property demands that, other things equal, the addition of a poor person to a population should increase poverty. This, on the face of it, appears reasonable enough, but it is not proof against controversy. For one thing, Axiom PG, by itself, could rule out the use of the most widely-employed of all poverty measures, the headcount ratio

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H. This becomes clear in the singular case of a situation wherein the *entire* population, to begin with, is poor, so that the addition of a poor person to the population would leave the value of H unchanged, at unity. To avoid such a drastic outcome, Axiom WPG incorporates the restriction that there be at least one nonpoor person in the population (as captured by the requirement, in the statement of the axiom, that $x^N{}_z = y^N{}_z \neq \phi$). Secondly, Axiom PG, by itself, could also rule out the use of the rather ''basic'' poverty index R (the so-called per capita income-gap ratio) which, as we have seen earlier, is the product of the headcount ratio H and the income-gap ratio I. To see this, note that with the addition of one poor income to any income distribution H would increase, while if the additional poor income happens to be higher than the initial average income of the poor I would decline; and it is conceivable that the reduction in I might swamp the increase in H, leading to an overall decline in the value of the measure R. (By way of example, consider a situation in which $z = 2$, $a = (1,3)$, and $b = (1,3,1.8)$. Then, in moving from a to b, H increases from 1/2 to 2/3; I declines from 1/2 to 3/10; and $R = HI$ declines from 1/4 to 1/5. Poverty, as measured by the index R, has *declined* with the addition of a poor person to the population.) In such a situation, the Poverty Growth Axiom would be violated, and it is not immediately clear that the violation is exceptionable. To avoid such cases, wherein it is difficult to pronounce judgement on the outcome in unambiguous terms, it may be in the interests of reasonable caution to commit oneself to a less demanding form of the Poverty Growth Axiom. Hence Axiom WPG. It is, of course, possible to weaken Axiom WPG even further, even though the motivational case for rejecting WPG would appear to be hard to establish. One possible weakening, leading to a property which one could call Diluted Poverty Growth (Axiom DPG), would be along the following lines. Axiom DPG is the same as WPG, save for the dilution that the sole addition to the poor population should have an income, $x'$, which is *less than* the income $x^o$ shared by the q poor people in the initial situation.

Finally, the Replication Invariance Axiom. This property is concerned with a consideration of how a poverty measure should respond to a *replication* of incomes. Axiom RI demands that any k-fold replication of an income distribution, for a given poverty line, should leave the value of the poverty index unchanged. Axiom RI has been widely regarded as being a fundamental property of poverty indices. Shorrocks (1988, p. 433) refers to it as ''perhaps the least controversial of the 'subsidiary' properties [of inequality indices]''. Foster and Shorrocks (1991, p. 690) state: ''Replication invariance ensures that the [poverty] index views poverty in per capita terms, *so that comparisons across different-sized populations are meaningful*'' (emphasis added). It should be stressed here that the appeal of Axiom RI stems from viewing the "extent of poverty" as being best captured by its "*compassed conception*", that is, in terms that allow for normalization with respect to population size. For the present, and in view of the near universal acceptance this view commands, we shall stick with Axiom RI. However, it is useful to underline that an alternative interpretation of the "extent of poverty" – in terms of what one may call its "*uncompassed*" conception, wherein there is no resort to normalization for population size – is also possible. This distinction could prove crucial and we shall return to the issue in Section 4.

What is the class of poverty indices that satisfy the three basic axioms presented in this section? This question is addressed in what follows.

## 4. SOME GENERAL POSSIBILITY RESULTS

We begin with an impossibility result. The following proposition is true.

**Proposition 4.1**. There exists no poverty index P satisfying Strong Focus, Replication Invariance and Weak Poverty Growth.
**Proof**. Consider the following three income distributions a = (1,3), b = (1,3,3), and c = (1,1,3,3), and let the poverty line z be 2. By virtue of Strong Focus, one has: P(b;z) = P(a;z); and by virtue of Replication Invariance – note that c is just a (2-) replication of a – one has: P(a;z) = P(c;z). Consequently, one must have: P(b;z) = P(c;z). However, the Weak Poverty Growth Axiom dictates that P(c;z) > P(b;z), and we have a contradiction. This completes the proof of the proposition. ■

Proposition 4.1 is a very simple impossibility result. The inevitability of the result can be clearly seen by spelling out the "mechanics" of its unfolding. Notice first that Strong Focus and Replication Invariance together imply an axiom which one may call "*Poor Population-Restricted Replication Invariance (Axiom RI*)*". As its name implies, this property demands that a poverty index should be invariant with respect to any replication of the income distribution of the poor, so long as the income distribution of the nonpoor remains unchanged. (That Axioms SF and RI together imply RI* can be easily seen by reconsidering the example provided in the proof of Proposition 4.1. By SF, P(a;z) = P(b;z), and by RI, P(a;z) = P(c;z), whence P(b;z) = P(c;z) – which is precisely what Axiom RI* demands). It is now immediate that Axiom RI* is in stark opposition to the Weak Poverty Growth Axiom: this swift and direct conflict between the two principles precipitates the impossibility result of Proposition 4.1.

This impossibility result is not remotely startling, nor revelatory, nor counter-intuitive; nor is it intended to be any of these things. What it does is point to a basic choice that a poverty analyst must make at the

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outset, namely, whether to take a ''compassed'' or an ''uncompassed'' view of ''the extent of poverty'' in a society. If the analyst should settle for the former conception, that is, exhibit a fundamental commitment to Axiom RI, then it would appear to be plainly mistaken to insist on a poverty index satisfying Axioms SF and WPG jointly: for it is obvious that in an approach where the extent of poverty is normalized for population size, one cannot simultaneously disregard additions to the nonpoor population (SF) *and* want to take account of the relative size of the poor subpopulation (WPG). Something has to give.

In terms of the ''basic choice'' alluded to above, one specific aspect of ''give'' would reside in interpreting ''the extent of poverty'' after an ''uncompassed'' rather than ''compassed'' fashion. In such an event, and in a spirit which is diametrically opposed to that of Replication Invariance, one's concern would have to be with a property that upholds a poverty measure's sensitivity to a population's ''scale factor''. In line with such a view, one may be disposed to advance some such axiom as ''*Replication Scaling*'', which demands that a poverty index should replicate the amount of poverty by the extent to which any income distribution is replicated, income by income. Formally, one has:

**Axiom RS (Replication Scaling)**. For any pair of income vectors x,y, any poverty line z, and any positive integer k, if y = (x,...,x) [k times], then P(y;z) = k.P(x;z).

It is easy to verify that there does exist a poverty index satisfying Axioms SF, WPG and RS: the aggregate headcount measure A satisfies all three properties. It is worth noting, though, that a poverty measure such as A is insensitive to considerations relating to the ''likelihood'' of encountering poverty in a society, in the following sense. Imagine a situation in which, for some given poverty line z, all the poor people have an income of $x_1$ and all the nonpoor an income of $x_2$ in each of two income vectors a and b; suppose further that there are 99 poor people in a total population of 100 in the first case, and 100 poor people in a total population of 10,000 in the second case. Then, since $A(b;z) = 100 > 99 = A(a;z)$, the index A will certify that there is more poverty associated with the vector b than with the vector a, even though the likelihood of encountering a poor person is 1% in the former case, and 99% in the latter. Any intuitive discomfort this judgement may occasion one is the price that has to be paid for accepting a thoroughgoing ''uncompassed'' view of poverty.

Alternatively, one could retain a ''compassed'' view of poverty, *via* Axiom RI, and look for appropriately weakened versions of Axioms SF and WPG which yield existence results. As it turns out, by first weakening Strong Focus to Focus, and then Weak Poverty Growth to Diluted Poverty Growth, we can obtain a couple of elementary possibility theorems. First, there exists a poverty index which satisfies Focus, Weak Poverty Growth, and Replication Invariance: it is easy to verify that both the headcount ratio H and the per capita income-gap ratio R satisfy these properties. Second, there also exists a poverty index which satisfies Strong Focus, Diluted Poverty Growth, and Replication Invariance: again, it is simple to verify that the income-gap ratio I satisfies these properties.

It is possible to argue that the above existence results have been purchased at a price, which is that the weakened versions of the relevant axioms are not wholly convincing. In this connection, consider, first, a possible argument relating to the relative merits of the Focus and Strong Focus Axioms. According to this argument, while it may be held that Focus is a *necessary* property of any poverty index whose job is seen as that of describing the condition of the *poor* (considered as the only relevant constituency that needs to be reckoned), one may also ask if it really goes far enough. For reasons that have been discussed in Section 2, there is, it can be held, a strong case for strengthening Axiom F to Axiom SF, if one wishes to capture the motivation of a certain sort of ''constituency principle'' (see Broome, 1996) in its entirety: in particular, it is problematic to buy ''income-focus'' while rejecting ''population-focus''. One specific consequence of violating Axiom SF is brought out in the following. Consider a two-person distribution $a=(1,3)$, given that the poverty line z is 2. Suppose poverty is measured by the headcount ratio H (which, of course, violates Strong Focus). It is immediate that $H(a;z)=1/2$. Now consider the vector $a'=(1,3,\ldots,3)$, where the income level 3 is replicated k times over. Then, the headcount ratio for this distribution is given by $H(a';z)=1/(k+1)$. Notice that as k becomes indefinitely large, H becomes vanishingly small. By simply going on supplementing the nonpoor population, one can make H as arbitrarily small as one chooses. If poverty can be pretty nearly eradicated simply by adding indefinitely to the nonpoor population, without doing anything to alleviate the condition of the poor, then such a view of poverty could be seen to be less than compellingly persuasive.

Next, in considering the relative merits of the Weak and the Diluted Poverty Growth Axioms, it is open to one to ask the following question: is it readily conceivable that a case can be established to the effect that Axiom WPG is an *impermissibly* strengthened version of Axiom DPG, and that the somewhat anaemic stand advocated by the latter is all that can reasonably be expected of a poverty measure? In line with such a view, it could be held that it seems to call for an unacceptable degree of conservatism to fail to declare that poverty has increased when, with all poor persons having the same income, an additional poor person with that income joins the community. Indeed, of a measure like the income-gap ratio I, which satisfies DPG, Sen (1981, p. 33) cites as one of its "damaging limitations" the fact that it "pays no attention whatever to the number or proportion of ... people below the poverty line, concentrating only on the aggregate short-fall". Refusing to go beyond DPG is, thus, to cast serious doubt on a long-established practice in poverty measurement, whereby some headcount of the poor is factored in as a component of the poverty measure. Briefly, then, this line of argumentation would suggest that violating Axiom WPG is not an intuitively appealing way of measuring poverty.

The preceding discussion is also open to a more "constructive" interpretation than it seems to allow. Specifically, it suggests that, even *within* a "compassed" view of "the extent of poverty", there are again certain basic choices which the poverty analyst must make. Depending on whether one finds Axiom SF or Axiom WPG relatively easier to dispense with, one is confronted with certain elementary options that must be evaluated and exercised. If one is rather more comfortable with relaxing WPG than SF, one is expressing a preferential option for measuring how poor, on average, the poor are, considering only the distribution of income and disregarding the relative size of the poor subpopulation; further, in settling for an index like the income-gap ratio I, one is taking a particular view of the notion of a "per capita" gap: the "per capita", here, is in terms of "per *poor* person". Alternatively, if one is rather more comfortable with relaxing SF than WPG, one is expressing a basic choice in favour of measuring the incidence of poverty (*via* H), which is also related to the notion (discussed earlier) of "the likelihood of encountering poverty in a society"; or of measuring the per capita depth of poverty (*via* R), where, by "per capita", one now means "per member of the general population" rather than just "per member of the poor population". These basic choices call for a measure of clarity in the conceptualization of alternative notions of poverty; and also for the recognition that even seemingly innocuous terms such as "per capita" are not devoid of ambiguity.

## 4. CONCLUDING DISCUSSION

This note has dealt with a very basic difficulty in the measurement of poverty, which can be explicated as follows. It appears that in measuring poverty, there are two ways of interpreting the notion of "extent" of poverty. One, the "compassed" view, normalizes for the size of the population; the other, the "uncompassed" view, does not so normalize. The former is the interpretation that has largely prevailed in the poverty measurement literature, and accounts for the wide general appeal of the *Replication Invariance Axiom*. The *Replication Scaling Axiom*, on the other hand, endorses an "uncompassed" view of the "extent" of poverty. A couple of other (arguably basic and unexceptionable) properties of poverty indices are those subsumed in the Strong Focus Axiom and the Weak Poverty Growth Axiom.

The poverty analyst now has two options. The first is to take a "compassed" view of the "extent" of poverty, that is, to endorse Replication Invariance, which, in the interests of ensuring the existence of a poverty index, must entail a weakening of either Strong Focus or Weak Poverty Growth (depending on which of these two axioms one finds less persuasive) and, therefore, a further fundamental choice between indices such as H and R on the one hand, and indices such as I on the other. The second option is to take an "uncompassed" view of the "extent" of poverty, that is, to endorse Replication Scaling, which, happily, sits easily with both Strong Focus and Weak Poverty Growth, though a possible price one may have to pay here is to experience some intuitive reservation in dissociating judgements regarding the extent of poverty in a society from any notion of the "likelihood" of encountering poverty in it. These basic choices confronting the poverty analyst are simple ones; arguably, they also make the measurement of poverty a somewhat complicated enterprise.

**REFERENCES**

Broome, J. 1996. The welfare economics of population. *Oxford Economic Papers*, 48:177–93

Foster, J. and A. F. Shorrocks. 1991. Subgroup consistent poverty indices. *Econometrica*, 59:687–709

Kundu, A. and T. E. Smith. 1983. An impossibility theorem on poverty indices. *International Economic Review*, 24:423–34

Sen, A. K. 1976. Poverty: an ordinal approach to measurement. *Econometrica*, 44:219–31

Sen, A. K. 1981. *Poverty and Famines: An Essay on Entitlement and Deprivation*. Clarendon Press

Shorrocks, A. F. 1988. Aggregation issues in inequality measurement. In *Measurement in Economics*, pp. 429–51. W. Eichhorn (ed.). Physica-Verlag