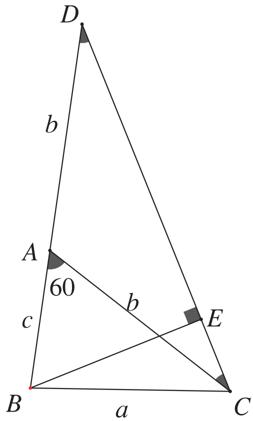
106.13 A triangle inequality

In the triangle ΔABC, $\angle A = 60°$. It is clear that, if ΔABC is equilateral, $a = \frac{1}{2}(b + c)$. If, however, ΔABC is not equilateral, then $a > \frac{1}{2}(b + c)$.

Suppose $\angle A = 60°$ and, without loss of generality, let $\angle B > \angle C$.



Extend BA to D with $AD = AC = b$.

Now $\angle ADC = \angle ACD = 30°$.

Let E be the foot of the perpendicular from B to DC.

From ΔBED, $BE = \frac{1}{2}(b + c)$ and, from ΔBCE, $a > BE$.

Hence $a > \frac{1}{2}(b + c)$ as required.

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106.14 Exarc radii and the Finsler-Hadwiger inequality

An exarc circle is a circle tangent to two sides of a triangle ABC and externally to the circumcircle of the triangle. We will call the radii of the exarc circles *exarc radii* and denote them by R_A, R_B, R_C. We denote by I_A, I_B, I_C and r_A, r_B, r_C the excentres and the exradii of ABC respectively, see Figure 1.

In the standard notation let a, b, c, Δ, R, r, s, represent respectively the sides, area, circumradius, inradius and semiperimeter of a triangle ABC.

Let X and Y be the the touching points of the A-exarc circle and the sides AB and AC of the triangle respectively. It is known [1] that the midpoint of XY is the excentre I_A of the triangle. Hence, the centre of the A-exarc circle is the intersection of the bisector of $\angle CAB$ and the perpendicular through the point X on the side AB, which is obtained as the intersection of AB and the perpendicular to this bisector through I_A, see Figure 1. We have