A New Garber-Style Solution to the Problem of Old Evidence

Stephan Hartmann and Branden Fitelson*†

In this discussion note, we explain how to relax some of the standard assumptions made in Garber-style solutions to the Problem of Old Evidence. The result is a more general and explanatory Bayesian approach.

1. Background: Garber, Jeffrey, and Earman. By the time Einstein had formulated the general theory of relativity (H), the evidence regarding the perihelion of Mercury (E)—which Newtonian theory was unable to adequately explain—had long been known (Roseveare 1982). Indeed, it is not implausible to suppose that Einstein was certain (in 1915) that E was true. Nonetheless, it is widely accepted that Einstein learned some proposition X (in 1915) that had the effect of confirming H (i.e., rationally raising Einstein's credence in H).

Garber (1983) proposes that what Einstein learned was a logical fact (i.e., that $X =$ "H entails E"). By adding an additional atomic statement X to the H, E language (and interpreting X extrasystematically as "H entails E"), Garber showed how it was possible to write down Bayesian models of this sort, having the following desired confirmation-theoretic property:

$$\Pr(H \mid X) > \Pr(H). \tag{1}$$

Received February 2015; revised May 2015.

*To contact the authors, please write to: Stephan Hartmann, Munich Center for Mathematical Philosophy, Ludwig Maximilians-Universität, München, Ludwigstraβe 31, 80539 München, Germany. Branden Fitelson, Department of Philosophy, Rutgers University, 106 Somerset Street, New Brunswick, NJ 08901.

†We would like to thank Fabrizio Cariani, Vincenzo Crupi, Kenny Easwaran, Aidan Lyon, Jan Sprenger, Mike Titelbaum, and two anonymous referees of this journal for useful feedback and comments on this article.

Philosophy of Science, 82 (October 2015) pp. 712–717. 0031-8248/2015/8204-0009$10.00

Garber did not, however, endorse specific constraints on $\Pr(\cdot)$ that would ensure (1). Subsequently, Earman (1992) and Jeffrey (1992) offered such constraints. Their approaches require the addition of X and a fourth atomic statement Y to the H, E language, where Y is extrasystematically interpreted as "H entails $\neg E$" (i.e., H refutes E). In these traditional Garber-style approaches, the background extrasystematic constraints on $\Pr(\cdot)$ consist of the following pair of *modus ponens* principles for the extrasystematic entailment relation.[1]

$$\Pr(E \mid H \& X) = 1. \tag{2}$$

$$\Pr(E \& H \& Y) = 0. \tag{3}$$

In addition to (2) and (3), Jeffrey offers the following extrasystematic constraint:

$$\Pr(X \vee Y) = 1. \tag{4}$$

Informally, (4) expresses certainty in the disjunction: either H entails E or H refutes E. Given the background assumption that $\Pr(E) = 1$, it is straightforward to show that (2)–(4) jointly entail the desired confirmation-theoretic conclusion (1). This, in essence, was Jeffrey's Garber-style approach to the old evidence problem.

As Earman rightly points out, Jeffrey's constraint (4) is not very plausible as a rational stricture on Einstein's credences, before his learning X (in 1915). There was no good reason for Einstein to be certain (before learning X) that H either entails or refutes E. Earman offers the following alternative additional extrasystematic constraint:

$$\Pr(H \mid X) > \Pr(H \mid \neg X \& \neg Y). \tag{5}$$

Earman shows that—given the background assumption that $\Pr(E) = 1$—(2), (3), and (5) jointly entail (1), and he argues that (5) is a plausible assumption regarding Einstein's credences (in 1915). We agree that (5) is more plausible than Jeffrey's (4). For one thing, (5) is not a numerical constraint but merely an ordinal constraint on Einstein's 1915 credences. Moreover, because Einstein was (antecedently) certain of E, it is reasonable to suppose that he would have judged that X confers a greater probability on H than $\neg X \& \neg Y$ does. Having said that, we would prefer a more general approach, which (*a*) does not

1. Traditionally, Garber-style approaches also rest on the background assumption that $\Pr(E) = 1$. In standard (Kolmogorovian) probability theory, this background assumption entails (2). We choose to state (2) explicitly here, however, since it reflects the fact that X is (in traditional Garber-style approaches) couched in terms of entailment. In the next section, we explain how to relax both of these assumptions.

https://doi.org/10.1086/682916 Published online by Cambridge University Press

presuppose that $\Pr(E) = 1$ and (b) does not require interpreting X and Y in terms of entailment relations.

2. A New Garber-Style Approach. We like Garber's idea of adding a pair of extrasystematic statements (and extrasystematic credal constraints) to the H, E language. But, we think the existing implementation of this general strategy has two main shortcomings. First, we think interpreting X and Y as "H entails E" and "H refutes E" is unduly restrictive. It is more plausible to suppose that what is learned in cases of old evidence (i.e., X) may not (always) be a logical fact. To be more precise, let X and Y be interpreted as follows:

- $X =_{\text{df}} H$ adequately explains (or accounts for) E.[2]
- $Y =_{\text{df}} H$'s best competitor (H') adequately explains (or accounts for) E.[3]

What really matters here is not whether H entails E (or $\neg E$) but whether H adequately explains (or accounts for) E and whether some alternative theory H' (which is H's best competitor; see n. 3) adequately explains (or accounts for) E. It may be that H adequately explains E in a deductive-nomological sense. But why not allow for the possibility that H (or H') explains E in a non-deductive-nomological way? In this regard, we think that the original Garber-style approaches are too narrow in their explanatory scope.

A second problem with the traditional Garber-style approaches is that they have required extrasystematic credal constraints (e.g., $\Pr(E) = 1$ and eq. [4]) that are implausibly strong. This defect is also remedied by moving to our alternative, explanatory extrasystematic interpretation of X and Y. Consider the following four ordinal constraints on $\Pr(\cdot|\cdot)$.

$$\Pr(H \mid X \& \neg Y) > \Pr(H \mid \neg X \& \neg Y). \tag{6}$$

2. We are not the first to consider this sort of generalization of Garber's approach. Garber himself (1983, 112) considers some alternative interpretations of X and Y that have a more general explanatory flavor. However, all of the alternatives Garber mentions involve some pattern of entailment relations between the salient propositions. So, Garber's account(s) would still be restricted to forms of explanation that supervene on deductive entailment relations. Moreover, Garber never works out any of these alternatives in any detail. Hartmann (2014) uses general explanatory language to interpret X and Y. Our approach is intended as a simplification of Hartmann's original idea (which is more complex, theoretically). A very recent article by Jan Sprenger (2015) also appeals to explanatory relations, but in a different way.

3. When we say that H' is H's "best competitor," what we mean is that H' is H's best competitor with respect to explaining/predicting phenomenon E—e.g., in our Mercury example, H was general relativity, H' was Newtonian theory, and E was the evidence (available in 1915) regarding the perihelion of Mercury.

$$\Pr(H \mid X\&\neg Y) > \Pr(H \mid \neg X\&Y). \tag{7}$$

$$\Pr(H \mid X\&Y) > \Pr(H \mid \neg X\&Y). \tag{8}$$

$$\Pr(H \mid X\&Y) \geq \Pr(H \mid \neg X\&\neg Y). \tag{9}$$

Let's examine each of four constraints, in turn. Suppose that H adequately explains E, but its best competitor H' does not. Constraints (6) and (7) assert that H is more probable, given this supposition ($X\&\neg Y$), than it is given either the supposition that neither H nor H' adequately explains E (i.e., given $\neg X\&\neg Y$) or the supposition that H's best competitor (H') adequately explains E but H does not ($\neg X\&Y$). These two constraints seem uncontroversial.

Constraint (8) also seems quite plausible. It asserts that H is less probable, given the supposition that its best competitor (H') adequately explains E but H does not ($\neg X\&Y$), than it is given the supposition that both H and its best competitor (H') adequately explain E (i.e., given $X\&Y$).

The fourth and final credal comparison (9) says that H is at least as probable, given the supposition that both H and its best competitor (H') adequately explain E (i.e., given $X\&Y$), as it is given the supposition that neither H nor H' adequately explains E (i.e., given $\neg X\&\neg Y$). One might maintain that it would be reasonable to rank $\Pr(H \mid X\&Y)$ strictly higher in one's comparative confidence ranking than $\Pr(H \mid \neg X\&\neg Y)$. After all, $X\&Y$ implies that H does adequately explain E (which is already known), whereas $\neg X\&\neg Y$ implies that H does not adequately explain E. But one might also reasonably argue that these two suppositions ($X\&Y$ and $\neg X\&\neg Y$) place H and H' on a par with respect to explaining E, and so they should not confer different probabilities on H. Both of these positions are compatible with (9). The only thing (9) rules out is the claim that H is more probable given $\neg X\&\neg Y$ than it is given $X\&Y$. As such, (9) is also eminently reasonable.

As it happens, the desired (Garberian) confirmation-theoretic conclusion (1) follows from (6) to (9) alone. To be more precise, we can prove the following general result.

Theorem. Constraints (6)–(9) jointly entail (1).

Proof. Let $\mathfrak{a} =_{df} \Pr(H \mid X\&\neg Y)$, $\mathfrak{b} =_{df} \Pr(H \mid X\&Y)$, $\mathfrak{c} =_{df} \Pr(H \mid \neg X \& \neg Y)$, $\mathfrak{d} =_{df} \Pr(H \mid \neg X\&Y)$, $x =_{df} \Pr(\neg Y \mid X)$, and $y =_{df} \Pr(\neg Y \mid \neg X)$. Given these assignments, (6)–(9) are as follows.

$$\mathfrak{a} > \mathfrak{c}. \tag{6}$$

$$\mathfrak{a} > \mathfrak{d}. \tag{7}$$

$$\mathfrak{b} > \mathfrak{d}. \tag{8}$$

$$\mathfrak{b} \geq \mathfrak{c}. \tag{9}$$

https://doi.org/10.1086/682916 Published online by Cambridge University Press

Suppose that $x > 0$ and $y < 1$. Then, (6)–(9) jointly entail

$$\mathfrak{a}x + \mathfrak{b}(1 - x) > \mathfrak{c}y + \mathfrak{d}(1 - y).$$

And, by the law of total probability, we have

$$\Pr(H \mid X) = \mathfrak{a}x + \mathfrak{b}(1 - x).$$

$$\Pr(H \mid \neg X) = \mathfrak{c}y + \mathfrak{d}(1 - y).$$

Thus, (6)–(9) jointly entail $\Pr(H \mid X) > \Pr(H \mid \neg X)$, which entails $\Pr(H \mid X) > \Pr(H)$. QED

We think our theorem undergirds a superior Garber-style approach to the problem of old evidence. Specifically, our approach has the following two distinct advantages over traditional Garber-style approaches.

i) Unlike previous Garber-style approaches, ours does not require the assumption that $\Pr(E) = 1$. It may be true that our constraints (6)–(9) are most plausible given the background assumption that E is (antecedently) known with certainty. But we think (6)–(9) retain enough of their plausibility, given only the weaker assumption that E is (antecedently) known with high credence.[4]

ii) Our approach is not restricted to cases in which H (or H') explains E in a deductive-nomological way. That is, our approach covers all cases in which scientists come to learn that their theory adequately explains E, not only those cases in which scientists learn that their theory entails E (or explains E deductive nomologically).[5]

4. If we relax the standard Garberian assumption that $\Pr(E) = 1$, then we will need to specify what happens to the probability of E when X is learned. Intuitively, the probability of E should not change during this learning process. As a result, models of such a learning event may need to be more sophisticated than our present (naive strict conditionalization) approach would suggest.

5. We have not said anything here about the nature of scientific explanation. This is intentional, as we would prefer to remain as neutral as possible on this score. Having said that, all we really need to presuppose (dialectically) is that our approach is compatible with some nondeductive approaches to scientific explanation. If that presupposition is correct, then we will have succeeded in pushing Garber's main ideas further than previous authors (except for Hartmann 2014) who have written (Garber style) on the old evidence problem. We think this presupposition is quite plausible. For instance, we think it is clear that our approach is compatible with various (objective) nondeductive theories of scientific explanation, e.g., so-called inductive-statistical and statistical-relevance approaches (Salmon 1989). One might (still) worry that our approach is not compatible with (subjective) Bayesian accounts of

REFERENCES

Earman, John. 1992. *Bayes or Bust? A Critical Examination of Bayesian Confirmation Theory*. Cambridge, MA: MIT Press.

Garber, Daniel. 1983. "Old Evidence and Logical Omniscience in Bayesian Confirmation Theory." In *Testing Scientific Theories*, ed. John Earman, 99–132. Minneapolis: University of Minnesota Press.

Hartmann, Stephan. 2014. "A New Solution to the Problem of Old Evidence." Unpublished manuscript, PhiSci Archive. http://philsci-archive.pitt.edu/10887/.

Jeffrey, Richard. 1992. "Bayesianism with a Human Face." In *Probability and the Art of Judgment*, 77–107. Cambridge: Cambridge University Press.

Roseveare, N. T. 1982. *Mercury's Perihelion from Le Verrier to Einstein*. Oxford: Clarendon.

Salmon, Wesley. 1989. "Four Decades of Scientific Explanation." In *Minnesota Studies in the Philosophy of Science*, vol. 13, ed. Wesley Salmon and Philip Kitcher, 3–219. Minneapolis: University of Minnesota Press.

Skyrms, Brian. 1980. "Higher Order Degrees of Belief." In *Prospects for Pragmatism*, ed. D. H. Mellor, 109–37. Cambridge: Cambridge University Press.

Sprenger, Jan. 2015. "A Novel Solution to the Problem of Old Evidence." *Philosophy of Science* 82 (3): 383–401.

scientific explanation, on the grounds that such an interpretation of our approach would require "higher order subjective probabilities" (since we would have to assign subjective probabilities to explanatory relations that themselves involve subjective probabilities). This is an interesting worry, but we suspect that our approach can be made compatible even with subjective Bayesian accounts of scientific explanation, provided that proper care is taken to model higher-order credences (Skyrms 1980).