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PHYSICAL CAPITAL, HUMAN CAPITAL, AND THE HEALTH EFFECTS OF POLLUTION IN AN OLG MODEL

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In a standard overlapping generations model, we show how the health effects of pollution impact the balanced-growth path (BGP) and the transition dynamics of the economy. The key driver is the differential between physical and human capital accumulation. The differential occurs because pollution alters the incentives to save and to invest in education via reductions in longevity and alters the effectiveness of education expenditures via impaired cognitive learning. Two predictions of the model are noteworthy. The first prediction is the existence of two stable BGPs with a separating saddle path. One BGP is desirable featuring high economic growth and low pollution, whereas the other should be avoided because it is associated with low economic growth and high pollution. The second prediction is that economic and environmental cycles may emerge, implying inequality between generations. These theoretical results are supported by empirical evidence and imply a role for government to steer the economy toward the desirable BGP and eliminate the cycles.

Keywords: Endogenous Growth, Overlapping Generations, Pollution, Health Effects

1. INTRODUCTION

Pollution leads to severe health consequences. Air, water, soil, heavy metal, chemical, and lead pollution has combined to cause 9 million early deaths in 2015 around the globe [Landrigan et al. (2018)]. Lead pollution, for example, also inflicts other detrimental effects on children's health. It is estimated that approximately one-third of children in Central and South America, 7% of children in North America, and 25% of children in China have elevated levels of lead in their bloodstream [Payne (2008) and Li et al. (2015)]. By damaging children's nervous systems, lead pollution has been shown to cause intelligence

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quotient (IQ) deficits, attention-deficit hyperactivity disorder, and conduct disorders [Hwang et al. (2019)]. These and other academic studies motivate our focus on two health effects imposed by pollution, namely, reducing longevity and impeding cognitive learning.[1]

Why should economists study these two health effects of pollution? Pollution reduces longevity, thus limiting the accumulation of physical and human capital. Pollution also impedes cognitive learning, thus limiting the accumulation of human capital. These channels by which pollution affects physical and human capital have been well established both empirically and theoretically. First, the empirical literature shows pollution reduces longevity [Wen and Gu (2012) and Ebenstein et al. (2015)] and a decreased longevity in turn lowers savings [Bloom et al. (2003), Zhang and Zhang (2005), and De Nardi et al. (2009)]. Motivated by the empirical evidence, several theoretical papers incorporate the longevity effect into their models, such that pollution endogenously modifies people's incentive to save [Pautrel (2009), Jouvet et al. (2010), Varvarigos (2010, 2013a), Raffin and Seegmuller (2014), and Fodha and Seegmuller (2014)]. Second, empirical research shows that longer lives promote education and enhance human capital [Hoque et al. (2019)]. In a similar vein, theoretical research establishes that pollution lowers the rate of human capital accumulation through longevity [Pautrel (2008)]. Third, the empirical evidence shows that pollution impedes children's cognitive learning in multiple ways. Pollution increases school absences [Currie et al. (2009), Mohai et al. (2011), and Chen et al. (2018a)], reduces years of schooling [Nilsson (2009)], enters and damages the brain [Maher et al. (2016)], causes a significant decline in cognitive performance [Ebenstein et al. (2016) and Zhang et al. (2018)], and jeopardizes mental health [Zhang et al. (2017), Kim et al. (2017), and Chen et al. (2018b)]. The health impact of pollution on cognitive learning and consequently on human capital has also inspired other theoretical studies [see, e.g., Raffin (2012), Aloi and Tournemaine (2013), and Sapci and Shogren (2017)].

The literature cited above often deals exclusively with one health aspect of pollution and thus only studies the effect of pollution on physical capital or on human capital. To the best of our knowledge, the literature is silent about the joint impact of pollution on the ratio of physical-to-human capital. This is an important gap because the growth literature points out that the capital ratio is a key indicator for economic growth [see, e.g., Mulligan and Sala-i-Martin (1993), Ladrón-de-Guevara et al. (1997), Barro (2001), and Duczynski (2002)]. If pollution only negatively affects the accumulation of physical capital, the physical-to-human capital ratio unambiguously decreases in pollution. In contrast, if we only analyze the negative effect of pollution on human capital, the physical-to-human capital ratio unambiguously increases in pollution. It is reasonable, however, to postulate that if pollution imposes negative effects on physical and human capital in an unbalanced way, the ratio of physical-to-human capital may increase, decrease, or stay the same, and the subsequent dynamics and policy implications may differ from past studies. By allowing pollution to have adverse impacts on both types of capital through health, our model closes the gap between the two strands of literature that analyze only one aspect of pollution health effects on capital accumulation. The modeling modification can lead to interesting transitional dynamics and balanced-growth paths (BGPs) in terms of economic and environmental consequences, and these results are supported by empirical evidence.

To study the negative impacts of pollution on health, and consequently on physical and human capital, we establish an otherwise standard overlapping generations (OLG) model. The novel aspect of our model is that by damaging health, pollution causes a differential impact on the accumulation of physical and human capital, which in turn affects the physical-to-human capital ratio. Pollution hampers the accumulation of physical capital because pollution decreases savings via reducing longevity. We call this mechanism *the physical capital effect* of pollution. Because pollution is detrimental to the accumulation of physical capital, the physical capital effect of pollution is always negative. Pollution also hampers the accumulation of human capital because pollution decreases private education expenditures via shortening longevity and hampers cognitive learning. Thus, pollution decreases both the amount and effectiveness of education expenditures. We call this mechanism *the human capital effect* of pollution, which is also negative. The magnitudes of these two effects of pollution will likely be different, both across countries and across time. The difference of the physical and human capital effects of pollution, *the capital accumulation differential*, determines how pollution affects the capital ratio and functions as the key mechanism we explore in this paper.

The physical-to-human capital ratio also affects pollution. Since the use of physical capital in production is associated with pollution, whereas human capital provides solutions to alleviate pollution, the capital ratio influences pollution. This is a standard assumption in the literature [e.g., Gradus and Smulders (1993), Smulders and Gradus (1996), and Pautrel (2008, 2009, 2012)]. Therefore, there exists a feedback loop between pollution and the physical-to-human capital ratio. The feedback loop works in two plausible ways depending on the capital accumulation differential caused by pollution.

In the first case, the capital accumulation differential is relatively large. Thus, the difference between the physical capital effect and the human capital effect indicates that pollution more adversely affects human capital relative to physical capital. We derive analytical results that on the BGP, pollution and the physical-to-human capital ratio are positively correlated, and the BGP is generally stable. The intuition is as follows. As pollution reduces the accumulation of human capital more than that of physical capital, physical capital becomes abundant relative to human capital and the ratio of physical-to-human capital tends to increase in pollution. A high ratio of physical-to-human capital leads to a high stock of pollution, which in turn reinforces the high capital ratio. Therefore, the BGP features a positive relationship between the pollution stock and the capital ratio, as well as a stable transition path. The economic growth rate on the BGP is negatively correlated with the pollution stock and the capital ratio, as higher pollution hampers physical and human capital, which in turn hinders economic growth.

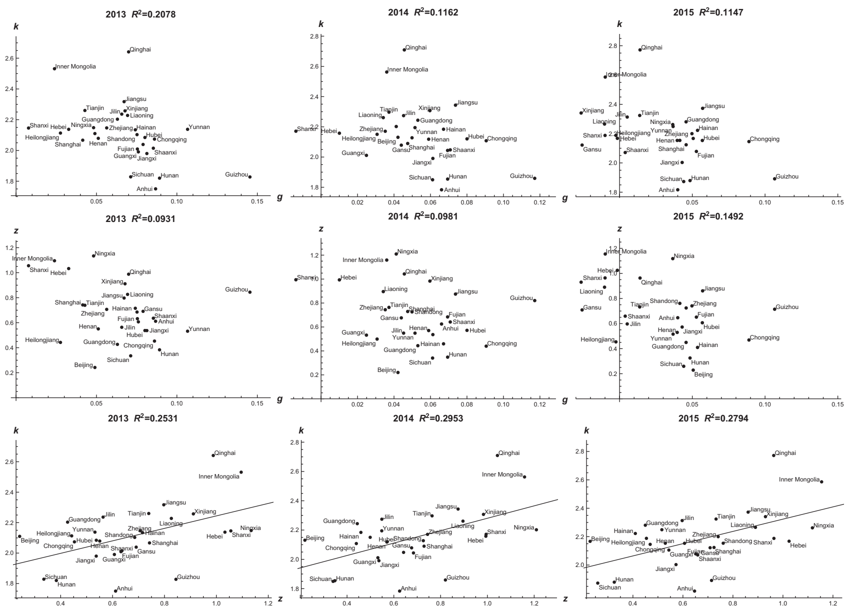
Interestingly, the positive relationship between the capital ratio and the pollution stock can be sustained at both low and high levels of pollution. We find that two BGPs and a middle one separating the former two can exist simultaneously. The former two BGPs are “sinks.” One is desirable in the sense that it features a high economic growth rate and a low stock of pollution, whereas the other should be avoided as it is associated with a low economic growth rate and a high stock of pollution. The BGP in the middle exhibits saddle stability, and it gives rise to a saddle path demarcating the two “sink” regions. The emergence of multiple BGPs is intuitive and related to the positive relationship between the capital ratio and pollution, as well as the agent’s decision of whether to invest in private education. A low stock of pollution and thus high longevity prompts the agent to invest in private education. More human capital is accumulated and physical capital becomes relatively scarce. The low ratio of physical-to-human capital leads to a low stock of pollution, which in turn maintains the low capital ratio. Thus, a virtuous circle continues with a high economic growth rate. In contrast, when facing a high stock of pollution and thus short longevity, the agent finds it optimal to not invest in private education. Without private education expenditures, less human capital is accumulated and physical capital becomes relatively abundant. The high ratio of physical-to-human capital generates more pollution, which conversely reinforces the high capital ratio. A vicious circle is at work with a low economic growth rate because high pollution severely limits the accumulation of physical and human capital. In this latter scenario, the government can potentially steer the economy away from the inferior BGP toward the desirable one.

Our theoretical results are supported with Chinese provincial and international data. We collect industrial waste gas emission data from the 1991 to 2016 China Statistical Yearbooks on the Environment [Wen et al. (2017)], and economic and demographic data from the China Center for Human Capital and Labor Market Research (2018) to construct proxies for pollution, the capital ratio, and economic growth. Pollution is measured by the natural logarithm of the population-weighted emission of industrial waste gas. The capital ratio is calculated by dividing the per-capita value of physical capital by that of human capital. Economic growth is the annual growth rate of real GDP per capita. To illustrate our theoretical results, Figure 1 exhibits the scatter plots of 30 Chinese provinces for the years from 2013 to 2015. Pairwise relations among pollution, the capital ratio, and the GDP growth rate are consistent with our theoretical results that arise from the case of a relatively large capital accumulation differential. To further illustrate the two stable BGPs, we align the comparison of economic growth rates associated with the virtuous and vicious circles with existing empirical evidence in Osang and Sarkar (2008), which shows that the average growth rate in countries with public education only is lower than in countries with both public and private education. We also follow Goenka and Liu (2020) and use K-means algorithm to conduct cluster analyses based on pollution, the capital ratio, and the GDP growth rate with Chinese provincial data in 1990 and in 2015, which are the beginning and the end of the sample period.[2] In Panel A of Table 1, 30 Chinese provinces with

TABLE 1. Cluster analyses for two groups of Chinese provinces

	Growth rate	Pollution	Capital ratio
Panel A: Cluster analysis in 1990			
Group 1	9.97%	−0.44	1.47
(15 provinces)	[7.14% 12.80%]	[−0.64 −0.24]	[1.34 1.60]
Group 2	4.67%	0.10	1.70
(15 provinces)	[2.90% 6.45%]	[−0.01 0.20]	[1.63 1.77]
Panel B: Cluster analysis in 2015			
Group 1	4.85%	0.49	2.12
(19 provinces)	[2.55% 7.15%]	[0.38 0.60]	[2.03 2.20]
Group 2	1.41	0.92	2.33
(11 provinces)	[−0.49% 3.31%]	[0.83 1.02]	[2.20 2.46]

Notes: (1) The growth rate is measured by the annual growth rate of real GDP per capita, pollution is measured by the natural logarithm of the population-weighted emission of industrial waste gas, and the capital ratio is calculated by dividing the per-capita value of physical capital by that of human capital. (2) Each cell reports the group mean and 95% confidence interval in the bracket.



Notes: (1) The relations among pollution (z), the capital ratio (k), and the economic growth (g) are visualized pairwise. The first row of figure cells plots the economic growth (g) against the capital ratio (k), the second row plots the economic growth (g) against pollution (z), and the third row plots pollution (z) against the capital ratio (k).
(2) The trend lines are drawn based on least-squares fits to scatters of 30 Chinese provinces from 2013 to 2015, and the values for R^2 are reported above each figure cell.

FIGURE 1. The pairwise relations among pollution, capital ratio, and economic growth in China.

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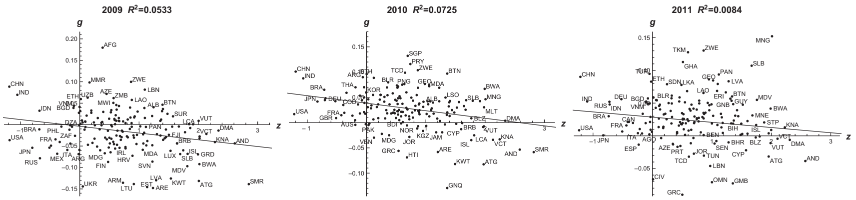

Notes: (1) In each figure cell, pollution (z) on the horizontal axis is represented by air stock pollutants, which are the logged values of population-weighted PM10 (2009–2011). Economic growth (g) on the vertical axis is the annual growth rate of real GDP per capita.
(2) The country codes can be found in World Bank Group (2018a).
(3) The trend lines are drawn based on least-squares fits to scatters of countries and regions in each year, and the values for R^2 are reported above each figure cell.

FIGURE 2. The negative relationship between pollution and growth in the world (PM10).

complete data in 1990 are endogenously divided into two groups. Group 1 lies on the desirable BGP with higher economic growth rate, lower pollution, and lower ratio of physical-to-human capital on average, whereas Group 2 lies on the inferior BGP. In Panel B of Table 1, qualitatively similar results hold for the same 30 Chinese provinces in 2015. Then, we use the Chinese provincial data in 1990 and 2015 to provide a dynamic view of how provinces transition between the two stable BGPs by constructing a Markov transition matrix based on Table 1. Panel A of Table 3 shows that the majority of Chinese provinces that lie on the desirable BGP in 1990 remain there in 2015, and 2/3 of the provinces that are on the inferior BGP in 1990 are still trapped there in 2015, thus lending empirical support for the stable property of the two BGPs.

We also collect panel data on air stock pollutants (mean annual exposure to PM2.5 and PM10 at the national level),[3] population, and real GDP per capita from the World Bank database [World Bank Group (2018a,b)].[4] Pollution is measured by the natural logarithm of population-weighted PM2.5 and PM10, and economic growth is measured by the annual growth rate of real GDP per capita. To save space in the main text, we present the empirical evidence based on PM10 but relegate the evidence based on PM2.5 to Appendix A. The relationship between pollution and economic growth is shown using scatter plots of the countries for each year (see Figure 2 for 3 years of data on PM10).[5] The negatively sloped trend lines indicate that economic growth and pollution are negatively correlated.[6] The pattern of two stable BGPs also emerges for international data, implying some countries experience both robust economic growth and more favorable environmental quality, while others suffer from both lower economic growth and less favorable environmental quality. Table 2 shows that in 1990 and in 2011, 141 countries with complete data are endogenously divided into two groups. Group 1 lies on the desirable BGP featuring higher growth rate and lower pollution, while Group 2 lies on the inferior BGP featuring lower growth rate and higher pollution. As we conduct cluster analyses for the same 141 countries in 1990 and in 2011, we are able to identify how a country transitions between groups and create

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TABLE 2. Cluster analyses for two groups of countries based on PM10

	Growth rate	Pollution
Panel A: Cluster analysis in 1990		
Group 1	1.25%	0.34
(77 countries)	[−0.45% 2.96%]	[0.24 0.44]
Group 2	1.05%	1.53
(64 countries)	[−1.13% 3.23%]	[1.41 1.66]
Panel B: Cluster analysis in 2011		
Group 1	2.70%	0.10
(80 countries)	[1.98% 3.41%]	[0.0003 0.20]
Group 2	2.24%	1.27
(61 countries)	[1.09% 3.40%]	[1.15 1.39]

Notes: (1) The growth rate is measured by the annual growth rate of real GDP per capita and pollution is measured by the natural logarithm of population-weighted PM10.
(2) Each cell reports the group mean and 95% confidence interval in the bracket.

TABLE 3. Markov transition matrices for Chinese provinces and countries

	Desirable BGP	Inferior BGP
Panel A: Chinese provinces from 1990 to 2015		
Desirable BGP	0.9333	0.0667
Inferior BGP	0.3333	0.6667
Panel B: Countries based on PM10 from 1990 to 2011		
Desirable BGP	0.9740	0.0260
Inferior BGP	0.0781	0.9219

Notes. (1) Panel A presents the 2 × 2 Markov transition matrix for Chinese provinces from 1990 to 2015, which is calculated based on how Chinese provinces transition between Groups 1 and 2 in Table 1. Panel B presents the 2 × 2 Markov transition matrix for countries from 1990 to 2011 with PM10 serving as a proxy of pollution. It is calculated based on how countries transition between Groups 1 and 2 in Table 2.
(2) The Markov transition matrix shows the probability that a province or a country transitions from one BGP to another. For example, the table cell indexed (1,1) in Panel A says the probability that a Chinese province remains on the desirable BGP from 1990 to 2015 is 93.33%, and the table cell indexed (2,1) in Panel B says the probability that a country transitions from the inferior BGP to the desirable BGP from 1990 to 2011 is 7.81%.

the Markov transition matrix. Panel B of Table 3 shows that from 1990 to 2011, 97.4% of the countries remain in the desirable BGP group and 92.19% of the countries remain in the inferior BGP group. The results are qualitatively similar using PM2.5 as another proxy of pollution.

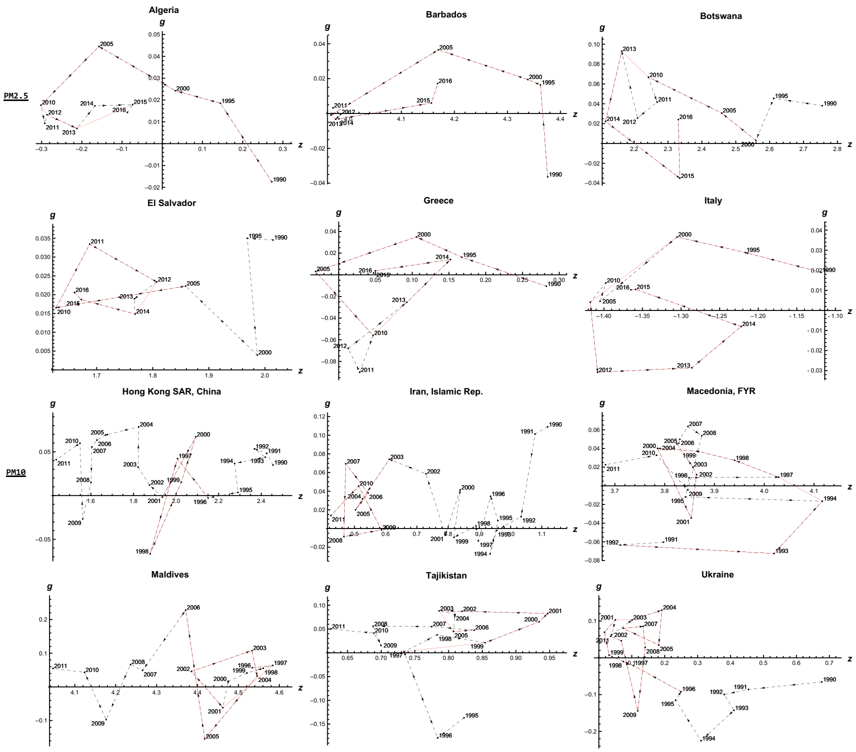
In the second, the capital accumulation differential is sufficiently small and becomes negative. The economy and the environment may exhibit cyclical movements. The intuition is as follows. A negative capital accumulation differential implies that pollution adversely affects physical capital more than human capital, and the effect is worsening in pollution. Thus, the change in the stock of pollution causes the ratio of physical-to-human capital to move in the opposite direction. Suppose initially as the stock of pollution increases, the capital ratio decreases. Because "clean" human capital becomes relatively more abundant than "dirty" physical capital, less pollution is discharged into the environment, which in turn leads to a higher capital ratio. Then the stock of pollution rises again. The back-and-forth dynamic relationship between the capital ratio and pollution thus leads to cycles in the economy and the environment. These cycles represent inequality between generations [Schumacher and Zou (2008, 2015)] because some generations may enjoy higher economic growth and better environmental quality, whereas others suffer from lower economic growth and worse environmental quality. We show that the government policy can eliminate the cycles.

The extant empirical research documents cyclical movements of economic and environmental variables. For example, there exists a cyclical relationship between mortality and the economy [Tapia Granados (2005), Rolden et al. (2014)] as well as evidence on cycles of urban air pollutants [Mayer (1999)]. Also based on the time series data on pollution and economic growth in each country and region from the World Bank database [World Bank Group (2018a,b)], we provide additional empirical support for the joint cycles of the economy and the environment. We check the time paths of pollution and economic growth combinations for each country and region and depict some observed cycles with red continuous lines in Figure 3.

The rest of the paper is organized as follows. Section 2 presents a literature review. Section 3 sets up the model and describes the equilibria under two regimes that are divided by the pollution stock and the capital ratio. Section 4 derives analytical results about the BGP and the transition dynamics based on the capital accumulation differential. Section 5 provides numerical examples to complement the analytical results. Section 6 concludes.

2. RELATED LITERATURE

We argue that the accumulation differential between physical and human capital caused by pollution through the health effects is a new mechanism that gives rise to economic and environmental cycles. This relates our paper to a strand of literature that theoretically demonstrates the emergence of cycles and explores the mechanisms behind the cycles. Therefore, we review the various mechanisms established in the literature. First, cycles may arise due to the relative magnitudes of technical parameters. Zhang (1999) and Seegmuller and Verchère (2004) utilize similar models to that developed by John and Pecchenino (1994) where agents engage in environmental maintenance, and they derive similar conditions that a sufficiently large environmental degradation rate relative to environmental maintenance efficiency gives rise to cycles. In addition, Varvarigos (2013b) finds that the emission rate of pollution above a threshold level results in dampened cycles. Second, the emergence of cycles can originate from the representative agent's subjective factors. Schumacher and Zou (2008) introduce behavioral economics



Notes:(1) In each figure cell, pollution (z) on the horizontal axis is represented by air stock pollutants, which are the logged values of population-weighted PM2.5 (the first two rows) and PM10 (the last two rows). Economic growth (g) on the vertical axis is the annual growth rate of real GDP per capita. (2) The cycles are colored red after some points are smoothed if necessary. For example, to highlight the cycle of PM2.5 and economic growth in Algeria (the upper-left corner), we smooth the cycle by omitting the points for 2011 and 2014.

FIGURE 3. The economic and environmental cycles in selected countries and regions.

into an otherwise standard OLG model and show that the deviation of the perceived level of pollution from the actual level can generate cycles. Schumacher and Zou (2015) and Constant and Davin (2019) highlight the role of endogenous environmental preferences. In the former model, a threshold environmental quality alters generations' preferences for the environment over consumption, which consequently leads to cycles. In the latter, high sensitivity of preferences for the environment and human capital causes cycles. Third, cycles can be attributed to government interventions. Palivos and Varvarigos (2017) compare models with and without public pollution abatement and show that cycles arise in the absence of public pollution abatement. Goenka et al. (2020) study a second-best optimal taxation scheme that is contingent on physical capital. Interestingly, this optimal taxation scheme can be a source of cycles. Fourth, cycles can stem from the health impacts of pollution relative to health expenditures. Raffin and Seegmuller (2017)

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develop an OLG model where longevity is jointly determined by pollution, as well as private and public health expenditures. The authors show that if the damaging effect of pollution on longevity outweighs the effect of health expenditures, cycles can emerge. Similar to this strand of the literature, our paper is an application of the OLG model to study the dynamic interplay of the economy and the environment. However, we study the relationship between pollution and both physical and human capital, whereas the literature exclusively focuses on one type of capital. Using this modeling framework, we identify the capital accumulation differential imposed by pollution as another source of economic and environmental cycles.

One exception in the literature is Motoyama (2016), who also touches on the dynamic interactions of pollution and the ratio of physical-to-human capital and therefore merits a careful comparison with our paper. Motoyama (2016) shows that multiple equilibria may emerge in an OLG model with physical capital being the source of pollution. If the capital ratio is below a threshold value, the productivity of education is moderately damaged by pollution. Households invest in education, and both physical and human capital accumulate. The economy converges to a low capital ratio. However, if the capital ratio surpasses the threshold, the productivity of education is reduced. Households stop investing in education and only physical capital accumulates through savings. The economy converges to a high capital ratio. The key difference between Motoyama (2016) and this paper is twofold. First, in Motoyama (2016), pollution only negatively affects human capital. As pollution increases, the capital ratio unambiguously rises. In contrast, we emphasize the interaction of health effects of pollution on physical and human capital. We not only derive multiple BGPs similar to Motoyama (2016) but also show that if the capital ratio decreases in pollution, a possibility absent from Motoyama (2016), economic and environmental cycles may emerge. We also show that the government has a role in eliminating the cycles. Second, in Motoyama (2016), the government does not invest in education and pollution is implicitly modeled as being associated with physical capital. In our model, however, the government provides public education even if agents may not invest in private education. We also explicitly model the dynamics of the pollution stock where unlimited growth of pollution can be checked by pollution abatement financed by government spending. Our model thus allows us to discuss policy implications in terms of government expenditures.

Another point to emphasize is that our results are not driven by altruism in the context of uncertain mortality. Chakraborty and Das (2019) point out that parents' altruism toward their offspring affects the capital ratio when mortality changes. When an agent passes away, her intangible human capital easily dissipates, while tangible physical capital can be readily inherited by her child. In the event of early death, the altruistic agent favors the heritage in the form of physical capital over human capital. Thus, as longevity declines due to pollution, the ratio of physical-to-human capital rises. Our basic model also introduces the agent's altruism toward her child's human capital because it motivates the agent to invest in private education. The decision of whether to invest in private education is endogenously determined by the pollution stock and the capital ratio and gives rise to two regimes featuring two stable BGPs. However, our model deviates from Chakraborty and Das (2019) because the results are driven not by the agent's altruism and her preference for investment in physical capital in the event of premature death, but by the capital accumulation differential caused by the health impacts of pollution. To highlight the difference between our model and that of Chakraborty and Das (2019), we consider a simplified model in Appendix J that *completely* shuts down altruism.[7] Our primary results continue to hold in the alternative model with the capital accumulation differential of pollution still acting as the driving force behind the BGP variable relations and the transition dynamics.

3. THE MODEL

3.1. Firms

The production factors in this model are physical capital K_t and labor L_t augmented by human capital H_t. Denote r_t as the rental price of physical capital, and w_t as wage rate paid per unit of labor. The production function that a typical competitive firm employs to produce a final good is $Y_t = AK_t^{\alpha}(H_tL_t)^{1-\alpha}$, where $A > 0$ is a production scalar, $\alpha \in (0, 1)$ is physical capital's share in production, and $1 - \alpha$ is augmented labor's share in production. The price of the final good is normalized to 1. The firm pays a proportional tax, τ, on the final good to the government [Barro (1990), Devarajan et al. (1996), and Vella et al. (2015)]. The representative firm hires physical capital K_t and labor L_t to maximize its profits π_t. The profit maximization problem remains the same in each period:

$$\max_{K_t,L_t} \pi_t = (1-\tau)AK_t^{\alpha}(H_tL_t)^{1-\alpha} - r_tK_t - w_tL_t.$$

Define $k_t = {}^{K_t}/_{H_t}$ as the ratio of physical-to-human capital. Since each input is paid its marginal product, the first-order conditions are

$$r_t = (1-\tau)\alpha Ak_t^{\alpha-1}L_t^{1-\alpha}, \tag{1a}$$

$$w_t = (1-\tau)(1-\alpha)Ak_t^{\alpha}H_tL_t^{-\alpha}. \tag{1b}$$

3.2. Government

The government collects fiscal revenues through the proportional tax on the final good, $\tau AK_t^{\alpha}(H_tL_t)^{1-\alpha} = \tau Ak_t^{\alpha}H_tL_t^{1-\alpha}$. The government allocates a portion of the fiscal revenues, $\Delta \in (0, 1)$, to finance pollution abatement a_t, and the remaining portion, $1 - \Delta \in (0, 1)$, to finance public education m_t. We assume that the government engages in public pollution abatement a_t because the net pollution flow can be treated as a function of the ratio of physical-to-human capital. We can thus establish the channel by which the capital ratio affects the pollution stock. This modeling specification is widely used in the literature and is important in our following analyses on the two-way interactions of the pollution stock and the capital ratio. We also assume that the government provides public education because even if there is no private education expenditures, human capital still accumulates. The government runs a balanced budget in each period, which requires

$$a_t = \Delta \tau A k_t^{\alpha} H_t L_t^{1-\alpha}, \qquad \textbf{(2a)}$$

$$m_t = (1-\Delta) \tau A k_t^{\alpha} H_t L_t^{1-\alpha}. \qquad \textbf{(2b)}$$

The stock of pollution increases due to production activities. But as the government abates pollution, the stock of pollution decreases. Because it is "too good to be true" that emissions cease to grow [Economides and Philippopoulos (2008)], we assume that emissions are ρK_t, where $\rho > 0$ represents the polluting capacity of physical capital. Thus, as long as physical capital accumulates, emissions that are proportional to physical capital also grow. Due to pollution abatement financed by government expenditures, the net pollution flow becomes $\rho K_t / a_t$ [similar specifications also can be found in Gradus and Smulders (1993), Smulders and Gradus (1996), and Pautrel (2009, 2012)].[8] The stock of pollution, z_t, evolves according to:

$$z_{t+1} = (1-\theta) z_t + \frac{\rho K_t}{a_t}, \qquad \textbf{(3)}$$

where $\theta \in (0, 1)$ represents the dissipation rate of pollution. The stock of pollution adversely affects the economy by inflicting two types of health effects on the representative agent, which will be fully explained in the next section.

3.3. Agents

The time length in each period is 1. The representative agent lives three periods, that is, childhood, adulthood, and elderhood. In childhood, the agent receives education to accumulate human capital. In adulthood, the agent gives birth to one child, inelastically supplies one unit of labor to earn wage income and makes decisions in terms of consumption during adulthood, savings, and private education expenditures on her child to maximize lifetime utility. In elderhood, the agent enjoys consumption financed by her savings and her child's human capital as a result of her private education expenditures.

The agent lives the entirety of her childhood and adulthood but lives only a fraction of her elderhood, $\phi \in (0, 1)$. The representative agent born at the beginning of period $t-1$ thus has a lifetime equal to $2 + \phi_{t+1}$. As longevity decreases in pollution [Varvarigos (2013b) and Fodha and Seegmuller (2014)], we introduce z_t as a determinant of longevity, which captures the first type of health effect caused by pollution. Longevity also increases in human capital because an individual possessing the necessary knowledge to improve her health tends to live longer [Cutler et al. (2006)].[9] However, two parallel arguments suggest that the pathway by which human capital increases longevity is adjusted by physical capital, which impairs health and reduces longevity. First, there are risks to longevity that come with production that involves intensive physical capital usage. Examples of those longevity risks include chronic health damages due to exposure to occupational

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carcinogens and work-related fatal injuries [World Health Organization (2009)]. The fatal injury rates of fishers, loggers, pilots, farmers, and truck drivers are well above the national average in the United States [Pegula and Janocha (2013)]. Second, the adverse effects on longevity associated with increased economic activities can be proxied by more physical capital [e.g., Blackburn and Cipriani (1998) and Osang and Sarkar (2008)]. See Goenka and Liu (2020) for a fine summary of those adverse health effects of more physical capital and increased economic activities. The above two arguments indicate that physical capital can counterbalance the positive effects of human capital on longevity. Thus, we introduce the ratio of physical-to-human capital k_t as another determinant of longevity to capture the idea that human capital alleviates the negative health effects of physical capital and serves as the key driving force that improves longevity.[10] Based on the above considerations, the longevity of the representative agent born at the beginning of period $t-1$ is $\phi_{t+1} = \phi(k_t, z_t) \in (\underline{\phi}, \overline{\phi})$, where $\underline{\phi} \geq 0$ and $\overline{\phi} \leq 1$ are the lower and upper bounds, and the longevity function satisfies $\frac{\partial \phi(k_t,z_t)}{\partial k_t} < 0$ and $\frac{\partial \phi(k_t,z_t)}{\partial z_t} < 0$.

Education expenditures are necessary for human capital accumulation in childhood. Denote e_t as private education expenditures by the agent. Total education expenditures are $\mu m_t + e_t$, where $\mu > 0$ measures the relative strength of public-to-private education expenditures [Buiter and Kletzer (1995) and Osang and Sarkar (2008)]. One can think of total education expenditures as the sum of what the government pays for compulsory education, plus what the parents choose to pay in the form of college tuition for her child. However, pollution impedes learning because a worsened environmental quality reduces schooling time due to absenteeism, undermines cognitive ability, or damages a child's mental health and nervous system. Consequently, each dollar spent on education becomes less effective in the accumulation of human capital than it would be without pollution. We follow Raffin (2012) and introduce the effectiveness of education expenditures $\lambda_t = \lambda(z_t) \in (\underline{\lambda}, \overline{\lambda})$, where $\underline{\lambda} \geq 0$ and $\overline{\lambda} \leq 1$ are the lower and upper bounds, and $\lambda'(z_t) < 0$. The effectiveness of education expenditures as a function of pollution thus captures the second type of pollution health effect that hampers learning.

With $\lambda(z_t)$ adjusting total education expenditures, effective education expenditures thus become $\lambda(z_t)(\mu m_t + e_t)$. Besides education expenditures, the evolution of human capital also depends on parents' human capital (e.g., parental expectations and guidance).[11] Both effective education expenditures and parents' human capital are subject to constant returns to scale in human capital formation. As the agent born at the beginning of period $t-1$ has human capital H_t in period t and gives birth to a child at the beginning of period t, the child born at the beginning of period t has human capital in period $t+1$ equal to:

$$H_{t+1} = B\left[\lambda_t\left(\mu m_t + e_t\right)\right]^{\beta} H_t^{1-\beta}, \tag{4}$$

where $B > 0$ is a scalar, $\beta \in (0, 1)$ is the share of effective education expenditures in the formation of human capital, and $1 - \beta \in (0, 1)$ is the share of parents' human capital.

Taking her longevity ϕ_{t+1} and human capital H_t as given, the agent born at the beginning of period $t-1$ makes decisions at the beginning of period t. The agent derives utility from her adulthood consumption c_t, elderhood consumption d_{t+1}, and her child's stock of human capital H_{t+1} due to altruism [Osang and Sarkar (2008)]. Because the agent cares about her elderhood consumption and the child's human capital, the agent has motives to save and invest in private education [de la Croix and Doepke (2003)].[12] We will show later that the agent's decision of whether to engage in private education expenditures can lead to two stable BGPs. Assuming a logarithmic function that is additively separable, the agent's lifetime utility is

$$U_{t-1} = \ln c_t + \phi_{t+1} \left(\ln d_{t+1} + \chi \ln H_{t+1} \right), \quad \textbf{(5)}$$

where the parameter $\chi > 0$ represents the agent's altruism toward her child's human capital.

During adulthood, the representative agent uses her wage income w_t to cover her adulthood consumption c_t, savings s_t, and private education expenditures for the child e_t. When the agent is old, she uses the remunerated savings $r_{t+1}s_t$ to finance her elderly consumption d_{t+1}. In adulthood and elderhood, the agent follows the same budget principle that equates her income to the total expenditures in that period.[13] The budget constraints for adulthood and elderhood are

$$w_t = c_t + s_t + e_t, \quad \textbf{(6a)}$$

$$\frac{r_{t+1}s_t}{\phi_{t+1}} = d_{t+1}. \quad \textbf{(6b)}$$

The representative agent takes her longevity as given and maximizes lifetime utility (5). The agent chooses c_t, s_t, e_t, and d_{t+1} subject to (4), (6a), and (6b), as well as an additional nonnegativity constraint $e_t \geq 0$.

The agent may or may not invest in private education based on the Kuhn–Tucker conditions. If the agent invests in private education, $e_t > 0$, the functions for adulthood consumption, savings, and private education expenditures are

$$c_t = \frac{\Phi_{t+1}}{\phi_{t+1}}(w_t + \mu m_t), \quad \textbf{(7a)}$$

$$s_t = \Phi_{t+1}(w_t + \mu m_t), \quad \textbf{(7b)}$$

$$e_t = \Omega_{t+1}(w_t + \mu m_t) - \mu m_t, \quad \textbf{(7c)}$$

where $\Phi_{t+1} = \Phi(\phi_{t+1}) = \frac{1}{1+\chi\beta+(1/\phi_{t+1})}$ is the agent's propensity to save because $\Phi_{t+1} = \frac{\partial s_t}{\partial w_t}$, and $\Omega_{t+1} = \Omega(\phi_{t+1}) = \frac{\chi\beta}{1+\chi\beta+(1/\phi_{t+1})}$ is the propensity to invest in private education because $\Omega_{t+1} = \frac{\partial e_t}{\partial w_t}$. The propensities satisfy $\Phi'(\phi_{t+1}) > 0$ and $\Omega'(\phi_{t+1}) > 0$.

However, if the following condition holds:

$$\frac{\chi\beta\phi_{t+1}}{\mu m_t} < \frac{1}{w_t - s_t}, \quad \textbf{(8)}$$

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the agent does not invest in private education, that is, $e_t = 0$. Condition (8) says that starting from no private education expenditures, if the marginal utility gained from the first dollar invested in private education is smaller than the utility lost due to the foregone young consumption, the agent does not invest in private education. Given the ratio of physical-to-human capital, the pollution stock plays a role in the agent's decision of whether to invest in private education because pollution reduces longevity, rendering the marginal utility gained from the first dollar invested in private education even smaller. If the representative agent does not invest in private education, human capital accumulation only depends on public education expenditures. The functions for adulthood consumption and savings become

$$c_t = \frac{\overline{\Phi}_{t+1}}{\phi_{t+1}} w_t, \tag{9a}$$

$$s_t = \overline{\Phi}_{t+1} w_t, \tag{9b}$$

where $\overline{\Phi}_{t+1} = \overline{\Phi}(\phi_{t+1}) = \frac{1}{1+(1/\phi_{t+1})}$ is the agent's propensity to save when $e_t = 0$ and the propensity satisfies $\overline{\Phi}'(\phi_{t+1}) > 0$. All else equal, if the agent lives longer, she increases savings and cuts back on adulthood consumption.

3.4. Equilibria

For ease of exposition, we assume no population growth and normalize the labor size to unity, that is, $L_t = 1$ for all t, and hereafter we do not specifically separate per-capita and aggregate variables. We also assume full depreciation of physical capital within one period. Savings in period t become physical capital in period $t + 1$, so we have $s_t = K_{t+1}$.[14]

The evolution of the economy varies due to the representative agent's decision of whether to invest in private education. Whether or not the agent invests in private education alters the accumulation of physical and human capital. We hereafter denote *PE* (Private Education) as the regime where private education expenditures are positive, $e_t > 0$, and *NPE* (No Private Education) as the regime where $e_t = 0$. We establish in the following proposition that the representative agent's decision to invest in private education is endogenously dictated by the stock of pollution z_t and capital ratio k_t.

PROPOSITION 1. *(The PE and NPE Regimes) In the case* $\frac{\underline{\phi}}{1+\underline{\phi}} < \frac{\mu\tau(1-\Delta)}{\chi\beta(1-\alpha)(1-\tau)} < \frac{\overline{\phi}}{1+\overline{\phi}}$, *the* (z_t, k_t) *space is divided by the downward-sloping boundary* $\overline{\Phi}(k_t, z_t) = \frac{\mu\tau(1-\Delta)}{\chi\beta(1-\alpha)(1-\tau)}$ *into the PE and NPE regimes.*

(1) The PE regime is defined by the combinations of z_t *and* k_t *satisfying* $\overline{\Phi}(k_t, z_t) > \frac{\mu\tau(1-\Delta)}{\chi\beta(1-\alpha)(1-\tau)}$. *The agent's investment in private education is positive,* $e_t > 0$.

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(2) The NPE regime is defined by the combinations of z_t and k_t satisfying $\overline{\Phi}(k_t, z_t) < \frac{\mu\tau(1-\Delta)}{\chi\beta(1-\alpha)(1-\tau)}$. The agent's investment in private education is zero, $e_t = 0$.

Proof. See Appendix B. ■

Proposition 1 says that the representative agent's decision of whether or not to invest in private education is endogenously determined by z_t and k_t. As shown in Figures 5, 6, and 7, the downward-sloping boundary divides the (z_t, k_t) space into two regions. The two regions correspond to the *PE* and *NPE* regimes and make intuitive sense. The *PE* regime lies to the bottom left of the (z_t, k_t) space and is characterized by lower z_t and k_t. The agent's longevity is high, and thus the increased return to human capital creates the incentive for the agent to invest in private education. In contrast, the *NPE* regime lies to the upper right of the (z_t, k_t) space and is characterized by higher z_t and k_t. The agent's longevity is low, and thus the return to human capital decreases to the level at which the agent finds it optimal not to invest in private education.

Next, we describe and compare the laws of motion for physical and human capital under the *PE* and *NPE* regimes. Using (2b) and (7b) under the *PE* regime, (9b) under the *NPE* regime, as well as (1b) and $s_t = K_{t+1}$, yields the difference equations for physical capital under the two regimes:

$$PE: \frac{K_{t+1}}{K_t} = A\left[(1-\tau)(1-\alpha) + \mu\tau(1-\Delta)\right]\Phi(k_t, z_t)k_t^{\alpha-1}, \quad \textbf{(10a)}$$

$$NPE: \frac{K_{t+1}}{K_t} = A(1-\tau)(1-\alpha)\overline{\Phi}(k_t, z_t)k_t^{\alpha-1}. \quad \textbf{(10b)}$$

Under the *PE* regime, given the capital ratio k_t, the term $\Phi(k_t, z_t)$ captures the negative impact of pollution on physical capital accumulation. The propensity to save increases in longevity, and longevity decreases in pollution, implying the propensity to save decreases in pollution, $\frac{\partial\Phi(k_t,z_t)}{\partial z_t} < 0$. Pollution thus reduces the agent's savings that transform into physical capital. This establishes the link describing how pollution damages physical capital by reducing longevity. Under the *NPE* regime, similarly, pollution lowers the agent's propensity to save as $\frac{\partial\overline{\Phi}(k_t,z_t)}{\partial z_t} < 0$, thus hampering the accumulation of physical capital.

Substituting (1b), (2b), and (7c) into (4) under the *PE* regime and substituting (2b) into (4) and setting $e_t = 0$ under the *NPE* regime yield the difference equations for human capital under the two regimes:

$$PE: \frac{H_{t+1}}{H_t} = BA^{\beta}\left[(1-\tau)(1-\alpha) + \mu\tau(1-\Delta)\right]^{\beta}\left[\Omega(k_t, z_t)\lambda(z_t)\right]^{\beta} k_t^{\alpha\beta}, \quad \textbf{(11a)}$$

$$NPE: \frac{H_{t+1}}{H_t} = BA^{\beta}\left[\mu\tau(1-\Delta)\right]^{\beta}\left[\lambda(z_t)\right]^{\beta} k_t^{\alpha\beta}. \quad \textbf{(11b)}$$

Under the *PE* regime, the terms $\Omega(k_t, z_t)$ and $\lambda(z_t)$ reveal that pollution harms human capital accumulation. Pollution reduces longevity, which in turn decreases the agent's propensity to invest in private education as $\frac{\partial\Omega(k_t,z_t)}{\partial z_t} < 0$. Pollution

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also decreases the effectiveness of education expenditures as $\lambda'(z_t) < 0$. Thus, pollution reduces both the private education expenditures and their effectiveness in the formation of human capital, which establishes the link describing how pollution damages human capital. Under the *NPE* regime, in contrast, the agent does not invest in private education, and thus there is no propensity to invest in private education in (11b). The stock of pollution reduces the accumulation of human capital by weakening the effectiveness of education expenditures, and education expenditures are funded by the government only.

Because $k_{t+1}/k_t = (K_{t+1}/K_t)/(H_{t+1}/H_t)$, from (10a) and (11a), and from (10b) and (11b), we derive the key components of our model, the difference equations for the ratio of physical-to-human capital under the *PE* and *NPE* regimes:

$$PE: \ k_{t+1} = \frac{A^{1-\beta}}{B}\left[(1-\tau)(1-\alpha)+\mu\tau(1-\Delta)\right]^{1-\beta}\frac{\Phi(k_t, z_t)}{\left[\Omega(k_t, z_t)\lambda(z_t)\right]^{\beta}}k_t^{\alpha-\alpha\beta}, \tag{12a}$$

$$NPE: \ k_{t+1} = \frac{A^{1-\beta}}{B}\frac{(1-\tau)(1-\alpha)}{\left[\mu\tau(1-\Delta)\right]^{\beta}}\frac{\overline{\Phi}(k_t, z_t)}{\lambda(z_t)^{\beta}}k_t^{\alpha-\alpha\beta}. \tag{12b}$$

Equations (12a) and (12b) state that the capital ratio in period $t+1$ depends on four terms. The first terms are the same under both regimes, with $A^{1-\beta}/B$ consisting of the scalars in the production function and the human capital formation function. The second terms capture the role of policy parameters τ and Δ in directly affecting capital. The third terms, other things equal, capture the effects of pollution on the capital ratio and highlight one of our primary departures from the literature. We focus on the whole fraction, while the literature examines either the numerator or the denominator. In $\Phi(k_t,z_t)/[\Omega(k_t,z_t)\lambda(z_t)]^{\beta}$ under the *PE* regime, the numerator shows that pollution reduces physical capital through the propensity to save $\Phi(k_t, z_t)$, such that the physical-to-human capital ratio declines in pollution. The denominator reveals that pollution decreases human capital through the propensity to invest in private education $\Omega(k_t, z_t)$ and through the effectiveness of education expenditures $\lambda(z_t)$, such that the physical-to-human capital ratio increases in pollution. In contrast, in $\overline{\Phi}(k_t,z_t)/\lambda(z_t)^{\beta}$ under the *NPE* regime, pollution damages physical capital by reducing the propensity to save $\overline{\Phi}(k_t, z_t)$ and undermines human capital by decreasing the effectiveness of education expenditures $\lambda(z_t)$. Also note that the third and fourth terms together show the persistence of the capital ratio over time.

From equations (12a) and (12b), we get the *kk* loci under the *PE* and *NPE* regimes:

$$kk \text{ locus under } PE: \frac{A^{1-\beta}}{B}\left[(1-\tau)(1-\alpha)+\mu\tau(1-\Delta)\right]^{1-\beta}\times \frac{\Phi(k_t, z_t)}{\left[\Omega(k_t, z_t)\lambda(z_t)\right]^{\beta}}k_t^{\alpha-\alpha\beta} = k_t, \tag{13a}$$

$$kk \text{ locus under } NPE: \frac{A^{1-\beta}}{B}\frac{(1-\tau)(1-\alpha)}{\left[\mu\tau(1-\Delta)\right]^{\beta}}\frac{\overline{\Phi}(k_t, z_t)}{\lambda(z_t)^{\beta}}k_t^{\alpha-\alpha\beta} = k_t. \tag{13b}$$

The kk loci define all the combinations of k_t and z_t where the ratio of physical-to-human capital is in steady state. The slope of the kk locus reflects how pollution affects the economy, but the relationship is complicated because of the various health impacts of pollution, which will be discussed in the next section.

Nonetheless, the evolution of the pollution stock takes the same form in the *PE* and *NPE* regimes. Substituting (2a) into (3) gives the difference equation for the pollution stock:

$$z_{t+1} = (1-\theta)z_t + \frac{\rho}{\Delta\tau A}k_t^{1-\alpha}. \tag{14}$$

From equation (14), we get the zz locus:

$$zz \text{ locus}: \ -\theta z_t + \frac{\rho}{\Delta\tau A}k_t^{1-\alpha} = 0. \tag{15}$$

The zz locus defines all the combinations of k_t and z_t where the stock of pollution is in steady state. The slope of the zz locus reflects how the economy affects the environment. On the zz locus, the stock of pollution increases in the ratio of physical-to-human capital because human capital is "clean" and physical capital is "dirty" in production, which implies that an economy with abundant physical capital relative to human capital tends to have a higher stock of pollution.

4. THE *PE* AND *NPE* REGIMES

4.1. Capital Accumulation Differential

To proceed in our analyses, we define the central expression of our model, the capital accumulation differential of pollution $\Psi_{t,i}$ ($i = PE, NPE$):

$$\Psi_{t,i} = \begin{cases} \underbrace{E_{\Phi_{t+1},z_t}}_{\text{physical capital effect}} - \underbrace{\beta(E_{\Omega_{t+1},z_t} + E_{\lambda_t,z_t})}_{\text{human capital effect}}, & i = PE \\ \underbrace{E_{\overline{\Phi}_{t+1},z_t}}_{\text{physical capital effect}} - \underbrace{\beta E_{\lambda_t,z_t}}_{\text{human capital effect}}. & i = NPE \end{cases} \tag{16}$$

The capital accumulation differential summarizes and compares the adverse effects of pollution on the accumulation of physical and human capital but differs under the two regimes. Under the *PE* regime, we call the elasticity $E_{\Phi_{t+1},z_t} = \frac{\partial\Phi(k_t,z_t)}{\partial z_t}\frac{z_t}{\Phi(k_t,z_t)} < 0$ the physical capital effect, which reflects how sensitive the propensity to save Φ_{t+1} is to changes in pollution. Because pollution reduces longevity, which in turn decreases the propensity to save, the accumulation of physical capital is lower. We call the term $\beta(E_{\Omega_{t+1},z_t} + E_{\lambda_t,z_t})$ the human capital effect, which is the summation of two parts and is adjusted by the share of effective education expenditures in human capital formation β. The elasticity $E_{\Omega_{t+1},z_t} = \frac{\partial\Omega(k_t,z_t)}{\partial z_t}\frac{z_t}{\Omega(k_t,z_t)} < 0$ reflects how sensitive the propensity to invest in private education Ω_{t+1} is to changes in pollution. Pollution reduces longevity, the propensity to invest in private education decreases, and the accumulation of human capital is lessened. Pollution also impedes cognitive learning. The elasticity $E_{\lambda_t,z_t} = \frac{\lambda'(z_t)}{\lambda(z_t)} z_t < 0$ reflects how sensitive the effectiveness of education expenditures is to changes in pollution. By subtracting the human capital effect from the physical capital effect, we get the capital accumulation differential caused by pollution under the *PE* regime. As the relative magnitudes of the physical and human capital effects are unknown, $\Psi_{t,PE}$ can vary in value and sign. In contrast, under the *NPE* regime when the agent does not invest in private education, we call $E_{\overline{\Phi}_{t+1},z_t} = \frac{\partial \overline{\Phi}(k_t,z_t)}{\partial z_t} \frac{z_t}{\overline{\Phi}(k_t,z_t)} < 0$ the physical capital effect. By reducing longevity, pollution decreases the propensity to save $\overline{\Phi}_{t+1}$ and thus savings, such that physical capital is reduced. We call $\beta E_{\lambda_t,z_t}$ the human capital effect. Because the agent does not invest in private education, the accumulation of human capital is supported by the government only. As a result, pollution damages the accumulation of human capital only by impeding cognitive learning, thus reducing the effectiveness of education expenditures.

As longevity also depends on the capital ratio k_t, we define the own effect of the capital ratio $\Lambda_{t,i}$ ($i = PE, NPE$) associated with the capital accumulation differential:

$$\Lambda_{t,i} = \begin{cases} E_{\Phi_{t+1},k_t} - \beta E_{\Omega_{t+1},k_t} < 0, & i = PE \\ E_{\overline{\Phi}_{t+1},k_t} < 0. & i = NPE. \end{cases} \tag{17}$$

Equation (17) states that the capital ratio affects itself in the next period through longevity and then through the propensities to save and to invest in private education. Under the *PE* regime, the elasticity of the propensity to save with respect to the capital ratio is $E_{\Phi_{t+1},k_t} = \frac{\partial \Phi(k_t,z_t)}{\partial k_t} \frac{k_t}{\Phi(k_t,z_t)} < 0$, which captures how the capital ratio affects physical capital. The elasticity of the propensity to invest in private education with respect to the capital ratio is $E_{\Omega_{t+1},k_t} = \frac{\partial \Omega(k_t,z_t)}{\partial k_t} \frac{k_t}{\Omega(k_t,z_t)} < 0$, which captures how the capital ratio affects human capital and also is adjusted by β. It can be verified that $\Lambda_{t,PE} < 0$ by noting $\Phi_{t+1} = \Phi(\phi_{t+1})$, $\Omega_{t+1} = \Omega(\phi_{t+1})$, and $\phi_{t+1} = \phi(k_t, z_t)$. Therefore, the capital ratio always decreases physical capital more than human capital, thus lowering the future capital ratio. Under the *NPE* regime, however, the own effect of the capital ratio only consists of one term, $E_{\overline{\Phi}_{t+1},k_t} = \frac{\partial \overline{\Phi}(k_t,z_t)}{\partial k_t} \frac{k_t}{\overline{\Phi}(k_t,z_t)} < 0$. Because the agent does not invest in private education, the capital ratio affects itself only by influencing the propensity to save $\overline{\Phi}(k_t, z_t)$, and the term describing how the capital ratio affects the propensity to invest in private education is absent. As a result, in $\Lambda_{t,NPE}$ the capital ratio affects physical capital but not human capital and thus tends to decrease the ratio. Finally, comparing (16) and (17), we see that through health, the capital ratio can be influenced by the pollution stock and last period's capital ratio.

For regime $i = PE, NPE$, conditional on $\Lambda_{t,i}$, the capital accumulation differential $\Psi_{t,i}$ reveals the asymmetry of pollution health effects on the accumulation of physical and human capital. This asymmetry leads the physical-to-human capital ratio k_t to increase or decrease in the pollution stock z_t on the *kk* locus. We summarize the relationship between the capital accumulation differential caused by pollution and the slope of the *kk* locus in the following proposition.

PROPOSITION 2. *(Slope of the kk Locus) Under regime i ($i = PE, NPE$), the capital accumulation differential $\Psi_{t,i}$ dictates the slope of the kk locus in the (z_t, k_t) space. The kk locus slopes up if $\Psi_{t,i} > 0$ and slopes down if $\Psi_{t,i} < 0$.*

Proof. See Appendix C. ■

Proposition 2 indicates that under the *PE* and *NPE* regimes, the slope of the *kk* locus depends on the capital accumulation differential $\Psi_{t,i}$. Note that both the physical and human capital effects caused by pollution are negative. If $\Psi_{t,i} > 0$, the human capital effect is larger than the physical capital effect, which in turn leads physical capital to become relatively abundant. Thus, the physical-to-human capital ratio rises in pollution and the *kk* locus slopes up in the (z_t, k_t) space. In contrast, if $\Psi_{t,i} < 0$, the physical capital effect is larger than the human capital effect, and human capital becomes relatively abundant. The physical-to-human capital ratio declines in pollution and the *kk* locus slopes down in the (z_t, k_t) space. It is also possible that the *kk* locus is hump-shaped. In a numerical example to be illustrated in Figure 6, the *kk* locus under the *PE* regime first slopes up and then down, indicating the capital accumulation differential may switch sign as pollution evolves.

4.2. Balanced Growth Path

We now turn our attention to the BGP. Along the BGP under regime i, where $i = PE, NPE$, the physical-to-human capital ratio k_t, the stock of pollution z_t, longevity $\phi(k_t, z_t)$, and the effectiveness of education expenditures $\lambda(z_t)$ remain constant. We denote the BGP capital ratio as $k_i^* = k_{t+1,i} = k_{t,i}$ and the BGP stock of pollution as $z_i^* = z_{t+1,i} = z_{t,i}$. As indicated by Proposition 1, (z_{PE}^*, k_{PE}^*) and (z_{NPE}^*, k_{NPE}^*) satisfy $\overline{\Phi}(k_{PE}^*, z_{PE}^*) > \frac{\mu\tau(1-\Delta)}{\chi\beta(1-\alpha)(1-\tau)}$ and $\overline{\Phi}(k_{NPE}^*, z_{NPE}^*) < \frac{\mu\tau(1-\Delta)}{\chi\beta(1-\alpha)(1-\tau)}$. Also along the BGP under regime i, where $i = PE, NPE$, physical capital K_t, human capital H_t, final output Y_t, pollution abatement expenditures a_t, public education expenditures m_t, young consumption c_t, and elderly consumption d_t grow at the same rate g_i^* for all t. We define the growth rate along the BGP as:

$$g_i^* = \ln\left(\frac{K_{t+1}}{K_t}\right)\bigg|_i = \ln\left(\frac{H_{t+1}}{H_t}\right)\bigg|_i = \ln\left(\frac{Y_{t+1}}{Y_t}\right)\bigg|_i = \ln\left(\frac{a_{t+1}}{a_t}\right)\bigg|_i$$
$$= \ln\left(\frac{m_{t+1}}{m_t}\right)\bigg|_i = \ln\left(\frac{c_{t+1}}{c_t}\right)\bigg|_i = \ln\left(\frac{d_{t+1}}{d_t}\right)\bigg|_i, \text{ where } i = PE, NPE. \quad \textbf{(18)}$$

Graphically, a BGP under the *PE* regime is represented by the intersection of the *kk* locus (13a) and the *zz* locus (15), and the BGP under the *NPE* regime is represented by the intersection of (13b) and (15). Both intersections are subject to Proposition 1. Mathematically, under the *PE* regime, the three key BGP variables, k_{PE}^*, z_{PE}^*, and g_{PE}^*, can be solved from (15), (10a), and (11a). Taking natural logs on both sides and applying the definition of the BGP growth rate (18) gives the following three equations:

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$$g^*_{PE} = \ln A + \ln\left[(1-\tau)(1-\alpha) + \mu\tau(1-\Delta)\right] + \ln \Phi(k^*_{PE}, z^*_{PE}) - (1-\alpha)\ln k^*_{PE}, \tag{19a}$$

$$g^*_{PE} = \ln BA^{\beta} + \beta \ln\left[(1-\tau)(1-\alpha) + \mu\tau(1-\Delta)\right] + \beta \ln\left[\Omega(k^*_{PE}, z^*_{PE})\lambda(z^*_{PE})\right] + \alpha\beta \ln k^*_{PE}, \tag{19b}$$

$$\ln\theta + \ln z^*_{PE} = \ln \tfrac{\rho}{\Delta\tau A} + (1-\alpha)\ln k^*_{PE}. \tag{19c}$$

Equations (19a)–(19c) together determine k^*_{PE}, z^*_{PE}, and g^*_{PE}. Similarly, under the *NPE* regime, by (10b), (11b), (15), and (18), we have the following three equations that together yield k^*_{NPE}, z^*_{NPE}, and g^*_{NPE}:

$$g^*_{NPE} = \ln A + \ln(1-\tau)(1-\alpha) + \ln \overline{\Phi}(k^*_{NPE}, z^*_{NPE}) - (1-\alpha)\ln k^*_{NPE}, \tag{20a}$$

$$g^*_{NPE} = \ln BA^{\beta} + \beta \ln \mu\tau(1-\Delta) + \beta \ln \lambda(z^*_{NPE}) + \alpha\beta \ln k^*_{NPE}, \tag{20b}$$

$$\ln\theta + \ln z^*_{NPE} = \ln \tfrac{\rho}{\Delta\tau A} + (1-\alpha)\ln k^*_{NPE}. \tag{20c}$$

4.3. Transition Dynamics

For regime $i = PE, NPE$, the capital accumulation differential (16) and the own effect of the capital ratio (17) evaluated on the BGP are denoted as Ψ^*_i and Λ^*_i. We establish in the following proposition that conditional on Λ^*_i, the transition dynamics surrounding the BGP under regime *i* change as Ψ^*_i increases.

PROPOSITION 3. *(Dynamic Properties around the BGP) Given a sufficiently small absolute value for Λ^*_i ($i = PE, NPE$), ranges for local transition dynamics arise. The ranges are characterized by multiple thresholds with signs varying from negative to positive. As Ψ^*_i increases in value and falls in different ranges, the local dynamic property goes through the following sequential pattern: outward cycles, dampened cycles, stability, saddle stability, and instability.*

Proof. See Appendix D. ■

The mathematical exposition behind Proposition 3 is relegated to Appendix D. To establish the transition dynamics, Proposition 3 disentangles two effects on the ratio of physical-to-human capital under regime *i* ($i = PE, NPE$).

The term Λ^*_i represents the own effect of the capital ratio through longevity. As Λ^*_i is always negative, the capital ratio adversely affects physical capital more than human capital. Thus, Λ^*_i tends to decrease the capital ratio if the capital ratio increases, and to increase the capital ratio if the capital ratio decreases. As the absolute value of Λ^*_i increases, the influence of the capital ratio on itself grows, and the capital ratio gradually switches from being a stabilizing force to being a destabilizing one. As a result, for a BGP to be reached requires the absolute value of Λ^*_i to be sufficiently small.

The term Ψ^*_i represents the effect of pollution on the capital ratio through health. Proposition 3 states that given Λ^*_i, the nature of the transition dynamics is

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determined by Ψ_i^*. The attainability of the BGP depends on the magnitude of Ψ_i^*. Failure of convergence toward the BGP (outward cycles and instability) arises from a sufficiently large absolute value for the capital accumulation differential Ψ_i^*. In contrast, convergence toward the BGP (dampened cycles, stability, and saddle stability) occurs with a sufficiently small absolute value for Ψ_i^*. In the following analyses, we assume that the condition for convergence is always satisfied, such that the BGP is attainable. It is equally important to understand how the BGP is reached. The transitional dynamics leading to the BGP may entail policy implications. Among the transitional dynamic patterns that may arise, dampened cycles of the economy and the environment are of policy interest. Because some generations enjoy both low pollution stock and high economic growth, while others suffer from both high pollution stock and low economic growth, cycles represent inequality between generations [Schumacher and Zou (2008, 2015)]. The emergence of cycles depends on the sign of Ψ_i^*. According to the mathematical exposition of Proposition 3 in Appendix D, for cycles to arise, Ψ_i^* must be negative. To understand the intuition behind BGP attainability and cycles due to Ψ_i^*, note that the dynamic interactions between pollution and the physical-to-human capital ratio are at work. Pollution influences the capital ratio due to the capital accumulation differential, and the capital ratio conversely affects pollution depending on the abundance of "dirty" physical capital relative to "clean" human capital. Further, the movement of the physical-to-human capital ratio is also determined by Λ_i^*. If the absolute value for Λ_i^* is sufficiently small, Λ_i^* functions as a force that pulls back the capital ratio if the capital ratio deviates from the BGP. The key intuition we would like to shed light on is that as Ψ_i^* moves from the left to the right of the ranges for transitional dynamics, Ψ_i^* becomes larger in value, and pollution has a relatively larger impact on human capital.

When pollution harms physical capital less than human capital, the capital accumulation differential becomes positive. Suppose initially pollution is above its BGP value. A positive capital accumulation differential tends to increase k_t. Whether k_t ultimately returns to the BGP depends on the size of the capital accumulation differential relative to the own effect of the capital ratio. If the capital accumulation differential is sufficiently large, k_t increases, which in turn generates higher pollution. The process then repeats, causing monotonic divergence away from the BGP. In contrast, if the capital accumulation differential is sufficiently small, the own effect of the capital ratio is strong enough to pull k_t back toward BGP. The value for k_t becomes smaller, which in turn generates lower pollution. Then, the positive capital accumulation differential initiates monotonic convergence toward the BGP.

But when pollution harms physical capital more than human capital, the capital accumulation differential becomes negative. A pollution stock that is initially larger than its BGP value, through the negative capital accumulation differential, generates a k_t below its BGP value. This lower k_t drives pollution below its BGP value, but pollution then increases k_t through the negative capital accumulation differential. The increased k_t is then followed by higher pollution. The back-and-forth movements of the capital ratio and pollution repeat and cycles emerge. The type of cycles depends on the relative size of the capital accumulation differential and the own effect of the capital ratio. If the absolute value of the capital accumulation differential is relatively large, outward cycles occur. Otherwise, dampened cycles are observed.

4.4. Comparative Statics

On the BGP for regime i ($i = PE, NPE$), we derive the relationships among the physical-to-human capital ratio k_i^*, the pollution stock z_i^*, and the economic growth rate g_i^* to provide a yardstick to align our theory with the empirical evidence. These relationships are written in the form of elasticities and summarized in the following proposition.

PROPOSITION 4. *(Pairwise Relationships Among the BGP Variables) On the BGP for regime i (i = PE, NPE),* $\frac{\partial z_i^*}{\partial g_i^*}\frac{g_i^*}{z_i^*} < 0$. *However, the relationship between* k_i^* *and* g_i^* *depends on the sign of the capital accumulation differential* Ψ_i^*. *If* $\Psi_i^* > 0$, $\frac{\partial k_i^*}{\partial g_i^*}\frac{g_i^*}{k_i^*} < 0$, *and thus* $\frac{\partial k_i^*}{\partial z_i^*}\frac{z_i^*}{k_i^*} > 0$. *In contrast, if* $\Psi_i^* < 0$, *we have* $\frac{\partial k_i^*}{\partial g_i^*}\frac{g_i^*}{k_i^*} > 0$, *and thus* $\frac{\partial k_i^*}{\partial z_i^*}\frac{z_i^*}{k_i^*} < 0$.

Proof. See Appendix E. ■

Proposition 4 states that the relationship between z_i^* and g_i^* is negative, which can be related to the empirical evidence based on the Chinese provincial data and international data shown in Figures 1, 2, and A.1. But as Ψ_i^* can be positive or negative, the relationship between k_i^* and g_i^*, and thus the relationship between k_i^* and z_i^* cannot be determined. The pairwise relationships among k_i^*, z_i^*, and g_i^* with scatter plots of Chinese provinces in Figure 1 match the case where $\Psi_i^* > 0$. Moreover, we find no economic and environmental cycles in Chinese provinces, which is consistent with Proposition 3 that cycles cannot emerge when $\Psi_i^* > 0$.

Next, we derive comparative statics to reveal the policy effects on k_i^*, z_i^*, and g_i^* while taking into account the transition dynamics around the BGP. The policy parameters in the model are the tax rate τ and the share of fiscal revenues devoted to pollution abatement Δ. The policy effects on the BGP are reported in the form of elasticities of the BGP fundamental variables with respect to τ and Δ under both regimes. The results are summarized in the following proposition.

PROPOSITION 5. *(The Policy Effects on the BGP Variables) Define the composite parameter:*

$$\Theta_i = \begin{cases} \dfrac{\beta(1-\alpha)(1-\tau)+\mu\tau(1-\Delta)}{(1-\alpha)(1-\tau)+\mu\tau(1-\Delta)} \in (\beta, 1), & i = PE \\ 0. & i = NPE \end{cases} \tag{21}$$

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Suppose the BGP under regime i ($i = PE, NPE$) features locally dampened cycles or local stability, the policy effects on z_i^*, k_i^*, and g_i^* are as follows.

(1) *The effect of τ on z_i^* is $\frac{dz_i^*}{d\tau}\frac{\tau}{z_i^*} < 0$. The effect of Δ on z_i^* is conditional on the regime. Under the PE regime, $\frac{dz_{PE}^*}{d\Delta}\frac{\Delta}{z_{PE}^*} < 0$. But under the NPE regime, if $\Delta \lesseqgtr \frac{1-\alpha+\alpha\beta-\Lambda_{NPE}^*}{1-\alpha+\beta-\Lambda_{NPE}^*}$, $\frac{dz_{NPE}^*}{d\Delta}\frac{\Delta}{z_{NPE}^*} \lesseqgtr 0$.*

(2) *The effects of τ and Δ on k_i^* depend on Ψ_i^*. If $\Psi_i^* \gtreqless -\beta - \frac{1}{1-\tau}(\tau - \Theta_i)$, $\frac{dk_i^*}{d\tau}\frac{\tau}{k_i^*} \lesseqgtr 0$. If $\Psi_i^* \gtreqless -\frac{\Delta}{1-\Delta}(\Theta_i - \beta)$, $\frac{dk_i^*}{d\Delta}\frac{\Delta}{k_i^*} \lesseqgtr 0$.*

(3) *The effects of τ and Δ on g_i^* depend on the relationship between z_i^* and g_i^*, the relationship between k_i^* and g_i^*, and the policy effects on k_i^* and on z_i^*.*

Proof. See Appendix F. ∎

Proposition 5 states that an increase in τ unambiguously reduces pollution in both regimes. The reason is that more tax revenues contribute to more pollution abatement that directly reduces pollution and contribute to more public education that indirectly reduces pollution via a lower capital ratio. The policy effect of Δ on pollution differs in the two regimes. Given the total taxes, a larger share of fiscal revenues devoted to pollution abatement implies a smaller share devoted to public education, which in turn entails two competing forces on pollution. More pollution abatement reduces pollution, but less public education tends to raise the physical-to-human capital ratio that induces more pollution. Under the *PE* regime, the agent's private education compensates the decrease in public education, pollution abatement always prevails in reducing pollution, and thus an increase in Δ unambiguously decreases pollution. Under the *NPE* regime, however, the private education is absent. When Δ is small, an increase in Δ decreases pollution because the effect of more pollution abatement surpasses that of a higher capital ratio. But when Δ becomes sufficiently large, an increase in Δ increases pollution because the effect of more pollution abatement is outweighed by the effect of a higher capital ratio due to a large decrease in public education.

Proposition 5 also highlights the importance of Ψ_i^*, the capital accumulation differential caused by pollution through health. The physical-to-human capital ratio k_i^* is endogenous and is ultimately determined by the exogenous policy parameters. Our theory states that Ψ_i^* is the key mechanism that influences k_i^*. Therefore, how the policy parameters affect the capital ratio must be through the capital accumulation differential.[15] As stated by the second point of Proposition 5, Ψ_i^* relative to the parameters within the model dictates the effects of τ and Δ on k_i^* in both regimes. Proposition 5 further states that the policy parameters can affect the economic growth rate g_i^* indirectly through the policy effects on k_i^* and z_i^*. Although Ψ_i^* does not play a role in deciding the policy effects on z_i^*, Ψ_i^* matters for the policy effects on k_i^*. Therefore, the capital accumulation differential Ψ_i^* also dictates how policies influence g_i^*. To conclude, the capital accumulation differential modifies the responses of the BGP variables to policy changes.

We thus demonstrate the importance of understanding both the absolute and relative health effects of pollution on physical and human capital accumulation in policy-making.

4.5. Multiple BGPs

As stated in Proposition 1, the *PE* and *NPE* regimes are separated by a downward-sloping boundary. From (13a) and (13b), the *kk* loci are different in the two regimes. The *zz* locus may simultaneously intersect the *kk* loci under both regimes, leading to an interesting case with multiple BGPs. To match our theory with the cluster analyses in the Introduction, we establish a necessary condition for the emergence of two stable BGPs in the following proposition.

PROPOSITION 6. *(A Necessary Condition for the Emergence of Two Stable BGPs) For two stable BGPs to arise, there must exist a BGP on which the slope of the kk locus is larger than that of the zz locus. Around this BGP, the transition dynamics can be saddle or unstable.*

Proof. See Appendix G. ■

Proposition 6 can be explained as follows. Evaluated on a stable BGP, the slope of the *kk* locus is smaller than that of the upward-sloping *zz* locus. After the *kk* locus intersects the *zz* locus and gives rise to the first stable BGP, the *kk* locus lies below the *zz* locus. Because the *kk* loci are everywhere continuous, before the *kk* locus intersects the *zz* locus from above to form the second stable BGP, the *kk* locus must intersect the *zz* locus from below. Thus, a BGP must exist on which the slope of the *kk* locus is larger than that of the *zz* locus. By the mathematical exposition of Proposition 3 in Appendix D, this BGP exhibits local saddle stability or instability. As we have established a necessary condition for the emergence of two stable BGPs, we numerically explore this possibility in Section 5.

Next, we investigate which of the multiple BGPs should be chosen by policymakers. To evaluate and compare the BGP welfare, we define the intergenerational welfare improvement as the difference in lifetime utility (5) between the agents born at the beginning of periods t and $t-1$. On the BGP, adulthood consumption c_t, elderhood consumption d_t, and human capital H_t grow at the same rate by (18), while the ratio of physical-to-human capital k_t and the stock of pollution z_t remain constant, such that longevity $\phi(k_t, z_t)$ also remains constant. Therefore, the intergenerational welfare improvement on the BGP is

$$W_i^* = U_{t,i}^* - U_{t-1,i}^* = \left[1 + (1+\chi)\phi\left(k_i^*, z_i^*\right)\right] g_i^*, \quad i = PE, NPE. \qquad \mathbf{(22)}$$

We summarize the guidelines in the following proposition.

PROPOSITION 7. *(Ranking of BGPs) When multiple BGPs emerge, a BGP that features a lower stock of pollution is preferred by policymakers because the associated economic growth rate and intergenerational welfare improvement are higher. Policymakers rank BGPs in the following two scenarios:*

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(1) For BGPs under the same regime, the BGP with a lower stock of pollution is preferred.

(2) For BGPs under different regimes, the BGP under the PE regime is preferred.

Proof. See Appendix H. ■

Proposition 7 ranks BGPs from an environmental and economic perspective. Pollution hampers the accumulation of physical and human capital through health, so a lower stock of pollution leads to a higher economic growth rate. The lower stock of pollution is also associated with a lower ratio of physical-to-human capital, so by (22) the intergenerational welfare improvement is higher. Proposition (7) also indicates that the stock of pollution is a guideline for policymakers to choose the desirable BGP when multiple BGPs emerge. If two candidate BGPs lie under the same regime, policymakers would pick the BGP with a lower stock of pollution. If two BGPs, respectively, fall into the *PE* and *NPE* regimes, the pollution stock associated with the BGP under the *PE* regime is lower than that under the *NPE* regime, and policymakers thus prefer the BGP under the *PE* regime. The policy implication is that the government should steer the economy to converge toward the BGP with a lower stock of pollution, which is discussed further in the next section.

5. NUMERICAL EXAMPLES

In this section, we provide three empirically plausible numerical examples to complement the analytical results in the previous sections. In the process, we choose explicit functional forms for the health effects of pollution and calibrate the parameters to match the real-world data.

5.1. Health Functions and Calibration

We specify the functions reflecting the health effects of pollution, $\phi(k_t, z_t)$ and $\lambda(z_t)$. Empirical evidence has documented nonlinearity of the pollution health effects [Chay and Greenstone (2003), Stieb et al. (2015), Chen et al. (2018a)], but there is a lack of empirical research on how pollution reduces longevity and damages the effectiveness of education expenditures. We adopt a flexible functional form that satisfies the basic properties and encompasses a wide range of possibilities for the shapes of $\phi(k_t, z_t)$ and $\lambda(z_t)$. The functional forms we utilize are $\phi(k_t, z_t) = \left(\overline{\phi}+\underline{\phi} k_t^{\gamma_{\phi k}} z_t^{\gamma_{\phi z}}\right)/\left(1+k_t^{\gamma_{\phi k}} z_t^{\gamma_{\phi z}}\right)$ and $\lambda(z_t) = \left(\overline{\lambda}+\underline{\lambda} z_t^{\gamma_{\lambda z}}\right)/\left(1+z_t^{\gamma_{\lambda z}}\right)$, where the curvature parameters for the pollution stock are $\gamma_{jz} > 0$ $(j = \phi, \lambda)$, and $\bar{j}$ and $\underline{j}$ $(j = \phi, \lambda)$ are the upper and lower bounds. For simplicity, we set $\bar{j} = 1$ and $\underline{j} = 0$. Figure 4 exhibits how longevity $\phi(k_t, z_t)$ and the effectiveness of education expenditures $\lambda(z_t)$ decrease in the stock of pollution z_t, and how the shape depends on the curvature parameter γ_{jz} $(j = \phi, \lambda)$ while holding capital ratio k_t and its curvature $\gamma_{\phi k}$ constant.

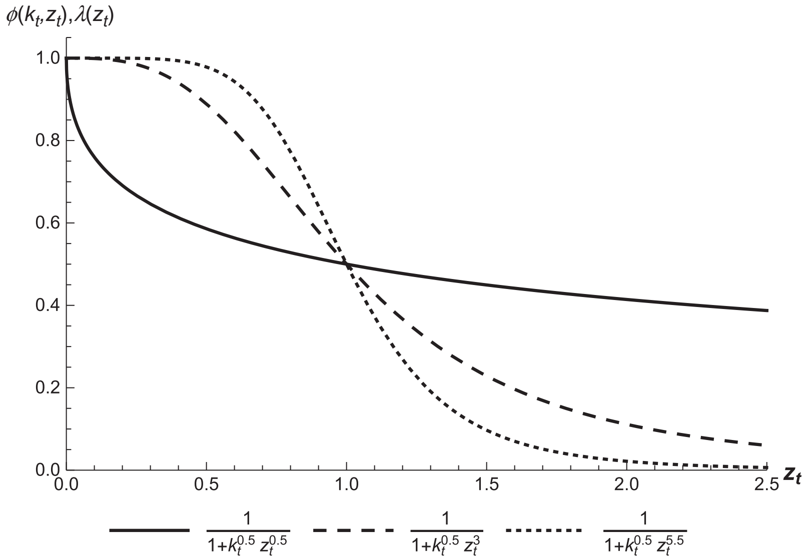

FIGURE 4. Functions simulating the health effects of pollution ($k_t = 1$).

To illustrate how our model responds to changes in the curvature parameters for the pollution stock in the longevity and education expenditures functions, we introduce the differential between the two curvature parameters for the pollution stock $\varepsilon = \gamma_{\lambda z} - \gamma_{\phi z}$. We fix the pollution stock curvature in the longevity function $\gamma_{\phi z} = 3$ and vary the value of ε, such that we can experiment with the pollution stock curvature for the effectiveness of education expenditures $\gamma_{\lambda z}$. Based on ε, we construct three illustrative numerical examples. In the benchmark example, $\varepsilon = 0$. In the second and third examples, $\varepsilon = -2.5$ and 2.5.

The other parameters in the model are calibrated as follows. The capital ratio curvature in longevity $\gamma_{\phi k}$ is set to 0.5 because by Proposition 3, this value ensures a sufficiently small absolute value for the own effect of the capital ratio on the BGP. In the production function, the share of physical capital α is set to 1/3, a value in line with empirical research [Mankiw et al. (1992), Gollin (2002)]. In the law of motion for the pollution stock (3), the dissipation rate of the pollution stock θ is set to 0.5. This value is consistent with Pautrel (2012) and can be supported by Economides and Philippopoulos (2008), where the annual regeneration rate of the environment is 0.015. This implies that environmental quality with a starting value of 1 will improve to $(1 + 0.015)^{30} = 1.56$ in one period (30 years), which is equivalent to a 56% improvement in environmental quality or a 56% reduction of the pollution stock. In the human capital formation function (4), we refer to the literature on the intergenerational transmission of human capital to calibrate the elasticity of children's human capital with respect to parents' human capital $1 - \beta$. Years of schooling is a commonly used indicator of human capital. Dearden et al. (1997) find that if fathers' schooling increases by 1 year, schooling

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years increase 0.443 for sons and 0.369 for daughters. Black et al. (2005) find that if a parent's education increases by 1 year, the child's schooling years increase by 0.2–0.25. As the human capital of children is usually higher than their parents' human capital, the value for $1-\beta$ tends to be lower than the above estimates. On average, we set $1-\beta=0.3$, and thus $\beta=0.7$. The relative strength of public-to-private education μ is set to 0.538 because in Osang and Sarkar (2008), the weights on public and private education expenditures in total education expenditures are 0.35 and 0.65, which translates into $\mu=0.538$ in our model. In the lifetime utility function (5), the agent's altruism toward her child's human capital χ is set to 0.65, a value consistent with Osang and Sarkar (2008). The policy parameters, the tax rate on final output τ, and the share of fiscal revenues devoted to pollution abatement Δ are calibrated together. The balanced budget implies that $\tau\Delta$ is the percentage of public pollution abatement expenditures in GDP and $\tau(1-\Delta)$ is the percentage of public education expenditures in GDP. The world average government expenditures on education as a percentage of GDP in 2015 is 4.81% [World Bank Group (2018a)]. Although we do not have world data on government expenditures on pollution abatement, we refer to Brock and Taylor (2010) and calculate the average percentage of pollution abatement expenditures in GDP for OECD countries, which is 1.33%. Solving $\tau\Delta=1.33\%$ and $\tau(1-\Delta)=4.81\%$ together yields $\tau=0.061$ and $\Delta=0.217$. These parameters satisfy the condition in Proposition 1, and thus a downward-sloping boundary arises and separates the (z_t, k_t) space into the *PE* and *NPE* regimes. Lastly, we adjust the scalar in production A, the scalar in human capital formation B, and the polluting capacity of physical capital ρ in accordance with the different values for ε in each numerical example, such that the calculated BGP values roughly match the world average growth rate of real GDP per capita (1.66%) and life expectancy at birth (71.86 years) in 2015. The benchmark parameters are summarized in Table 4.

5.2. Benchmark Example

In the benchmark example, $\varepsilon=0$ and all of the other parameter values are shown in Table 4. The longevity function is $\phi(k_t, z_t)=\frac{1}{1+k_t^{0.5}z_t^3}$ and the education expenditures effectiveness function is $\lambda(z_t)=\frac{1}{1+z_t^3}$. Figure 5 shows that there exists a boundary separating the *PE* and *NPE* regimes. The intersection of the zz locus with the kk locus under the *PE* regime determines the BGP. On the BGP, the calculated annual economic growth rate is 1.85% and longevity is 71.83 years, close to the world data in 2015. By Proposition 2, the capital accumulation differential is positive, pollution damages human capital more than physical capital, and the kk locus slopes up along the BGP. In this numerical example, the eigenvalues associated with the BGP are 0.91 and −0.37, indicating the BGP is locally stable.

One point is worth highlighting. Even if the two curvature parameters reflecting the health effects of pollution in $\phi(k_t, z_t)$ and $\lambda(z_t)$ are identical ($\gamma_{\phi z}=\gamma_{\lambda z}=3$ and

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TABLE 4. The benchmark parameters

Category	Description	Parameter	Value
Production	Production function scalar	A	7
	Physical capital's share in production	α	1/3
Environment	The dissipation rate of the pollution stock	θ	0.5
	The polluting capacity of physical capital	ρ	0.09
Longevity*	Lower bound of longevity	$\underline{\phi}$	0
	Upper bound of longevity	$\overline{\phi}$	1
	Curvature parameter in longevity for k_t	$\gamma_{\phi k}$	0.5
	Curvature parameter in longevity for z_t	$\gamma_{\phi z}$	3***
Education spending effectiveness**	Lower bound of the effectiveness	$\underline{\lambda}$	0
	Upper bound of the effectiveness	$\overline{\lambda}$	1
	Curvature parameter in the effectiveness function for z_t	$\gamma_{\lambda z}$	3***
Human capital	Scalar in the evolution of human capital	B	7
	Education expenditure's share in human capital	β	0.7
	The relative strength of public to private education	μ	0.538
Utility	The agent's altruism	χ	0.65
Government	Proportional tax on final output	τ	0.061
	Pollution abatement's share in fiscal revenues	Δ	0.217

Notes. *The longevity function is $\phi(k_t, z_t) = \frac{\overline{\phi}+\underline{\phi}k_t^{\gamma_{\phi k}}z_t^{\gamma_{\phi z}}}{1+k_t^{\gamma_{\phi k}}z_t^{\gamma_{\phi z}}}$.

**The education expenditures effectiveness function is $\lambda(z_t) = \frac{\overline{\lambda}+\underline{\lambda}z_t^{\gamma_{\lambda z}}}{1+z_t^{\gamma_{\lambda z}}}$.

***The differential between the two curvature parameters for z_t, ε, is equal to 0 in the benchmark because $\varepsilon = \gamma_{\lambda z} - \gamma_{\phi z} = 3 - 3 = 0$.

thus $\varepsilon = 0$), the capital accumulation differential imposed by pollution still exists, which can be revealed by the upward slope of the *kk* locus under the *PE* regime and the downward slope of the *kk* locus under the *NPE* regime. This example demonstrates that it is not the health effects of pollution that directly drive the capital accumulation differential. Rather, by impairing health, pollution modifies the decision to save and invest in education,[16] thus giving rise to an accumulation differential between physical and human capital. Therefore, it is important to understand both the health effects of pollution and people's responses to pollution health damages.

5.3. An Example with Cycles

In this example, $\varepsilon = -2.5$, $A = 7$, $B = 6$, $\rho = 0.1$, and the other parameters are the same as in Table 4. The longevity function is $\phi(k_t, z_t) = \frac{1}{1+k_t^{0.5}z_t^3}$ and the education expenditures effectiveness function is $\lambda(z_t) = \frac{1}{1+z_t^{0.5}}$. Recall that Proposition 2

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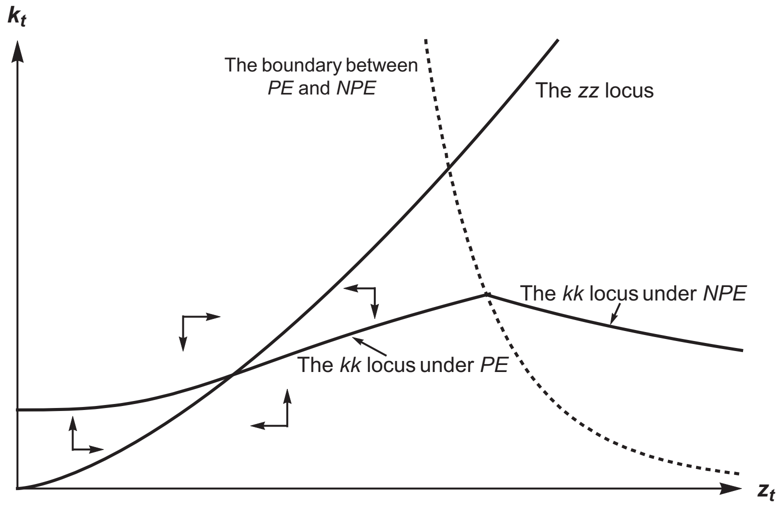

Note: In this and the following phase diagrams, we plot the stock of pollution z_t on the horizontal axis, and the physical-to-human capital ratio k_t on the vertical axis.

FIGURE 5. The benchmark phase diagram, $\varepsilon = 0$.

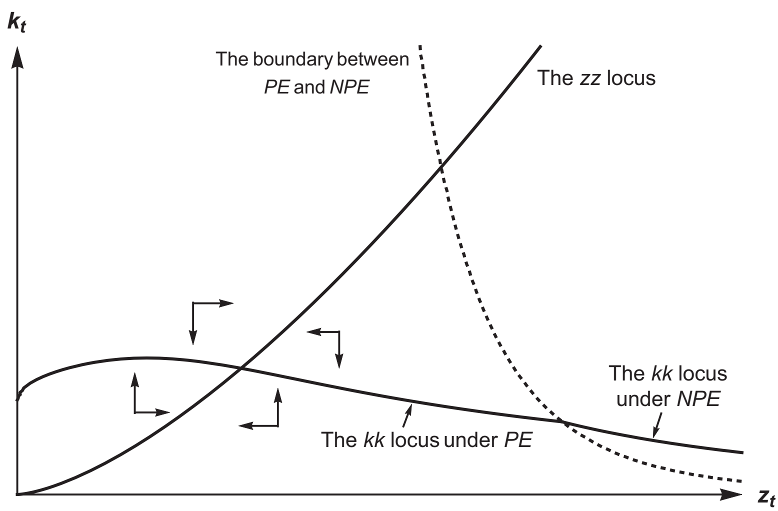

FIGURE 6. The phase diagram when cycles emerge, $\varepsilon = -2.5$.

establishes the connection between the slope of the *kk* locus and the sign of capital accumulation differential caused by pollution. In Figure 6, as the stock of pollution rises, the *kk* locus is hump-shaped because the capital accumulation differential switches sign from positive to negative. More specifically, the *kk* locus first rises for a lower pollution stock because the negative effect of pollution on

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human capital is larger than that on physical capital. The kk locus then falls for a higher pollution stock because the negative effect of pollution on physical capital becomes larger than that on human capital. The switch in sign for the capital accumulation differential can be explained by the relative shapes of pollution health effect functions. Given $k_t = 1$, for a lower pollution stock, $\lambda(z_t)$ is steeper than $\phi(k_t, z_t)$. As the pollution stock increases, however, $\phi(k_t, z_t)$ eventually becomes steeper than $\lambda(z_t)$. Thus, as the pollution stock rises, the propensity to save initially declines at a slow rate and physical capital is relatively abundant. But then the propensity to save declines faster and physical capital becomes relatively scarce.

Figure 6 shows the phase diagram. The zz locus intersects the kk locus when the kk locus slopes down, thus leading to a unique BGP that lies under the PE regime. On the BGP, the calculated annual economic growth rate is 2.08% and longevity is 71.66 years. Proposition 3 implies that cycles emerge only when the capital accumulation differential is negative, and by Proposition 2 the kk locus slopes down. Consistent with our analytical solution, the eigenvalues associated with the BGP are $0.27 \pm 0.05i$, indicating that the BGP features locally dampened cycles. We have presented the empirical relevance of these economic and environmental cycles in Figure 3. As mentioned earlier, the cycles represent inequality between generations, and government policy is required to eliminate the cycles.

Table 5 shows the effects of government interventions aimed at eliminating the cycles. The purpose of the exercises in Table 5 is threefold.[17] First, we compare two policy tools τ and Δ to evaluate the policy effects on eliminating cycles. Second, by comparing how different values for ρ affect the policy to eliminate cycles, we check how effective cleaner technology can be in eliminating cycles. Third, we compare the growth rates and intergenerational welfare improvements on the BGP before and after the policy change. The comparison allows us to answer the question whether the policy aimed at eliminating a short-run problem can also deliver long-run benefits.

The BGP in Table 5 lies under the PE regime. The first column shows different values of ρ to reflect the change in clean technology. The third column reports the BGP growth rate g^*_{PE} given in (18), intergenerational welfare improvement W^*_{PE} given in (22), and whether cycles emerge before the policy change. Cycles do not emerge when $\rho = 0.09$, but emerge when $\rho = 0.1$ and 0.11. The fourth column reports the minimum τ that is required to eliminate cycles while holding Δ constant at the benchmark level,[18] as well as the associated g^*_{PE} and W^*_{PE} after the policy change. Similarly, the fifth column reports the minimum Δ that is required to eliminate cycles. The reason why the government policy works to eliminate cycles also depends on the capital accumulation differential and on the dynamic interactions between k_t and z_t. As the government increases τ or Δ and shifts the capital accumulation differential to the right in the ranges of transition dynamics by Proposition 3, pollution increasingly harms human capital more than physical capital, and thus the physical-to-human capital ratio k_t begins to move in the same direction as the pollution stock z_t. In addition, as k_t measures the abundance of "dirty" physical capital relative to "clean" human capital, z_t moves in the same

TABLE 5. Policy effects on eliminating cycles

		(Benchmark) $\tau = 0.061, \Delta = 0.217$	–	–
$\rho = 0.09$	g^*_{PE}	0.7113	–	–
	W^*_{PE}	1.2482	–	–
	Cycles	No	–	–
		(Benchmark) $\tau = 0.061, \Delta = 0.217$	$\tau = 0.0619, \Delta = 0.217$	$\tau = 0.061, \Delta = 0.2202$
$\rho = 0.1$	g^*_{PE}	0.6198	0.6330	0.6331
	W^*_{PE}	1.0175	1.0488	1.0489
	Cycles	Yes	No	No
		(Benchmark) $\tau = 0.061, \Delta = 0.217$	$\tau = 0.0681, \Delta = 0.217$	$\tau = 0.061, \Delta = 0.2421$
$\rho = 0.11$	g^*_{PE}	0.5282	0.6318	0.6320
	W^*_{PE}	0.8180	1.0474	1.0470
	Cycles	Yes	No	No

Notes. (1) The BGP lies under the *PE* regime. In column 2, g^*_{PE} is the economic growth rate by (18) and W^*_{PE} is the intergenerational welfare improvement by (22). "Cycles" report whether economic and environmental cycles emerge. Column 3 reports the scenario of benchmark policy parameters (before the policy change). For example, for the polluting capacity of physical capital $\rho = 0.1$, and for the benchmark tax rate $\tau = 0.061$ and the benchmark fiscal share of pollution abatement $\Delta = 0.217$, the calculated BGP values are $g^*_{PE} = 0.6198$ and $W^*_{PE} = 1.0175$, and cycles emerge.
(2) Column 4 reports the scenario where the minimum τ is required to eliminate cycles while holding Δ constant. Column 5 reports the scenario where the minimum Δ is required to eliminate cycles while holding τ constant. For example, in Column 4 when $\rho = 0.1$ and holding Δ constant, $\tau = 0.0619$ is the minimum tax rate required to eliminate cycles, implying a $^{(0.0619-0.061)}/_{0.061} \times 100\% = 1.48\%$ increase in τ. The associated BGP values are $g^*_{PE} = 0.633$ and $W^*_{PE} = 1.0488$, and no cycles emerge.
(3) When $\rho = 0.09$, no cycles emerge for the benchmark policy parameters. Thus, no policy change is required to eliminate cycles and we fill the table cells in columns 4 and 5 with "–".

direction as k_t changes. Therefore, the two-way interactions of k_t and z_t eventually operate in the same direction, and cycles are eliminated.

We draw three conclusions from Table 5. First, the comparison of column 3 and 4 and the comparison of column 3 and 5 reveal that for a given ρ, approximately similar percentage increases in τ and Δ are required to eliminate cycles, and both policy changes deliver similar increase in the BGP growth rate and intergenerational welfare improvement relative to the benchmark. Second, when the production technology is clean enough ($\rho = 0.09$), the benchmark policy parameters do not give rise to cycles. So a clean technology can substitute for policy changes to eliminate cycles. Further, the comparison of different values for ρ reveals that an advance of cleaner technology, a lower ρ, generates a higher BGP growth rate and a larger intergenerational welfare improvement and allows for a smaller policy change to avoid cycles. Third, the comparison of BGP growth rates and welfare improvements before and after the policy change reveals that the policy change aimed at eliminating cycles, whether it is τ or Δ, can deliver both short-run and long-run benefits. The policy change not only gets rid of intergenerational welfare inequality in the short run but also boosts economic growth and welfare improvement in the long run. This finding echoes Raffin and Seegmuller (2017) and Constant and Davin (2019) on the compatibility of the policy effects in the short and long runs. Therefore, policies eliminating cycles are desirable based on the benchmark parameters.

5.4. An Example with Multiple BGPs

In this example, $\varepsilon = 2.5$, $A = 10$, $B = 6$, $\rho = 0.09$, and the other parameters are the same as listed in Table 4. The longevity function is $\phi(k_t, z_t) = \frac{1}{1+k_t^{0.5} z_t^3}$ and the education expenditures effectiveness function is $\lambda(z_t) = \frac{1}{1+z_t^{5.5}}$. In Figure 7 under the *PE* regime, the *kk* locus convexly increases because human capital is more severely damaged and the severity is increasingly intensified by higher pollution. The convex shape of the *kk* locus is justified by the shapes of the longevity function $\phi(k_t, z_t)$ and the education expenditure effectiveness function $\lambda(z_t)$ in Figure 4, in which $\lambda(z_t)$ declines faster than $\phi(k_t, z_t)$ in z_t. Under the *NPE* regime, the *kk* locus is flatter for two reasons. First, the adverse effect of pollution on physical capital accumulation is smaller under the *PE* regime than under the *NPE* regime because the decline in the propensity to save is smaller for the same increase in pollution. Second, the adverse effect of pollution on human capital accumulation is larger under the *PE* regime because pollution additionally reduces the propensity to invest in private education.[19] Thus, the shapes of the *kk* loci under the *PE* and *NPE* regimes give rise to the possibility of multiple BGPs. We can also see that the agent's decision of whether to invest in private education also plays a role in the emergence of multiple BGPs. Suppose the agent always invests in private education and only the *PE* regime exists, after the *kk* locus intersects with the *zz* locus the second time, the *kk* locus cannot become flatter or intersect with the *zz* locus again.

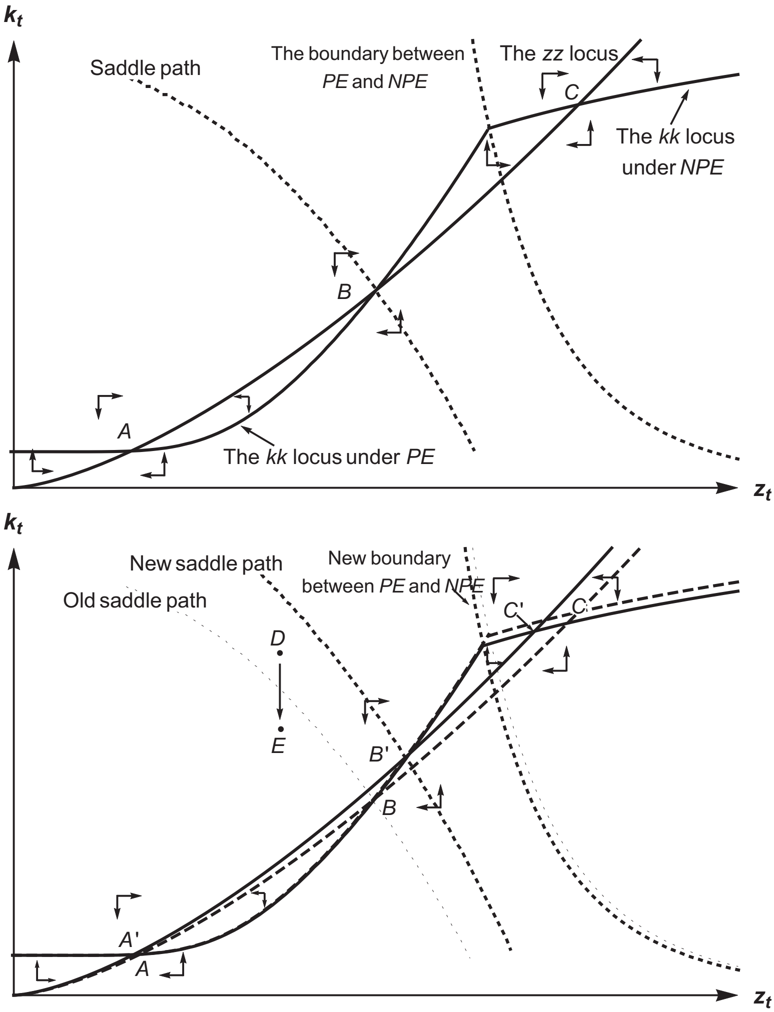

FIGURE 7. The emergence of multiple BGPs and the effects of government policies, $\varepsilon = 2.5$.

The upper panel of Figure 7 shows that under the *PE* regime, the *kk* locus intersects the *zz* locus twice at *A* and *B*. BGP *A* is locally stable with eigenvalues 0.73 and −0.14. BGP *B* exhibits locally saddle stability with eigenvalues 1.21 and −0.67. Under the *NPE* regime, the *kk* locus intersects the *zz* locus again at *C*. BGP *C* is also locally stable with eigenvalues 0.76 and −0.65. The emergence of two stable BGPs *A* and *C* is consistent with Proposition 6 that there must exist a BGP *B* on which the slope of the *kk* locus is larger than that of the *zz* locus. Due to the local dynamics, BGP *B* gives rise to a saddle path. This saddle path separates the

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first quadrant into two "sink" regions. The points to the lower left of the saddle path will converge to BGP A, whereas the points to the upper right of the saddle path will converge to BGP C. Among the three BGPs, BGP A features the highest economic growth rate and the lowest stock of pollution, whereas BGP C lies at the opposite extreme, consistent with Proposition 7. Because countries tend to operate either on BGP A or on BGP C, when calibrating the parameters in this example, we roughly match the calculated annual growth rate (1.99%) and longevity (71.49 years) on BGP B with the world average in 2015. The empirical relevance of this result is shown by Chinese provincial and international data. In the cluster analyses of Tables 1 and 2, group 1 corresponds to the desirable BGP A and group 2 corresponds to the inferior BGP C. Further, the Markov transition matrices of Table 3 illustrate the stability property of BGP A and C.

To policymakers, BGP A is desirable while BGP C should be avoided. But what if the economy lies to the right of the saddle path, such that the economy will eventually converge to the undesirable BGP C? Policymakers should steer the economy to the left of the saddle path and the economy will converge to the desirable BGP A. The lower panel of Figure 7 illustrates how the policy interventions work. The dashed lines represent the old loci when $\tau = 0.061$, and the solid lines represent the new loci when $\tau = 0.064$ (about 5% increase in the tax rate). As the BGP in the middle shifts from B to B', a new saddle path is generated. Point D initially lies to the right of the old saddle path and to the left of the boundary separating the *PE* and *NPE* regimes. If there were no government intervention, the local dynamics are dictated by (12a) and (14), and z_t will increase until it jumps over the boundary. As the agent finds it optimal not to invest in private education, the local dynamics are dictated by (12b) and (14) instead. Eventually, the economy converges to the undesirable BGP C. If the government raises the tax rate, however, point D lies to the left of the new saddle path and the economy will converge to the desirable BGP A'. As an alternative measure, the government can decrease the ratio of physical-to-human capital by encouraging agents to increase private education expenditures, decrease savings, or both. Point D will jump over the old saddle path and reach point E. The economy will converge to the desirable BGP A without the government modifying the tax rate τ.

6. CONCLUSIONS

Pollution reduces the accumulation of physical and human capital through negative health effects. Although the existing research has extensively studied the pollution health effects on physical capital or on human capital, little has been written on the simultaneous effects of pollution on both types of capital. Thus, the literature has all but ignored the consequences of pollution on the capital ratio, an important indicator for economic growth. By incorporating the health effects of pollution on both physical and human capital, we establish a link connecting the two strands of literature that focus either on physical capital or on human capital. Further, economic and environmental cycles are an empirical reality, and the literature has found mechanisms explaining the cycles. Cyclical movements also arise in our model. We, therefore, contribute to the literature on cycles by identifying the accumulation differential between physical and human capital caused by pollution as a new source of economic and environmental cycles.

Our analysis is based on a standard OLG model that depicts a decentralized economy. We investigate two types of pollution health effects. One is pollution reducing longevity and the other is pollution impeding children's cognitive learning. Longevity is directly built into the agent's lifetime utility, and the idea of pollution hampering cognitive learning is modeled as pollution reducing the effectiveness of education expenditures. The introduction of these two pollution health effects creates an accumulation differential between physical and human capital. Thus, pollution influences the ratio of physical-to-human capital in two possible scenarios. One scenario is that the capital accumulation differential is sufficiently large, pollution more negatively affects human capital, physical capital accumulates faster, and thus an increase in pollution raises the capital ratio. The other scenario is that the capital accumulation differential is sufficiently small, pollution more adversely affects physical capital, human capital accumulates faster, and thus an increase in pollution reduces the capital ratio. The capital ratio in turn affects pollution because physical capital generates pollution, while human capital does not. The above-described dynamic interactions between pollution and the capital ratio portray the basic operation of our model.

In characterizing the BGP and the associated transition dynamics, we show that the capital accumulation differential modifies the way that fundamental variables respond to policy changes on the BGP. When the capital accumulation differential is sufficiently large, the pollution stock and the capital ratio move in the same direction, pollution and the capital ratio reinforce each other, and the BGP is stable. A numerical example reveals the emergence of two stable BGPs separated by a boundary. One BGP is strictly preferred over the other because the superior one features low pollution and high economic growth. In contrast, when the capital accumulation differential is sufficiently small and becomes negative, pollution and the capital ratio move in opposite directions. As a result, cyclical movements in the economy and environment may arise and lead to inequality between generations. Empirical evidence based on the panel data from China and the world lends support in favor of the theoretical results. We have also discussed policy interventions that can move the economy toward the desirable BGP and eliminate the economic and environmental cycles.

As a final note, our theoretical results highlight the importance of understanding precisely how the accumulation of physical capital is negatively affected by pollution relative to that of human capital. However, there is a lack of empirical evidence documenting the relative pollution health effects, so we call for future research that estimates the relative health effects of specific pollutants.

SUPPLEMENTARY MATERIAL

To view supplementary material for this article, please visit https://doi.org/10.1017/S1365100520000668.

NOTES

1. Besides the two health effects of pollution we emphasize, the extant literature also highlights other health effects, such as increasing morbidity [Gutiérrez (2008) and Wang et al. (2015)] and decreasing labor supply [Hanna and Oliva (2015) and Jhy-hwa et al. (2015)]. We acknowledge that potentially interesting results may arise from the interactions of these pollution health effects but reserve this idea for future research.

2. We thank an anonymous referee for proposing this method.

3. PM2.5 and PM10 are particulate matter of less than 2.5 and 10 microns of diameter.

4. Compared with the Chinese data, the World Bank database is more comprehensive but lacks data on physical and human capital that can be used to calculate the capital ratio. The world panel data cover a wide range of countries, including both developing and developed ones. The data on PM2.5 cover 190 countries and regions in 11 years (1990, 1995, 2000, 2005, and 2010–2016). The data on PM10 cover 177 countries and regions in 22 years (1990–2011). The data on population and real GDP per capita go with those on PM2.5 and PM10 air pollutants. However, in each year, some countries' data are missing, so we delete those countries when we present cross-sectional evidence for that year.

5. Due to space limitations, we do not show all of the years with data available. For PM2.5, negatively sloped trend lines appear in 9 years, accounting for 81.8% of the total years. For PM10, negatively sloped trend lines appear in 11 years, accounting for 50% of the total years.

6. Note that a BGP describes the tendency of how a country will eventually operate. The empirical evidence we gather does not necessarily imply the countries are standing on their BGPs but only illustrates that the countries can operate in a way following the tendencies characterized by their BGPs.

7. By completely, we mean the agent does not care about the child's human capital, such that she never invests in private education for her child and thus the accumulation of human capital is only supported by public education expenditures. By completely, we also mean both physical and human capital fully depreciate within one period, such that children cannot directly inherit either type of capital from their parents. Deriving utility from children and leaving heritage for children are two key characteristics of altruism illustrated by Chakraborty and Das (2019). As we assume the two components away, altruism can no longer interfere with our results. Our primary results qualitatively survive the alternative model and the capital accumulation differential of pollution still functions as the key driving force behind the BGP variable relations and transition dynamics. Not surprisingly, the emergence of two stable BGPs under different regimes is gone from the simplified model because the representative agent never invests in private education for her child due to complete egoism and thus only one regime exists. At last, because parents favor heritage in the form of physical over human capital, altruism in Chakraborty and Das (2019) drives the ratio of physical-to-human capital to monotonically increase in pollution when pollution is introduced to cause premature death. In contrast, the capital accumulation differential we study allows the capital ratio to flexibly change in pollution, thus giving rise to rich and interesting results in terms of the BGP and transition dynamics.

8. There are two advantages associated with this specification. First, Gradus and Smulders (1993) show that even when investment activities (e.g., the use of cleaner fuels, which allows for a reduction in the amount of pollution per unit of capital in the production process) and abatement activities (e.g., "end-of-pipe measures," which aim at cleaning up existing pollution) are distinguished, this function for net emissions still qualitatively holds. So although we use the term "abatement," we cover both cases of reducing the flow of pollution and the existing stock of pollution. Second, Pautrel (2012) argues that the linear specification of the net emissions (e.g., $\rho k_t - a_t$) is "not constant along the BGP, and therefore the stock of pollution explodes in the long run."

9. In an early version of our paper circulated online, we consider a longevity function that is dependent on the stock of pollution only. The introduction of effective human capital as another determinant of longevity makes our model more in line with the real world. We thank an anonymous referee for pointing this out.

10. A careful reader may argue that the modeling specification ignores the positive effects of more physical capital on improving longevity, such as directly through the building of public sanitation and medical facilities, and indirectly through the building of schools. Suppose a fraction ξ of physical capital K_t increases longevity, while the other fraction $1-\xi$ proxies the negative effects of increased economic activities on longevity. Longevity is increased by the combination of ξK_t and human capital H_t (e.g., hospitals and doctors, universities and professors), and this combination is represented by a function of physical and human capital $n(\xi K_t, H_t)$ that is homogeneous of degree 1. The introduction of ${}^{(1-\xi)K_t}/_{n(\xi K_t,H_t)}$ as an alternative determinant of longevity thus captures both the positive and negative effects of physical capital on longevity. But after manipulating the alternative determinant, we get ${}^{(1-\xi)k_t}/_{n(\xi k_t,1)}$, where $k_t = {}^{K_t}/_{H_t}$, and find that k_t still determines longevity in a qualitatively similar way. Besides, the introduction of the capital ratio in the longevity function is consistent with the literature [Blackburn and Cipriani (1998), Osang and Sarkar (2008), and Goenka and Liu (2020)]. At last, this modeling specification also benefits the definition of the BGP, on which the capital ratio remains the same, but both physical and human capital grow at the same rate.

11. This specification implies that human capital does not fully depreciate and parents' human capital as a form of heritage can be directly left for their children. In the alternative model presented in Appendix J, we drop this assumption that parents' human capital matters for the formation of children's human capital and check the robustness of our model. We thank an anonymous referee for pointing this out.

12. In the alternative model presented in Appendix J, we also drop altruism to check the robustness of our results. As the agent has no incentive to invest in private education for her child, education expenditures are publicly supported only. We thank an associate editor of this journal and an anonymous referee for pointing this out.

13. Longevity ϕ_{t+1} can be interpreted in two equivalent ways, but the elderhood budget constraints are the same. First, ϕ_{t+1} can be interpreted as the living time length in elderhood, so the entire lifetime of the representative agent is $2+\phi_{t+1}$. We employ the first interpretation. Second, ϕ_{t+1} can be interpreted as the survival probability in elderhood, so the life expectancy of the representative agent is still $2+\phi_{t+1}$. In the second interpretation, an assumed mutual fund is used. The mutual fund operates in a perfectly competitive annuities market, receives savings from the agent paying return $\hat{r}_{t+1}$, and invests the savings in physical capital with return r_{t+1}. Perfect competition in the annuities market implies $\hat{r}_{t+1} = {}^{r_{t+1}}/_{\phi_{t+1}}$. Similar details can be found in Chakraborty (2004, p. 122). The choice of which interpretation to use for longevity ϕ_{t+1} does not alter the elderhood budget constraint (6b).

14. Compared with equation (4), a careful reader may notice that human capital does not fully depreciate within one period and might wonder if the difference in the depreciation rates of physical and human capital is driving our results. We address this issue in Appendix J and show that when physical and human capital depreciate at the same rate, our primary result carries over. We thank an anonymous referee for pointing this out.

15. We thank the comment of an anonymous referee for helping the organization of our thinking.

16. Recall from (7b) and (7c) that the propensities to save and invest in private education under the *PE* regime, $\Phi(\phi_{t+1})$ and $\Omega(\phi_{t+1})$, depend on how pollution reduces longevity, $\phi_{t+1} = \phi(k_t, z_t)$. Also recall from (9b), the propensity to save under the *NPE* regime, $\overline{\Phi}(\phi_{t+1})$, also relies on $\phi_{t+1} = \phi(k_t, z_t)$. In addition, equation (8) says that $\phi_{t+1} = \phi(k_t, z_t)$ is decisive in ascertaining whether or not the agent invests in private education, which in turn distinguishes the *PE* and *NPE* regimes.

17. We thank an associate editor and an anonymous referee for proposing this exercise.

18. We get the minimum values through grid searches.

19. The difference in the slopes of the *kk* loci when the regime switches is not due to the functional forms or parameter values we choose. It is a mathematical fact that evaluated on the boundary, when the *kk* locus under the *PE* regime slopes up, the slope of the *kk* locus under the *PE* regime is larger than that under the *NPE* regime. The proof is relegated to Appendix I.

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REFERENCES

Aloi, M. and F. Tournemaine (2013) Inequality, growth, and environmental quality trade-offs in a model with human capital accumulation. *Canadian Journal of Economics/Revue Canadienne d'économique* 46(3), 1123–1155.

Barro, R. J. (1990) Government spending in a simple model of endogenous growth. *Journal of Political Economy* 98(5), S103–S125.

Barro, R. J. (2001) Human capital and growth. *American Economic Review* 91(2), 12–17.

Black, S. E., P. J. Devereux and K. G. Salvanes (2005) Why the apple doesn't fall far: Understanding intergenerational transmission of human capital. *American Economic Review* 95(1), 437–449.

Blackburn, K. and G. P. Cipriani (1998) Endogenous fertility, mortality and growth. *Journal of Population Economics* 11(4), 517–534.

Bloom, D. E., D. Canning and B. Graham (2003) Longevity and life-cycle savings. *Scandinavian Journal of Economics* 105(3), 319–338.

Brock, W. and M. S. Taylor (2010) The green Solow model. *Journal of Economic Growth* 15(2), 127–153.

Buiter, W. H. and K. M. Kletzer (1995) Capital mobility, fiscal policy, and growth under self-financing of human capital formation. *Canadian Journal of Economics/Revue Canadienne d'Economique* 28, S163–S194.

Chakraborty, S. (2004) Endogenous lifetime and economic growth. *Journal of Economic Theory* 116(1), 119–137.

Chakraborty, S. and M. Das (2019) Death and capital. *Macroeconomic Dynamics,* 1–23. doi: 10.1017/S1365100519000749.

Chay, K. Y. and M. Greenstone (2003) The impact of air pollution on infant mortality: Evidence from geographic variation in pollution shocks induced by a recession. *The Quarterly Journal of Economics* 118(3), 1121–1167.

Chen, S., C. Guo and X. Huang (2018a) Air pollution, student health, and school absences: Evidence from China. *Journal of Environmental Economics and Management* 92, 465–497.

Chen, S., P. Oliva and P. Zhang (2018b) Air pollution and mental health: Evidence from China. Retrieved from SSRN: https://ssrn.com/abstract=3178649 or http://dx.doi.org/10.2139/ssrn.3178649.

China Center for Human Capital and Labor Market Research. (2018) China human capital measurement and human capital index project. Retrieved from http://humancapital.cufe.edu.cn/en/Human_Capital_Index_Project.htm.

Constant, K. and M. Davin (2019) Environmental policy and growth when environmental awareness is endogenous. *Macroeconomic Dynamics* 23(3), 1102–1136.

Currie, J., E. A. Hanushek, E. M. Kahn, M. Neidell and S. G. Rivkin (2009) Does pollution increase school absences? *Review of Economics and Statistics* 91(4), 682–694.

Cutler, D., A. Deaton and A. Lleras-Muney (2006) The determinants of mortality. *Journal of Economic Perspectives* 20(3), 97–120.

de la Croix, D. and M. Doepke (2003) Inequality and growth: Why differential fertility matters. *American Economic Review* 93(4), 1091–1113.

De Nardi, M., E. French and J. B. Jones (2009) Life expectancy and old age savings. *American Economic Review* 99(2), 110–115.

Dearden, L., S. Machin and H. Reed (1997) Intergenerational mobility in Britain.*Economic Journal* 107(440), 47–66.

Devarajan, S., V. Swaroop and H. Zou (1996) The composition of public expenditure and economic growth. *Journal of Monetary Economics* 37(2), 313–344.

Duczynski, P. (2002) Adjustment costs in a two-capital growth model. *Journal of Economic Dynamics and Control* 26(5), 837–850.

Ebenstein, A., M. Fan, M. Greenstone, G. He, P. Yin and M. Zhou (2015) Growth, pollution, and life expectancy: China from 1991-2012. *American Economic Review* 105(5), 226–231.

Ebenstein, A., V. Lavy and S. Roth (2016) The long-run economic consequences of high-stakes examinations: Evidence from transitory variation in pollution. *American Economic Journal: Applied Economics* 8(4), 36–65.

Economides, G. and A. Philippopoulos (2008) Growth enhancing policy is the means to sustain the environment. *Review of Economic Dynamics* 11(1), 207–219.

Fodha, M. and T. Seegmuller (2014) Environmental quality, public debt and economic development. *Environmental and Resource Economics* 57(4), 487–504.

Goenka, A., S. Jafarey and W. Pouliot (2020) Pollution, mortality and time consistent abatement taxes. *Journal of Mathematical Economics* 88, 1–15.

Goenka, A. and L. Liu (2020) Infectious diseases, human capital and economic growth. *Economic Theory* 70, 1–47.

Gollin, D. (2002) Getting income shares right. *Journal of Political Economy* 110(2), 458–474.

Gradus, R. and S. Smulders (1993) The trade-off between environmental care and long-term growth–Pollution in three prototype growth models. *Journal of Economics* 58(1), 25–51.

Gutiérrez, M. J. (2008) Dynamic inefficiency in an overlapping generation economy with pollution and health costs. *Journal of Public Economic Theory* 10(4), 563–594.

Hanna, R. and P. Oliva (2015) The effect of pollution on labor supply: Evidence from a natural experiment in Mexico City. *Journal of Public Economics* 122, 68–79.

Hoque, M. M., E. M. King, C. E. Montenegro and P. F. Orazem (2019) Revisiting the relationship between longevity and lifetime education: Global evidence from 919 surveys. *Journal of Population Economics* 32(2), 551–589.

Hwang, Y. H., C. K. Hsiao and P. W. Lin (2019) Globally temporal transitions of blood lead levels of preschool children across countries of different categories of Human Development Index. *Science of the Total Environment* 659(17), 1395–1402.

Jhy-hwa, C., S. Jhy-yuan and C. Juin-jen (2015) Environmental policy and economic growth: The macroeconomic implications of the health effect. *The B.E. Journal of Macroeconomics* 15, 223.

John, A. and R. Pecchenino (1994) An overlapping generations model of growth and the environment. *Economic Journal* 104(427), 1393–1410.

Jouvet, P.-A., P. Pestieau and G. Ponthiere (2010) Longevity and environmental quality in an OLG model. *Journal of Economics* 100(3), 191–216.

Kim, Y., J. Manley and V. Radoias (2017) Air pollution and long-term mental health. Retrieved from SSRN: https://ssrn.com/abstract=3028930 or http://dx.doi.org/10.2139/ssrn.3028930.

Ladrón-de-Guevara, A., S. Ortigueira and M. S. Santos (1997) Equilibrium dynamics in two-sector models of endogenous growth. *Journal of Economic Dynamics and Control* 21(1), 115–143.

Landrigan, P. J., R. Fuller, N. J. R. Acosta, O. Adeyi, R. Arnold, N. Basu, A. B. Baldé, R. Bertollini, S. Bose-O'Reilly, J. I. Boufford, P. N. Breysse, T. Chiles, C. Mahidol, A. M. Coll-Seck, M. L. Cropper, J. Fobil, V. Fuster, M. Greenstone, A. Haines, D. Hanrahan, D. Hunter, M. Khare, A. Krupnick, B. Lanphear, B. Lohani, K. Martin, K. V. Mathiasen, M. A. McTeer, C. J. L. Murray, J. D. Ndahimananjara, F. Perera, J. Potočnik, A. S. Preker, J. Ramesh, J. Rockström, C. Salinas, L. D. Samson, K. Sandilya, P. D. Sly, K. R. Smith, A. Steiner, R. B. Stewart, W. A. Suk, O. C. P. van Schayck, G. N. Yadama, K. Yumkella and M. Zhong (2018) The lancet commission on pollution and health. *The Lancet* 391(10119), 462–512.

Li, M., J. Cao, Z. Gao, X. Shen and C. Yan (2015) The trend of lead poisoning rate in Chinese population aged 0-18 years old: A meta-analysis. *BMC Public Health* 15(1), 756.

Maher, B. A., I. A. M. Ahmed, V. Karloukovski, D. A. MacLaren, P. G. Foulds, D. Allsop, D. M. A. Mann, R. Torres-Jardón and L. Calderon-Garciduenas (2016) Magnetite pollution nanoparticles in the human brain. *Proceedings of the National Academy of Sciences* 113(39), 10797–10801.

Mankiw, N. G., D. Romer and D. N. Weil (1992) A contribution to the empirics of economic growth. *The Quarterly Journal of Economics* 107(2), 407–437.

Mayer, H. (1999) Air pollution in cities. *Atmospheric Environment* 33(24–25), 4029–4037.

Mohai, P., B.-S. Kweon, S. Lee and K. Ard (2011) Air pollution around schools is linked to poorer student health and academic performance. *Health Affairs* 30(5), 852–862.

Motoyama, T. (2016) From physical to human capital accumulation with pollution. Retrieved from http://www2.econ.osaka-u.ac.jp/library/global/dp/1603.pdf.

Mulligan, C. B. and X. Sala-i-Martin (1993) Transitional dynamics in two-sector models of endogenous growth. *The Quarterly Journal of Economics* 108(3), 739–773.

Nilsson, J. (2009) The long-term effects of early childhood lead exposure: Evidence from the phase-out of leaded gasoline. Retrieved from http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.212.3578.

Osang, T. and J. Sarkar (2008) Endogenous mortality, human capital and economic growth. *Journal of Macroeconomics* 30(4), 1423–1445.

Palivos, T. and D. Varvarigos (2017) Pollution abatement as a source of stabilzation and long-run growth. *Macroeconomic Dynamics* 21(3), 644–676.

Pautrel, X. (2008) Reconsidering the impact of the environment on long-run growth when pollution influences health and agents have a finite-lifetime. *Environmental and Resource Economics* 40(1), 37–52.

Pautrel, X. (2009) Pollution and life expectancy: How environmental policy can promote growth. *Ecological Economics* 68(4), 1040–1051.

Pautrel, X. (2012) Pollution, private investment in healthcare, and environmental policy. *Scandinavian Journal of Economics* 114(2), 334–357.

Payne, M. (2008) Lead in drinking water. *Canadian Medical Association Journal* 179(3), 253–254.

Pegula, S., and J. Janocha (2013) Death on the job: Fatal work injuries in 2011. Retrieved from https://digitalcommons.ilr.cornell.edu/cgi/viewcontent.cgi?article=2148&context=key_workplace.

Raffin, N. (2012) Children's environmental health, education, and economic development. *Canadian Journal of Economics/Revue Canadienne d'économique* 45(3), 996–1022.

Raffin, N. and T. Seegmuller (2014) Longevity, pollution and growth. *Mathematical Social Sciences* 69, 22–33.

Raffin, N. and T. Seegmuller (2017) The cost of pollution on longevity, welfare and economic stability. *Environmental and Resource Economics* 68(3), 683–704.

Rolden, H. J. A., D. van Bodegom, W. B. van den Hout and R. G. J. Westendorp (2014) Old age mortality and macroeconomic cycles. *Journal of Epidemiology and Community Health* 68(1), 44–50.

Sapci, O. and J. F. Shogren (2017) Environmental quality, human capital and growth. *Journal of Environmental Economics and Policy* 7(2), 184–203.

Schumacher, I. and B. Zou (2008) Pollution perception: A challenge for intergenerational equity. *Journal of Environmental Economics and Management* 55(3), 296–309.

Schumacher, I. and B. Zou (2015) Threshold preferences and the environment. *Journal of Mathematical Economics* 60, 17–27.

Seegmuller, T. and A. Verchère (2004) Pollution as a source of endogenous fluctuations and periodic welfare inequality in OLG economies. *Economics Letters* 84(3), 363–369.

Smulders, S. and R. Gradus (1996) Pollution abatement and long-term growth. *European Journal of Political Economy* 12(3), 505–532.

Stieb, D. M., S. Judek, K. Brand, R. T. Burnett and H. H. Shin (2015) Approximations for estimating change in life expectancy attributable to air pollution in relation to multiple causes of death using a cause modified life table. *Risk Analysis* 35(8), 1468–1478.

Tapia Granados, J. A. (2005) Increasing mortality during the expansions of the US economy, 1900-1996. *International Journal of Epidemiology* 34(6), 1194–1202.

Varvarigos, D. (2010) Environmental degradation, longevity, and the dynamics of economic development. *Environmental and Resource Economics* 46(1), 59–73.

Varvarigos, D. (2013a) Endogenous longevity and the joint dynamics of pollution and capital accumulation. *Environment and Development Economics* 19(04), 393–416.

Varvarigos, D. (2013b) Environmental dynamics and the links between growth, volatility and mortality. *Bulletin of Economic Research* 65(4), 314–331.

Vella, E., E. V. Dioikitopoulos and S. Kalyvitis (2015) Green spending reforms, growth, and welfare with endogenous subjective discounting. *Macroeconomic Dynamics* 19(6), 1240–1260.

Wang, M., J. Zhao and J. Bhattacharya (2015) Optimal health and environmental policies in a pollution-growth nexus. *Journal of Environmental Economics and Management* 71, 160–179.

Wen, J., Y. Luo and B. Liu (eds.) (2017) *China Statistical Yearbook on Environment (1991-2016)*. Beijing: China Statistics Press (in Chinese).

Wen, M. and D. Gu (2012) Air pollution shortens life expectancy and health expectancy for older adults: The case of China. *Journals of Gerontology - Series A Biological Sciences and Medical Sciences* 67(11), 1219–1229.

World Bank Group. (2018a) The world development indicators. Retrieved from https://datacatalog.worldbank.org/dataset/world-development-indicators.

World Bank Group. (2018b) The world development indicators database archives. Retrieved from https://databank.worldbank.org/data/reports.aspx?source=1277&series=EN.ATM.PM10.MC.M3.

World Health Organization. (2009) Global health risks: Mortality and burden of disease attributable to selected major risks. Retrieved from https://apps.who.int/iris/handle/10665/44203.

Zhang, J. (1999) Environmental sustainability, nonlinear dynamics and chaos. *Economic Theory* 14(2), 489–500.

Zhang, J., and J. Zhang, (2005) The effect of life expectancy on fertility, saving, schooling and economic growth: Theory and evidence. *Scandinavian Journal of Economics* 107(1), 45–66.

Zhang, X., X. Chen and X. Zhang (2018) The impact of exposure to air pollution on cognitive performance. *Proceedings of the National Academy of Sciences* 115(37), 9193–9197.

Zhang, X., X. Zhang and X. Chen (2017) Happiness in the air: How does a dirty sky affect mental health and subjective well-being? *Journal of Environmental Economics and Management* 85, 81–94.