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A NOTE ON $[a, b]$-COMPACT SPACES

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Introduction. In this note we present an array of results which deals with the question "When is the product of two $[a, b]$-compact spaces an $[a, b]$-compact space".

In section 1, we give some essential terminology. In section 2, we define some new classes of functions and then obtain some product theorems. In section 3, we give some applications of the product theorems obtained in section 2. For example the theorem 3.3 generalizes the results of Nóvak [17], Mrówka [15], Gál [5], Dieudonné [3], Dowker [4], Hayashi [9], and Smith and Krajewski [19]; and the theorem 3.4 generalizes the results of Banerjee [2], Nardzewski [16], and Hanai [7].

1. **Preliminaries.** Here the letters a, b and m stands for infinite cardinal numbers. ω_0 will denote the cardinal number of the integers and $|X|$ will denote the cardinal number of the set X. Furthermore, for any family U, $|U|$ will denote the cardinal number of the indexing set of U.

DEFINITION 1.1. A space X is called $[a, b]$-compact if every open cover U of X with $|U| \leq b$ has a subcover of cardinality $<a$. If X is $[a, b]$-compact for all $b \geq a$, then it is called $[a, \infty]$-compact.

DEFINITION 1.2 [12]. A family $\mathbf{A}$ of subsets of a space X is called a point-m (or locally-m) family if each point x of X (or a suitable neighbourhood of each point of X) meet $<m$ members of A.

A space X is called m-metacompact (m-paracompact) if each open cover of cardinality $\leq m$ has an open point-ω_0 (locally-ω_0) refinement.

DEFINITION 1.3 [19]. A space X is called m-expandable (almost m-expandable) if for every locally-ω_0 collection $\{F_s \mid s \in S\}$ where $|S| \leq m$, of subsets of X, there is an open locally-ω_0 (point-ω_0) collection $\{G_s \mid s \in S\}$ of X such that for each s in S, $F_s \subset G_s$.

DEFINITION 1.4. A function $f: X \to Y$ is called a perfect function if f is continuous, closed and $f^{-1}y$ is compact for each y in Y.

A function $f: X \to Y$ is called a compact function if for each compact subset K of Y, $f^{-1}K$ is compact.

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DEFINITION 1.5 [18]. A space X is called weakly-m-ω_0-compact if for every open cover $\mathbf{U}$ with $|\mathbf{U}|\le m$, there is a finite subfamily $\mathbf{V}$ of $\mathbf{U}$ such that $X=\mathrm{cl}(\bigcup\{V \mid V\in\mathbf{V}\})$.

2. **Strong and weak [a, b]-functions.** In this section we define the notions of strong $[a, b]$-functions, weak $[a, b]$-functions and $[\omega_0, m]$-pseudo functions. Also, we prove some product theorems.

DEFINITION 2.1. A function $f:X\to Y$ is called a strong $[a, b]$-function if for every open cover $\mathbf{U}=\{U_s \mid s\in S\}$ where $|S|\le b$, of X there is an open cover $\mathbf{V}=\{V_t \mid t\in T\}$, where $|T|\le b$, of Y such that $f^{-1}V\subset\bigcup\{U_s \mid s\in S_v\subset S\}$ where $|S_v|<a$, for each $V\in\mathbf{V}$.

DEFINITION 2.2. A function $f:X\to Y$ is called a weak $[a, b]$-function if for each open cover $\mathbf{U}=\{U_s \mid s\in S\}$ where $|S|\le b$, of X there is an open cover $\mathbf{V}$ of Y such that $f^{-1}V\subset\bigcup\{U_s \mid s\in S_v\}$ where $|S_v|<a$, for each V in $\mathbf{V}$. If f is a weak $[a, b]$-function for all $b\ge a$, then it is called an $[a,\infty]$-function.

A continuous strong (weak) $[a, b]$-function will be called strong (weak) $[a, b]$-perfect function, and a continuous $[a,\infty]$-function will be called $[a,\infty]$-perfect function.

DEFINITION 2.3. A function $f:X\to Y$ is called $[\omega_0, m]$-pseudo function if for every open cover $\mathbf{U}$ of X with $|\mathbf{U}|\le m$ there is an open cover $\mathbf{V}$ of Y with $|\mathbf{V}|\le m$ such that for each V in $\mathbf{V}$, $f^{-1}V\subset\mathrm{cl}(\bigcup_{i=1}^{k}U_i)$. A continuous $[\omega_0, m]$-pseudo function will be called $[\omega_0, m]$-psedo perfect function.

PROPOSITION 2.4. *Let a and b be infinite cardinals such that $\sum_{c<a}b^c=b$. Then $f:X\to Y$ is a strong $[a, b]$-function if and only if f is a weak $[a, b]$-function.*

Proof. If f is a strong $[a, b]$-function, then it is trivially a weak $[a, b]$-function. Conversely, let f be a weak $[a, b]$-function and let $\mathbf{U}=\{U_s \mid s\in S\}$ be an open cover of X where $|S|\le b$. Then there is an open cover $\mathbf{V}=\{V_t \mid t\in T\}$ of Y such that $f^{-1}V\subset\bigcup\{U_s \mid s\in S_v\subset S\}$ where $|S_v|<a$, for each V in $\mathbf{V}$. Now, for each subset H of S of cardinality $<a$ define $V_H=\bigcup\{V_t \mid f^{-1}V_t\subset\bigcup\{U_s \mid s\in H\}\}$. Since $\sum_{c<a}b^c=b$, it follows that $|\mathbf{W}|$, where $\mathbf{W}=\{V_H \mid H\subset S$ with $|H|<a\}$, has cardinality $\le b$ and each $f^{-1}V_H$ is in the union of $<a$ members of U. Hence f is a strong $[a, b]$-function.

Now we state some facts, most of them without proof.

2.5. Any function from an $[a, b]$-compact space onto an arbitrary space is a srong $[a, b]$-function.

2.6. Let $f:X\to Y$ be a closed function such that $f^{-1}y$ is $[a, b]$-compact for each y in Y. Then f is a weak $[a, b]$-function.

2.7. A strong (weak) $[a, b]$-function need not be a closed function.

2.8. Let $f:X \to Y$ be a strong (weak) $[a, b]$-function. Then for any closed subset C of X, $f|_C : C \to Y$ is a strong (weak) $[a, b]$-function.

2.9. Let $f:X \to Y$ be an $[m, \infty]$-function. Then for each locally-m family $\mathbf{A} = \{A_t \mid t \in T\}$ of subsets of X, $f\mathbf{A}$ is a locally-m family in Y.

2.10. Let $f:X \to Y$ be a strong $[a, b]$-function. Then X is $[a, b]$-compact if Y is $[a, b]$-compact.

2.11. Let $f:X \to Y$ be a weak $[a, b]$-function. Then X is $[a, b]$-compact if Y is $[a, \infty]$-compact.

2.12. Let $f:X \to Y$ be an open $[\omega_0, m]$-pseudo perfect function. Then X is a weakly-m-ω_0-compact if Y is a weakly-m-ω_0-compact.

2.13. Let a and b be infinite cardinals where $\sum_{c<a} b^c = b$. Then the following holds:

(i) In an $[a, b]$-compact space, every locally-a family has cardinality $<a$.

(ii) Every weak $[a, b]$-function maps every locally-a family of cardinality $\leq b$ into a locally-a family.

(1) The proof of 2.13(i): Let $\mathbf{A}$ be a locally-a family. If $|\mathbf{A}| < a$, then there is nothing to prove. Suppose $|\mathbf{A}| \geq a$. Now A has a subfamily B of cardinality a, i.e., $B = \{A_t \mid t \in S\}$ such that $|S| = a$. Define $P_a(S) = \{S_1 \subset S \mid |S_1| < a\}$. It is easy to see that $|P_a(S)| \leq \sum_{c<a} a^c \leq \sum_{c<a} b^i = b$. Since B is a locally-a family there is an open cover $\mathbf{W}$ of X such that each member of $\mathbf{W}$ intersects $<a$ members of $\mathbf{B}$. For each S_1 in $P_a(S)$, define $R(S_1) = \{w \in \mathbf{W} \mid w \subset \bigcup \{A_t \mid t \in S_1\} \cup \{(X - \bigcup \{A_t \mid t \in S\})\}$. Clearly $R = \{\boldsymbol{R}(S_1) \mid S_1 \in P_a(S)\}$ is an open cover of X such that $|R| \leq b$. We claim that no subcover of $\mathbf{R}$ has a cardinality $<a$. Let $\mathbf{U}$ be a subcover of $\mathbf{R}$ of cardinality $<a$. By the construction of members of R it is clear that U covers $<a$ members of $\mathbf{B}$ which contradicts the fact that U is a cover of X, i.e., X is not an $[a, b]$-compact space. Hence the result follows.

The proof of 2.13(ii): Let $\mathbf{A} = \{A_t \mid t \in T\}$ be a locally-a family of subsets of a space X such that $|T| \leq b$. Let f be a weak $[a, b]$-function from X onto a space Y. Define $P_a(T) = \{S \subset T \mid |S| < a\}$. Then $|P_a(T)| \leq \sum_{c<a} b^c = b$. Since $\mathbf{A}$ is a locally-a family there is an open cover W of X such that each member of W intersects $<a$ members of $\mathbf{A}$. Now define $R(S) = \bigcup \{w \in W \mid w \subset \bigcup \{A_t \mid t \in S \cup (X - \{A_t \mid t \in T\})$ for each S in $P_a(T)$. Evidently, $\mathbf{R} = \{R(S) \mid S \in P_a(T)\}$ is an open cover of X of cardinality $\leq b$. But f is a weak $[a, b]$-function implies that there is an open cover V of Y such that inverse image of each member of $\mathbf{V}$ is contained in $<a$ members of $\mathbf{R}$. Now, it is easy to see that each member of $\mathbf{V}$ intersects $<a$ members of $f\mathbf{A}$. Hence the result follows.

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2.14. Let $f:X\to Y$ be a strong (weak) $[a, b]$-perfect function and let K be a closed $[a, b]$-compact ($[a, \infty]$-compact) subset of Y. Then $f^{-1}K$ is $[a, b]$-compact.

2.15. Let $f:X\to Y$ be a weak $[a, b]$-perfect function and let Y be a T_1 space. Then $f^{-1}y$ is $[a, b]$-compact for all y in Y.

2.16. Let $f:X\to Y$ be a compact function and let Y be a locally compact space. Then f is an $[\omega_0, \infty]$-function.

2.17. Let $f:X\to Y$ be an $[\omega_0, \infty]$-function and let Y be a T_2 space. Then f is a compact function.

Here, we might mention that Y is T_2 is a necessary condition. For consider, X a compact T_2 with one open set U which is not closed and $Y=\{a, b\}$ with topology $T=\{\phi, \{a\}, Y\}$. Define a function $f:X\to Y$ by setting $f(U)=a$ and $f(X-U)=b$. Clearly, f is an $[\omega_0, \infty]$-function. However, $f^{-1}\{a\}$ is not compact.

2.18. Let $f:X\to Y$ be any function and let Y be a locally compact T_2 space. Then the following are equivalent:

(a) f is an $[\omega_0, \infty]$-perfect function.
(b) f is a compact and continuous function.
(c) f is a perfect function.

(a)$\to$(b) by 2.16, and (b)$\to$(c) by Helfer [6].

THEOREM 2.19. *Let X be any space and let Y be a T_1 space. Then the following are equivalent*:

(a) *X is compact.*
(b) *The projection map $p_Y:X\times Y\to Y$ is an $[\omega_0, \infty]$-perfect function.*
(c) *The projection map $p_Y:X\times Y\to Y$ is a perfect function.*

Proof. (a)$\to$(c) see Mrówka [15] and Scarborough [20]. (c)$\to$(b) by 2.6, and (b)$\to$(a) by 2.15.

THEOREM 2.20. (a). *Let Y be a T_1 space such that each point has a neighbourhood base of cardinality $\leq m$ where m is a regular cardinal. Then X is $[\omega_0, m]$-compact if and only if $p_Y:X\times Y\to Y$ is a weak $[\omega_0, m]$-function.*

(b) *Let X be a weakly-m-ω_0-compact space and let Y be a space such that each point has a neighbourhood base of cardinality $\leq m$ where m is a regular cardinal. Then $p_Y:X\times Y\to Y$ is a weak $[\omega_0, m]$-pseudo perfect function.*

Proof. (a). Suppose X is $[\omega_0, m]$-compact. Let $\mathbf{U}$ be an open cover of $X\times Y$ of cardinality $\leq m$. Let $\{V(t)\mid t\in T_y\}$ where $|T_y|\leq m$, be a neighbourhood base at any point y in Y. Let $y\in Y$ be fixed. For each x in X there is a neighbourhood $O_t(x, U)$ such that $O_t(x, U)\times V_t(y)\subset U$ for some t in T_y and some U in $\mathbf{U}$. Put $O_t(U)=\bigcup\{O_t(x, U) \mid O_t(x, U)\times V_t(y)\subset U\}$ for all U in $\mathbf{U}$. Then $\{O_t(U) \mid U\in\mathbf{U}, t\in T_y\}$ is an open cover of X of cardinality $\leq m$. Therefore, it has a finite subcover say $O_{t_1}(U_1),\ldots, O_{t_k}(U_k)$. Consequenly, $P_Y^{-1}(\bigcap_{j=1}^k V_{t_j}(y))\subset\bigcup_{j=1}^k O_{t_j}(U_j)\times V_{t_j}(y)\subset\bigcup_{j=1}^k U_j$. Now, it is easy to show that p_Y is a weak $[\omega_0, m]$-function.

(b). The proof is similar.

LEMMA A. *Let a and b be infinite cardinals such that $\sum_{c<a} b^c=b$ and let X and Y be $[a, b]$-compact spaces. Then $p_Y: X\times Y\to Y$ is a strong $[a, b]$-perfect function if and only if for each y in Y there is a neighbourhood U_y of y such that $X\times\mathrm{cl}_y(U_y)$ is $[a, b]$-compact.*

Proof. Suppose p_Y is a strong $[a, b]$-perfect function. Therefore, by 2.10, $X\times Y$ is $[a, b]$-compact. Hence the "only if" part of the proof follows.

Conversely, suppose that for each y and Y there is a neighbourhood U_y of y such that $X\times\mathrm{cl}_Y(U_y)$ is $[a, b]$-compact. Let $\mathbf{U}$ be an open cover of $X\times Y$ of cardinality $\leq b$. Then by the hypothesis, for each y in Y, $X\times\mathrm{cl}_Y(U_y)$ is contained in the union of $<a$ members of U. Hence p_y is a weak $[a, b]$-function and by 2.4 it is a strong $[a, b]$-function.

LEMMA B. *Let X be an $[a, b]$-compact space and let Y be an $[a, \infty]$-compact space. Then $p_Y: X\times Y\to Y$ is a weak $[a, b]$-function if and only if for each y in Y there is a neighbourhood U_y of y such that $X\times\mathrm{cl}_Y(U_y)$ is $[a, b]$-compact.*

The proof is similar.

THEOREM 2.21. *Let a and b be infinite cardinals such that $\sum_{c<a} b^c=b$. Let X and Y be $[a, b]$-compact spaces. Then $X\times Y$ is $[a, b]$-compact if for each y in Y there is a neighbourhood U_y of y such $X\times\mathrm{cl}_Y(U_y)$ is $[a, b]$-compact.*

The proof follows by lemma A and 2.10.

THEOREM 2.22. *Let X be an $[a, b]$-compact space and let Y be an $[a, \infty]$-compact space. Then $X\times Y$ is $[a, b]$-compact if and only if for each y in Y there is a neighbourhood U_y of y such that $X\times\mathrm{cl}_Y(U_y)$ is $[a, b]$-compact.*

The proof follows by the lemma B and 2.11.

3. **Applications.** In this section some applications of the notions defined in section 2 will be given. We also give easy proofs of some known product theorems with the help of theorems 2.21 and 2.22.

THEOREM 3.1. *Let $f: X\to Y$ be a weak $[\omega_0, m]$-perfect function. Then*:

(i) *X is m-paracompact (m-metacompact) if Y is m-paracompact (m-metacompact).*

(ii) *X is m-expandable (almost m-expandable) if Y is m-expandable (almost m-expandable).*

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(iii) *X is pseudo-compact if f is open and Y is pseudo-compact where X and Y are completely regular.*

Proof. (i). We shall only show that if Y is m-paracompact then so is X. Let Y be m-compact and let f be a weak $[\omega_0, m]$-perfect function from X onto Y. By 2.4, f is a strong $[\omega_0, m]$-perfect function. Let $\mathbf{U}$ be an open cover of X of cardinality $\leq m$. Since f is a strong $[\omega_0, m]$-perfect function there exists an open cover $\mathbf{V}$ of Y cardinality $\leq m$ such that $f^{-1}V$ is contained in the union of some finite subfamily of $\mathbf{U}$ for each V in $\mathbf{V}$. But Y is m-paracompact implies $\mathbf{V}$ has an open locally-ω_0 refinement $\mathbf{W}$ say. Let $\mathbf{S}=\{f^{-1}W\cap U_i \mid i=1,\ldots,k,$ $W\in\mathbf{W}$, where $f^{-1}W\subset\bigcup_{i=1}^{k} U_i\}$. Evidently, $\mathbf{S}$ is an open locally-ω_0 refinement of $\mathbf{U}$. Hence X is m-paracompact.

(ii). We shall only show that if Y is m-expandable then so is X. Let $\mathbf{A}=\{A_t \mid t\in T\}$ where $|T|\leq m$, be a locally-ω_0 collection of subsets of X. Since f is a $[\omega_0, m]$-perfect function by 2.13 $f\mathbf{A}=\{fA_t \mid t\in T\}$ is a locally-ω_0 collection of subsets of Y. But Y is m-expandable so there is an open locally-ω_0 collection $\{G_t \mid t\in T\}$ of subsets of Y such that for each t in T, $fA_t\subset G_t$. Therefore, $A_t\subset f^{-1}G_t$ for t in T and it is easy to see that $\{f^{-1}G_t \mid t\in T\}$ is an open locally finite collection of subsets of X.

(iii). Let $\mathbf{U}$ be an open cover of X with $|\mathbf{U}|\leq\omega_0$. Then there is an open cover $\mathbf{V}$ of Y with $|\mathbf{V}|\leq\omega_0$ such that for each V in $\mathbf{V}$, $f^{-1}V\subset\bigcup_{i=1}^{k} U_i$. Since Y is completely regular and pseudo-compact $\mathbf{V}$ has a finite subfamily $V_1,\ldots,V_k$ such that $Y=\bigcup_{j=1}^{k}\mathrm{cl}(V_j)$. The rest follows easily.

COROLLARY 1.3.2. *Let $f: X\to Y$ be a closed continuous function. Then*:

(a) *X is countably metacompact if Y is countably metacompact and $f^{-1}y$ is countably compact for each y in Y.* (Banerjee [2]).

(b) *X is m-paracompact (m-compact) if Y is m-paracompact (m-compact) and $f^{-1}y$ is m-compact for each y in Y.* (Hanai [7]).

(c) *X is paracompact if Y is paracompact and $f^{-1}y$ is compact for each y in Y.* (Hanai 8, and Henriksen and Isbell [10]).

(d) *X is countably paracompact and collectionwise normal if Y is countably paracompact, collectionwise normal and $f^{-1}y$ is compact for each y in Y.* (Hanai [7]).

The proof of (a), (b), and (c) follows from theorem 3.1 and 2.6. The proof of (d) follows from theorem 3.1, 2.6 and [19].

THEOREM 3.3. *Let X be a compact space and let Y be a T_1 space. Then*

(A) *$X\times Y$ is $[a, b]$-compact if and only if Y is $[a, b]$-compact.*

(B) *$X\times Y$ is m-paracompact if and only if Y is m-paracompact.*

(C) *$X\times Y$ is m-metacompact if and only if Y is m-metacompact.*

(D) *$X\times Y$ is m-expandable if and only if Y is m-expandable.*

(E) $X\times Y$ *is almost* m*-expandable if and only if* Y *is almost* m*-expandable.*

(F) $X\times Y$ *is pseudo-compact if and only if* Y *is pseudo-compact, where* X *and* Y *are completely regular.*

The proof of the above theorem follows from theorem 3.1; 2.19, 2.10, and 2.11.

Theorem 3.3 gives as corollaries the results of Nóvak [17], Dowker [4], Mrówka [15], Gál [5], Dieudonné [3], Hayashi [9], and Smith and Krajewski [19].

In the above theorem we can replace the compactness of X by X is $[\omega_0, m]$-compact and each point of Y has a neighbourhood base of cardinality $\leq m$.

THEOREM 3.4. *Let* X *be an* $[\omega_0, m]$*-compact space and let* Y *be a space in which each point has a neighbourhood base of cardinality* $\leq m$. *Then*

(1) $X\times Y$ *is* m*-paracompact if and only if* Y *is* m*-paracompact.*

(2) $X\times Y$ *is* m*-metacompact if and only if* Y *is* m*-metacompact.*

(3) $X\times Y$ *is* m*-expandable if and only if* Y *is* m*-expandable.*

(4) $X\times Y$ *is almost* m*-expandable if and only if* Y *is almost* m*-expandable.*

(5) $X\times Y$ *is pseudo-compact if and only if* Y *is pseudo-compact, where* X *and* Y *are completely regular.*

(6) $X\times Y$ *is* $[\omega_0, m]$*-compact if and only if* Y *is* $[\omega_0, m]$*-compact.*

The proof follows from theorems 2.20, 3.1, 2.10, and 2.11.

The theorem 3.4 gives as corollaries the results of Banerjee [2] Nardzewski [16], Hanai [7], and Bagley, Connell and Mcknight Jr. [1].

COROLLARY 3.5. *Let* Y *be a locally compact* T_2 *space. Then*:

(*) $X\times Y$ *is* $[a, b]$*-compact if and only if* X *and* Y *are* $[a, b]$*-compact where* a *and* b *are infinite cardinals such that* $\sum_{c<a} b^c = b$.

(**) $X\times Y$ *is* $[a, b]$*-compact if* X *is* $[a, b]$*-compact and* Y *is* $[a, \infty]$*-compact.*

The proof follows from 2.21 and 2.22.

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