A LIBERAL EGALITARIAN PARADOX*

ALEXANDER W. CAPPELEN† AND BERTIL TUNGODDEN‡

A liberal egalitarian theory of justice seeks to combine the values of equality, personal freedom, and personal responsibility. It is considered a much more promising position than strict egalitarianism, because it supposedly provides a fairness argument for inequalities reflecting differences in choice. However, we show that it is inherently difficult to fulfill this ambition. We present a liberal egalitarian paradox which shows that there does not exist any robust reward system that satisfies a minimal egalitarian and a minimal liberal requirement. Moreover, we demonstrate how libertarianism may be justified in this framework if we drop the egalitarian condition.

1. INTRODUCTION

Liberal egalitarian theories of justice seek to combine the values of equality, personal freedom, and personal responsibility. The contemporary focus on this relationship can be traced back to the seminal work of Rawls (1971), but it has historical roots in both the US Declaration of Independence (1776) and the French Declaration of the Rights of Man and Citizen (1789). These societies developed in rather different directions, though, and as noted by Nagel (2002: 88), "what Rawls has done is to combine the very strong principles of social and economic equality associated with European socialism with the equally strong principles of pluralistic toleration and personal freedom associated with American liberalism, and he has done so in a theory that traces them to a common foundation." Rawls's theory inspired the development of other forms

393

of egalitarianism, notably by Dworkin (1981), Arneson (1989), Cohen (1989), Roemer (1993, 1996, 1998), and Fleurbaey (1995a,b), where the main achievement has been to include considerations of personal responsibility in egalitarian reasoning. The dominating modern egalitarian view is liberal egalitarianism, which holds that people, within a framework offering equal opportunities and respecting personal freedom, should be held responsible for their accomplishments.[1]

Interestingly, this perspective is also very much in line with moral intuitions present in modern societies. By way of illustration, in a recent study of moral opinions on distributive justice in Norway, the statement that we should accept inequalities due to personal choices gained the support of 87% of respondents. Moreover, 88% of respondents agreed to the claim that people exercising the same labor effort should receive the same income, and close to half of the sample (48%) endorsed the view that inequalities due to factors beyond a person's control should be eliminated. In contrast, only 12% of respondents supported the view that income should be distributed on the basis of needs.[2] In a similar vein, based on several surveys of attitudes to welfare policies, Bowles and Gintis (2000: 47) conclude that "egalitarian policies that reward people independently of whether and how much they contribute to society are considered unfair and are not supported, even if the intended recipients are otherwise worthy of support."[3] Such opinion polls do of course not validate the ethical reasoning underlying liberal egalitarianism, but the strong support for this position should make it even more interesting to study the coherency of this framework.

The liberal egalitarian view is considered to represent a much more appealing distributive ideal than strict (or outcome) egalitarianism. Strict egalitarians do not believe that there is a fairness argument for inequality, and argue that inequalities can only be justified in order to avoid Pareto inefficiency. Liberal egalitarians object to strict egalitarianism because they believe that *fairness* requires that people should be held responsible for their choices. The difference between liberal egalitarianism and strict egalitarianism is most easily seen in situations where there are no efficiency problems. Consider a situation where individuals' choice of labor effort is unaffected by the redistributive mechanism, for example because individuals primarily are motivated by non-monetary considerations. In this case, there is no efficiency argument for an unequal distribution of income, and strict egalitarians would thus insist on an equal distribution.

---

[1] For critical reviews of this position, see Fleurbaey (1995c) and Anderson (1999).

[2] The study was organized by Alexander Cappelen, Tone Ognedal, and Steinar Strøm at the University of Oslo. The sample consisted of 1,062 individuals in all age groups above 15 years. Further details of the study are available upon request.

[3] See also Fong (2001).

A liberal egalitarian, on the other hand, will claim that people should be held responsible for their choices even in this situation, and thus may find an unequal income distribution fair if it reflects differences in labor effort. In this paper, however, we present a liberal egalitarian paradox, showing that it is inherently difficult to justify rewarding effort in a robust manner within a liberal egalitarian framework.

In the discussion of this paradox, we will not limit ourselves to any particular view on what should be the appropriate equalisandum (see, for example, Sen 1992 and Fleurbaey 1995a). It may be welfare, income, primary goods, capabilities, or something else. To make things simple, though, we will refer to the equalisandum as income. The important point is that the equalisandum is determined both by factors people are held responsible for and by factors people are not held responsible for, where we leave open the question of which factors should fall into each of the two categories.[4] Again for simplicity's sake, let us refer to responsibiliy factors as *effort* and non-responsibilty factors as *talent*, where effort and talent may be seen as indices constructed on the basis of a broader set of variables.[5]

So what should a liberal egalitarian redistribution mechanism look like? In order to answer this question, we have to define more precisely our understanding of liberal egalitarianism. Inspired by Rawls (1971), we claim that any liberal egalitarian redistributive mechanism should (at least) satisfy the following two minimal conditions, one egalitarian and one liberal. The minimal egalitarian requirement is that the redistributive mechanism should eliminate income differences due to differences in talent (but it might still allow inequalities due to differences in effort); the minimal liberal requirement is that no one should be forced to work. It turns out that, given a very weak reward robustness condition, there is only one general redistributive mechanism that satisfies both these demands, namely strict egalitarianism. This result may be viewed as a strengthening of the impossibility result presented by Bossert and Fleurbaey (1996). They prove that marginal productivity reward is incompatible with the minimal egalitarian condition (that inequalities due to talent should be eliminated). The paradox presented in this paper extends this result by showing that *any* redistribution mechanism that rewards effort in a robust manner violates either a minimal liberal or a minimal egalitarian condition.

---

[4] The concept of personal responsibility is compatible with a naturalistic and deterministic account of the world (see, for example, Dennett 2003).

[5] By using these definitions we do not intend to imply anything about the extent to which individuals should be held responsible for effort and talent as they are used in everyday language. In Cappelen and Tungodden (2005), we analyze how the location of the responsibility cut effects redistributive policy in a liberal egalitarian framework. For a discussion of a how this approach can be operationalized in empirical analysis, see Roemer (2002).

After describing the framework in section 2, we present the liberal egalitarian paradox in section 3. As a response to the paradox, some liberals may consider weakening their egalitarian ambitions. In section 4, we show that if, at the same time, they endorse another commonly held moral intuition and slightly strengthen the reward robustness requirement, they have to endorse libertarianism. Section 5 contains some further discussion of the results. A formal statement of the framework and the results (including proofs) are presented in the appendix.

2. THE FRAMEWORK

We will here only provide an informal presentation of the framework; the formal statements are relegated to the appendix.[6] Consider a population where people are characterized by their talent level, and where there are differences in talent. Moreover, assume that there exist some unproductive persons in society (that is, persons with a talent that is so low that they for any effort level have no pre-tax income). This should be an uncontroversial assumption to make in the study of redistribution in large societies, but we will briefly comment on the implications of relaxing this assumption in the final section of the paper. The pre-tax income of each person is determined by her talent level and her choice of effort. We assume that people at least can choose between working and not working, but we do not impose any further restrictions on the set of possible effort levels. If a person does not work, then she has no pre-tax income.

The aim of the analysis is to see whether it is possible to establish a fairness argument for holding people responsible for their effort that is independent of efficiency considerations. For this purpose, it is necessary to rule out efficiency considerations from the analysis, which we do by assuming that people's choice of effort is unaffected by the design of the redistribution mechanism. This assumption implies that no redistribution mechanism has any distortionary effects and hence that all redistribution mechanisms generate Pareto-optimal allocations (as long as we assume that people have self-interested preferences and a positive marginal utility of income). However, note that the assumption does not imply that all people make the same choices or that an individuals's effort is the same in all situations covered by the redistribution mechanism.

More precisely, our object of study is a redistribution mechanism that for every distribution of talent and effort assigns a post-tax income to each person in society. We assume that this redistribution mechanism satisfies the following no-waste condition.

---

[6] The framework of this paper is a modified version of the framework introduced by Bossert (1995) and Bossert and Fleurbaey (1996).

No-Waste (NW): *For any distribution of talent and effort, the sum of post-tax income assigned by the redistribution mechanism always equals the sum of pre-tax income.*

Initially, given our specification of the set of possible effort and talent levels, we will impose no further restrictions on the distribution of effort and talent in society, that is, we will assume unrestricted domain richness.

Unrestricted Domain Richness (UDR): *The redistribution mechanism covers all possible distributions of talent and effort.*

We will not state explicitly that the redistribution mechanism satisfies NW and UDR when reporting the results. In the discussion of the core results of our paper, however, we also show the implications of slightly restricting the set of possible redistribution mechanisms. More precisely, we then make the following assumption, which should be uncontroversial in the analysis of redistribution in large, pluralistic societies.

Restricted Domain Richness (RDR): *The redistribution mechanism only covers those distributions of talent and effort where for any talent level, there is at least one person with such a talent who does not work.*

Finally, our analysis will focus on two specific redistribution mechanisms, strict egalitarianism and libertarianism. In the present framework, they are defined as follows.

Strict Egalitarianism: *For any distribution of talent and effort, the redistribution mechanism assigns the same post-tax income to everyone.*

Libertarianism: *For any distribution of talent and effort, the redistribution mechanism assigns to each person a post-tax income equal to her pre-tax income.*

These are two extreme positions, and the liberal egalitarian approach may be seen as an attempt to establish a reasonable compromise between them. However, as the analysis will reveal, this is no easy task.

3. THE PARADOX

A liberal egalitarian redistributive mechanism should satisfy some minimal egalitarian and liberal requirements. Inspired by Rawls (1971), the core egalitarian intuition underlying the liberal egalitarian approach is the idea that a redistributive mechanism should eliminate effects due to non-responsibility factors.[7] In our setting this implies that all individuals

---

[7] Interestingly, a similar view is expressed by Mirrlees (1971: 120) in his seminal paper on optimal income taxation, where he points out the great desirability of finding some effective redistribution mechanism that offsets "the unmerited favours that some of us receive from our genes and family advantages."

exercising the same effort, independent of talent, should have the same post-tax income (Bossert and Fleurbaey 1996: 346).

Equal Income for Equal Effort (EIEE): *For any distribution of talent and effort, if two persons exercise the same level of effort, then the redistribution mechanism assigns the same post-tax income to them.*

Notice that EIEE is consistent with huge inequalities in post-tax income as long as these inequalities correspond to differences in effort, and hence it is a substantial weakening of the strict egalitarian requirement of distributing income equally in all situations. However, as shown by Bossert and Fleurbaey (1996), EIEE is in general inconsistent with the following condition of natural reward.

Natural Reward (NR): *For any two distributions of talent and effort where the only difference between the two distributions is the difference in one person's effort, the difference in this person's post-tax income should be equal to the difference in his pre-tax income and everyone else should receive the same post-tax income in both distributions.*

The intuition behind Bossert and Fleurbaey's impossibility result can be illustrated by considering a two-person case with one talented and one untalented person and where the marginal productivity of effort is 10 for the talented and 5 for the untalented. Suppose that they both initially exercise 10 units of effort, which implies that the pre-tax income of the talented is 100 and the pre-tax income of the untalented is 50. In accordance with EIEE, both receive the same post-tax income of 75. Then the talented doubles his effort while the effort of the untalented remains the same. NR implies that in the new situation, the talented should have a post-tax income of 175 and the untalented should have a post-tax income of 75. Finally, let the untalented increase his effort such that he also exercises 20 units of effort. NR now requires that the untalented should get an increase in post-tax income of 50, which implies that the talented has a higher post-tax income (175) than the untalented (125) in a situation where both exercise the same level of effort. However, this violates EIEE and the result follows.

The implication of this result is that it is impossible to reward people's effort with their marginal productivity if one wants to remain within an egalitarian framework. However, NR is not a very appealing requirement in situations where the marginal productivity depends on a person's talent. In such situations, NR implies that individuals are held responsible not only for their effort but also partly for their talent (Tungodden 2005). Thus NR seems to violate one of the core intuitions underlying liberal egalitarian ethics, namely that inequalities due to differences in non-responsibility factors should be eliminated. In this paper we show, however, that a similar impossibility result arises also with conditions that should be very appealing to liberal egalitarians.

A basic liberal intuition is that society should not formally restrict the choices available to people as long as these choices do not place restrictions on other people's freedom to choose (see the first principle of justice in Rawls (1971) for a general expression of this intuition). A very weak version of this requirement is to demand that no one should be forced to work. In a liberal society, if people want to dedicate themselves to non-income-generating activities, then they should be formally free to do so and the tax system should not eliminate this possibility. In our framework this can be captured by demanding that people who do not work should not pay taxes, because imposing a tax on these people would eliminate the possibility of choosing the non-work option.

No Forced Labor (NFL): *For any distribution of talent and effort, the redistribution mechanism assigns a non-negative post-tax income to each person who does not work.*

Notice that NFL does not rule out the possibility of positive transfers to individuals who do not exercise effort. For example, strict egalitarianism satisfies NFL because it ensures all individuals a non-negative income in all situations.

The liberal egalitarian approach can now be seen as an attempt to establish a robust reward structure within the constraints of EIEE and NFL, where a robust reward structure should satisfy the following two minimal requirements.

Minimal Reward (MR): *There exists some distribution of talent and effort, where there are two persons with (a) the same talent, and (b) different effort levels, and where the redistribution mechanism assigns a strictly higher post-tax income to the person with a higher effort level.*

Reward Robustness (RR): *For any two distributions of effort and talent and any two persons with the same talent who do not change their effort when moving from one distribution to the other, if the redistribution mechanism assigns a strictly higher post-tax income to one of the two in one of the distributions, then the redistribution mechanism also assigns a strictly higher post-tax income to this person in the other distribution.*

MR states that a robust reward structure should at least sometimes reward effort, while RR states that such a reward should not be completely contingent on what others do. The content of the RR requirement can be illustrated by considering a three-person economy with individuals A, B, and C. Assume that initially A exercises 5 units of effort, B exercises 10 units of effort, and C exercises 15 units of effort and that individuals B and C have the same talent. Assume furthermore that society has decided to reward C for the additional effort she exercises in comparison with B and that C therefore receives a post-tax income that is higher than the

post-tax income of B. Consider then a new situation where individual A has changed her effort, while B and C exercise the same effort levels as in the initial situation. RR then tells us that society should not completely eliminate the reward C is given in comparison with B just because A has changed her effort.

Any reward structure that satisfies NR must satisfy these two requirements, but there are many reward structures that satisfy MR and RR, but not NR. However, it turns out that no robust reward structure is compatible with a liberal egalitarian framework.

PROPOSITION 1. There does not exist any redistribution mechanism satisfying EIEE, NFL, RR, and MR.

PROOF. See appendix.[8] ■

There is a straightforward intuition behind this liberal egalitarian paradox. If we want to reward effort in a robust manner and at the same time want to reward individuals who make the same effort equally, then there will be situations where some people will be rewarded with more than they actually produce. This will create a deficit that has to be financed by someone. However, unless we are willing to force people to work, we cannot be sure that we are able to finance this deficit. There is thus a conflict between our desire to treat people as equals and our desire to respect their freedom.

If we compare Proposition 1 with the impossibility result established by Bossert and Fleurbaey (1996), then we should notice that we have made one weakening and one strengthening. We have weakened the framework substantially by replacing NR with MR and RR, while we have strengthened the framework by imposing no forced labor (NFL). However, given that NFL is an almost uncontroversial condition in liberal egalitarian reasoning, we believe that Proposition 1 illustrates a more fundamental conflict than the impossibility result of Bossert and Fleurbaey (1996). It is not only the case that redistribution mechanisms satisfying the natural reward condition (NR) are in conflict with a minimal egalitarian requirement (EIEE). The liberal egalitarian paradox shows that *any* robust

---

[8] Notice that all the conditions are needed in order to establish an impossibility. Strict egalitarianism satisfies EIEE, NFL, and RR; libertarianism satisfies NFL, RR, and MR; the so-called egalitarian equivalent mechanism, introduced by Bossert and Fleurbaey (1996), giving people the same fixed reward for each effort level and then distributing the net deficit (surplus) equally among all individuals, satisfies EIEE, MR, and RR; and, finally, the proportional egalitarian equivalent mechanism, introduced by Cappelen and Tungodden (2003), which is equal to the egalitarian equivalent mechanism except for the fact that it distributes the net deficit (surplus) proportional to post-tax income, satisfies EIEE, NFL, and MR.

reward structure violates either a minimal liberal or a minimal egalitarian condition.

It is easy to see that strict egalitarianism satisfies EIEE, NFL, and RR.[9] Moreover, if we strengthen our domain restriction slightly, strict egalitarianism is the *only* redistribution mechanism that satisfies all of them.

PROPOSITION 2. Given RDR, a redistribution mechanism satisfies EIEE, NFL, and RR if and only if it is Strict Egalitarianism.

PROOF. See appendix. ∎

In sum, the two propositions show that it is not possible to provide a robust fairness argument for rewarding effort within a framework satisfying a minimal liberal and a minimal egalitarian requirement. Hence, if egalitarians do not want to restrict people's freedom, then they have to consider strict egalitarianism as the fair solution, and any deviation from strict egalitarianism has to be justified on the basis of efficiency considerations.

4. LIBERTARIANISM

Liberals are not necessarily committed to egalitarianism, and hence may, on the basis of the paradox, consider weakening their egalitarian ambitions and seek alternative redistributive mechanisms within the framework of NFL. But what alternatives are there? The minimal liberal condition is consistent with a wide range of redistribution mechanisms, including strict egalitarianism and libertarianism. It turns out, however, that if we combine this framework with a commonly shared moral intuition, that individuals not exercising any effort should not be subsidized, we can provide a characterization of libertarianism.

No Subsidy Without Effort (NSWE): *For any distribution of talent and effort and any person exercising no effort, the redistribution mechanism assigns a non-positive post-tax income to this person.*

It follows immediately that by combining NSWE and NFL, we have the following condition.

No Income Without Effort (NIWE): *For any distribution of talent and effort and any person exercising no effort, the redistribution mechanism assigns zero post-tax income to this person.*

---

[9] It satisfies EIEE because it distributes income equally to all individuals, including those at the same effort level. It satisfies NFL because everyone always gets the average income, which is non-negative. Finally, it trivially satisfies RR, since no one ever gets a higher post-tax income than someone else.

NIWE is clearly compatible with a wide range of liberal positions, including a number of egalitarian redistribution mechanisms satisfying EIEE. However, consider now the following strengthening of RR.

Strong Reward Robustness (SRR): *For any two distributions of effort and talent and any two persons with the same talent, if they do not change their effort when moving from one distribution to the other, then the difference between their post-tax income should be the same in the two distributions.*

The weak reward robustness condition requires that a reward for effort should not be completely contingent on what others do, while SRR strengthens this by requiring that the extra reward given to one person in comparison with someone else exercising less effort should be independent of what others do.

PROPOSITION 3. If the redistribution mechanism satisfies SRR and NIWE, then for any distribution of talent and effort and any two persons, the difference in post-tax income equals the difference in pre-tax income.

PROOF. See appendix. ∎

It follows immediately that any redistribution mechanism satisfying SRR and NIWE violates EIEE when people differ in their marginal productivity. More interestingly, by adopting restricted domain richness, we can see that the two conditions leave us with no other option than libertarianism.

PROPOSITION 4. Given RDR, a redistribution mechanism satisfies SRR and NIWE if and only if it is Libertarianism.

PROOF. See appendix. ∎

The intuition behind this result is straightforward. By the restricted domain richness assumption, for any talent represented in society, there is always someone who chooses not to work. According to NIWE, this group should not be affected by a change in the effort level of someone else. SRR, however, demands equal treatment of people with the same talent in these cases and thus no-one can be affected when another person changes her effort. Consequently, it follows from no-waste and NIWE that all individuals must receive exactly what they produce.

Notice that the propositions in this section provide a justification for the libertarian position without taking into account efficiency considerations. If we find it reasonable to neither subsidize nor tax people who do not exercise any effort and endorse the stronger reward robustness condition, then we have to endorse libertarianism.

5. CONCLUDING REMARKS

The liberal egalitarian paradox illustrates a basic tension between liberal and egalitarian ideals. It shows that if we combine the egalitarian ideal of equal income for equal effort with the liberal ideal of holding people responsible for their choices, then we sometimes will have to limit people's freedom. The fact that there exists no robust reward system that satisfies the minimal egalitarian and the minimal liberal requirement shows that it is inherently difficult to establish an independent fairness argument for rewarding effort within a liberal egalitarian framework. Notice that each of the two conditions, independently, is consistent with a wide range of robust reward structures. It is only in combination, as a minimal expression of liberal egalitarianism, that they leave us with no other choice than strict egalitarianism.

In the analysis, we have assumed that there are some unproductive individuals in the economy. This assumption should be reasonable within large societies, but is not at all essential for the general message of this paper. If we drop it, then the marginal productivity of the least talented will still place strong restrictions on the structure of a robustness reward structure. Given the two minimal liberal egalitarian requirements, it will not be possible to guarantee a reward larger than the marginal productivity of the least talented (see also Tungodden 2005). Moreover, we have disregarded efficiency considerations, which is typically seen as a limitation of the analysis. But this is not the case in the present context. On the contrary, in order to study the possibility of establishing an *independent* fairness argument for rewarding effort, it is necessary to ignore efficiency considerations.

There are different ways of responding to the liberal egalitarian paradox. First, one might see the result as a confirmation of the claim made by Rawls (1971), that only efficiency considerations can provide a rationale for deviating from strict egalitarianism within a liberal egalitarian framework. Alternatively, one might insist that the reward system should satisfy both EIEE and NFL, and reject our robustness requirement. Several redistribution mechanism are then possible. However, even if one chooses the second strategy, the paradox is important because it shows that we can only achieve the goal of incorporating responsibility considerations in an egalitarian framework at a cost. In sum, the analysis shows that it is inherently difficult to fulfil the liberal egalitarian ambition of establishing a middle way between strict egalitarianism and libertarianism.

6. APPENDIX

6.1 The formal framework

Consider a society with a population $N = \{1, \ldots, n\}$, $n \geq 6$, where agent i's effort is e_i and her talent is t_i. We assume that $e_i, t_i \in \Re$, where $\Re$ is the set of

real numbers. Let $\Omega^E \subseteq \Re$ be the set of possible effort levels and $\Omega^T \subseteq \Re$ be the set of possible talent levels. The pre-tax income function $f : \Omega \to \Re_+$, where $\Omega = \Omega^E \times \Omega^T$, is assumed to be increasing in effort and talent. Let $a_i = (a_i^E = e_i, a_i^T = t_i)$ be a characteristics vector of i and $a = (a_1, \ldots, a_n)$ a characteristics profile of society.

Define $\Omega_i \subset \Re^2$ as the set of possible characteristics vectors of person i, where for any $i \in N$ and $a_i, \tilde{a}_i \in \Omega_i$, $a_i^T = \tilde{a}_i^T$. In other words, we do not consider interprofile conditions with respect to talent, but assume that there is a single characteristics profile of talent in society.[10] We assume that there are differences in talent and that there are always more than two people at each talent level, i.e., for any $j \in N$ there exists $k, l, m \in N$ such that $a_j^T = a_k^T = a_l^T \neq a_m^T$. We also assume that there exist some unproductive persons in society, and refer to their talent level as $t^{\min}$, where $t^{\min} \leq t$ for every $t \in \Omega^T$ and $f(t^{\min}, e_i) = 0, \forall e_i \in \Omega^E$.

Let Ω_i^E be the set of possible effort levels for person i, where we assume that for any $j, k \in N$, $\Omega_j^E = \Omega_k^E \subseteq \Omega^E$. Moreover, define $e^{\min}$ as the effort level reflecting that a person does not work, where $e^{\min} \leq e$ for every $e \in \Omega^E$ and $f(t_i, e^{\min}) = 0, \forall t_i \in \Omega^T$. We assume that everyone can, at least, choose between working and not working, i.e., $a_i = (e^{\min}, a_i^T), \tilde{a}_i = (\tilde{a}_i^E > e^{\min}, a_i^T) \in \Omega_i, \forall i \in N$, but do not impose any further restrictions on the set of effort levels. Hence, the framework covers both continuous and discrete cases.

Let $\Omega^N = \Omega_1 \times \Omega_2 \times \cdots \times \Omega_n$ be the set of possible characteristics profiles of society, where $\Omega^N \subset \Re^{2n}$. Our object of study is a redistribution mechanism F, which satisfies the no-waste condition.

No-Waste (NW): $\sum_{i=1}^n F_i(a) = \sum_{i=1}^n f(a_i), \forall a \in \Omega^N$.

In most of our analysis we shall assume unrestricted domain.

Unrestricted Domain Richness (UDR): $F : \Omega^N \to \Re^n$.

We will not state explicitly these two restrictions when reporting the results. In the discussion of the core results of this paper, we also show the implications of slightly restricting the domain of F. Define $\tilde{\Omega}^N \subset \Omega^N$, where for every $a \in \tilde{\Omega}^N$ and every $j \in N$, there exists some $k \in N$ such that $a_k^T = a_j^T$ and $a_k^E = e^{\min}$.

Restricted Domain Richness (RDR): $F : \tilde{\Omega}^N \to \Re^n$.

In the analysis, we are concerned with two specific redistribution mechanisms, strict egalitarianism and libertarianism. The strict egalitarian

---

[10] See Fleurbaey and Maniquet (1999) for a related analysis using interprofile conditions with respect to talent.

redistribution mechanism always assigns the same post-tax income to everyone,

$$F_k^{SE}(a) = \frac{1}{n} \sum_{i \in N} f(a_i), \forall k \in N, \forall a \in \Omega^N,$$

whereas the libertarian redistribution mechanism always gives individuals what they produce,

$$F_k^L(a) = f(a_k), \forall k \in N, \forall a \in \Omega^N.$$

6.2 Formal statements of the conditions

Equal Income for Equal Effort (EIEE): For any $a \in \Omega^N$ and $j, k \in N$, where $a_j^E = a_k^E$, $F_j(a) = F_k(a)$.

Natural Reward (NR): For any $a, \tilde{a} \in \Omega^N$ and $j \in N$, where $a_j^E \neq \tilde{a}_j^E$ and $a_i^E = \tilde{a}_i^E, \forall i \neq j$, $F_j(a) - F_j(\tilde{a}) = f_j(a) - f_j(\tilde{a})$ and $F_i(a) = F_i(\tilde{a}), \forall i \neq j$.

No Forced Labor (NFL): For any $a \in \Omega^N$ and $i \in N$, where $a_i^E = e^{\min}$, $F_i(a) \geq f(a_i) = 0$

Minimal Reward (MR): There exists some $a \in \Omega^N$ and $j, k \in N$, where $a_j^T = a_k^T$, $a_j^E > a_k^E$, and $F_j(a) > F_k(a)$.

Reward Robustness (RR): For any $a, \tilde{a} \in \Omega^N$ and $j, k \in N$, where $a_j^T = a_k^T$ and $j, k \in \{i \in N \mid a_i^E = \tilde{a}_i^E\}$, if $F_j(a) > F_k(a)$, then $F_j(\tilde{a}) > F_k(\tilde{a})$.

Strong Reward Robustness (SRR): For any $a, \tilde{a} \in \Omega^N$ and $j, k \in N$, where $a_j^T = a_k^T$ and $j, k \in \{i \in N \mid a_i^E = \tilde{a}_i^E\}$, $F_j(a) - F_k(a) = F_j(\tilde{a}) - F_k(\tilde{a})$.

No Subsidy Without Effort (NSWE): For any $a \in \Omega^N$ and $i \in N$, $F_i(a_i^T, e^{\min}) \leq 0, \forall i \in N$.

No Income Without Effort (NIWE): For any $a \in \Omega^N$ and $i \in N$, $F_i(a_i^T, e^{\min}) = 0, \forall i \in N$.

6.3 Proof of propositions

PROPOSITION 1. *There does not exist any redistribution mechanism F satisfying EIEE, NFL, RR, and MR.*

PROOF. We prove that any redistribution mechanism satisfying EIEE, NFL, and RR must violate MR.

(1) Suppose that MR is satisfied. Then there exists $a \in \Omega^N$ and $j, k \in N$, where $a_j^T = a_k^T$, $a_j^E > a_k^E$, and $F_j(a) > F_k(a)$.

(2) Consider $\tilde{a} \in \Omega^N$ and $l, m \in N$, where $a_l^T = a_m^T = t^{\min}$ and $\tilde{a}_l^E = a_j^E$, $\tilde{a}_m^E = a_k^E$, and $\tilde{a}_i^E = a_i^E, \forall i \neq l, m$. It follows from (1) and RR that $F_j(\tilde{a}) > F_k(\tilde{a})$. By EIEE, $F_l(\tilde{a}) = F_j(\tilde{a})$ and $F_m(\tilde{a}) = F_k(\tilde{a})$, which implies that $F_l(\tilde{a}) > F_m(\tilde{a})$.

(3) Consider $\hat{a} \in \Omega^N$, where $\hat{a}_l^E = \tilde{a}_l^E$, $\hat{a}_m^E = \tilde{a}_m^E$ and $\hat{a}_i^E = e^{\min}$, $\forall i \neq l, m$. By NFL and NW, $F_i(\hat{a}) = 0$, $\forall i \in N$. This implies that $F_l(\hat{a}) = F_m(\hat{a})$. But, taking into account (2), this violates RR and thus the supposition in (1) is not possible. The result follows. ∎

PROPOSITION 2. *Given RDR, a redistribution mechanism F satisfies EIEE, NFL and RR if and only if $F = F^{SE}$.*

PROOF. The if part is trivial, and hence we only prove the only-if part.

(1) Let us show that the proof of Proposition 1 is valid given RDR. In order to do this, we have to show that each of the characteristics profiles used in the proof also is a member of the restricted domain $\tilde{\Omega}^N$. By supposition, $a \in \tilde{\Omega}^N$. Consider now $\tilde{a}$. RDR requires that there exists some $r \in N$ such that $a_r^E = e^{\min}$ and $a_r^T = t^{\min}$. However, if we assume that $l, m \neq r$, which is admissible because by assumption there are more than two individuals at each talent level, it follows that there exists some $\tilde{a} \in \tilde{\Omega}^N$, where $a_l^T = a_m^T = t^{\min}$ and $\tilde{a}_l^E = a_j^E$, $\tilde{a}_m^E = a_k^E$, and $\tilde{a}_i^E = a_i^E$, $\forall i \neq l, m$. Finally, by the fact that the only difference between $\tilde{a}$ and $\hat{a}$ is that more people exercise minimum effort, it follows immediately that there exists some $\hat{a} \in \tilde{\Omega}^N$, where $\hat{a}_l^E = \tilde{a}_l^E$, $\hat{a}_m^E = \tilde{a}_m^E$ and $\hat{a}_i^E = e^{\min}$, $\forall i \neq l, m$.

(2) Proposition 1 shows that given EIEE, NFL and RR, it is not possible to satisfy MR. This implies that for any $a \in \tilde{\Omega}^N$ and $j, k \in N$, where $a_j^T = a_k^T$ and $a_j^E = e^{\min}$, $F_j(a) = F_k(a)$. In order to show that EIEE, NFL and RR imply strict egalitarianism given RDR, we also have to prove that for any $s \in N$, where $a_s^T \neq a_j^T$, $F_s(a) = F_j(a)$.

(3) By RDR, for any $s \in N$, there exists $u \in N$ such that $a_u^T = a_s^T$ and $a_u^E = e^{\min}$. Moreover, by RDR, there exists $r \in N$ such that $a_r^T = a_j^T$ and $a_r^E = e^{\min}$. By Proposition 1, $F_s(a) = F_u(a)$ and $F_j(a) = F_r(a)$. By EIEE, $F_u(a) = F_r(a)$. In sum, this implies that $F_s(a) = F_j(a)$, and the result follows. ∎

PROPOSITION 3. *If F satisfies SRR and NIWE, then for any $a \in \Omega^N$ and $j, k \in N$, where $a_j^T = a_k^T$, $F_j(a) - F_k(a) = f(a_j) - f(a_k)$.*

PROOF. (1) Suppose there exists $a \in \Omega^N$ and $j, k \in N$, where $a_j^T = a_k^T$ and $F_j(a) - F_k(a) \neq f(a_j) - f(a_k)$. In this case, by assumption, there exists some $l \neq j, k$ such that $a_l^T = a_j^T$.

(2) Consider $\tilde{a} \in \Omega^N$, where $\tilde{a}_j^E = a_j^E$, $\tilde{a}_k^E = a_k^E$, and $\tilde{a}_i^E = e^{\min}$, $\forall i \neq j, k$. By NIWE, $F_i(\tilde{a}) = 0$, $\forall i \neq j, k$, and hence $F_j(\tilde{a}) - F_l(\tilde{a}) = F_j(\tilde{a})$.

(3) Suppose $F_j(\tilde{a}) = f(a_j)$. From (2) and NW, $F_j(\tilde{a}) + F_k(\tilde{a}) = f(\tilde{a}_j) + f(\tilde{a}_k) = f(a_j) + f(a_k)$, and thus $F_k(\tilde{a}) = f(a_k)$. This implies that $F_j(\tilde{a}) - F_k(\tilde{a}) = f(a_j) - f(a_k)$. By SRR, $F_j(\tilde{a}) - F_k(\tilde{a}) = F_j(a) - F_k(a)$, which implies, $F_j(a) - F_k(a) = f(a_j) - f(a_k)$. This violates the supposition in (1). Hence, given the supposition in (1), the supposition in the beginning of this paragraph cannot be correct, and we have that $F_j(\tilde{a}) \neq f(a_j)$.

(4) Consider $\hat{a} \in \Omega^N$, where $\hat{a}_j^E = a_j^E$, $\hat{a}_i^E = e^{\min}$, $\forall i \neq j$. By NIWE, $F_i(\hat{a}) = 0$, $\forall i \neq j$, and hence from NW, $F_j(\hat{a}) - F_l(\hat{a}) = f(a_j)$. Taking into account (2) and (3), we have that $F_j(\tilde{a}) - F_l(\tilde{a}) = F_j(\tilde{a}) \neq f(a_j) = F_j(\hat{a}) - F_l(\hat{a})$. But this violates SRR, and hence the supposition in (1) is not possible. The result follows. ∎

PROPOSITION 4. *Given RDR, a redistribution mechanism F satisfies SRR and NIWE if and only if* $F = F^L$.

PROOF. The if part is trivial, and hence we prove only the only-if part.

(1) We will show that for any $a \in \tilde{\Omega}^N$ and $j \in N$, $F_j(a) = f(a_j)$. By RDR, there exists $k \in N$ such that $a_k^T = a_j^T$ and $a_k^E = e^{\min}$. By NIWE, $F_k(a) = 0$, and hence $F_j(a) - F_k(a) = F_j(a)$.

(2) Consider $\tilde{a} \in \tilde{\Omega}^N$, where $\tilde{a}_j^E = a_j^E$ and $\tilde{a}_i^E = e^{\min}$, $\forall i \neq j$. By NIWE, $F_i(\tilde{a}) = 0$, $\forall i \neq j$. Given NW, this implies $F_j(\tilde{a}) = f(\tilde{a}_j)$, and thus $F_j(\tilde{a}) - F_k(\tilde{a}) = f(\tilde{a}_j)$. By SRR, $F_j(\tilde{a}) - F_k(\tilde{a}) = F_j(a) - F_k(a)$. Taking into account (1), this implies $F_j(a) = f(\tilde{a}_j) = f(a_j)$, and the result follows. ∎

REFERENCES

Anderson, E. 1999. What is the point of equality?. *Ethics* 109:287–337

Arneson, R. 1989. Equality and equal opportunity for welfare. *Philosophical Studies* 56:159–94

Bossert, W. 1995. Redistribution mechanism based on individual characteristics. *Mathematical Social Sciences* 29:1–17

Bossert, W. and M. Fleurbaey. 1996. Redistribution and compensation. *Social Choice and Welfare* 13:343–55

Bowles, S. and H. Gintis. 2000. Reciprocity, self-interest and the welfare state. *Nordic Journal of Political Economy* 26:33–54

Cappelen, A. and B. Tungodden. 2003. How should the costs of compensation be shared?. Unpublished manuscript, Norwegian School of Economics and Business Administration

Cappelen, A. and B. Tungodden. 2005. Relocating the responsibility cut: should more responsibility imply less redistribution?. Forthcoming in *Politics, Philosophy and Economics*

Cohen, G. A. 1989. On the currency of egalitarian justice. *Ethics* 99:906–44

Dennett, D. C. 2003. *Freedom Evolves.* Penguin Books

Dworkin, R. 1981. What is equality? Part 2: equality of resources. *Philosophy and Public Affairs* 10:283–345

Fleurbaey, M. 1995a. Three solutions for the compensation problem. *Journal of Economic Theory* 6:96–106

Fleurbaey, M. 1995b. Equality and responsibility. *European Economic Review* 39:683–9

Fleurbaey, M. 1995c. Equal opportunity or equal social outcome. *Economics and Philosophy* 11:25–55

Fleurbaey, M. and F. Maniquet. 1999. Cooperative production with unequal skills: the solidarity approach to compensation. *Social Choice and Welfare* 16:569–83

Fong, C. 2001. Social preferences, self-interest, and the demand for redistribution. *Journal of Public Economics* 82:225–46

Mirrlees, J. A. 1971. An exploration in the theory of optimal taxation. *Review of Economic Studies* 38:175–208

Nagel, T. 2002. *Concealment and exposure & other essays.* Oxford University Press

Rawls, J. 1971. *A theory of justice.* Harvard University Press

Roemer, J. 1993. A pragmatic theory of responsibility for the egalitarian planner. *Philosophy and Public Affairs* 22:146–66

Roemer, J. E. 1996. *Theories of distributive justice*. Harvard University Press

Roemer, J. E. 1998. *Equality of opportunity.* Harvard University Press

Roemer, J. E. 2002. Equality of opportunity: a progress report. *Social Choice and Welfare* 19:455–71

Sen, A. 1992. *Inequality reexamined.* Harvard University Press

Tungodden, B. 2005. Responsibility and redistribution: the case of first best taxation. *Social Choice and Welfare* 24:33–44