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Adiabatic effects of electrons and ions on electro-acoustic solitary waves in an adiabatic dusty plasma

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Abstract. A dusty plasma consisting of negatively charged cold dust, adiabatic hot ions, and inertia-less adiabatic hot electrons has been considered. The adiabatic effects of electrons and ions on the basic properties of electro-acoustic solitary waves associated with different types of electro-acoustic (viz. ion-acoustic (IA), dust ion-acoustic (DIA), and dust acoustic (DA)) waves are thoroughly investigated by the reductive perturbation method. It is found that the basic properties of the IA, DIA, and DA waves are significantly modified by the adiabatic effects of ions and inertia-less electrons. The implications of our results in space and laboratory dusty plasmas are briefly discussed.

1. Introduction

The study of electro-acoustic (particularly ion-acoustic (IA), dust ion-acoustic (DIA), and dust acoustic (DA)) solitary waves in plasmas is very important not only from an academic point of view, but also from the view of its vital role in understanding the nonlinear features of localized electrostatic disturbances in laboratory and space environments.

The basic features of solitary waves associated with IA waves, in which electron thermal pressure gives rise to a restoring force and ion mass provides the inertia, were first theoretically predicted by Washimi and Tanuiti (1966) by assuming an ideal plasma containing cold ions and isothermal electrons. These basic features (Washimi and Taniuti 1966) were verified by a novel laboratory experiment of Ikezi et al. (1970). The basic features of IA solitary waves, which are relevant to laboratory (Ikezi et al. 1970; Lonngren 1983; Nakamura et al. 1993) and space (Temerin et al. 1982; Boström et al. 1988; Dovner et al. 1994) plasmas, were investigated by Schamel and Bujarbarua (1980), Nishihara and Tajiri (1981), and Cairns et al. (1995) who assumed cold ions and non-adiabatic electrons following a vortex-like distribution, a bi-Maxwellian distribution, and a non-thermal distribution, respectively. All these theoretical works (Washimi and Taniuti 1966; Schamel and Bujarbarua 1980; Nishihara and Tajiri 1981; Cairns et al. 1995) are only valid for a cold ion limit. The effects of finite ion-temperature were then included by different authors (Das 1977; Das 1979; El-Labany and El-Sheikh 1992a,b; Mamun 1997) who assumed adiabatic ions and non-adiabatic electrons following different types of distributions (Washimi and Taniuti 1966; Schamel and Bujarbarua 1980; Nishihara and Tajiri 1981; Cairns et al. 1995). It is obvious that these investigations (Das

1977; Das 1979; El-Labany and El-Sheikh 1992a,b; Mamun 1997) are concerned with different plasma models which are not consistent. The inconsistency of all these plasma models by the above authors arises from the fact that one component (ion fluid) is assumed to be adiabatic, but the other component (electron fluid) is assumed to be non-adiabatic.

It is now well-established theoretically as well as experimentally that dust, which is not neutral, but is negatively charged by electron and ion currents, is ubiquitous in most space and laboratory plasmas (Goertz 1989; Selwyn 1993; Mendis and Rosenberg 1994; Winter 1998; Shukla and Mamun 2002; Ishihara 2007). It has been shown both theoretically (Rao et al. 1990; Shukla and Silin 1992) and experimentally (Barkan et al. 1996; Nakamura et al. 1999; Barkan et al. 1995) that in an unmagnetized dusty plasma the dynamics of such charged dust introduces two types of new eigenmodes, namely DIA and DA waves (Rao et al. 1990; Barkan et al. 1995).

Shukla and Silin (1992) have first theoretically shown that due to the conservation of equilibrium charge density

$$n_{\mathrm{e}0}e + n_{\mathrm{d}0}Z_{\mathrm{d}}e - n_{\mathrm{i}0}e = 0$$

and the strong inequality $n_{\mathrm{e}0} \ll n_{\mathrm{i}0}$ (where n_{s0} is the particle number density of the species s with $s = \mathrm{e}, \mathrm{i}, \mathrm{d}$ for electrons, ions and dust particles, Z_{d} is the number of electrons residing on the dust grain surface, and $-e$ is the electronic charge), a dusty plasma (with negatively charged static dust grains) supports low-frequency DIA waves with phase speed much smaller (larger) than the electron (ion) thermal speed. The dispersion relation (a relation between the wave frequency ω and the wave number k) of the linear DIA waves is (Shukla and Silin 1992)

$$\omega^2 = (n_{\mathrm{i}0}/n_{\mathrm{e}0})k^2C_{\mathrm{i}}^2/\big(1 + k^2\lambda_{\mathrm{De}}^2\big),$$

where $C_{\mathrm{i}} = (k_{\mathrm{B}}T_{\mathrm{e}}/m_{\mathrm{i}})^{1/2}$ is the IA speed (with T_{e} being the electron temperature, k_{B} the Boltzmann constant, and m_{i} the ion mass) and $\lambda_{\mathrm{De}} = (k_{\mathrm{B}}T_{\mathrm{e}}/4\pi n_{\mathrm{e}0}e^2)^{1/2}$ is the electron Debye radius. For a long wavelength limit (viz. $k\lambda_{\mathrm{De}} \ll 1$), the dispersion relation for the DIA waves becomes

$$\omega = (n_{\mathrm{i}0}/n_{\mathrm{e}0})^{1/2}kC_{\mathrm{i}}.$$

This form of spectrum is similar to the usual IA wave spectrum (Lonngren 1983) for a plasma with $n_{\mathrm{i}0} = n_{\mathrm{e}0}$ and $T_{\mathrm{i}} \ll T_{\mathrm{e}}$ (where T_{i} is the ion-fluid temperature). DIA waves have been observed in laboratory experiments (Barkan et al. 1996; Nakamura et al. 1999). The linear properties of the DIA waves in dusty plasmas are now well understood from both theoretical and experimental points of view (Shukla and Silin 1992; Barkan et al. 1996; Nakamura et al. 1999; Shukla and Rosenberg 1999; Verheest 2000; Shukla and Mamun 2002). DIA solitary waves have been investigated by several authors (Bharuthram and Shukla 1992; Nakamura and Sharma 2001; Mamun and Shukla 2002a,b). However, all these latter investigations are limited to a cold ion-fluid limit ($T_{\mathrm{i}} = 0$).

Rao et al. (1990) first predicted the existence of DA waves, where the dust particle mass provides the inertia and the pressures of inertia-less electrons and ions give rise to the restoring force. The predictions of Rao et al. (1990) were conclusively verified by the laboratory experiment of Barkan (1995). Mamun et al. (1996) have investigated DA solitary waves in a two-component unmagnetized dusty plasma consisting of a negatively charged cold dust fluid and an inertia-less

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isothermal ion fluid. The work of Mamun et al. (1996) is only valid when a complete depletion of electrons onto the dust grain surface is possible. A number of theoretical investigations (Ma and Liu 1997; Mamun 1999; Mamun and Shukla 2001) have been carried out into DA solitary waves in order to generalize the work of Mamun et al. (1996) by assuming a three-component unmagnetized dusty plasma consisting of a negatively charged cold dust fluid and inertia-less isothermal electron and ion fluids. These works are only valid for cold dust and isothermal electrons and ions. Recently, the effects of the dust fluid temperature on DA solitary waves have been investigated by a number of authors (Mendoza-Briceño et al. 2000; Gill et al. 2004; Sayed and Mamun 2007). Mendoza-Briceño et al. (2000) assumed a two-component dusty plasma containing adiabatic dust fluid and non-adiabatic ions following the non-thermal distribution of Cairns et al. (1995), and they studied the effect of the dust fluid temperature on DA solitary waves by the pseudo-potential approach (Bernstein et al. 1957). Gill et al. (2004) assumed a dusty plasma containing adiabatic dust fluid and non-adiabatic ions following the bi-Maxwellian distribution of Nishihara and Tajiri (1981), and they also studied the effect of the dust fluid temperature on DA solitary waves by the pseudo-potential approach. Sayed and Mamun (2007) assumed a dusty plasma containing adiabatic dust fluid and non-adiabatic (isothermal) inertia-less electron and ion fluid, and they studied the effect of the dust fluid temperature on DA solitary waves by the reductive perturbation method (Washimi and Taniuti 1966). It is obvious that all these above investigations are concerned with different dusty plasma models, which are not consistent (appropriate). The inconsistency of all these dusty plasma models arises from the consideration of one component (dust) being adiabatic, and other components (electrons or ions or both) being non-adiabatic.

Therefore, in the present work, we consider a consistent and realistic dusty plasma system containing non-inertial adiabatic electrons, adiabatic ions, and negatively charged static dust in order to perform a proper investigation of the basic properties of small, but finite-amplitude IA, DIA, and DA solitary waves by the reductive perturbation method (Washimi and Taniuti 1966).

This paper is organized as follows. The basic equations governing the adiabatic plasma system under consideration are given in Sec. 2. The Korteweg–de Vries (KdV) equation is derived, and is analyzed in order to study small but finite-amplitude IA, DIA, and DA solitary waves in Sec. 3. A brief discussion is presented in Sec. 4.

2. Governing equations

We consider an unmagnetized dusty plasma consisting of negatively charged cold dust fluid, adiabatic hot ions, and inertia-less adiabatic hot electrons. The dynamics of the electro-acoustic waves in one-dimensional form in such a dusty plasma system is governed by

$$\frac{\partial n_s}{\partial t} + \frac{\partial}{\partial x}(n_s u_s) = 0, \tag{2.1}$$

$$\frac{\partial u_{\mathrm{d}}}{\partial t} + u_{\mathrm{d}} \frac{\partial u_{\mathrm{d}}}{\partial x} = \frac{Z_{\mathrm{d}} e}{m_{\mathrm{d}}} \frac{\partial \phi}{\partial x}, \tag{2.2}$$

$$\frac{\partial u_{\mathrm{i}}}{\partial t} + u_{\mathrm{i}} \frac{\partial u_{\mathrm{i}}}{\partial x} = -\frac{e}{m_{\mathrm{i}}} \frac{\partial \phi}{\partial x} - \frac{1}{m_{\mathrm{i}} n_{\mathrm{i}}} \frac{\partial P_{\mathrm{i}}}{\partial x}, \tag{2.3}$$

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$$\frac{\partial P_j}{\partial t} + u_j \frac{\partial P_j}{\partial x} + \gamma_j P_j \frac{\partial u_j}{\partial x} = 0, \tag{2.4}$$

$$e\frac{\partial \phi}{\partial x} - \frac{1}{n_{\rm e}} \frac{\partial P_{\rm e}}{\partial x} = 0, \tag{2.5}$$

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e(n_{\rm e} - n_{\rm i} + Z_{\rm d} n_{\rm d}), \tag{2.6}$$

where n_s is the number density of dusty plasma species s with $s = {\rm e, i, d}$ for electrons, ions and dust, u_s is the fluid speed of dusty plasma species s, ϕ is the wave potential, $Z_{\rm d}$ is the number of electrons residing on a dust grain surface, $m_{\rm d}$ is the mass of the dust, P_j is the thermal pressure of the plasma species j with $j = {\rm e, i}$ for electrons and ions, $m_{\rm i}$ is the mass of the ion, γ_j is the adiabatic constant for plasma species j, $m_{\rm e}$ is the mass of the electron, and e is the magnitude of the electronic charge.

3. Derivation of the KdV equation

Now, we derive the KdV equation from (2.1)–(2.6), by employing the reductive perturbation technique and the stretched coordinates $\zeta = \epsilon^{1/2}(x - v_0 t)$ and $\tau = \epsilon^{3/2} t$, where ϵ is a small parameter measuring the weakness of the dispersion, and v_0 is the phase speed of the electro-acoustic waves. We can express (2.1)–(2.6) in terms of ζ and τ as

$$\epsilon^{3/2}\frac{\partial n_s}{\partial \tau} - \epsilon^{1/2} v_0 \frac{\partial n_s}{\partial \zeta} + \epsilon^{1/2}\frac{\partial}{\partial \zeta}(n_s u_s) = 0, \tag{3.1}$$

$$\epsilon^{3/2}\frac{\partial u_{\rm d}}{\partial \tau} - \epsilon^{1/2} v_0 \frac{\partial u_{\rm d}}{\partial \zeta} + \epsilon^{1/2} u_{\rm d}\frac{\partial u_{\rm d}}{\partial \zeta} = \epsilon^{1/2}\frac{Z_{\rm d} e}{m_{\rm d}}\frac{\partial \phi}{\partial \zeta}, \tag{3.2}$$

$$\epsilon^{3/2}\frac{\partial u_{\rm i}}{\partial \tau} - \epsilon^{1/2} v_0 \frac{\partial u_{\rm i}}{\partial \zeta} + \epsilon^{1/2} u_{\rm i}\frac{\partial u_{\rm i}}{\partial \zeta} = -\epsilon^{1/2}\frac{e}{m_{\rm i}}\frac{\partial \phi}{\partial \zeta} - \epsilon^{1/2}\frac{1}{m_{\rm i} n_{\rm i}}\frac{\partial P_{\rm i}}{\partial \zeta}, \tag{3.3}$$

$$\epsilon^{3/2}\frac{\partial P_j}{\partial \tau} - \epsilon^{1/2} v_0 \frac{\partial P_j}{\partial \zeta} + \epsilon^{1/2} u_j\frac{\partial P_j}{\partial \zeta} + \epsilon^{1/2}\gamma_j P_j \frac{\partial u_j}{\partial \zeta} = 0. \tag{3.4}$$

$$\epsilon^{1/2} e\frac{\partial \phi}{\partial \zeta} - \epsilon^{1/2}\frac{1}{n_{\rm e}}\frac{\partial P_{\rm e}}{\partial \zeta} = 0, \tag{3.5}$$

$$\epsilon\frac{\partial^2 \phi}{\partial \zeta^2} = 4\pi e(n_{\rm e} - n_{\rm i} + Z_{\rm d} n_{\rm d}). \tag{3.6}$$

We can expand the variables n_s, u_s, P_j, and ϕ in a power series of ϵ as

$$n_s = n_{s0} + \epsilon n_s^{(1)} + \epsilon^2 n_s^{(2)} + \cdots, \tag{3.7}$$

$$u_s = \epsilon u_s^{(1)} + \epsilon^2 u_s^{(2)} + \cdots, \tag{3.8}$$

$$P_j = P_{j0} + \epsilon P_j^{(1)} + \epsilon^2 P_j^{(2)} + \cdots, \tag{3.9}$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots, \tag{3.10}$$

where n_{s0} is the number density of dusty plasma species s at equilibrium and P_{j0} is the thermal pressure of plasma species j at equilibrium. Now, substituting (3.7)–(3.10) into (3.1)–(3.6) and taking the coefficients of $\epsilon^{3/2}$ from (3.1)–(3.5) and ϵ from (3.6) we get

$$n_{\mathrm{d}}^{(1)} = \frac{n_{\mathrm{d0}}}{v_0} u_{\mathrm{d}}^{(1)} = -\frac{Z_{\mathrm{d}} e n_{\mathrm{d0}}}{m_{\mathrm{d}} v_0^2} \phi^{(1)}, \tag{3.11}$$

$$n_{\mathrm{i}}^{(1)} = \frac{n_{\mathrm{i0}}}{v_0} u_{\mathrm{i}}^{(1)} = -\left(\frac{n_{\mathrm{i0}}^2 e}{\gamma_{\mathrm{i}} P_{\mathrm{i0}} - m_{\mathrm{i}} n_{\mathrm{i0}} v_0^2}\right) \phi^{(1)}, \tag{3.12}$$

$$n_{\mathrm{e}}^{(1)} = \frac{n_{\mathrm{e0}}}{v_0} u_{\mathrm{e}}^{(1)} = \frac{n_{\mathrm{e0}}^2 e}{\gamma_{\mathrm{e}} P_{\mathrm{e0}}} \phi^{(1)}, \tag{3.13}$$

$$4\pi e \left[n_{\mathrm{e}}^{(1)} - n_{\mathrm{i}}^{(1)} + Z_{\mathrm{d}} n_{\mathrm{d}}^{(1)} \right] = 0. \tag{3.14}$$

Using (3.11)–(3.13) into (3.14), we have

$$n_{\mathrm{e0}}^2 m_{\mathrm{d}} v_0^2 X_p + \gamma_{\mathrm{e}} P_{\mathrm{e0}} n_{\mathrm{i0}}^2 m_{\mathrm{d}} v_0^2 - \gamma_{\mathrm{e}} P_{\mathrm{e0}} Z_{\mathrm{d}}^2 n_{\mathrm{d0}} X_p = 0, \tag{3.15}$$

where $X_p = (\gamma_{\mathrm{i}} P_{\mathrm{i0}} - m_{\mathrm{i}} n_{\mathrm{i0}} v_0^2)$. Equation (3.15) is the linear dispersion relation (a relation between ω and k, where ω is the wave frequency and k is the propagation constant) for the electro-acoustic waves propagating in the dusty plasma model. We assume that at equilibrium the ion-thermal (electron-thermal) pressure is isothermal, i.e. $P_{\mathrm{i0}} = n_{\mathrm{i0}} k_{\mathrm{B}} T_{\mathrm{i0}}$ ($P_{\mathrm{e0}} = n_{\mathrm{e0}} k_{\mathrm{B}} T_{\mathrm{e0}}$), where T_{i0} (T_{e0}) is the ion (electron) temperature at equilibrium and k_{B} is the Boltzman constant. We now consider three special cases, viz. IA waves, DIA waves, and DA waves.

3.1. IA waves

We first assume that no dust is present (i.e. $n_{\mathrm{d0}} \to 0$ and $n_{\mathrm{i0}} = n_{\mathrm{e0}}$) and $\gamma_{\mathrm{e}} = \gamma_{\mathrm{i}} = \gamma$ in (3.15), so we obtain

$$v_0 = \sqrt{\gamma(1+\sigma)} C_{\mathrm{i}}, \tag{3.16}$$

where $\sigma = T_{\mathrm{i0}}/T_{\mathrm{e0}}$ and $C_{\mathrm{i}} = (k_{\mathrm{B}} T_{\mathrm{e0}}/m_{\mathrm{i}})^{1/2}$ is the IA speed. For $\gamma = 1$, $T_{\mathrm{i0}} \to 0$ (i.e. $\sigma \to 0$), (3.16) becomes $v_0 = \omega/k = C_{\mathrm{i}}$. This is the well-known linear dispersion relation for the IA waves in electron–ion plasma. Therefore, IA waves are those waves where the restoring force comes from the electron thermal pressure, and inertia is provided by the ion mass.

3.2. DIA waves

We next assume that dust is static (i.e. $m_{\mathrm{d}} \to \infty$) and $\gamma_{\mathrm{e}} = \gamma_{\mathrm{i}} = \gamma$, so then from (3.15) we obtain

$$v_0 = \sqrt{\frac{\gamma(1+\mu\sigma)}{\mu}} C_{\mathrm{i}}, \tag{3.17}$$

where $\mu = n_{\mathrm{e0}}/n_{\mathrm{i0}}$ and m_{d} is the dust mass. For $\gamma = 1$ and $T_{\mathrm{i0}} \to 0$ (i.e. $\sigma \to 0$), (3.17) becomes $v_0 = \omega/k = (n_{\mathrm{i0}}/n_{\mathrm{e0}})^{1/2} C_{\mathrm{i}}$. This is the well-known linear dispersion relation for DIA waves propagating in dusty plasma consisting of Maxwellian electrons and ions, and stationary dust. This means that DIA waves are IA waves modified by the term $(n_{\mathrm{i0}}/n_{\mathrm{e0}})^{1/2}$, where the restoring force comes from the electron thermal pressure, and inertia is provided by the ion mass.

3.3. DA waves

We finally assume that electrons and ions are Maxwellian and dust is mobile (i.e. $m_{\rm i} \to 0$) and $\gamma_{\rm e} = \gamma_{\rm i} = \gamma$ in (3.15), so we get

$$v_0 = \sqrt{\gamma \left(\frac{1-\mu}{1+\mu\sigma} \right)} C_{\rm d}, \tag{3.18}$$

where $C_{\rm d} = (Z_{\rm d} k_{\rm B} T_{\rm i0}/m_{\rm d})^{1/2}$ is the DA speed. For $\gamma = 1$ and $n_{\rm e0} \to 0$ (i.e. $\mu \to 0$), (3.18) becomes $v_0 = \omega/k = C_{\rm d}$. This is the well-known linear dispersion relation for DA waves propagating in dusty plasma consisting of Maxwellian electrons and ions, and mobile dust. Therefore, we can say that DA waves are those waves where the restoring force comes from ion thermal pressure, and inertia is provided by the dust mass.

Similarly, substituting (3.7)–(3.10) into (3.1)–(3.6) and equating the coefficients of $\epsilon^{5/2}$ from (3.1)–(3.5) and ϵ^2 from (3.6), we obtain

$$\frac{\partial n_s^{(1)}}{\partial \tau} - v_0 \frac{\partial n_s^{(2)}}{\partial \zeta} + \frac{\partial}{\partial \zeta}\left[n_s^{(1)} u_s^{(1)}\right] + n_{s0} \frac{\partial u_s^{(2)}}{\partial \zeta} = 0, \tag{3.19}$$

$$\frac{\partial u_{\rm d}^{(1)}}{\partial \tau} - v_0 \frac{\partial u_{\rm d}^{(2)}}{\partial \zeta} + u_{\rm d}^{(1)} \frac{\partial u_{\rm d}^{(1)}}{\partial \zeta} = \frac{Z_{\rm d} e}{m_{\rm d}} \frac{\partial \phi^{(2)}}{\partial \zeta}, \tag{3.20}$$

$$n_{\rm i0} \frac{\partial u_{\rm i}^{(1)}}{\partial \tau} - v_0 n_{\rm i0} \frac{\partial u_{\rm i}^{(2)}}{\partial \zeta} - v_0 n_{\rm i}^{(1)} \frac{\partial u_{\rm i}^{(1)}}{\partial \zeta} + n_{\rm i0} u_{\rm i}^{(1)} \frac{\partial u_{\rm i}^{(1)}}{\partial \zeta}$$

$$= -\frac{e}{m_{\rm i}} n_{\rm i0} \frac{\partial \phi^{(2)}}{\partial \zeta} - \frac{e}{m_{\rm i}} n_{\rm i}^{(1)} \frac{\partial \phi^{(1)}}{\partial \zeta} - \frac{1}{m_{\rm i}} \frac{\partial P_{\rm i}^{(2)}}{\partial \zeta}, \tag{3.21}$$

$$\frac{\partial P_j^{(1)}}{\partial \tau} - v_0 \frac{\partial P_j^{(2)}}{\partial \zeta} + u_j^{(1)} \frac{\partial P_j^{(1)}}{\partial \zeta} + \gamma_j P_j^{(1)} \frac{\partial u_j^{(1)}}{\partial \zeta} + \gamma_j P_{j0} \frac{\partial u_j^{(2)}}{\partial \zeta} = 0, \tag{3.22}$$

$$e n_{\rm e}^{(1)} \frac{\partial \phi^{(1)}}{\partial \zeta} + e n_{\rm e0} \frac{\partial \phi^{(2)}}{\partial \zeta} - \frac{\partial P_{\rm e}^{(2)}}{\partial \zeta} = 0, \tag{3.23}$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} = 4\pi e\left[n_{\rm e}^{(2)} - n_{\rm i}^{(2)} + Z_{\rm d} n_{\rm d}^{(2)}\right]. \tag{3.24}$$

Now, using (3.11)–(3.15), and eliminating $n_s^{(2)}$, $u_s^{(2)}$, $P_j^{(2)}$ and $\phi^{(2)}$, we finally obtain

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \zeta} + B \frac{\partial^3 \phi^{(1)}}{\partial \zeta^3} = 0, \tag{3.25}$$

where the coefficients A and B are given by

$$A = \frac{e\left(\gamma_{\rm e} P_{\rm e0} Z_{\rm d}^2 n_{\rm d0} X_p^2 H - m_{\rm d} v_0^2 G I\right)}{2\gamma_{\rm e} P_{\rm e0} m_{\rm d} v_0^2 X_p \left(\gamma_{\rm e} P_{\rm e0} Z_{\rm d}^2 n_{\rm d0} X_p + n_{\rm i0} m_{\rm i} v_0^2 G\right)}, \tag{3.26}$$

$$B = \frac{\gamma_{\rm e} P_{\rm e0} m_{\rm d} v_0^3 X_p}{8\pi e^2 \left(\gamma_{\rm e} P_{\rm e0} Z_{\rm d}^2 n_{\rm d0} X_p + n_{\rm i0} m_{\rm i} v_0^2 G\right)}, \tag{3.27}$$

where

$$G = \left(\gamma_{\rm e} P_{\rm e0} Z_{\rm d}^2 n_{\rm d0} - n_{\rm e0}^2 m_{\rm d} v_0^2\right), \quad H = v_0\left[3\gamma_{\rm e} P_{\rm e0} Z_{\rm d} + n_{\rm e0} m_{\rm d} v_0^2 (2-\gamma_{\rm e})\right]$$

and

$$I = \gamma_e P_{e0} n_{i0} v_0 [3X_p - \gamma_i P_{i0}(1+\gamma_i)] + n_{e0} X_p^2 v_0 (2-\gamma_e).$$

Equation (3.25) is the KdV equation describing the nonlinear propagation of electro-acoustic waves in an unmagnetized dusty plasma consisting of negatively charged cold dust fluid, adiabatic hot ions, and inertia-less adiabatic hot electrons.

3.4. Stationary solution of the KdV equation

The stationary solution of this KdV equation is obtained by transforming the independent variables ζ and τ to $\xi = \zeta - u_0\tau$ and $\tau = \tau$, where u_0 is a constant velocity, and imposing the appropriate boundary conditions, viz. $\phi^{(1)} \to 0$, $\partial\phi^{(1)}/\partial\xi \to 0$, $\partial^2\phi^{(1)}/\partial\xi^2 \to 0$ at $\xi \to \pm\infty$. Thus, one can express the steady-state solution of the KdV equation as

$$\phi^{(1)} = \phi_m \operatorname{sech}^2\left(\frac{\xi}{\Delta}\right), \tag{3.28}$$

where the amplitude ϕ_m and the width Δ are given by

$$\phi_m = \frac{3u_0}{A}, \tag{3.29}$$

$$\Delta = \sqrt{\frac{4B}{u_0}}. \tag{3.30}$$

It is obvious from (3.28)–(3.30) that as u_0 increases, the amplitude (width) of the solitary waves increases (decreases). We now consider three special cases, viz. IA waves, DIA waves, and DA waves.

3.5. IA waves

We first assume that no dust is present (i.e. $n_{d0} \to 0$ and $n_{i0} = n_{e0}$ $(\mu = 1)$) and $\gamma_i = \gamma_e = \gamma$ in (3.26) and (3.27), so the coefficients A and B for IA waves become

$$A = \frac{e(1+\gamma)(1+\sigma)}{2m_i v_0}, \tag{3.31}$$

$$B = -\frac{\gamma^2 \lambda_{De}^2 C_i^2}{2v_0}, \tag{3.32}$$

where $v_0 = [\gamma(1+\sigma)]^{1/2} C_i$ is the phase speed of the IA waves and $\lambda_{De} = (k_B T_{e0}/4\pi n_{e0} e^2)^{1/2}$ is the electron Debye radius.

3.6. DIA waves

We next assume that dust is static (i.e. $m_d \to \infty$) and $\gamma_i = \gamma_e = \gamma$ in (3.26) and (3.27), so the coefficients A and B for DIA waves, in which both positive and negative solitary profiles exist, become

$$A = \frac{e[(1+\gamma)\sigma\mu^2 + 3\mu + \gamma - 2]}{2m_i v_0 \mu}, \tag{3.33}$$

$$B = -\frac{\gamma^2 \lambda_{De}^2 C_i^2}{2v_0 \mu}, \tag{3.34}$$

where $v_0 = [\gamma(1+\mu\sigma)/\mu]^{1/2} C_i$ is the phase speed of the DIA waves.

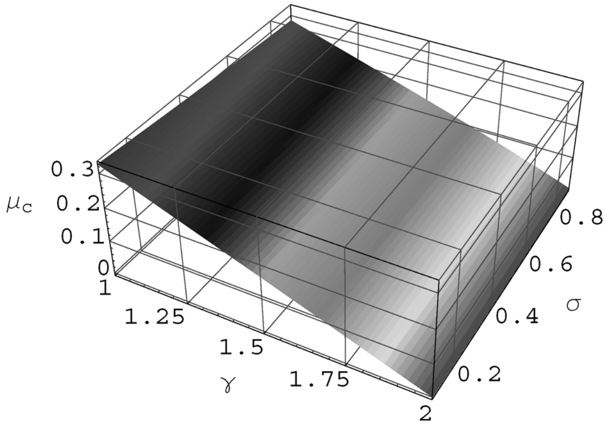

Figure 1. The variation of μ_c (obtained from $A(\mu = \mu_c) = 0$) with γ and σ for DIA waves.

3.7. DA waves

We finally assume that electrons and ions are Maxwellian and dust is mobile (i.e. $m_i \to 0$) and $\gamma_i = \gamma_e = \gamma$ in (3.26) and (3.27), so the coefficients A and B for DA waves, in which only negative solitary profiles exist, become

$$A = -\frac{Z_d e[3(1+\mu\sigma)^2 + (\mu\sigma^2 - 1)(2-\gamma)(1-\mu)]}{2m_d v_0(1+\mu\sigma)}, \tag{3.35}$$

$$B = \frac{\gamma v_0(1-\mu)\lambda_D^2}{2(1+\mu\sigma)^2}, \tag{3.36}$$

where $v_0 = [\gamma(1-\mu)/(1+\mu\sigma)]^{1/2} C_d$ is the phase speed of the DA waves and $\lambda_D = (k_B T_{i0}/4\pi Z_d n_{d0} e^2)^{1/2}$ is the modified ion Debye radius.

It is clear from (3.28), (3.29), and (3.33) that the solitary potential profile for DIA waves is positive (negative) if $A > 0$ ($A < 0$). Therefore, $A(\mu = \mu_c) = 0$, where μ_c is the critical value of μ above (below) which the solitary waves with a positive (negative) potential exist, gives the value of μ_c. To find the parametric regimes for which the positive and negative solitary potentials exist, we have numerically analyzed A, and obtained a $A(\mu = \mu_c) = 0$ surface plot, which is shown in Fig. 1. It is clear from (3.29) that $\phi_m = \infty$ at $\mu = \mu_c$. This means that the small-amplitude DIA solitary waves with a negative (positive) potential exist for a set of plasma parameters corresponding to any point that is much below (above) the $A(\mu = \mu_c) = 0$ surface shown in Fig. 1. Figure 1 shows that μ_c, which is a function of γ and σ, decreases with the increase of γ and σ. For $\gamma = 1$ and $\sigma = 0$, $\mu_c = 1/3$ which is similar to the results of earlier works. The basic properties (amplitude and width) of the small-amplitude DIA solitary structures are examined in Figs 2–5. The amplitude decreases with increasing γ and σ for $\mu = 0.5$, as shown in Fig. 2. Figures 3 and 4 show that the amplitude ϕ_m of the negative (positive) solitary profiles increases (decreases) rapidly with the increase of μ but slowly with the increase of σ for a constant γ. The width of the DIA waves increases linearly with the increase of γ but decreases slowly with σ for constant μ, as shown in Fig. 5. Similarly, it is found that the width of the DIA waves increases linearly with the increase of μ but decreases slowly with σ for constant γ. The basic features

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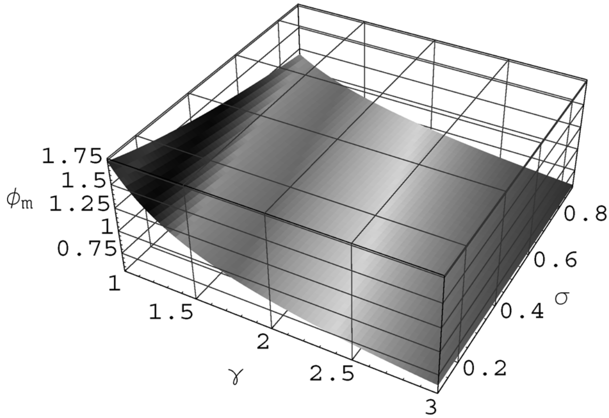

Figure 2. The variation of the amplitude ϕ_m with γ and σ for DIA waves and $\mu = 0.5$.

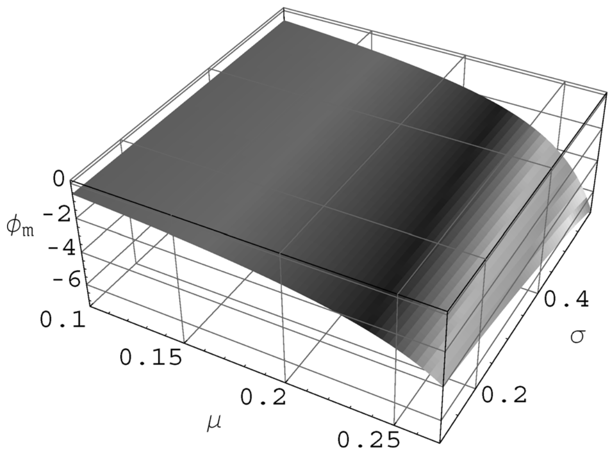

Figure 3. The variation of the amplitude ϕ_m of the negative solitary profile with μ and σ for DIA waves and $\gamma = 1$.

(amplitude and width) of DA waves in which only a negative solitary profile exists are shown in Figs 6 and 7. Figure 6 shows that the magnitude of the amplitude ϕ_m increases (decreases) with γ (σ) for constant μ. Similarly, we have found that the amplitude increases linearly (slowly) with the increase of μ (σ) for constant γ. Figure 7 shows that the width Δ of the DA waves increases linearly (decreases slowly) with γ (σ) for constant μ. Similarly, it is found that Δ decreases linearly (slowly) with μ (σ) for constant γ.

4. Discussion

The adiabatic effects of electrons and ions on the basic features of electro-acoustic solitary waves in an unmagnetized dusty plasma consisting of negatively charged cold dust, adiabatic hot ions, and inertia-less adiabatic hot electrons are investigated theoretically. The results that have been found from this investigation can be summarized as follows.

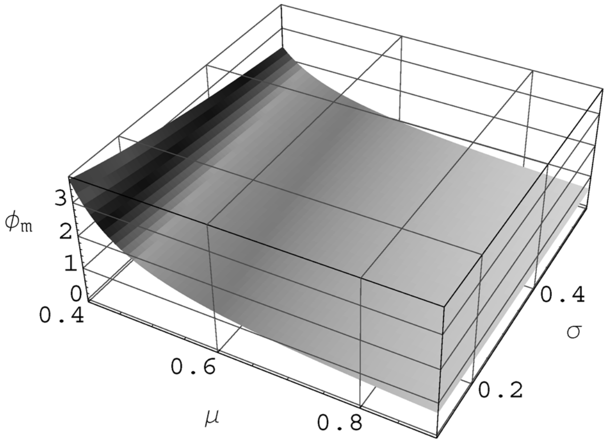

Figure 4. The variation of the amplitude ϕ_m of the positive solitary profile with μ and σ for DIA waves and $\gamma = 1$.

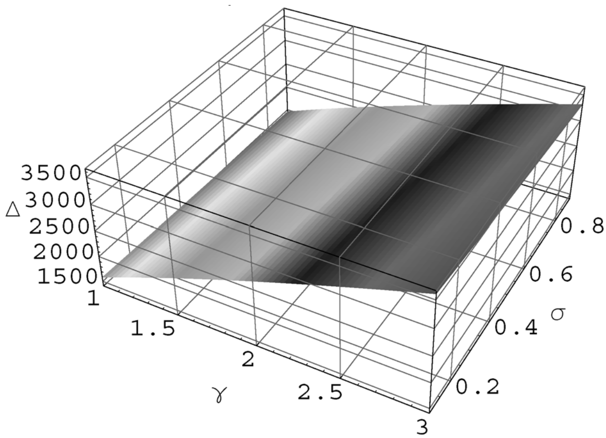

Figure 5. The variation of the width Δ with μ and σ for DIA waves and $\gamma = 0.5$.

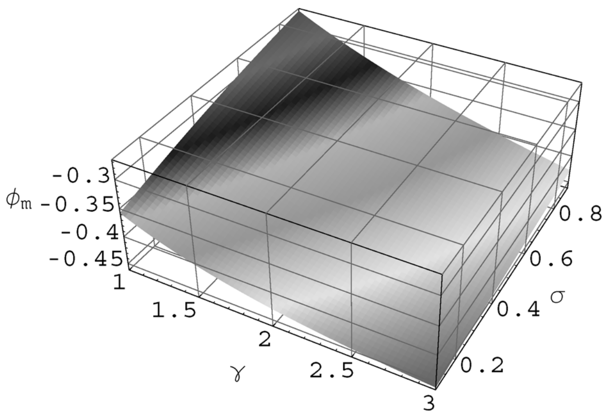

Figure 6. The variation of the amplitude ϕ_m with γ and σ for DA waves and $\mu = 0.5$.

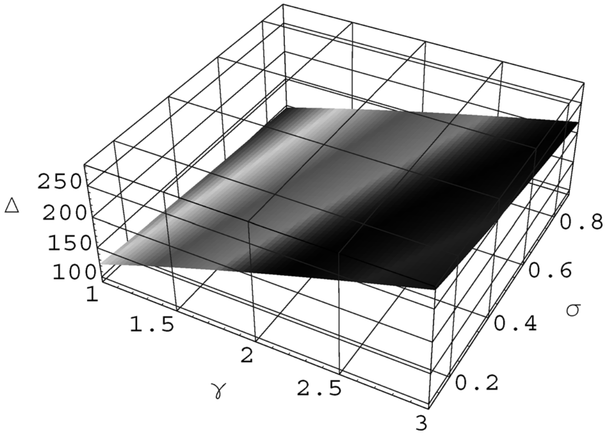

Figure 7. The variation of the width Δ with γ and σ for DA waves and $\mu = 0.5$.

(i) We have found that the DIA solitary waves with a positive (negative) potential can exist for $\mu > \mu_c$ ($\mu < \mu_c$) (as clearly shown in Fig. 1).

(ii) The amplitude of the solitary profiles for DIA waves decreases with increasing values of γ and σ (as clearly shown in Figs 2–4).

(iii) The magnitude of the amplitude of the negative (positive) solitary potential profiles increases (decreases) with increasing μ (as clearly shown in Figs 3 and 4).

(iv) The width of the solitary profiles for DIA waves increases with increasing values of γ but decreases with increasing σ (as clearly shown in Fig. 5).

(v) The magnitude of the amplitude of the solitary profiles for DA waves increases (decreases) with the increase of γ (σ) (as clearly shown in Fig. 6).

(vi) The width of the solitary profiles for DA waves increases with an increase of γ but decreases with increasing values of μ (as clearly shown in Fig. 7).

The ranges ($\gamma = 1 - 3$, $\sigma = 0.1 - 0.9$ and $\mu = 0.1 - 0.9$) of the dusty plasma parameters used in this numerical analysis are very wide. The parameters which we have chosen in our numerical analysis are appropriate for both space (Shukla and Mamun (2002), Mendis and Rosenberg (1994)) as well as laboratory (Barkan et al. (1996), Merlino et al. (1998)) dusty plasmas.

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