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Kinematic calibration of the parallel Delta robot

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SUMMARY

This article deals with the kinematic calibration of the Delta robot. Two different calibration models are introduced: The first one takes into account deviations of all mechanical parts except the spherical joints, which are assumed to be perfect ("model 54"), the second model considers only deviations which affect the position of the end-effector, but not its orientation, assuming that the "spatial parallelogram" remains perfect ("model 24"). A measurement set-up is presented which allows to determine the end-effector's position and orientation with respect to the base. The measurement points are later be used to identify the parameters of the two calibration model resulting in an accuracy improvement of a factor of 12.3 for the position and a factor of 3.7 for the prediction of the orientation.

KEYWORDS: Delta robot; Kinematic calibration; Two calibration models; Implicit calibration.

1. INTRODUCTION

Parallel mechanisms are generally regarded as being highly accurate due to the non-cumulative joint errors.[1] Programming of high precision assembly tasks by the traditional "teach-in" method becomes very expensive. Hence, off-line programming is needed which claims a robot with a small static pose error, or in other words, with a high accuracy. The accuracy of parallel robot can be improved by an appropriated calibration technique, which is the subject of this paper.

The major part of articles addressing calibration methods for parallel robots are based on the Stewart[2] Platform which is a fully parallel non-redundant manipulator with six degrees of freedom (DOF). Many authors[3–6] used a calibration model assuming universal (U-) and spherical (S-)joints to be perfect as well as prismatic (P-)actuators to be perfectly assembled, which leads to a model with 42 kinematic parameters ("model 42"). Not assuming U, S and P-joints to be perfect would lead to 138 kinematic parameters.

However, some of the problems encountered in improvement of accuracy of parallel robots cannot be demonstrated with the Stewart Platform. That is:

a) Model 42 can be established without the risk of introducing mathematical singularities in the parameterization. The 36 parameters used to describe the Cartesian coordinates of the attachment points of the U- and S-joints are free of singularities. The remaining 6 parameters, the transducers' off-set, describing a distance are free of singularities, too. The modeling process is simple due to the lack of rotative (R-)joints, the joint axis of which has to be modeled, and due to the very simple topology of the kinematic chains ("legs") which link the end-effector to the base.
b) Some errors in the pose of the end-effector which occur for manipulators with less than 6 DOF cannot be influenced by its actuators. These non-influenceable errors are imposed by the mechanical structure of the robot and cannot be corrected by a calibration procedure. Taking for instance a SCARA robot with its 4 DOF, it is obvious that the remaining, non-influenceable 2 DOF correspond to the perpendicularly of the end-effector with respect to its base.

A more sophisticated test vehicle for the calibration of parallel robots may be the Delta robot[7] with its 3 translational DOF and rotative actuators (Figure 1). Its kinematic chains as well are more complex than the "legs" of a Stewart Platform, since they are composed of an "arm" which branches into two parallel "forearms". Furthermore, calibration is of particular interest since this robot is marketed by Demaurex, Robotique & Microtechnique S.A.

The Delta robot maintains passively its end-effector's orientation with respect to the base. The concept was termed "spatial parallelogram". For calibration this particularity of the Delta robot introduces kinematic parameters which are nearly unobservable. If for a calibration model the spatial parallelogram is assumed to be perfect, the end-effector size as well as the distance between the two parallel forearms become totally unobservable and therefore unidentifiable. However, small mechanical deviations will disturb the spatial parallelogram and cause small changes of the orientation. The observability of the kinematic parameters mentioned above (end-effector size and distance between the forearms) will still be very small compared to other parameters such as the length of the arms for instance.

The aim of this paper is to present an experimentally verified calibration of the Delta robot using the methods developed in Vischer.[8]

According to Mooring[9] a calibration process consists of four different steps: modeling, measurement, identification and implementation. This paper is structured accordingly. Existing work on the calibration of the Delta robot is first reviewed.

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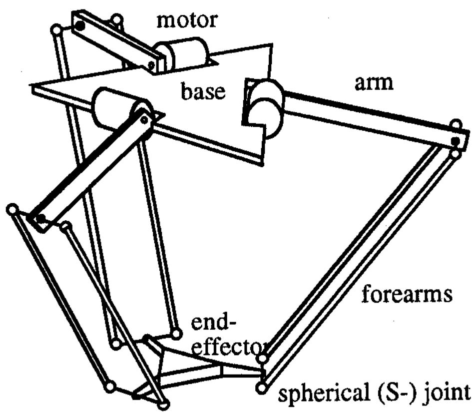

Fig. 1. The Delta robot with 3 translational DOF.

2. SURVEY OF LITERATURE

The first approach to calibrate a Delta robot was made by Zobel.[10] Based on the assumption that the end-effector remains perfectly parallel to the base, he proposed a calibration model containing 18 parameters. To identify these parameters a premeasured fixture (precision plate) with six touch points for full position measurement was used. The fixture could be placed in three different positions on the base plate of the robot. In a first experiment 3 parameters were identified using 3 error equations and it was shown that an error of 10 millimeters could easily be identified. For verification the 3 parameters have also been *directly measured.* However, the introduced test-error of 10 millimeters is *large* (5%) compared to the characteristical length of the robot (length of the arm: 205 mm). Standard manufacturing tolerances of such a piece are in the range of 0.5–0.01 millimeters (0.25–0.005%).

Maurine[11] proposed a recalibration procedure for a Delta robot based on a displacement measurement of a single Laser sensor (triangulation). He stated that a calibrated robot moved to a new work place has to be recalibrated with respect to its environment and that the offsets of the joint transducers must be reidentified. To identify 9 parameters he proposed a two-step method. In a first step a plane is precisely located in parallel to the base plate and a first set of 6 parameters is identified with this set-up. In a second step small cylinders are arranged in a circle on this plane and the remaining three parameters identified. Simulations were perfromed with 200 to 600 micrometers of measurement noise in order to show the robustness of the proposed method. For the experimental part of his work the orientation of the plane as well as the location of the cylinders were identified in a previous step. *Different* sets of parameters were identified *depending* on the *initial values* for the iterative non-linear least square algorithm (problem of multiple minima).

Lintott[12] simulated the calibration procedure of a Delta robot by investigating in a first step a Stewart Platform. He stated that the lower part (sub-structure) of the Delta robot (Figure 1) represents a general Stewart Platform when subjected to mechanical errors. He investigated the optimal choice of measurement points and observed that they migrate towards the edges of the workspace (inverse singularities) and towards the singular configurations within the workspace (direct singularities). However, measurement points located within singularities will cause problems during the identification phase. He adapted the method developed for the calibration of serial robots which requires solving of the direct problem (forward calibration, Figure 9) of the calibration model during identification. This is very time-consuming. Based on simulated noisy measurement data and using the Levenberg-Marquardt algorithm for non-linear least-squares estimation, the Euclidean norm of the error vector in the position could be improved by a factor of 50 (4.33 mm → 0.086 mm). For the orientation he reached an improvement factor of 417 (3.1 degrees → 27 arcseconds). Such high improvement factors are difficult to reach in an experimental calibration as opposed to a simulated one. As a rule of thumb the accuracy of the measurement unit must be a magnitude higher than the level which should be gained by calibration. Using current technology it is difficult and expansive to provide a 3D-orientation measurement unit with an accuracy of 2.7 arcseconds.

3. MODELING

A "good" calibration model must fulfill three criteria: *completeness, equivalence,* and *proportionality*:[13]

- A complete model contains a sufficient number of parameters to describe the mechanical structure of a robot without being redundant. According to Vischer[8] this number of independent parameters (C) can be calculated for a multi-loop parallel robot as follows:

$$C = 3R + P + SS + E + 6L + 6(F - 1) \tag{1}$$

  In addition to open-loop structures a multi-loop mechanism with L loops may contain unsensed spherical joints (S) as well as revolute (R) and primsatic joints (P), which can be either sensed or unsensed. SS counts the number of pairs of S-joints. The number of measurement transducers is counted by E whereas F is the number of arbitrarily located frames. F typically equals two for an arbitrarily located base frame {B} and moving frame {P}.
- A parameterization is proportional if small changes in the geometry of the robot are reflected by small changes in the parameters. Thus, proportionality addresses the problem of *mathematical singularities,* which can be introduced if not choosing carefully the parameterization. The classical example is the DH-(Denavit & Hartenberg) parameterization, which fails to be proportional for nearly parallel joint axes. According to Hayati[14] problems of unproportionality can be avoided by taking for nearly parallel joint axes the H-(Hayati) parameterization and for nearly perpendicular axes the DH-parameterization.
- Two calibration models are equivalent if they are complete and proportional.

A calibration model for the Delta robot considering all possible geometrical deviations would have 138 parameters. This can be verified by equation (1), where an

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S-pair is modeled as a 5R-joint-link train. ($R = 33$, $E = 3$, $L = 5$, $F = 2$). Based on a simulation of a Stewart Platform, Wang[4] concluded that errors in passive multi-DOF joints are negligible compared to other manufacturing errors. Furthermore, special design efforts were made in Vischer[8] to create S-joints which are as perfect as possible. If the S-joints are modeled as perfectly, the number of parameters drops to 54 ($R = 3$, $SS = 6$, $E = 3$, $L = 5$, $F = 2$). This calibration model will be referred to as "model 54". Assuming further that the "spatial parallelogram" remains perfect, model 54 can be reduced to a model containing 24 parameters, which is termed "model 24". In order to achieve proportionality special care has to be taken about three nearly parallel lines, which are the motor axis, the connecting line of the proximal and of the distal S-joints (Figure 2).

3.1. Parameterization

For better understanding of the parameterization the Delta mechanism is first shown without geometric deviations of its mechanical parts (Figure 2, nominal robot) whereas in Figure 3 deviations are introduced. The upside-down representation corresponds to the measurement set-up shown in Figure 6.

For parameterization the end-effector is considered to be fixed to the base-plate (Figure 3). This allows to parameterize model 24 and model 54 using 24 *identical parameters.*

According to Figure 2 and 3 the following points, lines and frames are defined:

$B_{i,1\cdots2}$: Center points of the S-joints attached to the end-effector → distal S-joints

$C_{i,1\cdots2}$: Center points of the S-joints attached to the arms → proximal S-joints

ℓ_d, ℓ_p: Connecting straight section from $B_{i,1}$ to $B_{i,2}$ and $C_{i,1}$ to $C_{i,2}$, respectively

B_iC_i: Mid-point of the section ℓ_d and ℓ_p, respectively

O_i: Projection point of C_i on the motor axis

$O_{i,1\cdots2}$: Points on the motor axis located at a distance of $\pm\ell_d/2$ of O_i

{B}: The base frame {B} is arbitrarily fixed to the base

{P}: The moving frame {P} is arbitrarily fixed to the end-effector

{0}: The z-axis of the distal S-joint frame {0} is parallel to ℓ_p

{1}: The z-axis of the motor frame {1} is parallel to the motor axis

{2}: The twisted motor frame {2} is the frame {1} twisted by the motor angle

- **6 world coordinates** (2)

End-effector's pose:

$${}^{B}\mathbf{P} = \{x, y, z\}^T$$

$${}^{B}_{P}\mathbf{R} = \mathrm{Rot}(z, \gamma) \cdot \mathrm{Rot}(y, \beta) \cdot \mathrm{Rot}(x, \alpha)$$

- **3 joint coordinates** $i = 1 \cdots 3$ (3)

Motor angles θ_i:

$${}^{1}_{2}\mathbf{Q} = \mathrm{Rot}(z, \theta_i)$$

- **54 kinematic parameters** $i = 1 \cdots 3$ (4)

DH-parameters for nearly perpendicular axes:

$${}^{B}_{0}\mathbf{T}_i = {}^{P}_{0}\mathbf{T}_i = \mathrm{Rot}(z, \vartheta_i) \cdot \mathrm{Rot}(x, \alpha_i)$$

H-parameters for nearly parallel axes:

$${}^{0}_{1}\Delta\mathbf{T}_i = \mathrm{Rot}(x, \Delta\alpha_i) \cdot \mathrm{Rot}(y, \Delta\beta_i)$$

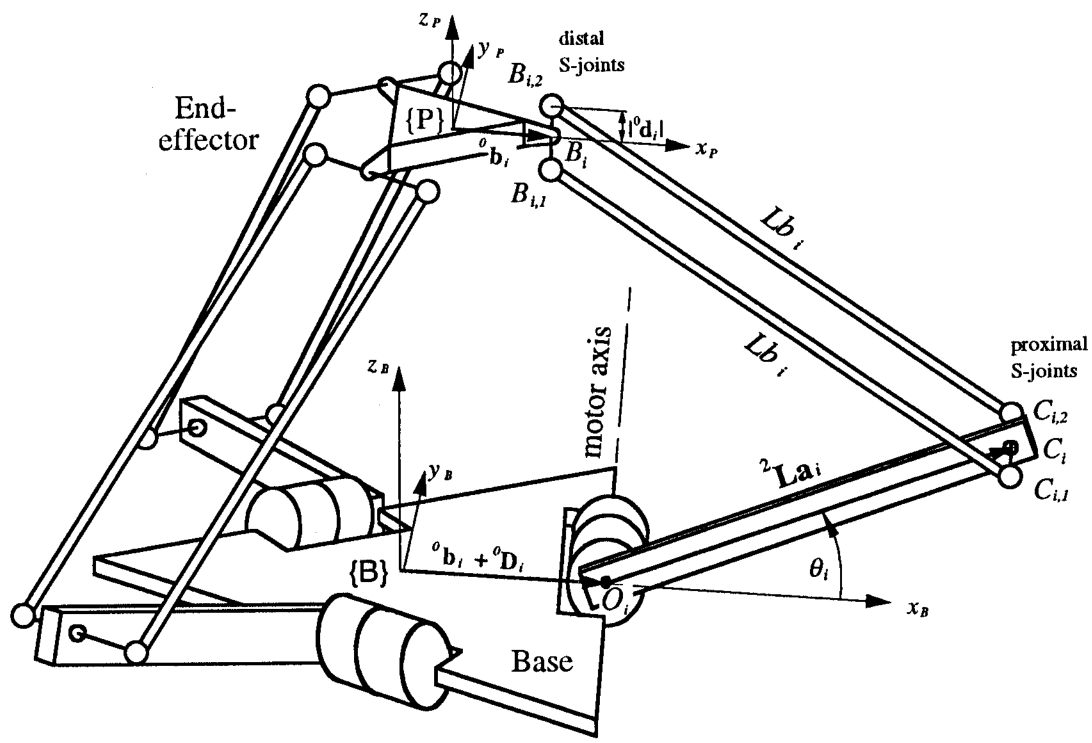

Fig. 2. Delta mechanism *without* deviations.

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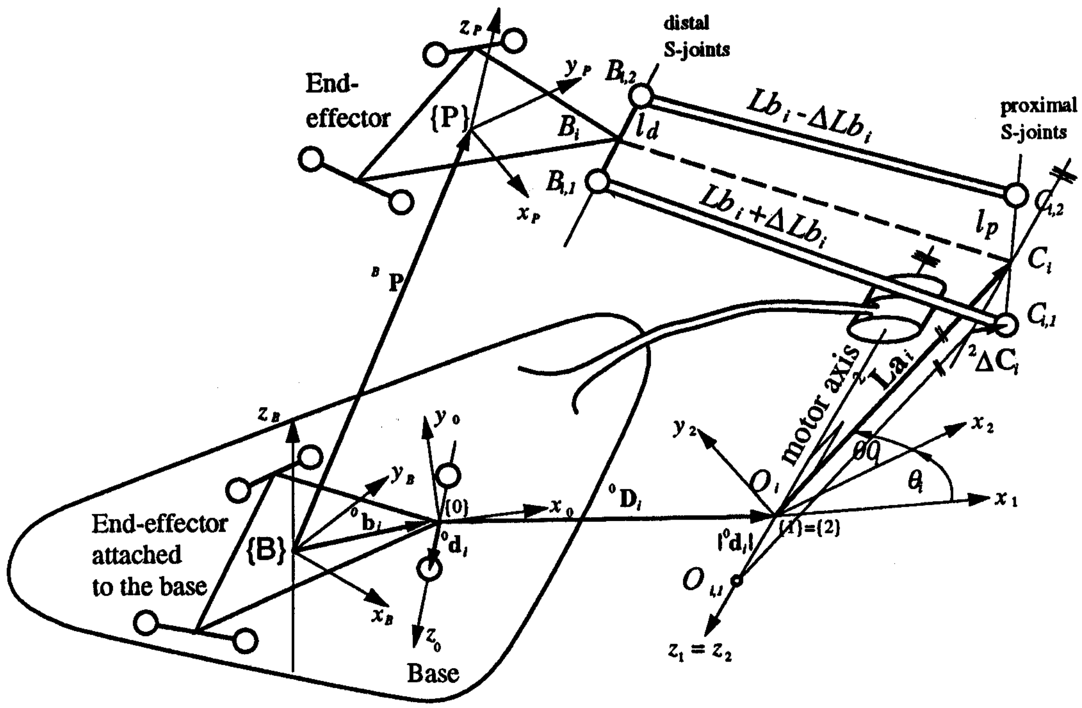

Fig. 3. Parameterization of one main joint-link train *with* geometric deviations.

Vector from B_i of the attached end-effector to the point O_i on the motor axis:

${}^0\mathbf{D}_i = \{D_{xi}, D_{yi}, D_{zi}\}^T$

Vector O_i to the C_i including the encoder offset θO_i and the arm length La_i:

${}^2\mathbf{La}_i = \{La_{xi}, La_{yi}, 0\}^T$

Vector from the origin of {P}-frame to B_i:

${}^0\mathbf{b}_i = \{b_{xi}, b_{yi}, b_{zi}\}^T$

Vector whose z-component is half as long as section ℓ_d:

${}^0\mathbf{d}_i = \{0, 0, d_{zi}\}^T$

Error vector: Difference between vector $\mathbf{OC}_{i,1}$ $(= C_{i,1} - O_{i,1})$ and vector $\mathbf{OC}_i$ $(= C_i - O_i)$:

${}^2\mathbf{\Delta C}_i = \{\Delta C_{xi}, \Delta C_{yi}, \Delta C_{zi}\}^T$

Average length of the forearms;

Lb_i

Half of the difference of the forearms' lengths:

ΔLb_i

These are the scalars, vectors and rotation matrices, which together parametrize the Delta robot completely.

3.2. *Model 54*

As model 42 of the Stewart Platform, model 54 must also satisfy six closure equations since the upper part of a Delta robot (the 6 forearms and the end-effector) is an immobile Stewart Platform (Figure 2). These six closure equations will be coupled in pairs in the three joint coordinates (motor angles). Such a pair of closure equations represents one of the three main joint-link trains (Figure 3). Since model 54 will later on be reduced to model 24, it is more convenient to describe one main joint-link train by the *sum* ($G1$) and the *difference* ($G2$) of these two closure equations. For simplicity the leading sub- and superscripts are dropped yields model 54:

$$G1:\ \mathbf{CB}_i^T \cdot \mathbf{CB}_i + \mathbf{\Delta d}_i^T \cdot \mathbf{\Delta d}_i = Lb_i^2 + \Delta Lb_i^2$$

$$G2:\ \mathbf{CB}_i^T \cdot \mathbf{\Delta d}_i = Lb_i * \Delta Lb_i$$

and

$$\mathbf{CB}_i = \mathbf{P} + \mathbf{R} \cdot \mathbf{T}_i \cdot \mathbf{b}_i - \mathbf{T}_i \cdot (\mathbf{b}_i + \mathbf{D}_i + \mathbf{\Delta T}_i \cdot \mathbf{Q}_i \cdot \mathbf{La}_i)$$

$$\mathbf{\Delta d}_i = \mathbf{R} \cdot \mathbf{T}_i \cdot \mathbf{d}_i - \mathbf{T}_i \cdot \mathbf{\Delta T}_i \cdot (\mathbf{d}_i + \mathbf{Q}_i \cdot \mathbf{\Delta C}_i)$$

$$i = 1 \cdots 3 \quad (5)$$

3.3. *Model 24*

To establish model 24, model 54 is simplified by assuming that the end-effector remains perfectly parallel to the base frame. In other words: The spatial parallelogram is modeled as being perfect:

$$\mathbf{R} = \mathbf{I} \quad \text{where } \mathbf{I} \text{ is the } 3 \times 3 \text{ identity matrix} \quad (6)$$

The number of world coordinates (equation (2)) is reduced from 6 to the 3 Cartesian coordinates describing the origin of the {P}-frame. This simplification is only valid if the three lines given by the axis of the motor, the connecting section of the proximal S-joints and the connecting section of the distal S-joints remain perfectly parallel to each other. In order to reach this 18 parameters are fixed on their nominal values.

$$P1:\ \mathbf{\Delta T}_i = \mathbf{I}, \quad \mathbf{\Delta C}_i = \mathbf{0}, \quad \Delta Lb_i = 0 \quad i = 1 \cdots 3 \quad (7)$$

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Substituting equation (6) and equation (7) into equation (5) shows that the $\mathbf{b}_i$- as well as the $\mathbf{d}_i$-Vector containing altogether another 12 parameters vanish.

$$P2: \ \mathbf{b}_i, \mathbf{d}_i \rightarrow vanish \quad i = 1 \cdots 3 \tag{8}$$

Geometrically, this corresponds to the reduction of the end-effector to a single point and the degeneration of the $R(2S/2S)$] joint-link train to a $R2S$ chain as shown in Figure 4.

A further consequence of equation (8) is the degeneration of the second set of equations $G2$ given in equation (5) to identity ($0 = 0$) whereas the first set $G1$ leads to model 24:

$$\boxed{\begin{aligned} &\mathbf{CB}_i^T \cdot \mathbf{CB}_i = Lb_i^2 \\ &\text{with} \\ &\mathbf{CB}_i = \mathbf{P} - \mathbf{T}_i \cdot (\mathbf{D}_i + \mathbf{Q}_i \cdot \mathbf{La}_i) \end{aligned}} \quad i = 1 \cdots 3 \tag{9}$$

This model can be applied to the Delta robot assuming that its end-effector always stays perfectly parallel to the base, which corresponds to assumption for the nominal model of Clavel.[7] It can therefore be said that model 24 is an extended nominal model.

3.4. Conclusion for model 24 and 54

Reducing model 54 to model 24 leads to the following characteristic properties of model 54:

a) The first set of closure equations $G1$ contains model 24, whereas the second set $G2$ will degenerate to identity (equation (5)).

b) Only 30 of the 54 parameters have an influence on the orientation of the end-effector. These 30 parameters could further be split into two subsets:

c) The generating set $P1$ contains 18 parameters of magnitude Δ reflecting small errors in the joint-link train (equation (7))

d) The amplifying set $P2$ contains 12 parameters of magnitude 1 describing the dimensions of the end-effector and the distance between the forearms (equation (8))

These two sets are related in an interesting way: If $P1$ is considered to be zero, $P2$ has no influence on the pose of the end-effector. The set $P2$ cannot generate pose errors by itself, but if $P1$ is not zero, it will amplify them. In Figure 5 this is represented symbolically by a triangle.

It can further be concluded:

- Variation of $P1$ affects the orientation much more than variation of $P2$. Thus, $P1$ is much more sensitive to orientation errors than $P2$. To build Delta robots with smallest possible orientation errors efforts have thus to be concentrated on the 18 parameters of set $P1$.
- A parameter set which causes small end-effector errors over the whole workspace is nearly unobservable in the identification phase. For identification the parameters of set $P2$ will be fixed to their *nominal values* and only the remaining 42 parameters will be identified.
- The bigger $P2$ becomes, the smaller is the orientation error. This is useful for the choice of the nominal parameters for an application of the Delta robot

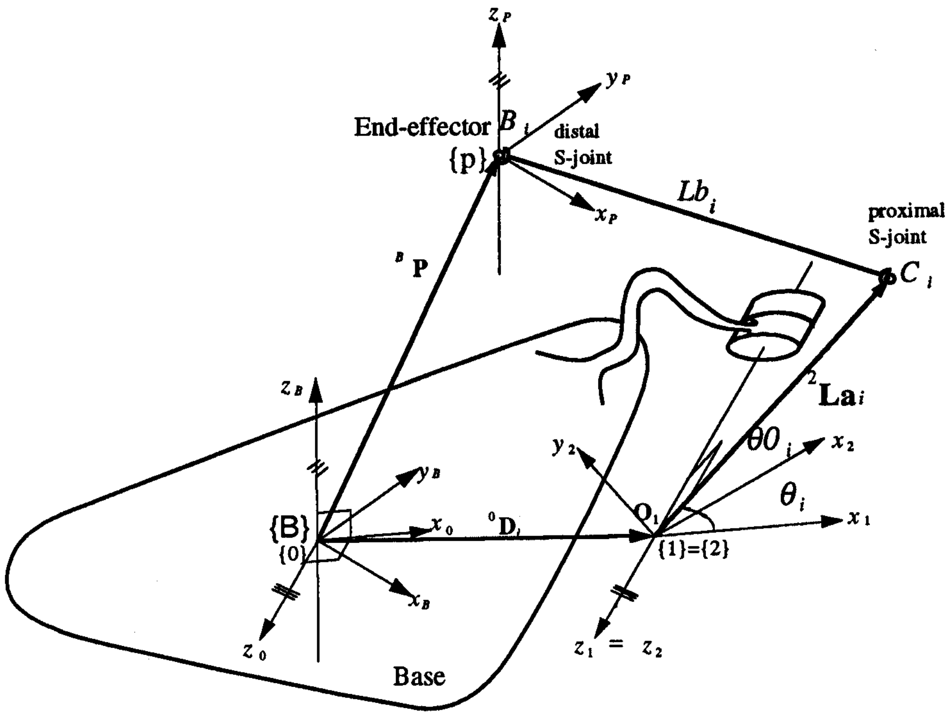

Fig. 4. Geometric interpretation of model 24 as a spatial 3[R2S] structure.

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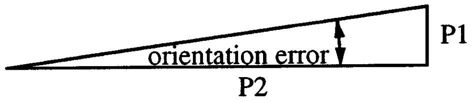

Fig. 5. Symbolical representation of the relation between the parameter set $P1$ and $P2$.

aiming at being very precise. In this case $P2$ should be chosen as large as possible with respect to the remaining parameters. A larger distance between the forearms for instance will decrease the resulting orientation error of the end-effector.

4. MEASUREMENT

The principle goal of the measurement step is to gain some redundant information about the robot to be calibrated. Measurement set-ups can be grouped into classes, depending on whether external measurement devices are used or not:

Set-ups *without* external measurement devices are very dependent on the robot's topology. They provide only *partial* information on the end-effector's pose (e.g. when sliding on a plane[15]). For parallel robots this is fatal since the direct problem of the calibration model has to be solved (in order to substitute x, y, z in the equation of the plane). This results not only in a cumbersome mathematical treatment, but also the problem of multiple minima arises.[8,11]

Set-ups with external measurement devices are more expensive since external sensors are added, but they offer the advantage that the end-effector's full pose (position and orientation) can be measured. By substituting these measurement points into the 3 pairs of closure equations of a main joint-link train, the pairs become decoupled and there is no need to solve the direct nor the inverse problem.

Figure 6 shows our full pose measurement set-up for the Delta robot. Its base is rigidly attached to a 3D measuring machine (TESA Validator 10) whereas the end-effector is fixed to the z-axis of the measuring machine by means of a spherical joint (Figure 7). The accuracy of the measuring machine is ±10 micrometers and the measurement volume $300 \times 300 \times 120$ millimeters.

The end-effector is able to twist about the spherical joint when the orientation changes with respect to the base. Three linear digital probes (TESA-GT22C) orthogonally arranged to each other are used to measure the orientation with an accuracy of ±15 arcseconds (within 1.9 degrees). The measurement volume is ±4.7 degrees for each of the three angles.

The joint angles are measured by high resolution Laser encoders (CANON M1) with an accuracy of 25 arcseconds (accumulated error per revolution).

With the full-pose measurement set-up shown in Figure 6 a set of 74 measurement points was acquired, which are about uniformly distributed within the workspace. This set is used in the next section for identification of the kinematic parameters.

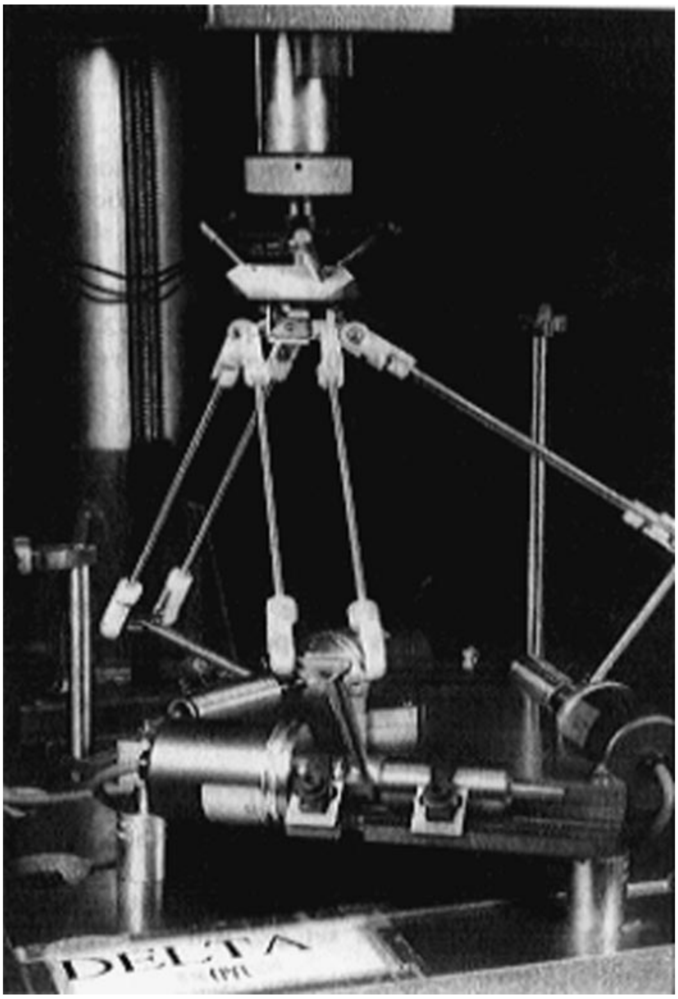

Fig. 6. Full pose measurement set-up.

5. IDENTIFICATION

Identification is the central step of calibration. The parameters of the calibration model are determined to match the measurement data most closely.

Based on simulations of the calibration of a single-loop structure, *implicit calibration* (Figure 8) was proposed as the standard calibration method for parallel mechanisms.[8]

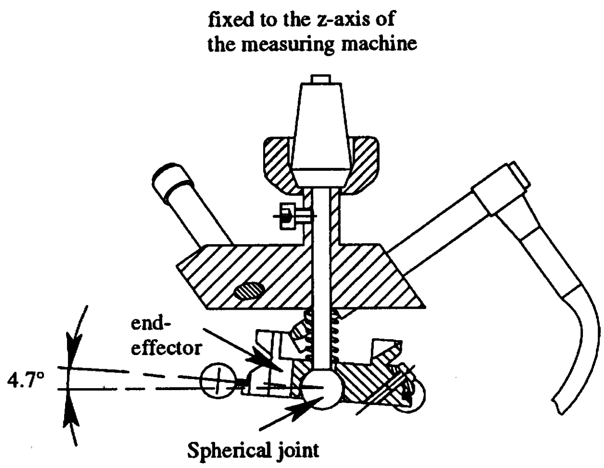

Fig. 7. Orientation measurement unit.

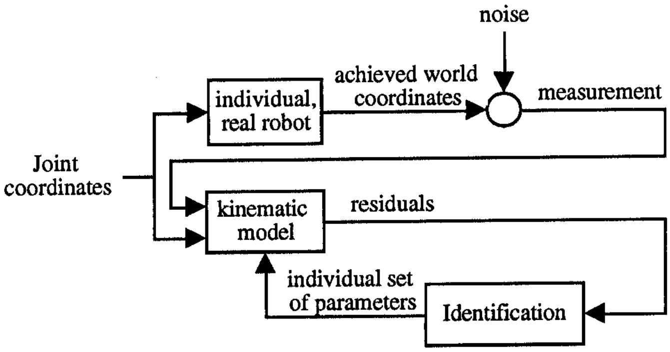

Fig. 8. Implicit calibration for parallel robots.

In contrast to forward calibration (Figure 9), which is the standard calibration method for serial robots, implicit calibration (Figure 8) doesn't require the resolution of the direct problem. This difference becomes important for parallel robots since solving the direct problem requires to sort out the multiple solutions.

In addition to implicit calibration a further interesting method is introduced in this section called "semi-parametric calibration".[7]

5.1. *Implicit calibration*

Implicit calibration of parallel robots starts from the closure equations. These equations are generally coupled in the world coordinates and *decoupled* in the joint coordinates and the kinematic parameters. (Only valid for not too complicated models such as model 24 and model 42. In contrast, model 54 is also pairwise coupled in the joint coordinates as well as the kinematic parameters.) Measuring all joint coordinates and especially *all* world coordinates the identification problem becomes decoupled for each chain. Taking for instance model 24 (equation (9)) the residual (r_j) of one of the three main chains can be written as:

$$r_j = \mathbf{CB}^T \cdot \mathbf{CB} - Lb^2$$
$$\text{with} \qquad j = 1 \cdots N \qquad (10)$$
$$\mathbf{CB} = \hat{\mathbf{P}}_j - \mathbf{T} \cdot (\mathbf{D} + \hat{\mathbf{Q}}_j \cdot \mathbf{La})$$

The ˆ-sign indicates a measured value, which is subjected to measurement noise. Due to this noise the number of measurement points taken (N) has to be larger than the number of kinematic parameters (n). It results an estimation problem where a merit function must be minimized. According to Schröer[16] the square of the residuals is well suited for robot calibration:

$$Q = \mathbf{r}^T \cdot \mathbf{r}$$
$$\text{with} \qquad (11)$$
$$\mathbf{r} = \{r_1, r_2, \ldots, r_j, \ldots, r_N\}^T$$

Equation (10) together with equation (11) yields a non-linear least-squares estimation problem since the kinematic parameters are contained in the model (equation (10)) in a non-linear way. Furthermore, the tolerances allocated to the mechanical parts are ignored, which allows to treat the estimation problem as unconstrained. It can be solved with the Levenberg-Marquardt (LM-)algorithm[17] which is implemented in the "optimization toolbox" of MatLab™. The LM-algorithm is a mixture between the Gauss–Newton and the steepest descent algorithm aiming at keeping the quadratic convergence rate of the Gauss–Newton algorithm by avoiding the problem of rank deficiency of the identification Jacobian.

The nominal parameters of the Delta robot shown in Figure 6 are listed in Table I. They are used as initial values for the iterative LM-algorithm.

- **Identification of model 24**

For the identification of the 8 parameters ($n = 8$) of one of the three main chains (equation (10)) of model 24 (equation (9)) only the position of the end-effector and joint angle are required from the 74 measurement ($N = 74$) points collected. The 74×8 identification Jacobian needed for the LM-algorithm can be analytically differentiated. The eight partial derivatives consist of only 61 different factors.

With 3×9 iterations the LM-algorithm has identified the following parameter set (Table II) in 3×29 seconds (Pentium processor running at 90 MHz) using the nominal parameters given in Table I as initial guess. In contrast to in here discussed implicit calibration (Figure 8), forward calibration (Figure 9) with a numerically

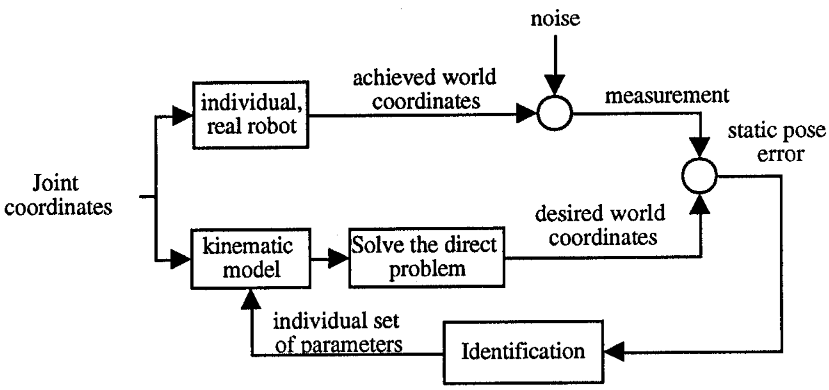

Fig. 9. Forward calibration for serial robots.

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Table I. Nominal parameters of the Delta robot

model 24								
main chain	D_x	D_y	D_z	ϑ	α	La_x	La_y	Lb
unit	[mm]	[mm]	[mm]	[°]	[°]	[mm]	[mm]	[mm]
1	76	−16.5	0	0	90	119.963	−3	240
2	76	−16.5	0	120	90	119.963	−3	240
3	76	−16.5	0	240	90	119.963	−3	240

additional 30 parameters for model 54										
main chain	set $P1$ (equation (7))						set $P2$ (equation (8))			
	$\Delta\alpha$	$\Delta\beta$	ΔC_x	ΔC_y	ΔC_z	ΔLb	d_z	b_x	b_y	b_z
unit	[°]	[°]	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]
1	0	0	0	0	0	0	20	24	0	0
2	0	0	0	0	0	0	20	24	0	0
3	0	0	0	0	0	0	20	24	0	0

derivated identification Jacobian takes about six times longer.

In order to check if there are other minima around this minimum, the initial guess was varied. By increasing the distance in the parameter space between the identified parameters (Table II) and the initial guess, it could be observed that the algorithm – if it converges – always finds the *same* minimum (the minimum listed in Table II). Furthermore, the algorithm is still converging for an initial guess which differs much more from the identified parameters than the nominal set (Table I).

The norm of the position error of the end-effector before and after calibration is shown in Figure 10, whereas the improvement in quantitative terms is given by the mean error and the standard deviation in the Table III.

In Figure 11 the mean as well as the standard deviation of the norm of the position error ($|\{\Delta x, \Delta y, \Delta z\}|$) is plotted versus the number of measurement points used for idefintification. Already 2×8 measurement points

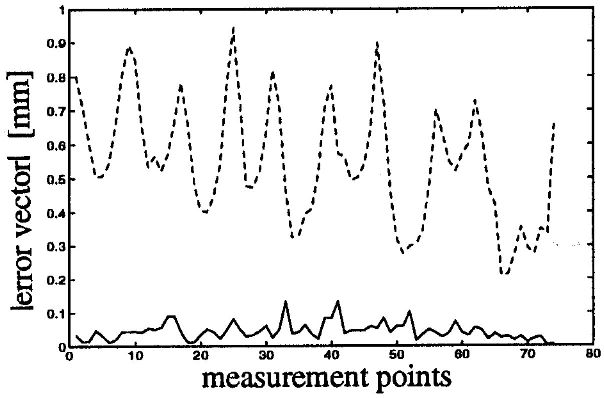

Fig. 10. Position error of the Delta's end-effector before (dashed line) and after (solid line) calibration of model 24 (equation (9)).

yields to a quite reliable calibration. This *rule of thumb* of taking *twice as much measurement points as parameters of the model* (one main chain has 8 kinematic parameters) is also supported by the work of Zhuang.[3]

- **Identification of model 54**

As shown in the modeling section, 12 parameters (set $P2$, equation (8)) of model 54 will be set to their nominal values since they are much less likely to cause pose errors than the remaining 42 parameters. The entire measured pose of the 74 collected measurement points is needed, including the measured deviations of the parallelism between the end-effector and the base.

Implicit calibration was performed by splitting the entire problem into three subproblems. Thus, each main joint-link train forming a double-loop structure together with the measurement device was identified separately. In 3×13 iterations using 3×740 seconds of calculation time, the LM-algorithm has identified the parameter set shown in Table IV using the nominal parameter set (Table I) as an initial guess.

The improvement of the end-effector's position is comparable to the result of model 24 (Figure 10). Furthermore, the calibrated model 54 allows a better prediction of the end-effector's orientation. In Figure 12 the norm of the error vector of the orientation is shown. It corresponds to the difference between the measured values and the values calculated based on model 54

Table II. Identified parameters of model 24 (equation (9))

model 24								
main chain	D_x	D_y	D_z	ϑ	α	La_x	La_y	Lb
unit	[mm]	[mm]	[mm]	[°]	[°]	[mm]	[mm]	[mm]
1	75.949	−16.410	0.404	0.064	89.881	119.960	−3.702	240.130
2	76.141	−16.709	0.182	120.008	90.044	119.933	−3.204	240.067
3	76.023	−16.624	0.062	239.998	90.010	119.990	−2.672	239.949

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Table III. Position error of the end-effector before and after calibration of model 24 (equation (9))

	Position error [μm]			
	Δx	Δy	Δz	$\lvert\{\Delta x, \Delta y, \Delta z\}\rvert$
before calibration				
mean	260	350	−230	550
deviation	220	99	190	180
after calibration				
mean	−0.7	−0.1	−3.6	44
deviation	37	19	29	26

Factor of position improvement

$$F^{\text{Pos}} = \frac{550}{44} = 12.3$$

before and after calibration. The improvement in quantitative terms is given by the mean error and the standard deviation in the Table V.

The standard deviation doesn't decrease a lot since it is very difficult to measure the orientation of a rigid body in space with sufficient accuracy.

However, the mean values have improved by a factor of 12 in the position and 3.7 in the orintation, which proofs that implicit calibration works well for parallel robots.

5.2. Semiparametric calibration

By expanding the closure equations and replacing each coefficient in front of a combination of joint and/or world coordinates by a *linear independent factor,* the calibration model becomes linear, which is referred to as *semiparametric calibration.* Joint coordinates and joint offset are separated by expanding the trigonometric function into a sum of trigonometric functions. The more non-linear the original kinematic parameters are, the more linear factors are needed (model 24→36, model 42→186, model 54→366).

For model 24 (equation (9)) semiparametric calibration becomes also decoupled for one main chain. Hence expanding one of these equations and replacing the 24 kinematic parameters ($p_{1\cdots3,1\cdots8}$) by 36 linear factors ($v_{1\cdots3,1\cdots12}$) yields:

$$\begin{aligned} &v_{i,1} + v_{i,2}*x + v_{i,3}*y + v_{i,4}*z + v_{i,5}*\cos\alpha_i \\ &\quad + v_{i,6}*\sin\alpha_i + v_{i,7}*x*\cos\alpha_i + v_{i,8}*y*\cos\alpha_i \\ &\quad + v_{i,9}*z*\cos\alpha_i + v_{i,10}*x*\sin\alpha_i \\ &\quad + v_{i,11}*y*\sin\alpha_i + v_{i,12}*z*\sin\alpha_i = 0 \\ &\qquad\qquad i = 1\cdots3 \quad (12) \end{aligned}$$

with

$$\begin{aligned} v_{i,1} &= p_{i,1\cdots3}\cdot p^T_{i,1\cdots3} + p_{i,6\cdots7}\cdot p^T_{i,6\cdots7} - p^2_{i,8}; \\ v_{i,2} &= -2(p_{i,1}*\cos p_{i,4} - p_{i,2}*\cos p_{i,5}*\sin p_{i,4} \\ &\quad + p_{i,3}*\sin p_{i,5}*\sin p_{i,4}); \\ v_{i,3} &= -2(p_{i,1}*\sin p_{i,4} + p_{i,2}*\cos p_{i,5}*\cos p_{i,4} \\ &\quad - p_{i,3}*\cos p_{i,4}*\sin p_{i,5}); \\ v_{i,4} &= -2(p_{i,3}*\cos p_{i,5} + p_{i,2}*\sin p_{i,5}); \\ v_{i,5} &= 2(p_{i,1}*p_{i,6} + p_{i,2}*p_{i,7}); \\ v_{i,6} &= 2(p_{i,2}*p_{i,6} - p_{i,1}*p_{i,7}); \\ v_{i,7} &= -2(p_{i,6}*\cos p_{i,4} - p_{i,7}*\cos p_{i,5}*\sin p_{i,4}); \\ v_{i,8} &= -2(p_{i,6}*\sin p_{i,4} + p_{i,7}*\cos p_{i,5}*\cos p_{i,4}); \\ v_{i,9} &= -2p_{i,7}*\sin p_{i,5}; \\ v_{i,10} &= 2(p_{i,7}*\cos p_{i,4} + p_{i,6}*\cos p_{i,5}*\sin p_{i,4}); \\ v_{i,11} &= 2(p_{i,7}*\sin p_{i,4} - p_{i,6}*\cos p_{i,5}*\cos p_{i,4}); \\ v_{i,12} &= -2*p_{i,6}*\sin p_{i,5}; \\ &\qquad\qquad i = 1\cdots3 \quad (13) \end{aligned}$$

Equation (13) shows the linear factor as a function of the kinematic parameters (Table II: $p_{i,1} = D_{i,x}$, $p_{i,2} = D_{i,y}$, $p_{i,3} = D_{i,z}$, $p_{i,4} = \vartheta_i$, etc.). However, since they are assumed to be independent, these geometric constraints are dropped, hence the name "semiparametric" calibration. From now on it is important to work consequently with the 36 linear factors. The direct and inverse problem for instance have to be solved directly from the semiparametric model (equation (12)).

By substituting the measurement points into the equation (12) the residuals of one main chain result in:

$$\begin{aligned} &v_1 + v_2*\hat{x}_j + v_3*\hat{y}_j + v_4*\hat{z}_j + v_5*\cos\hat{\alpha}_j \\ &\quad + v_6*\sin\hat{\alpha}_j + v_7*\cos\hat{\alpha}_j + v_8*\hat{y}_j*\cos\hat{\alpha}_j \\ &\quad + v_9*\hat{z}_j*\cos\hat{\alpha}_j + v_{10}*\hat{x}_j*\sin\hat{\alpha}_j \\ &\quad + v_{11}*\hat{y}_j*\sin\hat{\alpha}_j + v_{12}*\hat{z}_j*\sin\hat{\alpha}_j = \rho_i \\ &\qquad\qquad j = 1\cdots N \quad (14) \end{aligned}$$

Again the square of the residuals (ρ_i) is used as merit function. The same set of 74 measurement points as for implicit calibration is used for identification. The resulting 12×74 matrix was inverted by means of singular values decomposition.[17] Within 0.5 seconds the linear factors ($v_{1\cdots3,1\cdots12}$) of all three main chains were identified leading to the improvement presented in Table VI.

From Table VI and Table III it can be seen that

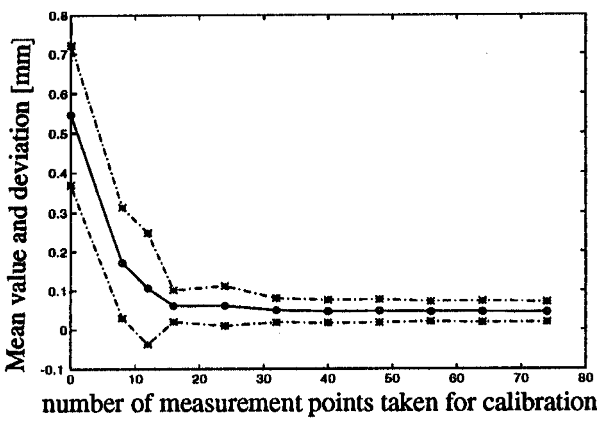

Fig. 11. Mean value (solid line) and standard deviation (dashed line) plotted versus the number of measurement points based on model 24 (equation (9)).

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Table IV. Identified parameters of model 54 (equation (5))

24 parameters influencing the position								
main chain	D_x	D_y	D_z	ϑ	α	La_x	La_y	Lb
unit	[mm]	[mm]	[mm]	[°]	[°]	[mm]	[mm]	[mm]
1	75.902	−16.32	−0.642	−0.517	89.513	119.97	−3.693	240.07
2	76.116	−16.72	−0.793	119.37	90.268	119.94	−3.236	240.11
3	76.063	−16.79	−0.658	239.57	119.98	119.98	−2.673	240.01

30 parameters affecting orientation & position										
main chain	set $P1$ (equation (7))						set $P2$ (equation (8))			
	$\Delta\alpha$	$\Delta\beta$	ΔC_x	ΔC_y	ΔC_z	ΔLb	d_z	b_x	b_y	b_z
unit	[°]	[°]	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]
1	0.343	0.582	−0.023	−0.005	−0.006	0.092	20	24	0	0
2	−0.235	0.610	−0.049	0.069	0.023	0.009	20	24	0	0
3	−0.080	0.404	−0.049	0.076	−0.012	−0.062	20	24	0	0

semiparametric calibration works better than implicit calibration being 174-times faster. However, semi-parametric calibration has the disadvantage of the linear factors not having a physical meaning anymore. This makes it impossible to replace mechanical parts which are out of tolerances. Such quality control is only possible by implicit calibration, which is the reason for proposing this method as standard method for calibration of parallel robots.

6. IMPLEMENTATION

The implementation step deals with the question of how to solve the direct and inverse problem of the calibration model. This is a very difficult task if trying to find the solution by reducing the non-linear system of equations to a univariate polynomial since simplifications (e.g. intersecting axes) which allow the reduction of the nominal model into a polynomial, may not be valid any more. Simple polynomial solutions can be found for model 24 whereas for model 54 this is not possible any more.[8]

However, numerical algorithms such as Newton-Raphson work generally well for such kind of problems. In order to reduce the calculation time solutions must be found which are based on solutions of low order polynomials.

Due to the splitting of model 54 (equation (5)) into two sets of equations ($G1$ and $G2$) a simple algorithm to solve the direct problem (Figure 13) could be found. It is polynomial based and faster than the Newton-Raphson algorithm.

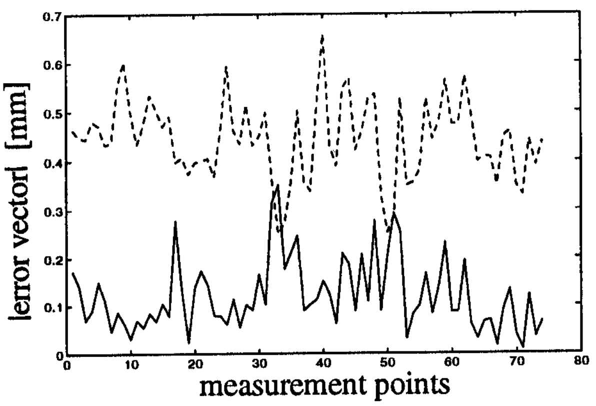

Fig. 12. Orientation error of the Delta end-effector before (dashed line) and after (solid line) calibration of model 54 (equation (5)).

Table V. Orientation error of the end-effector before and after calibration of model 54 (equation (5))

Orientation error [']				
	$\Delta\alpha$	$\Delta\beta$	$\Delta\gamma$	$\lvert\{\Delta\alpha, \Delta\beta, \Delta\gamma\}\rvert$
before calibration				
mean	13	−9.4	18	27
deviation	6.8	8.2	3.1	4.9
after calibration				
mean	−0.10	−0.25	−0.04	7.2
deviation	6.1	5.6	2.2	4.5
Factor of position improvement $F^{Ori} = \frac{27}{7.2} = 3.7$				

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Table IV. Position error of the end-effector before and after calibration of the semiparametric model (equation (12)) which is based on model 24

Position error [μm]				
	Δx	Δy	Δz	$\lvert\{\Delta x, \Delta y, \Delta z\}\rvert$
before calibration				
mean	260	350	−230	550
deviation	220	99	190	180
after calibration				
mean	−0.72	−0.36	0.01	36
deviation	31	18	24	23
Factor of position improvement $F^{Pos} = \frac{550}{36} = 15.2$				

The direct problem of model 54 is that of calculating no, one ore several sets of world coordinates (P, R) for a given set of joint angles (Q). As an initial guess (R^*) the orientation matrix can be set equal to the identity matrix since the end-effector is almost parallel to the base. By substituting this guess (R^*) and the joint angles (Q) into the first set of equations $G1$ an estimation of the end-effector's position (P_i) can be calculated based on a second order univariate polynomial. This estimation as well as the joint angle set are forwarded to the second set of equations $G2$. The rotation matrix can be linearized in $G2$ since the deviations in the parallelism are small. By doing so $G2$ can be linearly solved for the orientation (R_i). This procedure can be continued iteratively. It remains an error due to the linearization of the rotation matrix. However this error is small and the algorithm converging very fast.

Table VII shows a comparison of the calculation time (Motorola 68040 with co-processor) with an accelerated Newton-Raphson algorithm (no updating of the Jacobian, no precalculation of the position with the nominal model, stopping after five steps). The iteration of the cascaded iterative algorithm is stopped after one and a half steps.

The third column of Table VII gives the remaining error in the calculation of the end-effector's pose as compared to a Newton-Raphson solution of machine

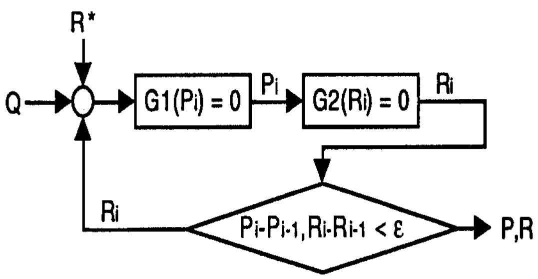

Fig. 13. Cascaded iterative algorithm to solve the direct probem of model 54 (equation (5)).

Table VII. Calculation time of a Newton-Raphson algorithm and the cascaded iterative algorithm (Figure 13)

Algorithm	Direct solution [s]	Remaining pose error {[μm], [arcseconds]}
accelerated Newton-Raphson	2.7	{0.02, 0.3}
Cascaded iterative (Figure 13)	2.0	{0.1, 1.3}

precision. The inverse problem of model 54 is not easy to solve since it is coupled by pairs in the joint coordinates and all six equations are coupled in the orientation angles. However, it can be solved similarly to the direct problem (Figure 13) leading to a calculation time of 1.9 seconds.

7. CONCLUSIONS

In this paper the kinematic calibration of the Delta robot was reported. Two parametric models were established. Model 24 takes only position errors of the end-effector into account, whereas in the model 54 models deviations from the parallelism between the base and the end-effector are also considered.

A measurement set-up was built with a measuring machine and linear probes capable to measure the full pose of the end-effector with respect to the base.

Based on the same set of 74 measurement points the parameters of model 24 and model 54 were identified. The parameter estimation problem was defined using directly the implicit closure equations, which is referred to as implicit calibration. This method enables to solve the direct and inverse problem during identification. Improvement in position of a factor of 12.3 was reached by the identified model 24. Prediction of the orientation could be improved by a factor of 3.4 for the identified model 54.

A second linear calibration method referred to as semiparametric calibration was tested. Starting by expansion of the closure equations and replacing all coefficients in front of the joint and world coordinates by linear factors leads to the semiparametric model. Its linear factors can be identified linearly which is very advantageous, compared to non-linear estimation: no initial guess is needed, no Jacobian is needed, there is no problem with multiple minima since the solution is unique, it is very fast since there is no need for iteration. And last but not least, improvement in the position is with a factor of 15.2 higher than the one of implicit calibration. However, semiparametric calibration allows no quality control of the mechanical parts of the robot and further tests will be necessary to check its reliability. Hence, implicit calibration is proposed as standard calibration method for parallel robots.

In a last section a new algorithm was presented allowing to solve the direct as well as the inverse problem of model 54 faster than with the Newton-Raphson algorithm.

The main contribution of this paper is the experimental verification of the proposed theoretical tool. It was

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shown that improved accuracy of the parallel Delta robot by means of calibration is possible.

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