Fulling Non-uniqueness and the Unruh Effect: A Primer on Some Aspects of Quantum Field Theory*

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We discuss the intertwined topics of Fulling non-uniqueness and the Unruh effect. The Fulling quantization, which is in some sense the natural one for an observer uniformly accelerated through Minkowski spacetime to adopt, is often heralded as a quantization of the Klein-Gordon field which is both physically relevant and unitarily inequivalent to the standard Minkowski quantization. We argue that the Fulling and Minkowski quantizations do not constitute a satisfactory example of physically relevant, unitarily inequivalent quantizations, and indicate what it would take to settle the open question of whether a satisfactory example exists. A popular gloss on the Unruh effect has it that an observer uniformly accelerated through the Minkowski vacuum experiences a thermal flux of Rindler quanta. Taking the Unruh effect, so glossed, to establish that the notion of particle must be relativized to a reference frame, some would use it to demote the particle concept from fundamental status. We explain why technical results do not support the popular gloss and why the attempted demotion of the particle concept is both unsuccessful and unnecessary. Fulling non-uniqueness and the Unruh effect merit attention despite these negative verdicts because they provide excellent vehicles for illustrating key concepts of quantum field theory and for probing foundational issues of considerable philosophical interest.

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1. Introduction. We propose here to explicate and assess the intertwined topics of Fulling non-uniqueness and the Unruh effect. There are good reasons to do so. The topics illustrate some important and problematic features of relativistic quantum field theory (QFT) on flat spacetime, features which are not present in ordinary quantum mechanics (QM) of systems with finitely many degrees of freedom. They lay the foundation for understanding features of QFT on curved spacetime—in particular, the Hawking effect and black hole evaporation. And they have been supposed to have negative implications for the particle concept.

QFT is also known as particle physics. Yet a variety of considerations suggest that the particle notion is not fundamental to QFT. One assault on the particle notion musters what's known as the *Unruh effect.* In this effect, or so the story goes, an observer uniformly accelerated through Minkowski spacetime in its quantum field theoretic vacuum state—a state in which there are no particles to detect, a state in which inertial observers detect no particles—nevertheless detects a thermal flux of particles! Holding that "we can't talk meaningfully about whether such-and-such a state contains particles except in the context of a specified particle detector measurement" (1984, 69), Davies invokes the Unruh effect to claim that there is no context-independent answer to questions about the particle content of quantum field theoretic states. The Unruh effect is supposed to sanction this claim by showing that detectors in different states of motion give different answers, among which there is nothing to choose. Davies' provocative conclusion is that "particles do not exist."

Wald also appeals to the democracy of observers who parse particle contents differently:

> How can an accelerating observer assert that "particles" are present . . . when any inertial observer would assert that, "in reality", all of Minkowski spacetime is devoid of particles? Which of these two observers is "correct" in his assertion? The answer is, of course, that both observers are correct. . . . (1994, 116)

Though less dramatically stated than Davies' moral, Wald's moral also demotes the *particle* concept.

> It simply happens that the natural notion of "particles" defined by accelerating observers . . . differs from the natural notion of particles defined by inertial observers. . . . No paradox arises when one views quantum field theory as, fundamentally, being a theory of local field observables, with the notion of "particles" merely being introduced as a convenient way of labeling states in certain situations. (*ibid*)

We agree wholeheartedly that the particle notion should be demoted in QFT from fundamental to derivative status. And we join the authors just discussed in supposing that such demotion must be accompanied by an account of how the particle notion nevertheless gets the purchase it does in the application of the theory. *But* we do not think that the demotion of the particle concept is supported in any straightforward and unproblematic way by the Unruh effect and the related phenomenon of Fulling non-uniqueness. Nor do we think that the demotion has to be based on these phenomena. In what follows, we attempt to justify these judgments.

So that we might frame an informal sketch of the remainder of this paper, let us cast the demotions in question as follows:

(A1) If the particle notion were fundamental to QFT, there would be a matter of fact about the particle content of quantum field theoretic states.

(A2) The accelerating and inertial observers differ in their attributions of particle content to quantum field theoretic states.

(A3) Nothing privileges one observer's attributions over the other's.

(C4) Therefore, there is no matter of fact about the particle content of quantum field theoretic states. (from (A2) and (A3))

(C5) Therefore, the particle notion is not fundamental. (from (A1) and (C4))

Thus presented, the inference to (C4) appears weak. The accelerating and inertial observers' assessments of particle content could after all differ in the way Mary's and Martha's assessments of the temperature of a vat of liquid differ, when Mary uses a Celsius and Martha uses a Fahrenheit thermometer. Or their assessments of particle content could differ in the way Mary's and Martha's assessments of the strengths of the electric and magnetic fields at a point differ, when Mary and Martha are moving relative to one another. In both the Mary and Martha cases, the translations between Mary's assessment and Martha's are so direct and straightforward that one needn't react to their divergence by denying the existence of the matters of fact they purport to assess. One might instead recognize that those matters of fact have a structure sufficient to account for the divergent assessments: temperature is measured on a scale whose gradation is a matter of conventional choice; **E** and **B** field strengths are tractably relative to observers' states of motion, and so on.

Sections 2-5 develop the technical apparatus for characterizing the relation between the inertial and accelerating observers' assessments of particle content. These sections lay the groundwork for a case that the difference between these assessments is more profound than the differences between Mary's and Martha's assessments, and so perhaps profound enough, given (A3), to sanction the move to (C4). The crux of this case is that the QFT construction associated with the accelerating observer's particle notion is *unitarily inequivalent* to the QFT construction associated with the inertial observer's particle notion.

The existence of unitarily inequivalent representations of the canonical commutation relations (CCRs) is a characteristic feature of QFT, and one that separates it from ordinary QM. Leaving aside the technical niceties, ordinary QM quantizes systems with finitely many degrees of freedom; in such cases, the representation of the CCRs is unique up to unitary equivalence.[1] An isomorphism U of Hilbert space identifies density matrices (states) in one representation with density matrices in the other; U likewise identifies self-adjoint operators (observables) on one representation with self-adjoint operators on the other. When representations are unitarily equivalent, this isomorphism U translates any attribution of physical content one can make in terms of one representation into an attribution of physical content one can make in terms of the other. Roughly speaking, the lesson of the Stone-von Neumann uniqueness theorem is that such translations will in general be available between representations quantizing a system with finitely many degrees of freedom.

When, as in QFT, an infinite number of degrees of freedom are in play, however, there are uncountably many unitarily inequivalent representations. Between such representations there are no isomorphisms U. Thus particle content attributions offered in terms of one representation lack translations mediated by such isomorphisms to particle content attributions offered in terms of unitarily inequivalent representations. *If* the accelerated and inertial observers subscribe to particle notions associated with unitarily inequivalent representations, this unitary inequivalence could secure the inference from (A2) via (A3) to (C4), above, from the deflationary evocation of Mary and Martha scenarios.

But perhaps such inequivalent representations exist merely as mathematical possibilities. If so, it is not evident that cognizance must be taken of them either in the practice or philosophical interpretation of physics—certainly they are not mentioned in many standard physics texts on QFT (e.g. Peskin and Schroeder (1995)), and they play only a passing role in Teller's (1995) interpretation of QFT. The supposed significance of *Fulling non-uniqueness* is that it provides a concrete example of (at least) two physically relevant ways to quantize the scalar Klein-Gordon field that turn out to be not only unitarily inequivalent but inequivalent in a much stronger sense to be made precise below. Moreover, one quantization, the standard Minkowski quantization, is in some sense the natural quantization for inertial observers to adopt, whereas the other, the Fulling quantization, is the natural quantization for some family of accelerating observers to adopt. Thus, on its standard interpretation, Fulling non-uniqueness sug-

1. Technical niceties do matter here. For a system with a finite number of degrees of freedom, the Stone-von Neumann theorem assures uniqueness, up to unitary equivalence, only for irreducible strongly continuous representations of the Weyl form of the CCRs; see Cavallero et al. (1999).

gests that the particle notions of inertial and accelerated observers are both physically relevant and profoundly different.

What makes the inequivalent constructions available is the fact that a portion of Minkowski spacetime, the right Rindler wedge (see Fig. 1), maintains two different timelike Killing fields (which are generators of symmetries of the spacetime metric), the trajectories of one corresponding to inertial motion, the trajectories of the other to uniformly accelerated motion. These different timelike isometries define different senses of "positive frequency" modes of the field and, hence, different vacuum states—the standard Minkowski vacuum state and the Rindler vacuum state. QFT associates with the right Rindler wedge an algebra of local observables, generated from the CCRs governing the field in that region. The Minkowski and Rindler vacuum states determine representations of this wedge algebra which are *disjoint*—not only do the representations fail to be unitarily equivalent, but also no state expressible as a density matrix on one is expressible as a density matrix on the other.

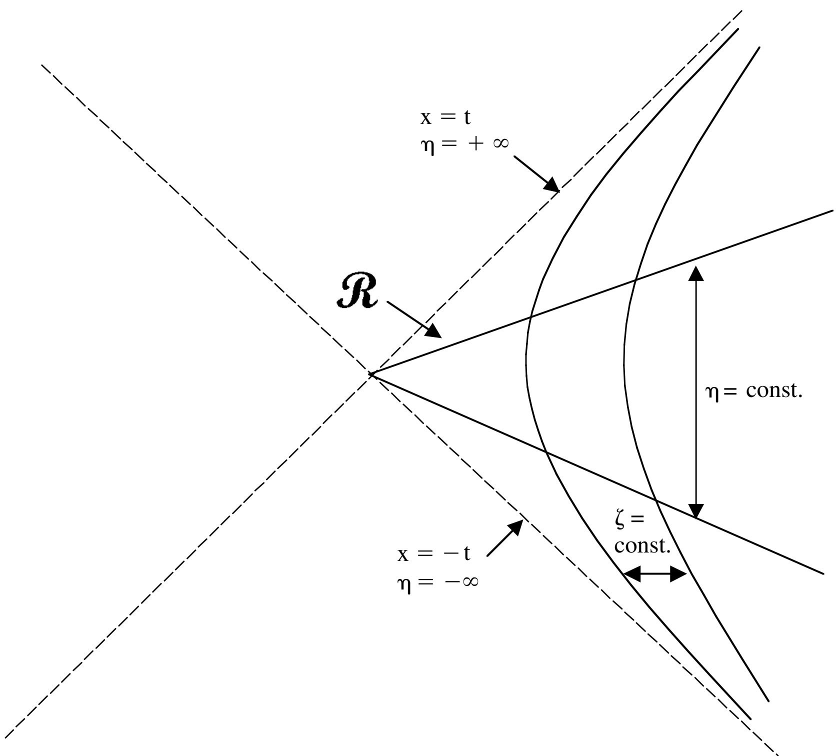

Fig. 1. Right Rindler wedge.

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To explicate Fulling non-uniqueness in more detail, Section 2 introduces the reader to Rindler coordinates, the Rindler frame, and Rindler spacetime. Section 3 describes a method for quantizing the Klein-Gordon field using a static frame on a globally hyperbolic spacetime. Rindler spacetime is a globally hyperbolic spacetime for which the Rindler frame is static. Thus this method of quantization applies, and yields the Fulling quantization. Section 4 sketches the algebraic formulation of QFT and a technique which, given a stationary frame of a certain kind, produces a vacuum state for the Klein-Gordon field. The algebraic formalism is used in Section 5 to characterize some results about the inequivalence of the Minkowski and Fulling representations.

But in order to wrest a demotion of the particle notion from this inequivalence, a premise (e.g., (A3)) is required to the effect that the irreconcilable particle notions associated with the Minkowski and Fulling quantizations have equal claim to physical significance. Section 6 questions this premise by questioning the physical realizability of the Rindler vacuum state. Concluding that Fulling non-uniqueness fails to constitute a satisfactory example of particle notions allied with physically relevant but unitarily inequivalent quantizations, Section 7 characterizes the conditions under which such an example would exist, and observes that these conditions have not been shown to be met.

Although the qualms developed in Sections 6-7 need to be taken seriously, they are waived for purposes of subsequent discussion. In Section 8 we underscore the obvious but neglected point that there is no need to use Fulling non-uniqueness and the Unruh effect to beat up on the particle concept in QFT. Indeed, what is needed is an explanation of how a theory which is couched in terms of local field observables can explain particle-like behavior. But supposing that one did want to use Fulling non-uniqueness and the Unruh effect to demote the particle concept in QFT to second class status, one needs an argument to support the claim that an observer uniformly accelerating through the Minkowski vacuum really does experience a thermal flux of Rindler quanta (cf. premise (A2) above). Sections 9-11 urge that the prospects for such an argument are dimmer than popular presentations of the Unruh effect might lead one to expect. Section 9 uses the algebraic approach to explicate a precise sense in which the restriction of the Minkowski vacuum state to the right Rindler wedge is a thermal state. But, owing precisely nothing to the Fulling representation, this sense does not justify assigning a Rindler particle content to the thermal state. Section 10 considers a standard attempt to express this thermal state in terms of Rindler modes. Ignoring the lessons of Sections 2-5, about just how different Fulling and Minkowski representations are, this attempt falls short of the mark. Finally, Section 11 offers a skeptical review of attempts to invest the thermal state with Rindler particle content by citing this investment as the best explanation of the behavior of particle detectors. Our conclusions are presented in Section 12. The Appendix outlines relevant concepts and results from algebraic QFT.

2. Rindler Coordinates and Rindler Spacetime. Consider the spacetime line element

$$ds^2 = d\xi^2 + dy^2 + dz^2 - \xi^2 d\eta^2 \tag{1}$$

where the velocity of light has been set to unity. At $\xi = 0$ the determinant of the metric components g_{ij} vanishes, so that the contravariant metric components g^{ij} are singular there. However, it is easily seen that this singularity is merely a coordinate artifact. A computation shows that the Riemann curvature tensor of the metric for (1) vanishes. This suggests that the line element (1) represents the Minkowski metric for a portion of Minkowski spacetime. That this is indeed the case is revealed by the coordinate transformation

$$x = \xi \cosh \eta,\ y = y,\ z = z,\ t = \xi \sinh \eta \tag{2}$$

In the new coordinate system the line element assumes the familiar Minkowski form

$$ds^2 = dx^2 + dy^2 + dz^2 - dt^2 \tag{3}$$

The apparent singularity at $\xi = 0$ is due to the fact that the *Rindler coordinates* (ξ, y, z, η) are only valid for the *right Rindler wedge* $\mathcal{R}$: $x > |t|$ of Minkowski spacetime. On the boundary $x = |t|$ these coordinates "go bad"; in particular, $\eta = \tanh^{-1}(t/x)$ so that $x = t$ is assigned the value $\eta = +\infty$ and $x = -t$ is assigned the value $\eta = -\infty$.

Rindler (1969) introduced these coordinates to motivate the idea that the $r = 2M$ singularity of the Schwarzschild solution to Einstein's gravitational field equations is a coordinate artifact that can be removed by the Kruskal extension of the exterior Schwarzschild solution (see also Wald (1984)). Ironically, the very same coordinate system had been used by Einstein and Rosen (1935) for the purpose of illuminating what they took to be a genuine physical singularity at $r = 2M$ (see Earman and Eisenstaedt (1999)).[2]

The right Rindler wedge, considered as a spacetime in its own right

2. Thus, there is a good precedent for speaking of *Einstein-Rosen coordinates* instead of *Rindler coordinates.* To avoid confusion, however, we will stick with the standard terminology.

(aka *Rindler spacetime*), is globally hyperbolic with Cauchy surfaces $\eta = const.$[3] The orthogonal trajectories of these hypersurfaces are the hyperbolas $\xi^2 = x^2 - t^2 = const$ (see Fig. 1). An observer whose world line is one of these hyperbolas undergoes constant proper acceleration of magnitude $a = \xi^{-1}$. These hyperbolas are also the orbits of isometries, namely Lorentz boosts; in particular, a translation in Rindler time $\eta_0 \mapsto \eta_0 + \eta$ is a Lorentz boost in the x-direction of Minkowski spacetime with speed $\tanh \eta$.

The smooth congruence of timelike curves given by the Rindler hyperbolas specifies a *reference frame* for Rindler spacetime. Alternatively, a reference frame can be specified by a smooth non-vanishing timelike vector field V^a, with the world lines of the frame being the integral curves of the vector field. V^a is said to define a *stationary frame* for the spacetime $\mathcal{M}$, g_{ab} just in case V^a is a Killing field, i.e. $\nabla_{(a}V_{b)} = 0$, where ∇_a is the derivative operator determined by the metric g_{ab}. This is the necessary and sufficient condition that there exists (at least locally) a coordinate system x^i, $i = 1$, 2, 3, 4, which is adapted to the frame in the sense that the integral curves of V^a are given by $x^\alpha = const$, $\alpha = 1, 2, 3$, and $V^a = (\partial/\partial x^4)$ (after rescaling). In such a frame, the metric components g_{ij} are independent of the time coordinate x^4. If $V_{[a}\nabla_b V_{c]} = 0$, the frame is *non-rotating* and the integral curves of V^a are hypersurface orthogonal. In a non-rotating frame, the coordinate system can be chosen (at least locally) so that the $g_{\alpha 4}$ components vanish. A frame that is both stationary and non-rotating is said to be *static.* The Rindler frame is in fact static.

In the next section we describe a procedure which associates with a static frame for a globally hyperbolic spacetime a quantization of the Klein-Gordon field. Since Rindler spacetime is globally hyperbolic and since the Rindler frame is a static frame for this spacetime, the procedure in question produces a quantization, first explicitly described by Fulling (1972, 1973). It turns out that this Fulling quantization is physically distinct, in the strongest possible way, from the quantization associated with any inertial frame of Minkowski spacetime.

3. Fulling Quantization. Consider a globally hyperbolic spacetime $\mathcal{M}$, g_{ab}, and let $\Sigma(t) : t = const$ be a one-parameter family of Cauchy surfaces that foliate the spacetime. Define the timelike vector field V^a by the condition $V^a\nabla_a t = -1$. If V^a defines a static frame, then there is a procedure for quantizing the Klein-Gordon field on the spacetime by using the t-time to pick out the positive frequency solutions. The basic idea is straightfor-

3. For a definition of global hyperbolicity, see Wald (1984). For our purposes, the key fact about such a spacetime is that it can be foliated by *Cauchy surfaces,* that is, spacelike hypersurfaces that intersect every maximally extended timelike curve exactly once.

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ward, but the details are tedious. We will review enough of them to convey a sense of what is going on.

In generally covariant form the Klein-Gordon equation reads

$$g^{ab}\nabla_a\nabla_b\phi - m^2\phi = 0 \tag{4}$$

where m is the mass of the field. In coordinates adapted to the static frame, (4) becomes

$$\frac{-\partial^2\phi}{\partial t^2} = g_{44}\left[\frac{1}{\sqrt{-g}}\partial_\alpha\left(\sqrt{-g}\,g^{\alpha\beta}\partial_\beta\phi\right) - m^2\phi\right] \equiv K\phi,\ \alpha\beta, = 1,2,3 \tag{5}$$

Since the differential operator K contains only spatial derivatives, (2) can be solved by separation of variables

$$\phi(\mathbf{x},t) = \psi(\mathbf{x})\chi(t) \tag{6}$$

to give

$$\frac{d^2\chi(t)}{dt^2} + \omega^2\chi(t) = 0 \tag{7a}$$

$$K\psi(\mathbf{x}) = \omega^2\psi(\mathbf{x}) \tag{7b}$$

Equation (7a) is the familiar equation for the harmonic oscillator; its solutions are linear combinations of the exponentials $\exp(\pm i\omega t)$.

The operator K is formally symmetric and positive on the Hilbert space $L^2(\Sigma, \rho d^3x)$ of complex valued square integrable functions on a Cauchy surface Σ, with the inner product given by

$$\langle f,g\rangle := \int_\Sigma \bar{f}(\mathbf{x})g(\mathbf{x})\rho(\mathbf{x})d^3x \tag{8}$$

(here $\rho = -g^{44}\sqrt{-g}$). Independent of the choice of Σ from the family $\Sigma(t)$, this inner product is not indexed with Σ. From the fact that K is positive and symmetric, it follows that K has a unique self-adjoint extension and, further, that the square root of this extension is a positive linear operator. This square root serves as the single particle Hamiltonian relative to the time t.

The next step is to diagonalize K, that is, to find a measure space $(\tilde{\Sigma}, d\mu)$, a unitary map $U : L^2(\Sigma, \rho d^3x) \to L^2(\tilde{\Sigma}, d\mu) : f \mapsto \tilde{f},$ and a function $\tilde{\Sigma} \to \mathbb{R}^+ : k \to \omega_k^2$ such that

$$\left(UKU^{-1}\tilde{f}\right)(k) = \omega_k^2 \tilde{f}(k) \tag{9}$$

(9) expresses the requirement that under the isomorphism U, the operator K corresponds to multiplication by ω_k^2. One can then choose an orthonormal basis $\{\psi_k\}$ for $L^2(\Sigma, \rho d^3x)$ consisting of solutions to

$$K\psi_k = \omega_k^2 \psi_k \tag{10}$$

The functions $u_k(\mathbf{x}, t) = (2\omega_k^2)^{-1/2}\psi_k(\mathbf{x})\exp(-i\omega_k t)$, together with their complex conjugates $\bar{u}_k$, constitute a complete set of mode solutions to (5). The u_k and $\bar{u}_k$ are called respectively the *positive* and *negative* frequency modes, as the heuristic identification of $i\partial/\partial t$ as the energy operator suggests. And so, the general solution to (5) can be written in the form

$$\phi(\mathbf{x},t) = \int_{\tilde{\Sigma}} \left[a_k \psi_k(\mathbf{x})\exp(-i\omega_k t) + a_k^{\dagger} \bar{\psi}_k(\mathbf{x})\exp(i\omega_k t)\right] \frac{d\mu(k)}{\sqrt{2\omega_k}} \tag{11}$$

where the a_k and $a_k^{\dagger}$ are arbitrary complex coefficients.

Functions of the form

$$\Phi(\mathbf{x},t) := \int_{\tilde{\Sigma}} a_k \psi_k(\mathbf{x})\exp(-i\omega_k t) \frac{d\mu(k)}{\sqrt{2\omega_k}} \tag{12}$$

make up the positive frequency (with respect to t) solutions $\mathcal{K}^+$. Applied to such solutions the bilinear form

$$\left\langle \Phi_1, \Phi_2 \right\rangle := i\int_{\Sigma} (\bar{\Phi}_1 \nabla_a \Phi_2 - \Phi_2 \nabla_a \bar{\Phi}_1) n^a d\Sigma \tag{13}$$

(where n^a is the unit normal to Σ) constitutes an inner product, which is independent of the choice of Σ. The completion of $\mathcal{K}^+$ in this inner product gives the "one-particle" Hilbert space $\mathcal{H}$ for the field.

The state space for the field is constructed as the symmetric Fock space $\mathcal{F}$ over $\mathcal{H}$. That is, $\mathcal{F}$ is the completed direct sum $\bigoplus_{i=0}^{\infty}(S[\otimes_i \mathcal{H}])$, where $S[\otimes_n \mathcal{H}]$ denotes the symmetrized n-fold tensor product of $\mathcal{H}$ and $\otimes_0 \mathcal{H}$ is stipulated to be $\mathbb{C}$. The objects a_k and $a_k^{\dagger}$ act as operators on $\mathcal{F}$ (and thus will be hatted), and are identified respectively as the annihilation and creation operators. The vacuum state $|0\rangle \in \mathcal{F}$ is defined by the condition that

$\hat{a}_k|0\rangle = 0$ for all k. $\hat{N} := \int_{\tilde{\Sigma}} d\mu(k)\hat{a}_k^\dagger \hat{a}_k$ has a natural interpretation as the total particle number operator. $|0\rangle$, $\hat{a}_k$, $\hat{a}_k^\dagger$, and $\hat{N}$, thus understood, affiliate a particle notion with this QFT construction.

Of course, the procedure outlined above applies to an inertial frame in Minkowski spacetime, and leads to the Minkowski Fock space $\mathcal{F}_M$ and its vacuum state $|0_M\rangle$.[4] The procedure also applies to the Rindler frame on Rindler spacetime, leading to the Rindler Fock space $\mathcal{F}_R$ and its vacuum state $|0_R\rangle$ (Fulling (1972, 1973)). It is conventional to call the particles (or better, quanta) associated with this representation *Rindler particles* (or better, *Rindler quanta*), and the state $|0_R\rangle$ from which they are absent the *Rindler vacuum.* In order to avoid confusion we will observe convention in what follows, even though the construction of $\mathcal{F}_R$ owes everything to Fulling and nothing to Rindler.

4. The Algebraic Approach. The algebraic approach to QFT provides a framework in which relations among different quantizations can be characterized precisely. We turn to it now. The algebraic approach codes observables as elements of a C^*-algebra $\mathcal{A}$. A state ω is a normalized positive linear map from $\mathcal{A}$ to $\mathbb{C}$. The Hilbert space formalism is recovered in the form of a representation $(\pi, \mathcal{H})$ of $\mathcal{A}$, where π is a homomorphism from $\mathcal{A}$ into the set $\mathcal{B}(\mathcal{H})$ of bounded operators on a separable Hilbert space $\mathcal{H}$ (see Appendix A). A fundamental theorem due to Gel'fand, Naimark, and Segal (GNS) guarantees that for any state ω on $\mathcal{A}$ there is a representation $(\pi_\omega, \mathcal{H}_\omega)$ of $\mathcal{A}$ and a cyclic vector $|\Psi_\omega\rangle \in \mathcal{H}_\omega$ (i.e. $\pi_\omega(\mathcal{A})|\Psi_\omega\rangle$ is dense in $\mathcal{H}_\omega$) such that $\omega(A) = \langle\Psi_\omega|\pi_\omega(A)|\Psi_\omega\rangle$ for all $A \in \mathcal{A}$. That is to say, to any abstract algebraic state there corresponds a concrete Hilbert space realization. The GNS representation is, moreover, the unique, upto unitary equivalence, cyclic representation of $\mathcal{A}$.

Algebraically the distinction between pure and mixed states corresponds to the distinction between states admitting irreducible and reducible GNS representations. (A representation $(\pi, \mathcal{H})$ of $\mathcal{A}$ is *irreducible* just in case the only closed subspaces of $\mathcal{H}$ that are invariant under $\pi(\mathcal{A})$ are $\varnothing$ and $\mathcal{H}$ (see Appendix A).) A reason often cited for focussing physical attention on cyclic representation is that every representation is a direct sum of cyclic representations. Cyclic representations needn't be irreducible. So there can exist representations resolvable as direct sums of other representations, none of which correspond to pure states. This will become important presently.

We will be concerned with a special type of C^*-algebra, called a *Weyl*

4. Talk of *the* Minkowski vacuum state is justified since the results of applying the procedure to any two inertial frames are unitarily equivalent representations of the canonical communication relations.

algebra, which is built over a symplectic vector space (S, Ω), where S is a topological vector space and $\Omega : S \times S \to \mathbb{C}$ is a non-degenerate, bilinear, and anti-symmetric form. In our application S will be the vector space of smooth real solutions to the Klein-Gordon equation (4) having compact support on some Cauchy surface Σ (and thus on all Cauchy surfaces) of a globally hyperbolic spacetime $\mathcal{M}$, g_{ab}. The symplectic form is given by

$$\Omega(\phi_1, \phi_2) := \int_\Sigma (\phi_1 \nabla_a \phi_2 - \phi_2 \nabla_a \phi_1) n^a d\Sigma \tag{14}$$

Because the symplectic current $(\phi_1\nabla_a\phi_2 — \phi_2\nabla_a\phi_1)$ is conserved, Ω is independent of the choice of Cauchy surface. The Weyl algebra $\mathcal{A}(\mathcal{M})$ over this symplectic space encodes an exponentiated version of the CCRs for the Klein-Gordon field. There are various isomorphic version of $\mathcal{A}(\mathcal{M})$, but for our purposes the most convenient one is the four-smeared field version constructed in Kay and Wald (1991, 72–73).[5] It leads to a net of C^*-algebras $\{\mathcal{A}(\mathcal{O})\}$ where $\mathcal{O} \subset \mathcal{M}$ is any open set of compact closure; $\mathcal{A}(\mathcal{O})$ is the Weyl algebra of the globally hyperbolic spacetime $\mathcal{O}$, $g_{ab}|\mathcal{O}$, and if $\mathcal{O} \subset \mathcal{O}'$ then $\mathcal{A}(\mathcal{O})$ is a subalgebra of $\mathcal{A}(\mathcal{O}')$.

Now let V^a be any smooth timelike vector field on the globally hyperbolic spacetime $\mathcal{M}$, g_{ab}, and let $\Sigma(t)$ be a foliation of Cauchy surfaces. If this vector field satisfies

$$-V^a V_a \geq \varepsilon V^a n_a \geq \varepsilon^2 \text{ for some } \varepsilon > 0 \tag{15}$$

on some member Σ of the foliation $\Sigma(t)$ (with n^a the unit normal to Σ), then there is a procedure that associates with Σ a pure quasi-free state ω^Σ on $\mathcal{A}(\mathcal{M})$ (see Chmielowski (1994) and Wald (1994, 4.3)).[6] Since the GNS representation of a quasi-free state has a natural Fock space structure in which the representing vector is the Fock space vacuum, ω^Σ is a candidate for the vacuum state. If V^a is defined by $V^a\nabla_a t = -1$ and is also a Killing field, then the state ω^Σ is independent of the choice of the member of the foliation $\Sigma(t)$. Indeed, in this case the usual dynamics is recovered in terms of a strongly continuous group of unitary transformations on a Fock space representation that leave the vacuum state vector invariant (see Kay (1978)). If in addition to being stationary, the frame V^a associated with the Cauchy foliation $\Sigma(t)$ is also static, then the algebraic quantization procedure is equivalent to the Hamiltonian diagonalization procedure of Section 3.

5. The constructions in Wald (1994, Ch. 4) and Dimock (1980) differ from each other and from Kay and Wald (1991) in details but lead to isomorphic global Weyl algebras.

6. We omit the technical definition of *quasi-free state* (see Wald (1994) for details), but we mention that for such a state the n-point functions are determined by the two point functions.

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If one applies this algebraic quantization procedure to Minkowski spacetime, taking the $t = const$ slices to be given by an inertial time coordinate, the result is the standard Minkowski vacuum state ω_M. One can properly speak of *the* Minkowski vacuum state since if another inertial coordinate t' is chosen, the corresponding vacuum state ω'_M is the same as ω_M. (This follows from a result of Chmielowski (1994) discussed below in Section 7.)

The algebra at issue in the Rindler case is the right Rindler wedge algebra $\mathcal{A}(\mathcal{R})$, i.e., the Weyl algebra over the symplectic space of solutions to the Klein-Gordon equation having compact support on the Rindler time slices of Rindler spacetime. Unfortunately, the rigorous algebraic quantization procedure just discussed is not guaranteed to work for $\mathcal{A}(\mathcal{R})$, because in the Rindler case, the condition (15) is violated. For in Minkowski coordinates, the Rindler frame is defined by $V^a = \text{const} \cdot (x\frac{\partial}{\partial t^a} + t\frac{\partial}{\partial x^a})$. Thus $-V^aV_a = const \cdot (t^2 - x^2)$, and near the edges of the Rindler wedge, the norm of V^a becomes arbitrarily small. Nevertheless, with some extra work it is possible to construct an algebraic state ω_R on $\mathcal{A}(\mathcal{R})$ that is the counterpart of the Rindler vacuum state $|0_R\rangle$ derived by the separation of variables procedure set out in Section 3 (see Kay (1985)).[7]

5. Fulling Non-uniqueness. In this Section we use the algebraic resources just mustered to describe how the Fulling quantization differs from the standard Minkowski quantization. The restriction of the Minkowski vacuum state ω_M to the right Rindler wedge algebra $\mathcal{A}(\mathcal{R})$ defines a state $\omega_M|_{\mathcal{A}(\mathcal{R})}$ on that algebra. $\omega_M|_{\mathcal{A}(\mathcal{R})}$ is a mixed state[8] whereas ω_R is a pure state. It follows trivially that $\omega_M|_{\mathcal{A}(\mathcal{R})}$ and ω_R determine unitarily inequivalent GNS representations of $\mathcal{A}(\mathcal{R})$ since the former representation is reducible while the latter is irreducible. But the two quantizations are different in a stronger way; namely, they are *disjoint*. These representations are nevertheless *locally* quasi-equivalent (in at least one of the two possible senses of that term). We now turn to explaining these concepts and their significance.

The *folium* $\mathfrak{F}(\omega)$ of a state ω on a C^*-algebra $\mathcal{A}$ is the set of all abstract

7. Kay's (1985) construction can be seen as a rigorous version of Fulling's (1972, 1973) somewhat heuristic construction.

8. This results from the following facts. 1) Let $\mathcal{O}$ and $\mathcal{O}'$ be relatively spacelike open regions of Minkowski spacetime. If the state ω implies correlations between the local observables belonging to $\mathcal{A}(\mathcal{O})$ and $\mathcal{A}(\mathcal{O}')$ in the sense that $\omega(OO') \neq \omega(O)\omega(O')$ for some $O \in \mathcal{A}(\mathcal{O})$ and $O' \in \mathcal{A}(\mathcal{O})$ and if $[\mathcal{A}(\mathcal{O}), \mathcal{A}(\mathcal{O}')] = 0$, then $\omega|_{\mathcal{A}(\mathcal{O})}$ and $\omega|_{\mathcal{A}(\mathcal{O}')}$ are mixed states. And 2) the Minkowski vacuum state ω_M implies correlations between the local observables associated with relatively spacelike regions of Minkowski spacetime. It will be seen in Section 9 that not only is $\omega_M|_{\mathcal{A}(\mathcal{R})}$ a mixed state, it is a thermal state at finite temperature.

states that can be expressed as density matrices on the Hilbert space of the GNS representation of $\mathcal{A}$ determined by ω. The states ω_1 and ω_2 (or the GNS representations determined by them) are said to be *quasi-equivalent* if and only if $\mathfrak{F}(\omega_1) = \mathfrak{F}(\omega_2)$, whereas they are said to be *disjoint* iff $\mathfrak{F}(\omega_1) \cap \mathfrak{F}(\omega_2) = \emptyset$. Additional characterizations of quasi-equivalence are given in Appendix B.

The following lemma gives some equivalent characterizations of disjointness.

Lemma 1. (from various results of Bratteli and Robinson (1996)). The following conditions are equivalent:
(1) ω_1 and ω_2 are disjoint
(2) no subrepresentation[9] of the GNS representation π_{ω_1} is quasi-equivalent to any subrepresentation of the GNS representation π_{ω_2}, and vice versa
(3) no subrepresentation of the GNS representation π_{ω_1} is unitarily equivalent to any subrepresentation of the GNS representation π_{ω_2}, and vice versa
(4) the GNS representation $\pi_{\omega_1+\omega_2}$ determined by $\omega_1 + \omega_2$ is the direct sum of the representations π_{ω_1} and π_{ω_2} determined by ω_1 and ω_2, i.e. $\pi_{\omega_1+\omega_2} = \pi_{\omega_1} \oplus \pi_{\omega_2}$, $\mathcal{H}_{\omega_1+\omega_2} = \mathcal{H}_{\omega_1} \oplus \mathcal{H}_{\omega_2}$ and $|\Psi_{\omega_1+\omega_2}\rangle = |\Psi_{\omega_1}\rangle \oplus |\Psi_{\omega_2}\rangle$, where as usual $(\pi_\omega, \mathcal{H}_\omega, |\Psi_\omega\rangle)$ is the GNS triple associated with ω
(5) ω_1 and ω_2 are orthogonal, i.e. there is a projection operator $\hat{P} \in \pi_{\omega_1+\omega_2}(\mathcal{A})'$ such that $\omega_1(A) = \langle\Psi_{\omega_1+\omega_2}|\hat{P}\pi_{\omega_1+\omega_2}(A)|\Psi_{\omega_1+\omega_2}\rangle$ and $\omega_2(A) = \langle\Psi_{\omega_1+\omega_2}|(\hat{I} - \hat{P})\pi_{\omega_1+\omega_2}(A)|\Psi_{\omega_1+\omega_2}\rangle$ for all $A \in \mathcal{A}$.

Every representation is a subrepresentation of itself. But irreducible representations have no proper subrepresentations. Thus, if ω_1 and ω_2 are pure, quasi-equivalence reduces to unitary equivalence and disjointness reduces to non-unitary equivalence. If, on the other hand, either ω_1 or ω_2 are mixed, the situation becomes more complicated. ω_1 and ω_2 can be quasi-equivalent without being unitarily equivalent—as when ω_2 is pure and quasi-equivalent to every subrepresentation of (mixed and factorial) ω_1 (see Appendix B). What's more, when ω_1 and ω_2 are mixed, they can fail to be either quasi-equivalent or disjoint—as when non-factorial ω_1 has disjoint subrepresentations, one of which is unitarily equivalent to ω_2. What Lemma 1 helps to make clear is that disjoint representations are about as different as can be imagined.

Lemma 2. The states $\omega_M|_{\mathcal{A}(\mathcal{R})}$ and ω_R on $\mathcal{A}(\mathcal{R})$ are disjoint.

9. Roughly, a *subrepresentation* of a representation π_ω of $\mathcal{A}$ is a closed subspace of $\mathcal{H}_\omega$ which itself admits a representation of $\mathcal{A}$; see Appendix A for the precise definition.

By singling out ω_R, Lemma 2 can give a misleading impression. For an inspection of the proof of Lemma 2 (see Appendix B) shows that the disjointness of the Fulling and Minkowski representations of $\mathcal{A}(\mathcal{R})$ has nothing to do with the Fulling representation *per se.* For that proof establishes $\omega_M|_{\mathcal{A}(\mathcal{R})}$ to be disjoint from *any* pure state θ on $\mathcal{A}(\mathcal{R})$, not just the Rindler vacuum. Looking ahead to the putative implications of Fulling non-uniqueness for the particle concept, we remark that the disjointness has nothing to do with *Rindler particles.* What's more, this disjointness obtains whether or not π_θ is unitarily equivalent to a Fock representation. Thus holding of irreducible representations of $\mathcal{A}(\mathcal{R})$ which sustain no particle notion, the disjointness seems to have nothing to do with *particles* at all. Disjointness follows rather from the fact that for any pure state θ on $\mathcal{A}(\mathcal{R})$, the von Neumann algebra associated with π_θ is Type I (see Appendix B) while the von Neumann algebra associated with $\pi_{\omega_M|\mathcal{A}(\mathcal{R})}$ is of Type III (see Araki (1964)). *Thus,* the disjointness result owes everything to the nature of the state $\omega_M|_{\mathcal{A}(\mathcal{R})}$. As will be seen below, other features discussed under the labels of Fulling non-uniqueness and the Unruh effect are attributable to the nature of $\omega_M|_{\mathcal{A}(\mathcal{R})}$ and have little or nothing to do with ω_R.

A consequence of the disjointness of the Minkowski and Fulling representations is worth noting. Correlations between spatially separated regions are a familiar and characteristic feature of quantum physics. The point is usually illustrated in ordinary QM by the singlet state $\Psi(I, II)$ on the Hilbert space $\mathcal{H}_I \otimes \mathcal{H}_{II}$ of two spin 1/2 particles, which we can imagine are well separated in space. What the singlet state says about the observables of $\mathcal{H}_{II}$ can be expressed by a density matrix on $\mathcal{H}_{II}$, obtained from the density matrix for $\mathcal{H}_I \otimes \mathcal{H}_{II}$ by tracing out over the degrees of freedom for system I. But in the relativistic QFT case, what ω_M says about the right Rindler wedge is given by the restriction of ω_M to $\mathcal{A}(\mathcal{R})$, and the disjointness result shows that the state $\omega_M|_{\mathcal{A}(\mathcal{R})}$ on $\mathcal{A}(\mathcal{R})$ defined by this restriction is not expressible as a density matrix in the GNS representation π_{ω_R} determined by ω_R—or for that matter in the GNS representation π_θ determined any pure state θ on $\mathcal{A}(\mathcal{R})$. This expressive incompleteness is not mysterious. It reflects the fact that the big algebra $\mathcal{A}(\mathcal{M})$ for Minkowski spacetime cannot be written as $\mathcal{A}' \otimes \mathcal{A}(\mathcal{R})$ for some $\mathcal{A}'$.

This expressive incompleteness vanishes on the local level, at least if "local" refers to open spacetime regions with compact closure. Suppose that ω_1 and ω_2 are quasi-free Hadamard states[10] on the algebra $\mathcal{A}(\mathcal{M})$ for Minkowski spacetime, and that $\mathcal{O} \subset \mathcal{M}$ is any open region with compact closure. Let $\pi_\omega|\mathcal{A}(\mathcal{O})$ denote the representation obtained from the GNS

10. Hadamard states are characterized in Section 6. The brief for restricting attention to them is that they support an expectation value assignment to the stress-energy tensor.

construction for ω by restricting to the image under π_ω of $\mathcal{A}(\mathcal{O})$ then completing in the natural topology of $\mathcal{H}_\omega$. Verch (1994) shows that

$$\pi_{\omega_1}|\mathcal{A}(\mathcal{O}) \text{ is quasi-equivalent to } \pi_{\omega_2}|\mathcal{A}(\mathcal{O}). \tag{16}$$

Since ω_M is a quasi-free Hadamard state on $\mathcal{A}(\mathcal{M})$ while ω_R is a quasi-free Hadamard state on $\mathcal{A}(\mathcal{R})$,[11] it follows that for any open region $\mathcal{O} \subset \mathcal{R}$ with compact closure

$$\pi_{\omega_M}|\mathcal{A}(\mathcal{O}) \text{ is quasi-equivalent to } \pi_{\omega_R}|\mathcal{A}(\mathcal{O}) \tag{17}$$

This establishes one sense in which ω_M and ω_R are locally quasi-equivalent. π_{ω_M} and π_{ω_R} are unitarily inequivalent—indeed, disjoint—representations of $\mathcal{A}(\mathcal{R})$. Verch's result shows that for any quasi-local algebra $\mathcal{A}(\mathcal{O})$ associated with a region $\mathcal{O} \subset \mathcal{R}$ of compact closure, the expectation values assigned any number of observables in $\mathcal{A}(\mathcal{O})$ by a density matrix in π_{ω_M}'s folium are exactly reproduced by the expectation values assigned those same observables by a density matrix in π_{ω_R}'s folium, and vice versa. Consequently, only the measurement of observables associated with regions of non-compact closure can empirically distinguish the Minkowski and Rindler representations (Wald (1994, 97)). This does not, however, ensure the local equivalence of the Fulling and Minkowski particle concepts. Indeed, "the local equivalence of particle concepts" is an incoherent notion: insofar as particle number operators do not belong to local algebras, the particle concept is a non-local concept in QFT.

It is worth noting that there is a second sense in which the Minkowski and Rindler representations may be locally quasi-equivalent; namely, for any open region $\mathcal{O} \subset \mathcal{R}$ with compact closure

$$\pi_{\omega_M|_{\mathcal{A}(\mathcal{O})}} \text{ is quasi-equivalent to } \pi_{\omega_R|_{\mathcal{A}(\mathcal{O})}} \tag{18}$$

Compare local quasi-equivalence in the sense of (17): there, the representations obtained by representing ω_M and ω_R, then restricting to concrete operators in the image under those representations of $\mathcal{A}(\mathcal{O})$, are claimed to be quasi-equivalent; here, the representations obtained by restricting ω_M and ω_R to $\mathcal{A}(\mathcal{O})$, then representing the restricted algebra, are claimed to be quasi-equivalent. It is not known whether the Minkowski and Rindler representations are locally quasi-equivalent in this second sense. But

11. The latter claim is found throughout the physics literature on Rindler quanta, but we do not know a specific proof of it.

we can get close to (18); namely, it can be shown (see Appendix C) that for any open region $\mathcal{O} \subset \mathcal{R}$ with compact closure

$$\pi_{\omega_M|_{\mathcal{A}(\mathcal{O})}} \text{ is quasi-equivalent to } \pi_{\omega_R}|\mathcal{A}(\mathcal{O}) \tag{19}$$

It follows that, if the second sense of local quasi-equivalence should fail, $\pi_{\omega_M|\mathcal{A}(\mathcal{O})}$ and $\pi_{\omega_R}|\mathcal{A}(\mathcal{O})$ would fail to be locally quasi-equivalent for some $\mathcal{O} \subset \mathcal{R}$ of compact closure. The physical significance of such a failure remains to be fathomed.

6. Is the Rindler Vacuum State Physically Realizable? From the very early days of relativistic QFT, it was known that the CCRs admitted unitarily inequivalent and, indeed, disjoint representations. Fulling's simple and vivid example brought this mathematical possibility to life. But the physical relevance of Fulling's example will remain obscure until the issue of the physical realizability of the Rindler vacuum state is settled. We believe that there are persuasive reasons to doubt that the Rindler vacuum state is physically realizable. This Section first states the case for our negative verdict, and then turns to the defense.

We base our case for a negative verdict on four premises.

(P1) Any candidate spacetime $\mathcal{M}$, g_{ab} for representing the actual universe must be inextendible (i.e., must not be isometrically embeddable as a proper subset of another spacetime).

Ontological justifications of this principle involve metaphysics that is controversial, e.g. Leibniz's principle of plenitude. Perhaps the best justification is methodological—without the restriction to inextendible spacetimes, the usual practice of science is not possible. For example, if spacetime can be truncated in the past, there is no way to defeat creationism; if spacetime can have "holes," determinism cannot be true (see Earman (1995)); etc.

(P2) For an inextendible spacetime $\mathcal{M}$, g_{ab}, a physically realizable state ω on the subalgebra $\mathcal{A}(\mathcal{O}) \subset \mathcal{A}(\mathcal{M})$, $\mathcal{O} \subset \mathcal{M}$, must be extendible to a state $\tilde{\omega}$ on the full algebra $\mathcal{A}(\mathcal{M})$; moreover, $\tilde{\omega}$ must satisfy whatever criteria govern the selection of physically realizable states on $\mathcal{A}(\mathcal{M})$.

The argument for (P2) is straightforward. (P2) is equivalent to the requirement that any physically realizable state ω on $\mathcal{A}(\mathcal{O})$ be of the form $\tilde{\omega}|_{\mathcal{A}(\mathcal{O})}$ for some physically realizable state $\tilde{\omega}$ on $\mathcal{A}(\mathcal{M})$. This in turn amounts to the requirement that, insofar as it makes sense to talk about the state of a subsystem of a big system, this subsystem state must say neither less nor more than what some global state says about the local observables of the subsystem. What the global state says about the local observables of the subsystem is given precisely by its restriction to the subalgebra of these local observables.

(P3) ω_R cannot be extended to a non-singular, i.e. Hadamard, state on Minkowski spacetime.

Indeed, ω_R becomes singular on the edges of the Rindler wedge.

(P4) To be physically realizable, a state on the global algebra for an inextendible spacetime must satisfy the Hadamard condition.

From (P1)–(P4) it follows that ω_R is not physically realizable.

To motivate the crucial fourth premise, note that Hadamard states are states for which $\langle\hat{\phi}(x)\hat{\phi}(x')\rangle_\omega$, the expectation value of two point functions of the field, exhibits a prescribed singularity structure—a singularity structure of Hadamard form—as the spacetime points x and x' approach one another (Wald (1994)). For such states, there exists a "point splitting" procedure for defining the renormalized expectation value $\langle T_{ab}\rangle_\omega$ of the stress-energy tensor

$$T_{ab} := \nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}(\nabla_c\phi\nabla^c\phi + m^2\phi^2) \tag{20}$$

for the Klein-Gordon field. In flat spacetime—the case at issue—the expectation values assigned T_{ab} by this procedure are the unique ones which satisfy the (generally accepted) axioms by which Wald (1994, Sec. 4.6) would govern the definition of $\langle T_{ab}\rangle_\omega$. In the more general setting of curved spacetime, Wald's axioms determine $\langle T_{ab}\rangle_\omega$ upto "local curvature terms."

In order to calculate the backreaction effect of quantum fields on the metric, semi-classical quantum gravity replaces T_{ab} on the right hand side of Einstein's gravitational field equations with $\langle T_{ab}\rangle_\omega$. Hawking bases his prediction of black hole radiation and black hole evaporation on just such semi-classical calculations. In light of the need in these important theoretical contexts for $\langle T_{ab}\rangle_\omega$, Wald (1994) proposes that states of the quantum field must be Hadamard to be physically acceptable (see also Kay and Wald (1991)). The fact that $\langle T_{ab}\rangle_{\omega_R}$ diverges as the edges of the Rindler wedge are approached underscores the unphysical nature of the Rindler vacuum state.[12]

If this line of argument succeeds in establishing that ω_R is physically unrealizable, it can be extended from the Rindler case to arbitrary globally hyperbolic spacetimes with *bifurcate* Killing horizons. Such spacetimes are characterized by the existence of two null hypersurfaces h_A and h_B, inter-

12. In fact, the expected energy density approaches minus infinity (Wald, private communication).

secting in a two dimensional spacelike hypersurface O, and by a Killing vector field V^a that vanishes on (and only on) O and becomes null on h_A and h_B. Provided that there is a Cauchy surface containing O, the horizons $h_A \cup h_B$ divide the spacetime up into four wedges $\mathfrak{F}$, $\mathcal{P}$, $\mathcal{L}$, $\mathcal{R}$, as indicated schematically in Fig. 2. In the Minkowski case, $\mathcal{L}$ and $\mathcal{R}$ are, of course, the left and right Rindler wedges. In the case of the Kruskal extension of the exterior Schwarzschild solution, $\mathcal{L}$ and $\mathcal{R}$ are the analogues of the left and right Rindler wedges, and for this case the analogue of the Rindler vacuum state is called the *Boulware vacuum state* ω_B. As the analogy leads us to expect, $\langle T_{ab}\rangle_{\omega_B}$ is singular on $h_A \cup h_B$.

Of the premises (P1)–(P4), only (P3) is invulnerable. However, the consequences of abandoning either (P1) or (P2) strike us as quite unpalatable. This leaves (P4) which, we readily admit (see Arageorgis, Earman, and Ruetsche (2002)), can be challenged. Thus, we do not claim to have shown

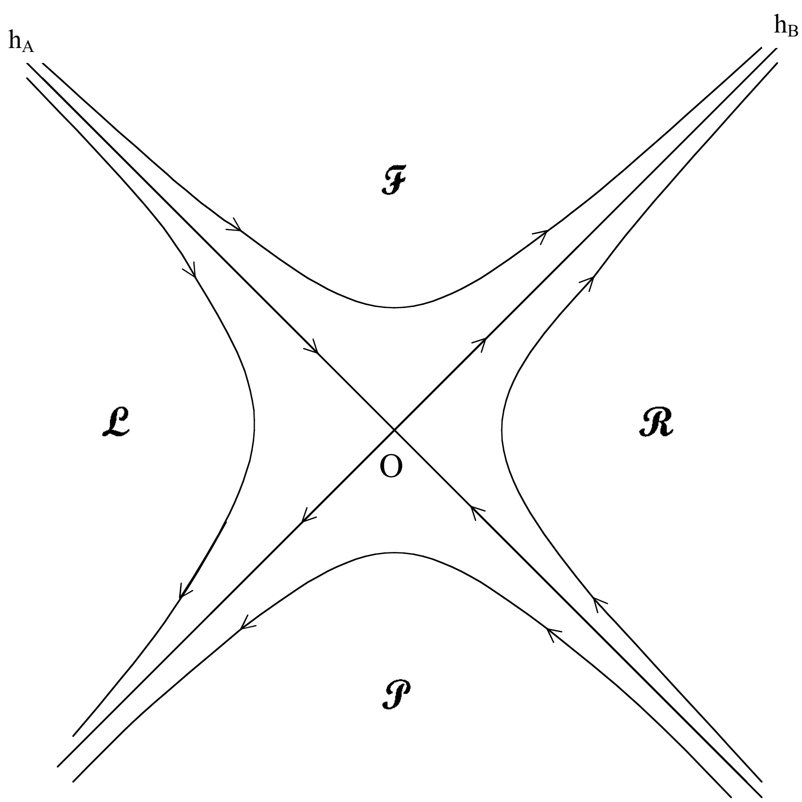

Fig. 2. Bifurcate Killing horizons along with the orbit structure of the isometry group.

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conclusively that ω_R is not a physically realizable state. But we hope to have given pause to those who have readily assumed the opposite.

7. Are There Examples of Inequivalent Global Quantizations? Setting aside the Hadamard condition and associated qualms about whether ω_R is physically realizable, we next focus on $\omega_M|_{\mathcal{A}(\mathcal{R})}$ and its suitability to serve in an example of genuine non-uniqueness. $\omega_M|_{\mathcal{A}(\mathcal{R})}$ is the *mixed state* that results when one restricts to $\mathcal{A}(\mathcal{R})$ the state ω_M which quantizes the Klein-Gordon field on a larger spacetime. Thus the states $\omega_M|_{\mathcal{A}(\mathcal{R})}$ and ω_R do not provide an example of inequivalent quantizations in the sense of two *pure states* that correspond to inequivalent vacua for the same spacetime. As will be seen in Section 9, $\omega_M|_{\mathcal{A}(\mathcal{R})}$ is a thermal state, from which a pure state can be derived by "cooling it down to temperature 0." Those pursuing genuine examples of non-uniqueness might hope that the pure state thus obtained and ω_R provide inequivalent quantizations for Rindler spacetime. But their hope is dashed, since the cooled down state is ω_R itself!

Of course, in some sense, inequivalent quantizations/particle concepts are all too easy to find. For any globally hyperbolic spacetime $\mathcal{M}$, g_{ab} there exist innumerably many pure quasi-free states on the Weyl algebra $\mathcal{A}(\mathcal{M})$ over the symplectic space of real solutions to the Klein-Gordon equation (see Wald (1994)). Among these states there are pairs whose GNS representations are not unitarily equivalent. Since the GNS representation of a quasi-free state has a natural Fock space structure, such pairs of states will be candidates for alternative vacuum states, affiliated with alternative particle concepts. However, a candidate vacuum state for a spacetime will be of little physical interest if the spacetime does not admit a timelike Killing field V^a under whose associated symmetries the candidate state is invariant. For, in the absence of such structure, we see little hope for distinguishing invidiously between "physical" vacuum states and "unphysical" ones.

For quantizations that follow the algebraic scheme outlined in Section 4, we can make our demand for genuine non-uniqueness precise. Consider a globally hyperbolic spacetime $\mathcal{M}$, g_{ab} that admits two different foliations by Cauchy surfaces $\Sigma(t)$: $t = const$ and $\Sigma'(t')$: $t' = const.$ Suppose that the associated vector fields given respectively by the conditions $V^a\nabla_a t = -1$ and $V'^a\nabla_a t' = -1$ are both Killing fields that satisfy (15). Then, by the algebraic quantization procedure outlined in Section 4, there are quasi-free states ω_{V^a} and $\omega_{V'^a}$ on the Weyl algebra $\mathcal{A}(\mathcal{M})$ corresponding to the two foliations. The GNS representations determined by these states both have a natural Fock space structure. Hence associated with the frame V^a is a particle concept, V^a-particles (or better, a quanta concept, V^a-quanta), and mutatis mutandis for the frame V'^a. Now, we would have a physically relevant example of inequivalent quantizations meeting our demand for genuine non-uniqueness if the V^a-particle and V'^a-particle concepts were

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different. But according to a result of Chmielowski, these particle concepts coincide if V^a and V'^a commute:

> **Theorem** (Chmielowski (1994)). If V^a and V'^a are timelike Killing fields (as defined above), if they each satisfy (15), and if $[V^a, V'^a] = 0$, then $\omega_{V^a} = \omega_{V'^a}$.

Chmielowski conjectures that the last conjunct of the antecedent is irrelevant:

> **(Con1)** The Theorem continues to hold even if $[V^a, V'^a] \neq 0$.

If (Con 1) holds, any pair of globally defined stationary frames for the same spacetime to which the algebraic quantization procedure outlined in Section 4 applies—these will be frames satisfying (15)—will yield *equivalent quantizations* via that procedure.

We must still investigate the possibility that a globally hyperbolic spacetime admits two globally defined stationary frames, at least one of which violates condition (15) and so foils the procedure of Section 4. One might think that imposing the reasonable condition that the spacetime be inextendible precludes this possibility, since (intuitively speaking) the failure of (15) signals that the spacetime is, like Rindler spacetime, embeddable in a larger spacetime in such a way that the boundary of its image in the larger spacetime is a horizon on which the Killing vector field V^a becomes null. However, this intuition is wrong. It is possible to construct examples where (15) fails in an inextendible spacetime because singularities in the metric develop as the (missing) horizon is approached.[13]

It is also possible to construct artificial examples of inextendible, globally hyperbolic spacetimes that admit two different foliations $\Sigma(t)$ and $\Sigma'(t')$ by Cauchy surfaces such that the associated vector fields defined respectively by $V^a\nabla_a t = -1$ and $V'^a\nabla_a t' = -1$ are Killing fields and at least one of them violates (15). In cases where both of these frames are static and both violate (15), it is possible that the separation of variables procedure of Section 3 will yield inequivalent quantizations when applied to the two frames. In cases where one frame is static and satisfies (15) and the other is stationary and violates (15), it is possible that applying the procedure of Section 3 to the former and the algebraic procedure of Section 4 to the latter will yield inequivalent quantizations. But until it is shown that one of these possibilities can be realized and that the spacetime on which it is realized is of some physical interest, a genuine example of

13. We are grateful to Robert Geroch for showing us how such examples are constructed.

inequivalent quantizations of the humble scalar field has not, in our opinion, been achieved.[14]

8. The Particle Concept in QFT. Physicists and philosophers have attempted to draw dire implications for the particle concept from Fulling non-uniqueness and the Unruh effect. We now turn to a critical examination of their efforts. Before delving into the details, we feel compelled to issue a brief sermonette.

In relativistic QFT there is no need to beat up on the particle concept. Indeed, the need lies in exactly the opposite direction. To repeat the sentiment of the Wald passage cited in Section 1, QFT is a theory whose fundamental concepts include local field observables, *n*-point functions, and the like. The concept of "particle" does not appear on the list. True, most presentations of QFT work with Fock representations, and so with the particle-friendly apparatus of vacuum states, number operators, and so on. But what these representations give directly are ways of counting quanta of excitation of the field. It remains to connect these quanta to a more robust concept of particle—e.g. something that is localized and endures through time, so as to leave a streak in a cloud chamber. (This is why we tried above to use the more neutral "quanta" in place of "particles," although in some contexts the standard usage requires the latter, as in "particle creation/annihilation operators.") Thus, quantum field theorists have their work cut out to show how particle-like behavior, such as tracks in cloud chambers, can be explained by a theory whose basic assumptions can be formulated without appeal to particles (see Steinmann (1968) for an attempt at such an explanation within the algebraic framework).[15]

Here is where the slogan "Particles are what particle detectors detect" makes sense—not as an endorsement of operationalism but as a reminder

14. Expanding universe models provide, so to speak, the next best thing. Consider, for example, the quantization of the Klein-Gordon field on a background spacetime given by a Friedmann-Walker-Robertson model with initial and final static phases. The portion of this spacetime up to some time t_1 admits a non-rotating timelike Killing vector field, while the portion of the spacetime after some $t_2 > t_1$ admits another such field. Associated with each of these Killing fields is a natural vacuum state for the Klein-Gordon field. But if the expansion of the universe produces "infinite particle creation," the two vacuum states correspond to unitarily inequivalent representations of the canonical commutation relations; see Fulling (1989, Ch. 7). However, the model in question will not satisfy Einstein's gravitational field equations for any physically reasonable stress-energy tensor.

15. For an argument for the claim that QFT must save the particle concept in a stronger sense than merely explaining particle-like behavior, see Barrett (2000). We will not attempt to rebut this claim here.

that particle-like behavior is characterized in terms of the response of a detector which is coupled in some appropriate way to the field. We will return to this point in Section 11 below.

Although a Fock space representation is not sufficient to underwrite the particle concept in QFT, it is necessary. For this reason, the particle concept becomes especially problematic when one attempts to do QFT on curved spacetime. In the first place there may be no timelike Killing field. For instance, in the expanding universe models used in current cosmology, such a field does not exist even locally. In these situations there is no motivated way to pick out a vacuum state. Secondly, in curved spacetimes one cannot expect the dynamics of the quantum field to be unitarily implementable in a fixed Fock space (see Arageorgis, Earman, and Ruetsche (2002)). So the folium of the vacuum one starts with may not be closed under dynamical evolution.

The use of Rindler quanta to beat up on the particle concept is doubly misguided. First because, as just explained, no such beating is needed. Second because the proposed beating is based on some dubious assumptions and some outright falsehoods. The beating is supposed to go as follows. Suppose that spacetime is Minkowskian and that the state of the world is given by the Minkowski vacuum state. Then the universe should be free of particles. But (the story goes) a uniformly accelerated observer will detect a thermal bath of Rindler quanta. As Sciama et al. (1981) write:

> Unruh's observation was that the theory that is thereby constructed [quantizing using the Rindler frame] is not unitarily equivalent to the usual free field theory on Minkowski spacetime. Of even greater surprise was the subsequently discovered fact that the usual Poincaré invariant vacuum state appropriate to Minkowski space . . . contains a thermal distribution with respect to the Fulling Fock space.[16]

Similarly, DeWitt (1979) writes that "The Minkowski vacuum is full of Rindler photons, although it is devoid of Minkowski photons" (694).[17] One might be tempted to draw the moral that the particle concept has to be relativized to a frame of reference or state of motion of the observer.

As noted in the Introduction, even if this moral is correct it would not show that particles are not "real," any more than the need to relativize the electric and magnetic fields to a reference frame would show that these

16. Sciama et al. are correct in using the terminology "Fulling Fock space" rather than "Rindler Fock space" since it was Fulling and not Rindler who performed the quantization. However, we have bowed to the common usage.

17. The use of "photons" here is not a slip, for as will be seen in Section 11, only for a massless Klein-Gordon field does a uniformly accelerated DeWitt-Unruh "particle detector" record a thermal spectrum.

fields are not real. But it would undermine the concept of particle inherited from classical physics and ordinary QM, in so far as that concept does not tolerate the consequence that the presence or absence of particles is relative to a reference frame.

In the next three sections we examine the claim on which the overstated moral is based; namely, that, as experienced by a uniformly accelerated observer, the Minkowski vacuum contains a thermal bath of Rindler quanta. From our examination, three conclusions emerge. First, there is a precise sense in which $\omega_M|_{\mathcal{A}(\mathcal{R})}$ is a thermal state. But it is not a sense that justifies assigning any particle content to $\omega_M|_{\mathcal{A}(\mathcal{R})}$ (Section 9). Second, attempts to establish the claim by expressing the $\omega_M|_{\mathcal{A}(\mathcal{R})}$ as a density matrix in the Fulling Fock space are ill-founded (Section 10). Third, the behavior of the DeWitt-Unruh "particle detector," which registers a thermal bath when placed in uniformly accelerated motion through the Minkowski vacuum, can be explained without any reference to Rindler quanta. Furthermore, there are a number of reasons to be suspicious of the claim that this device detects particles (Section 11).

9. KMS States. There is a rigorous sense in which an observer uniformly accelerated through the Minkowski vacuum will detect a thermal state. To explicate this sense requires the notion of a *KMS state,* which generalizes the more familiar notion of equilibrium state. Here we give a nonrigorous introduction, and review some relevant results about KMS states.[18]

A *Gibbs state* at inverse temperature β is written as a density matrix $\hat{\rho} = \exp(-\beta\hat{H})/Tr(\exp(-\beta\hat{H}))$, where $\hat{H}$ is the Hamiltonian operator. (From here on we neglect the normalization factor in the denominator.) This density matrix defines an algebraic state on the concrete algebra $\mathcal{B}(\mathcal{H})$ of bounded operators on $\mathcal{H}$ by setting $\omega(\hat{A}) := Tr(\hat{\rho}\hat{A})$, $\hat{A} \in \mathcal{B}(\mathcal{H})$. Further, $\hat{H}$ defines a one-parameter group of automorphisms on that algebra by $\alpha_t(\hat{A}) := \exp(it\hat{H})\hat{A}\exp(-it\hat{H})$. It is easy to verify that for the state so defined $\omega(\hat{A}\alpha_{i\beta}(\hat{B})) = \omega(\hat{B}\hat{A})$, or equivalently, $\omega(\alpha_{-i\beta/2}(\hat{A})\alpha_{i\beta/2}(\hat{B})) = \omega(\hat{B}\hat{A})$, for all $\hat{A}, \hat{B} \in \mathcal{B}(\mathcal{H})$.

Now consider a case where there may be no density matrix of the appropriate form—say because normalization fails, as will be the case when $\hat{H}$ has a continuous spectrum. Or suppose that we are not working in a representation but rather with an abstract C^*-algebra $\mathcal{A}$. Must we therefore abandon the notion of equilibrium state? Not if there exists a one-parameter group of automorphisms $\alpha_t : \mathcal{A} \rightarrow \mathcal{A}$. For then we can characterize an analog of equilibrium in terms of the property just derived. An algebraic (α_t, β)-KMS state, $0 \leq \beta < \infty$, on $\mathcal{A}$ is defined by the requirement

18. A detailed treatment of KMS states is to be found in Bratteli and Robinson (1979).

that $\omega(A\alpha_{i\beta}(B)) = \omega(BA)$, or equivalently, $\omega(\alpha_{-i\beta/2}(A)\alpha_{i\beta/2}(B)) = \omega(BA)$ for all $A,B \in \mathcal{A}$. There are a number of reasons for taking such KMS states to generalize the notion of equilibrium given by the standard Gibbs state. First, in the case that $(\mathcal{A},\alpha_t)$ admits a standard Gibbs state at inverse temperature β, the (α_t, β)-KMS state is unique and coincides with the Gibbs state (Bratelli and Robinson (1997), Ex. 5.3.31). So the KMS criterion coincides with the Gibbs identification of equilibrium in situations where that identification makes sense. Second, if ω is an (α_t, β)-KMS state, for $\beta \neq 0$, then ω is a *stationary state* with respect to $\alpha_t : \omega(\alpha_t(A)) = \omega(A)$ for all $A \in \mathcal{A}$. This is the sort of stability one would expect of an equilibrium state. Bratelli and Robinson (1997, §5.3.1) catalog other, more local stability properties of KMS states, and gloss them by appeal to entropy maximization. Third, it has been argued, states satisfying the KMS condition at inverse temperature β act like thermal reservoirs, in the sense that any finite system coupled to a system in such a state reaches thermal equilibrium at β (Sewell (1986)). Although these considerations are not universally received as conclusive (Thirring (1980) offers reservations), they are generally taken to establish the KMS condition as a suitable analysis of thermal equilibrium.

We can now state the main results concerning the restriction of Minkowski vacuum state ω_M to various subalgebras of observables.

> **Lemma 3.** (i) Let $\mathcal{A}(\mathcal{O})$ be the subalgebra associated with an open region $\mathcal{O}$ of Minkowski spacetime. If $\mathcal{O}$ has non-null causal complement, then the state $\omega_M|_{\mathcal{A}(\mathcal{O})}$ obtained by restricting ω_M to $\mathcal{A}(\mathcal{O})$ is a KMS state at finite temperature with respect to some automorphism group of $\mathcal{A}(\mathcal{O})$. (ii) In the case where $\mathcal{O} = \mathcal{R}$, $\omega_M|_{\mathcal{A}(\mathcal{R})}$ is a KMS state at temperature $T = 1/2\pi$ with respect to the automorphism group α_η generated by the Rindler isometries, where η is Rindler time and units have been chosen so that $\hbar = k = 1$.

Part (i) has an easy proof (see Appendix D) but may lack physical significance. For there is no guarantee that the local automorphism group with respect to which $\omega_M|_{\mathcal{A}(\mathcal{O})}$ is a KMS state is related in any nice way to the symmetries of Minkowski spacetime or to the motions of accelerated observers. Part (ii) is highly non-trivial and is based on a deep theorem of Bisognano-Wichmann (see Sewell (1982) and Kay (1985)). For our purposes, the significance of its derivation is that, making no use whatsoever of the Fulling representation, it can hardly be taken to impute Rindler particle content to the Minkowski vacuum. Part (ii) can be generalized, in a somewhat weakened form, to an arbitrary globally hyperbolic spacetime with bifurcate Killing horizons: if ω is any quasi-free Hadamard state invariant under the timelike Killing symmetries that generate the bifurcate horizons, then its restriction to a "large" subalgebra of the right wedge algebra is a KMS state at finite temperature (see Kay and Wald (1991)).[19]

The definition of KMS states can be extended to cover the case of zero temperature, i.e. $\beta = +\infty$. A (α_t, ∞)-KMS state is called a *ground state* (with respect to the automorphism group α_t). Such a state is stationary, and the generator of a unitary representation of α_t is positive. Also noteworthy is a procedure for "cooling down" a quasi-free KMS state at finite inverse temperature β to a ground state at inverse temperature $+\infty$ (i.e. temperature 0) (see Kay and Wald (1991) and Chmielowski (1994)). If this cooled down ground state exists and if its one-particle Hamiltonian has no zero modes (i.e., eigenvectors with eigenvalue 0), then it is unique. It follows that ω_R is the ground state that results from cooling down the KMS state $\omega_M|_{\mathcal{A}(\mathcal{R})}$. This is what precludes the possibility, discussed in Section 7, that Fulling non-uniqueness constitutes an example of unitarily inequivalent quantizations arising from different pure states for the same spacetime.

Many presentations of part (ii) of Lemma 3 characterize the equilibrium temperature as $T = a/2\pi$, rather than $T = 1/2\pi$. Such characterizations suggest that a uniformly accelerated observer experiences a temperature proportional to the magnitude a of her proper acceleration. But the characterization is justified only if it is qualified. Suppose that ω is a KMS state at inverse temperature β with respect to the automorphism group α_t. Then if C is a positive constant, ω is a KMS state at inverse temperature β/C with respect to $\alpha_{t'}$, where $t' = t/C$. Now if a is the magnitude of acceleration along a *particular Rindler trajectory,* the proper time τ_a along that trajectory is related to the Rindler time η by $\tau_a = \eta/a$. Thus, $\omega_M|_{\mathcal{A}(\mathcal{R})}$ is a KMS state at temperature $a/2\pi$ with respect to α_{τ_a}. The upshot is that one can say *either* that there is *a* KMS state at temperature $1/2\pi$ with respect to the automorphism group α_η, *or* that there is an infinity of KMS states, one for each value of $a \in (0, +\infty)$, at temperature $a/2\pi$ with respect to the automorphism group α_{τ_a}.

To render the popular version of the Unruh effect fully rigorous, one would still need to link the KMS formalism to (idealized) observation procedures. In particular, one would need an account of measurement procedures which underwrites the claim that an observer uniformly accelerating through the Minkowski vacuum experiences a temperature proportional to the magnitude of her acceleration. The "particle detector" approach considered in the Section 11 can be taken to supply at least a partial answer. But that approach must be considered in light of a warning we issue before closing this section. We emphasize that nothing in the

19. However, there is no guarantee that such a state exists for an arbitrary globally hyperbolic spacetime with bifurcate Killing horizons.

derivation of the result that $\omega_M|_{\mathcal{A}(\mathcal{R})}$ is a KMS state at temperature $a/2\pi$ with respect to α_{τ_a} requires or justifies assigning any particle content to this state, much less interpreting it as a thermal bath of Rindler quanta.

10. What Does the Minkowski Vacuum State Imply about the Presence of Rindler Quanta? One strategy for describing the experience of an observer uniformly accelerated through the Minkowski vacuum is to express the restriction $\omega_M|_{\mathcal{A}(\mathcal{R})}$ of the Minkowski vacuum to $\mathcal{A}(\mathcal{R})$ as a density matrix in the Fulling representation. A number of authors claim to do this and to find that this density matrix represents a thermal bath of Rindler quanta at temperature $a/2\pi$. For example, Unruh and Wald (1984) write

$$\left|0_M\right\rangle = \prod_j \left[N_j \sum_{n_j=0}^{\infty} \exp(-\pi n_j \omega_j/a) \left|n_j, \mathcal{L}\right\rangle \otimes \rangle \left|n_j, \mathcal{R}\right\rangle \right] \tag{21}$$

where $N_j = (1 - \exp(-2\pi\omega_j/a))^{1/2}$. The product is taken over a complete set of Fulling modes, and $|n_j, \mathcal{L}\rangle$ (respectively, $|n_j, \mathcal{R}\rangle$) denotes the state with n_j Rindler quanta in mode j in the left Rindler wedge $\mathcal{L}$ (respectively, the right Rindler wedge $\mathcal{R}$). Tracing out over the degrees of freedom in $\mathcal{L}$ produces the reduced density matrix for $\mathcal{R}$:

$$\hat{\rho}^{\mathcal{R}}_{2\pi/a} = \prod_j \left[N_j^2 \sum_{n_j} \exp(-2\pi n_j \omega_j/a) \left|n_j, \mathcal{R}\right\rangle \otimes \left\langle n_j, \mathcal{R}\right| \right] \tag{22}$$

Similar expressions are given in Unruh (1976), in Sciama et al. (1981), and Ginsburg and Frolov (1987). But strictly speaking, this density matrix expression is meaningless. The disjointness of $\omega_M|_{\mathcal{A}(\mathcal{R})}$ and ω_R means that $\omega_M|_{\mathcal{A}(\mathcal{R})}$ is not in the folium of ω_R, or of any other pure state on $\mathcal{A}(\mathcal{R})$.

Although some authors take the formulas (21) and (22) literally,[20] Wald (1994) acknowledges that they do not make strict mathematical sense. But

20. It may be worth tracing the source of confusions that lead various authors to assert the literal correctness of equations (21) and (22). Sciama et al. (1981) say that $\mathcal{R}$ and $\mathcal{L}$ taken together contain a global Cauchy surface. This is not strictly true since $\mathcal{R} \cup \mathcal{L}$ leaves out the origin of Minkowski spacetime. This might seem like a minor point; but two-dimensional Minkowski spacetime with the origin removed is not simply connected, and the change in topology can result in thermal effects (see Dowker (1978) and Troost and van Dam (1979)). But let this pass. Sciama et al. jump from the existence of a global Cauchy surface to saying that the quantization for the entire spacetime is determined by the quantizations for $\mathcal{R}$ and $\mathcal{L}$. That is not true. The rigorous definition of $\mathcal{A}(\mathcal{R})$ ($\mathcal{A}(\mathcal{L})$) is in terms of solutions with compact support on a Cauchy surface for $\mathcal{R}(\mathcal{L})$. The definition of $\mathcal{A}(\mathcal{M})$ uses solutions with compact support on a global Cauchy surface, and such solutions may have support on both $\mathcal{R}$ and $\mathcal{L}$. See also the criticisms of Belinskiĭ et al. (1997, 1999) and Fedetov et al. (1999).

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he appeals to Fell's theorem to justify using such expressions as approximations. According to that theorem, the expectation values $\omega_M|_{\mathcal{A}(\mathcal{R})}$ assigns to any finite number of observables in $\mathcal{A}(\mathcal{R})$ can be approximated to any desired finite degree of accuracy by a density matrix in the Fulling Fock space. That is, for any $A_1, A_2, \ldots, A_n \in \mathcal{A}(\mathcal{R})$ and any $\varepsilon_i > 0$, $i = 1, 2, \ldots, n$, there is a density matrix ρ_F in the Fulling Fock space such that $|\omega_M(A_i) - Tr(\rho_F \pi_{\omega_R}(A_i))| < \varepsilon_i$ for all i.

But this strategy for warranting talk of the quanta-content of the Minkowski vacuum is too indiscriminately successful. For the consequence just stated of Fell's theorem remains true when any other pure state θ on $\mathcal{A}(\mathcal{R})$ is substituted for the Rindler vacuum state ω_R. If θ is a quasi-free state, it will define an associated particle concept (though, perhaps, not a "natural" one if θ is not appropriately related to the symmetries of Rindler spacetime). So an appeal to Fell's theorem cannot by itself justify assertions about the presence of Rindler particles (or quanta) as opposed to θ-particles (or quanta).

It is not obvious how, or whether, to draw from the Minkowski vacuum state meaningful conclusions about the presence of Rindler quanta. One way to proceed would be to extend $\omega_M|_{\mathcal{A}(\mathcal{R})}$ from $\mathcal{A}(\mathcal{R})$ to the affiliated von Neumann algebra $\mathcal{V}_{\pi_{\omega_R}}(\mathcal{A}(\mathcal{R}))$, which is the algebra in which Rindler particle number operators occur. But there are many such extensions. And, unlike the case of a state in the folium of ω_R, no one of these can be singled out as the natural extension. Thus, to prove that $\omega_M|_{\mathcal{A}(\mathcal{R})}$ implies a conclusion about the probability of finding Rindler quanta, one must show that that conclusion holds for every extension of $\omega_M|_{\mathcal{A}(\mathcal{R})}$ to $\mathcal{V}_{\pi_{\omega_R}}(\mathcal{A}(\mathcal{R}))$. Pursuing this strategy, Clifton and Halvorson (2001) demonstrate that every such extension gives a probability of 0 of finding exactly n Rindler quanta for every $n \in \mathbb{N}^+$. (Such a probability distribution would be impossible if it were countably additive; however, the extension of $\omega_M|_{\mathcal{A}(\mathcal{R})}$ to $\mathcal{V}_{\pi_{\omega_R}}(\mathcal{A}(\mathcal{R}))$ does not give a countably additive state.) It follows from their result that, for the state $\omega_M|_{\mathcal{A}(\mathcal{R})}$, the probability is 1 of finding $n > 0$ Rindler quanta for any $n \in \mathbb{N}^+$. This might be glossed as "With probability 1, the Minkowski vacuum contains an infinite number of Rindler quanta." And if probability 1 signifies truth, the gloss can be shortened to "The Minkowski vacuum contains an infinite number of Rindler quanta."

But, as Clifton and Halvorson (2001) note (and as the reader may have begun to anticipate), this result has nothing to do with Rindler quanta per se. For an exactly parallel result holds if for ω_R is substituted any pure regular[21] state θ on $\mathcal{A}(\mathcal{R})$ that induces a natural Fock space representation.

21. A state ω on a Weyl algebra over a symplectic space (S, Ω) is said to be *regular* if its GNS representation π_ω is such that for the generators W of the algebra, $t \mapsto \pi_\omega(W(ty))$ is strongly continuous for all $y \in S$. The *quasi-free states* which carried the burden of discussion above are regular.

Thus, if the original gloss is sound, then so is "The Minkowski vacuum contains an infinite number of Rindler quanta, and it also contains an infinite number of __-quanta," where any pure regular state θ that has a natural Fock space representation can be substituted for the blank. So one might accept the gloss and do a *modus ponens* to conclude that the Minkowski vacuum is a very crowded place indeed. Or one might do a *modus tollens* and conclude that the gloss is an artifact of the peculiarity of disjoint representations, not a guide to the particle contents of the Minkowski vacuum.

In any case, even if the *modus ponens* route is taken, it does not follow that the Minkowski vacuum is full of Rindler quanta in the sense relevant to the Unruh effect—namely, that the Minkowski vacuum contains a *thermal bath* of Rindler quanta. Pronouncements of this sort are precluded by the fact that $\omega_M|_{\mathcal{A}(\mathcal{R})}$ cannot be expressed by a density matrix in the Fulling Fock space. Nevertheless, it might still prove useful to take on board the notion that the Minkowski vacuum literally contains an infinity of Rindler quanta *if* this notion were to figure in a plausible explanation of the experiences of observers who are uniformly accelerated through the Minkowski vacuum. We will take up this matter in the following section.

11. Particle Detectors. Another way to get a handle on what an accelerating observer experiences as she moves through the Minkowski vacuum is to appeal to "particle detectors." It is often claimed that such detectors register a thermal flux of particles. Then one need only consider this claim in light of Davies' (1984) slogan that "Particles are what particle detectors detect" to arrive at the conclusion that particles are "relative to the reference frame."

As we noted above, we agree with one way of reading Davies' slogan: not as an endorsement of operationalism but as a way of underscoring the fact that, unlike ordinary QM, QFT does not have the concept of "particle" among its basic concepts. In QFT, the particle concept must be constructed from the materials at hand. The construction can be described, at least in part, in terms of features we would like a "particle detector" to have. Unruh (1990) details some of the choices one has to make. Does one want the detector to give only discrete responses, or may it give a continuum of responses? Is the detector to interact locally with the quantum field, or may it be permitted to interact non-locally with the field? Does the detector couple instantaneously to the field, or over a finite time interval, or an infinite time interval? Is the detector to be designed so that in a one-quantum state of the field the probability is strictly 0 that two detectors, located in two relatively spacelike regions, both give positive responses? And so on. Different answers to these questions reflect different kinds of particle detectors—indeed, particle detectors that will behave differently when accelerated through the Minkowski vacuum. Unless it can be successfully argued that there is only one "right" answer to all of these questions, the slogan that "Particles are what particle detectors detect" does not yield a single, fundamental particle notion.

Unruh's own choices are satisfied by an *DeWitt-Unruh* box detector, consisting of a box containing a Schrödinger particle initially in its ground state. The particle is coupled to the quantum field. A transition of the detector particle to an excited state counts as the detection of a particle of the field. The form of the coupling to the Klein-Gordon field chosen for the DeWitt-Unruh detector implies, in the first-order perturbation approximation, that when the detector is moving through the Minkowski vacuum, the probability of transition from an energy E_o to E is proportional to

$$\int_{-\infty}^{+\infty} d\tau \int_{-\infty}^{+\infty} d\tau' \exp\big(-i(E - E_o)(\tau - \tau')\big)\langle 0_M \,|\hat{\phi}\big(x(\tau)\big)\hat{\phi}\big(x(\tau')\big)|\,0_M\rangle \qquad (23)$$

where $x(\tau)$ is the world line of the detector parameterized by proper time (see Birrell and Davies (1989, 48–59)). For a *massless* Klein-Gordon field in four-dimensional Minkowski spacetime, a computation of (23) shows that an DeWitt-Unruh detector which is always in inertial motion will experience the Minkowski vacuum as devoid of particles (null response) while an always uniformly accelerated detector will experience the same state as a thermal (Planck) distribution at temperature $T = a/2\pi$.

It is tempting to explain this result by saying that the DeWitt-Unruh detector gets excited by modes of the field that are positive frequency with respect to proper time along its world line. Thus if we suppose the Minkowski vacuum to be full of particles that are positive frequency with respect to proper time along a uniformly accelerating observer's world line—that is, Rindler particles—then we are on our way to explaining the predicted response of the DeWitt-Unruh detector. But such an explanation, however tempting, should be resisted. For there are grounds for resisting the suggestion that the uniformly accelerating DeWitt-Unruh detector detects a thermal distribution of particles or even that it detects particles at all.

For an *always* uniformly accelerated DeWitt-Unruh detector coupled to a *massive* ($m > 0$) Klein-Gordon field in Minkowski spacetime, the spectrum registered is *not thermal* (Planckian), and vanishes as $m \to \infty$ (Takagi (1986)). Takagi also showed that if the dimension of spacetime is odd, an always uniformly accelerated DeWitt-Unruh detector will register a *Fermi* distribution (Takagi (1986)). Thus, even supposing that the detector detects particles, its behavior is explained by populating the Minkowski vacuum with a thermal distribution of Rindler quanta only in the case of massless fields and spacetimes whose dimension is even.

The DeWitt-Unruh detector bears only a distant relationship to any apparatus that experimental physicists use to detect particles. That is to say, experimental physicists would answer Unruh's engineering questions differently and, hence, would design a different kind of particle detector, quite possibly one which behaves differently from the DeWitt-Unruh detector when accelerated through the Minkowski vacuum. To take just one example, the DeWitt-Unruh detector is coupled non-locally to the field—the transition probability in (23) is from $\tau = -\infty$ to $\tau = +\infty$—whereas actual detectors, and any detectors that experimenters could hope to construct, are "switched on" for only a finite amount of time. Sriramkumar and Padamanabhan (1996) have analyzed the response behavior of Dewitt-Unruh detectors with different window functions whose width T determines the period for which the detector is coupled to the field. They find that smooth window functions, such as Gaussians and exponentials, do not produce divergent detector responses and satisfy the natural constraints that the response vanishes for both inertial and uniformly accelerated motions when $T \to 0$ whereas the response in the $T \to \infty$ limit for a uniformly accelerated detector coincides with that of the idealized detector coupled to the field for all times. But, not surprisingly, a uniformly accelerated Dewitt-Unruh switched on for a finite T does not respond as if it were in a thermal bath of Rindler particles.

Even for idealized DeWitt-Unruh detectors that are switched on for all time, there is reason to be cautious about saying that a positive response represents the detection of particles. The integral curves of timelike Killing fields on Minkowski spacetime include not only inertial and linear motion with constant proper acceleration (the Rindler case) but also uniformly rotating motions as well. Since a uniformly rotating frame is stationary but not static, the procedure of Section 3 for quantizing the Klein-Gordon field does not apply; nor is the algebraic approach of Section 4 applicable since the region of spacetime on which the frame is rotating with a velocity less than that of light is not globally hyperbolic. Nevertheless, Letaw and Pfautsch (1980, 1981), Padamanabhan (1982), and Sriramkumar and Padamanabhan (2002) have provided a heuristic quantization procedure and find that the vacuum state associated with the rotating frame is identical with the Minkowski vacuum. However, these authors also find that a DeWitt-Unruh detector which is at rest in the rotating frame and which is switched on for all time detects a non-flat, non-thermal spectrum for the case of the Minkowski vacuum for the massless Klein-Gordon field. If their quantization procedure can be given a rigorous footing,[22] we would have a clear case where theoretical considerations dictate that the detector is not detecting particles. There are, of course, important differences be-

22. For some conjectures on this matter, see Chmielowski (1994).

tween the Rindler and the rotating frame. In particular, an event horizon bounds the domain of Minkowski spacetime on which the Rindler frame is defined, and work is required to maintain an observer at rest in this frame; but these features are absent for the uniformly rotating frame (Letaw and Pfautsch (1981)). But these differences do not seem to justify the notion that responses of detectors at rest in a static frame are reliable indicators of particles whereas responses of detectors at rest in stationary but non-static frames are not to be trusted.[23]

Finally, the particle explanation of response of the DeWitt-Unruh detector may not square with our best answer to the question of what energy-momentum of the Klein-Gordon field an observer uniformly accelerated through the Minkowski vacuum measures. The obvious way to answer this question is to take the components in the Rindler coordinates of the renormalized stress-energy tensor's expectation value in the Minkowski vacuum state ω_M. Since $\langle T_{ab}\rangle_{\omega_M} = 0$, a uniformly accelerating observer (or any observer for that matter) perceives no energy-momentum. Thus, *if* a uniformly accelerating observer is detecting particles, the particles she detects have a ghostly existence that does not manifest itself by contributing to $\langle T_{ab}\rangle$, as ordinary particles do.

If the accelerating DeWitt-Unruh detector is not detecting particles, what then is it detecting? Here is the beginning of one possible answer. For excitations of amount $\Delta = E - E_o$, the rate of response $I(\Delta)$ of the detector per unit proper time along the world line of a stationary frame in Minkowski spacetime is

$$I(\Delta) = \int_{-\infty}^{+\infty} d\tau \exp(-i\Delta\tau)\langle 0_M | \hat{\phi}(x(\tau))\hat{\phi}(x(0)) | 0_M \rangle \tag{24}$$

This expression for $I(\Delta)$ resembles the formula for the power spectrum of the noise of a stochastic process, suggesting that what an accelerating detector is detecting is not particles but the noise of the zero-point fluctuations of the Minkowski vacuum (see Sciama et al. (1981)). Notice that this approach explains the device's response no matter what its state of motion.

Whether or not this suggestion stands up to scrutiny, the point remains that a derivation of the response of a DeWitt-Unruh detector can be done entirely within the Minkowski representation. Thus, insofar as explanation consists of derivation from first principles, an explanation of the response of the detector need not use or refer to Rindler quanta, or any other non-Minkowski quanta. There is a simple reason why this style of

23. As Padamanabhan puts it: "[O]ne is forced to consider the results based on the model detector with suspicion—they have to be confirmed by actual quantization calculations" (1982, 262).

explanation is preferable to one involving Rindler or other quanta. When the DeWitt-Unruh detector is non-uniformly accelerated through the Minkowski vacuum, it will register a non-thermal spectrum. But there is no timelike symmetry of Minkowski spacetime associated with such a motion and, thus, no natural particle notion associated with the motion, making unavailable an explanation in terms of particles. If uniformity of explanation for different motions of the detector is a desideratum for good explanation, then a non-particle explanation is preferred.

The upshot of our discussion is that, unlike the considerations of the two preceding sections, the approach using a "particle detector" does give some positive support to the popular notion that an observer accelerating through the Minkowski vacuum experiences a thermal flux of particles (or quanta). However, the support is far from perfect and needs to be qualified and hedged in a number of ways. And those who subscribe to the uniformity-of-explanation principle announced in the last paragraph would prefer an alternative account that does not explain the responses of the detector in terms of particles (or any non-Minkowskian quanta).

12. Conclusion. Fulling non-uniqueness and the Unruh effect are marvelous vehicles for exploring the mysteries of QFT. But the physical significance of Fulling non-uniqueness turns on the physical realizability of the Rindler vacuum state ω_R, and we have argued that there are reasons to be dubious of the latter. And apart from the issue of physical realizability, ω_R does not team up with the the Minkowski vacuum state ω_M to give an interesting example of "alternative vacuum states." Such an example would involve a globally hyperbolic spacetime $\mathcal{M},g_{ab}$ admitting two independent timelike Killing fields V^a and V'^a, together with two pure quasi-free states ω_{V^a} and $\omega_{V'^a}$ such that ω_{V^a} (respectively, $\omega_{V'^a}$) is invariant under the symmetries associated with V^a (respectively, V'^a) and such that the GNS representations of the Weyl algebra $\mathcal{A}(\mathcal{M})$ determined by ω_{V^a} and $\omega_{V'^a}$ are unitarily inequivalent. Whether there are physically interesting examples of this sort remains an open question.

Attempts to link Fulling non-uniqueness to the Unruh effect require a stretch that is not supported by the formalism of QFT. What is key to the phenomena discussed under these labels has little to do with the Rindler representation and much to do with the peculiar nature of the restriction $\omega_M|_{\mathcal{A}(\mathcal{R})}$ of the Minkowski vacuum state to the right Rindler wedge algebra $\mathcal{A}(\mathcal{R})$. That $\omega_M|_{\mathcal{A}(\mathcal{R})}$ and ω_R give disjoint representations of $\mathcal{A}(\mathcal{R})$ has nothing to do with ω_R per se since the disjointness holds of any pure state on $\mathcal{A}(\mathcal{R})$. The properly thermal nature of $\omega_M|_{\mathcal{A}(\mathcal{R})}$ resides in the fact that $\omega_M|_{\mathcal{A}(\mathcal{R})}$ is a KMS state at finite temperature with respect to the automorphism group of $\mathcal{A}(\mathcal{R})$ generated by the Rindler isometries. If the "Unruh effect" designates this result, and analogous results for the general case of globally hyperbolic spacetimes with bifurcate Killing horizons, then provably the Unruh effect does exist. But if the "Unruh effect" means that the Minkowski vacuum is full of Rindler quanta, in the sense of containing a thermal distribution of such quanta, then that effect does not exist—or, more cautiously, the most straightforward ways to express the effect in the formalism of QFT do not work.[24] Nor can the popular version of the Unruh effect be founded on attempts to use "particle detectors" to operationalize the claim that, for a uniformly accelerated observer, the Minkowski vacuum contains a thermal flux of particles. For, in our opinion, the best explanation of the response of such detectors is not that they are detecting particles but that they are detecting the noise of the Minkowski vacuum.

The popular—and we believe, dubious—versions of the Unruh effect were promoted in part as a way of demoting the particle concept in QFT from fundamental status. But even unassailed by the Unruh effect, the particle concept is beleaguered, for reasons including those we have rehearsed in Sections 7 and 8. The challenge facing interpreters of QFT it not to free it from a fundamental particle concept, but rather to show how QFT can explain effects that were previously taken to indicate the presence of particles.

Although it seems obvious that Fulling non-uniqueness and the Unruh effect provide crucial test cases for any philosophical interpretation of QFT, philosophers have been slow to respond; for example, Teller's (1995) book on the foundations of QFT and Huggett's (2000) review article offer only cursory discussions of the issues, and while Clifton and Halvorson (2001) provide a rich and illuminating discussion of the Fulling quantization, they have little to say about the Unruh effect. Without pretending to offer any overall interpretative stance on QFT, we have staked out a position on Fulling non-uniqueness and the Unruh effect. This position can be summarized by the slogan that almost every interesting and correct assertion that can be made about these phenomena can be derived from properties of the state $\omega_M|_{\mathcal{A}(\mathcal{R})}$ without the need to invoke any sorts of particle contents or strike any operationalist stances. Whether or not our position proves to be tenable, we hope to have identified the relevant issues that must be addressed in any adequate treatment of these phenomena. And independently of these phenomena, we hope to have illustrated how QFT poses philosophically challenging interpretational issues that are not encountered in ordinary QM.

24. Thus, we partly agree and partly disagree with the assertion of Belinskiĭ et al. (1997, 1999) and Fedetov et al. (1999) that "the Unruh effect does not exist."

Appendix

A. Some Basics of C-Algebras.*

A *C*-algebra* $\mathcal{A}$ is an algebra, over the field $\mathbb{C}$ of complex numbers, with an involution * satisfying: $(A^*)^* = A$, $(A + B)^* = A^* + B^*$, $(\lambda A)^* = \bar{\lambda} A^*$ and $(AB)^* = B^* A^*$ for all $A, B \in \mathcal{A}$ and all complex λ (where the overbar denotes the complex conjugate). In addition, a C^*-algebra is equipped with a norm, satisfying $\|A^*A\| = \|A\|^2$ and $\|AB\| \leq \|A\| \, \|B\|$ for all $A, B \in \mathcal{A}$, and is complete in the topology induced by that norm. We also assume that $\mathcal{A}$ contains a unit 1 such that $1A = A1 = A$ for all $A \in \mathcal{A}$. Observables are identified with self-adjoint elements of $\mathcal{A}$, i.e. elements A such that $A^* = A$.

A *representation* of a C^*algebra $\mathcal{A}$ is a mapping $\pi : \mathcal{A} \to \mathcal{B}(\mathcal{H})$ from the abstract algebra into the concrete algebra $\mathcal{B}(\mathcal{H})$ of bounded linear operators on a Hilbert space $\mathcal{H}$ such that $\pi(\lambda A + \mu B) = \lambda\pi(A) + \mu\pi(B)$, $\pi(AB) = \pi(A)\pi(B)$, and $\pi(A^*) = \pi(A)^\dagger$ for all $A, B \in \mathcal{A}$ and all $\lambda, \mu \in \mathbb{C}$. A representation is *faithful* if $\pi(A) = 0$ implies $A = 0$. A representation $(\pi, \mathcal{H})$ of $\mathcal{A}$ is *irreducible* just in case the only closed subspaces of $\mathcal{H}$ that are invariant under $\pi(\mathcal{A})$ are $\{0\}$ and $\mathcal{H}$. If $(\pi, \mathcal{H})$ is a representation of $\mathcal{A}$ and $\mathcal{K} \subset \mathcal{H}$ is a non-zero closed subspace invariant under $\pi(\mathcal{A})$, then the mapping $\mathcal{A} \to \mathcal{B}(\mathcal{H}) : A \mapsto \pi(A)\hat{P}_{\mathcal{K}}$, where $\hat{P}_{\mathcal{K}}$ is the orthogonal projection onto $\mathcal{K}$, is a *subrepresentation* of π. Two representations $(\pi_1, \mathcal{H}_1)$ and $(\pi_2, \mathcal{H}_2)$ of a C^*-algebra $\mathcal{A}$ are said to be *unitarily equivalent* just in case there is an isomorphism $\hat{U}: \mathcal{H}_1 \to \mathcal{H}_2$ such that $\hat{U}\pi_1(A)\hat{U}^{-1} = \pi_2(A)$ for all $A \in \mathcal{A}$.

A *state* on $\mathcal{A}$ is a linear functional ω that is normed ($\omega(1) = 1$)) and positive ($\omega(A^*A) \geq 0$ for all $A \in \mathcal{A}$). An algebraic state is said to be *pure* (respectively, *mixed*) if it cannot (respectively, can) be written as a nontrivial convex linear combination of other states. A basic result is that the GNS representation determined by a state ω is irreducible just in case ω is pure. (Recall from Section 4 that the GNS representation determined by a state ω is the unique, upto unitary equivalence, cyclic representation.)

If $(\pi, \mathcal{H})$ is a representation of C^*-algebra $\mathcal{A}$, the *von Neumann algebra* $\mathcal{V}_\pi(\mathcal{A})$ associated with the representation is $[\pi(\mathcal{A})]''$ (i.e. the double commutant of $\pi(\mathcal{A})$). A von Neumann algebra is said to be *Type I* if its commutant is abelian. If ω is a pure state on $\mathcal{A}$, the von Neumann algebra associated with the GNS representation of ω is of Type I since $\mathcal{V}_{\pi_\omega}(\mathcal{A}) = \mathcal{B}(\mathcal{H}_\omega)$. A state ω on $\mathcal{A}$ (or the GNS representation $(\pi_\omega, \mathcal{H}_\omega)$ determined by ω) is said to be *factorial* iff $\mathcal{V}_{\pi_\omega}(\mathcal{A}) \cap \mathcal{V}_{\pi_\omega}(\mathcal{A})'$ consists of multiples of the identity. A state ω on $\mathcal{A}$ has a canonical extension to a state $\tilde{\omega}$ on the von Neumann algebra $\mathcal{V}_{\pi_\omega}(\mathcal{A})$ associated with the GNS representation determined by ω. ω is said to be *normal* iff $\tilde{\omega}$ on $\mathcal{V}_{\pi_\omega}(\mathcal{A})$ is countably additive.

B. The Disjointness of the Fulling and Minkowski Representations.

The following Lemma contains some characterizations of the concept of the quasi-equivalence of representations. (Recall from Section 5 that two representations are quasi-equivalent just in case their folia coincide.)

Lemma 4. (i) If π and π' are non-degenerate representations of a C^*-algebra $\mathcal{A}$, then they are quasi-equivalent iff there is a *-isomorphism $\alpha : \mathcal{V}_{\pi}(\mathcal{A}) \to \mathcal{V}_{\pi'}(\mathcal{A})$ such that $\alpha(\pi(A)) = \pi'(A)$ for all $A \in \mathcal{A}$. (ii) π and π' are quasi-equivalent iff π has no subrepresentation disjoint from π' and vice versa. (iii) A representation π of a C^*-algebra $\mathcal{A}$ is factorial iff every sub-representation of π is quasi-equivalent to π.

That π_{ω_R} and $\pi_{\omega_M|\mathcal{A}(\mathcal{R})}$ are not quasi-equivalent follows from part (i) of Lemma 4 since they determine non-isomorphic von Neumann algebras—$\mathcal{V}_{\pi_{\omega_R}}$ is Type I while $\mathcal{V}_{\pi_{\omega_M|\mathcal{A}(\mathcal{R})}}$ is Type III (see Clifton and Halvorson (2001)). Although in general a failure of quasi-equivalence does not entail disjointness, the implication does hold in the present case. ω_R is a pure state and, thus, π_{ω_R} is irreducible and has no non-trivial subrepresentations. Also $\pi_{\omega_M|\mathcal{A}(\mathcal{R})}$ is factorial and, thus, by part (iii) of Lemma 4 is quasi-equivalent to each of its subrepresentations. So from part (ii) it follows that π_{ω_R} and $\pi_{\omega_M|\mathcal{A}(\mathcal{R})}$ are not quasi-equivalent iff they are disjoint.

C. The Local Quasi-Equivalence of the Fulling and Minkowski Representations.

The two senses of local quasi-equivalence for the Minkowski and Rindler vacuum states—(17) and (18)—would coincide if the von Neumann algebras $[\pi_{\omega_M}(\mathcal{A}(\mathcal{O}))]''$ and $[\pi_{\omega_R}(\mathcal{A}(\mathcal{O}))]''$ were factors for any open region $\mathcal{O} \subset \mathcal{R}$ with compact closure. This follows from (iii) of Lemma 4 and

Lemma 5. Let ω be a state on a C^*-algebra $\mathcal{A}$, and let $\mathcal{B}$ be a C^*-algebra sub-algebra. Then $(\pi_{\omega|\mathcal{B}}, \mathcal{H}_{\omega|\mathcal{B}})$ and $(\pi_{\omega}|\mathcal{B}, \mathcal{H}_{\omega}^{\mathcal{B}})$ are unitarily equivalent, where $\mathcal{H}_{\omega}^{\mathcal{B}} \subset \mathcal{H}_{\omega}$ stands for the span of $\pi_{\omega}(\mathcal{B})$.

Thus in general $(\pi_{\omega|\mathcal{B}}, \mathcal{H}_{\omega|\mathcal{B}})$ is unitarily equivalent to a sub-representation of $(\pi_{\omega}|\mathcal{B}, \mathcal{H}_{\omega})$. But if the span of $\pi_{\omega}(\mathcal{B}) = \mathcal{H}_{\omega}$, then $(\pi_{\omega|\mathcal{B}}, \mathcal{H}_{\omega|\mathcal{B}})$ is unitarily equivalent to $(\pi_{\omega}|\mathcal{B}, \mathcal{H}_{\omega})$. Now $\pi_{\omega_M|\mathcal{A}(\mathcal{O})}$ is factorial for an open $\mathcal{O} \subset \mathcal{R}$ with compact closure. But is $\pi_{\omega_R}|\mathcal{A}(\mathcal{O})$ factorial? If so, (17) and (18) are equivalent. Verch (1994) proves the factorial character of $\pi_{\omega_R}|\mathcal{A}(\mathcal{O})$ only for special $\mathcal{O}$. If $\pi_{\omega_R}|\mathcal{A}(\mathcal{O})$ is not factorial, we can still make some progress. By the Reeh-Schlieder property of the Minkowski vacuum, $\pi_{\omega_M|\mathcal{A}(\mathcal{O})}$ and $\pi_{\omega_M}|\mathcal{A}(\mathcal{O})$ are unitarily equivalent for any open region $\mathcal{O} \subset \mathcal{R}$ with compact closure. So we get that $\pi_{\omega_M|\mathcal{A}(\mathcal{O})}$ is quasi-equivalent to $\pi_{\omega_R}|\mathcal{A}(\mathcal{O})$ for any open region $\mathcal{O} \subset \mathcal{R}$ with compact closure.

D. Thermalization by Restrictions.

Let $(\pi_\omega, \mathcal{H}_\omega, |\Psi_\omega\rangle)$ be the GNS triple determined by the state ω on $\mathcal{A}$, and let $\tilde{\omega}$ be the canonical extension of ω to the von Neumann algebra $\mathcal{V}_{\pi_\omega}(\mathcal{A})$ associated with π_ω. $\tilde{\omega}$ is said to be *faithful* iff $\tilde{\omega}(\hat{P}) = 0$ for any positive $\hat{P} \in \mathcal{V}_{\pi_\omega}(\mathcal{A})$ implies that $\hat{P} = 0$. A vector $|\psi\rangle \in \mathcal{H}$ is said to be *separating* for a von Neumann algebra $\mathcal{V} \subseteq \mathcal{B}(\mathcal{H})$ iff $\hat{R}|\psi\rangle = 0$, $\hat{R} \in \mathcal{V}$, implies that $\hat{R} = 0$.

Lemma 6. (i) $\tilde{\omega}$ is faithful iff the GNS vector $|\Psi_\omega\rangle$ is separating for $\mathcal{V}_{\pi_\omega}(\mathcal{A})$. (ii) $|\Psi_\omega\rangle$ is separating for $\mathcal{V}_{\pi_\omega}(\mathcal{A})$ iff $|\Psi_\omega\rangle$ is cyclic for $\mathcal{V}_{\pi_\omega}(\mathcal{A})'$.

Now let $\mathcal{O}$ be an open set of Minkowski spacetime with non-empty spacelike complement. We want to show that $\tilde{\omega}_M|_{\mathcal{A}(\mathcal{O})}$ is faithful on $\mathcal{V}_{\pi_{\omega_M}|_{\mathcal{A}(\mathcal{O})}}(\mathcal{A}(\mathcal{O}))$. Begin by noting that by the Reeh-Schlieder property, $\pi_{\omega_M|\mathcal{A}(\mathcal{O})}$ is unitarily equivalent to $\pi_{\omega_M}|\mathcal{A}(\mathcal{O})$. Thus, the von Neumann algebras $\mathcal{V}_{\pi_{\omega_M}|_{\mathcal{A}(\mathcal{O})}}(\mathcal{A}(\mathcal{O}))$ and $\mathcal{V}_{\pi_{\omega_M}}(\mathcal{A}(\mathcal{O}))$ are isomorphic, and $\tilde{\omega}_M|_{\mathcal{A}(\mathcal{O})}$ is faithful on $\mathcal{V}_{\pi_{\omega_M}|_{\mathcal{A}(\mathcal{O})}}(\mathcal{A}(\mathcal{O}))$ iff ω_M is faithful on $\mathcal{V}_{\pi_{\omega_M}}(\mathcal{A}(\mathcal{O}))$. By Lemma 6, ω_M is faithful on $\mathcal{V}_{\pi_{\omega_M}}(\mathcal{A}(\mathcal{O}))$ iff $\mathcal{V}_{\pi_{\omega_M}}(\mathcal{A}(\mathcal{O}))'|\Psi_\omega\rangle$ is dense in $\mathcal{H}_{\omega_M}$. The right hand side of the last iff is true by the Reeh-Schlieder property. Now since $\tilde{\omega}_{M|\mathcal{A}(\mathcal{O})}$ is normal as well as faithful, it follows from the Tomita-Takesaki theorem (see Bratteli and Robinson (1979)) that there is a weakly continuous one-parameter family of modular automorphisms $\alpha_t^{\omega_M|\mathcal{A}(\mathcal{O})}$ on $\mathcal{V}_{\pi_{\omega_M}|_{\mathcal{A}(\mathcal{O})}}(\mathcal{A}(\mathcal{O}))$. The modular condition is the KMS condition, so $\omega_M|_{\mathcal{A}(\mathcal{O})}$ is a KMS state with respect to $\alpha_t^{\omega_M|\mathcal{A}(\mathcal{O})}$.

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