Essay Review: *The Laws of Belief*

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Wolfgang Spohn, ***The Laws of Belief: Ranking Theory and Its Philosophical Applications***. Oxford: Oxford University Press (2012), 625 pp., £75.00 (cloth).

Wolfgang Spohn's book *The Laws of Belief: Ranking Theory and Its Philosophical Applications* is a detailed presentation of ranking theory and its many and varied applications in philosophy. It is a work in formal or mathematical philosophy, philosophy that uses formal or mathematical tools to tackle philosophical problems. The book uses one simple and elegant yet powerful formal tool, namely, the theory of ranking functions, and applies it fruitfully to a great variety of philosophical problems. These start with problems in epistemology such as the representation of belief and its objects as well as justification and the a priori. They go to problems in metaphysics such as causation and objectification and problems in the philosophy of science such as laws and their confirmation as well as ceteris paribus conditions and dispositions. And they include, at least indirectly, problems in the philosophy of language such as conditionals and problems in the philosophy of mind such as perception and consciousness. The result is a unified picture of the world and our relation to it as epistemic agents: a philosophy. It is arrived at in a constructive attempt to reduce all "natural" or alethic modalities to nonmodal facts on the one hand and doxastic modalities as represented by ranking functions (and subjective probability measures) on the other hand. And it does so by displaying the virtues of using formal tools to tackle philosophical problems. The result is a formal philosophy whose radical nature, however, becomes visible only by working through the details, not something one can do over an extended weekend.

What I consider to be one of its most problematic features is the downside of the comprehensiveness of Spohn's monumental piece: its length is scary, and many parts of the book are not an easy read, especially since it contains several new theorems that Spohn proves in the main body of text. At a minimum, Spohn should have put the proofs in an appendix. Ideally he would have outsourced the proofs to technical papers in order to make the book more accessible in both length and style. While, as readers, we should not cherish the illusion that philosophy is easy, as authors we should try our best to not add to the difficulty.

The plot of the book can be split up into three acts, and I think Spohn should have written three books rather than one, or at least he should have cut it up into three digestible volumes. That is, at least, how I want to perform the play: the first act corresponds to chapters 1–6 and 8–9; the second act roughly corresponds to chapters 10–13 and 16–17; the third act roughly corresponds to chapters 7 and 14–15 as well as some elements of chapter 12.

Ranking Theory. Spohn arrives at the central notion of a ranking function, originally called an "ordinal conditional function" in Spohn (1988), as the natural result of combining the frustrations of Bayesians to capture the notion of (categorical or outright or yes-or-no) belief (or belief simpliciter) in probabilistic terms and the frustrations of belief revision theorists in the AGM tradition (Alchourrón, Gärdenfors, and Makinson 1985) to cope with iterated belief changes. The first thesis Spohn submits is that subjective probability theory in the Bayesian tradition does not have a notion of belief that is capable of being true (44), although it has a dynamics that is complete (37ff.). The latter is the case if regularity is imposed as an additional constraint on probability measures and Jeffrey conditionalization is adopted as an update rule or if classical probabilities are replaced by Popper-Rényi measures (Popper 1955; Rényi 1955) and these are improved on as indicated in Spohn (1986; see also 202ff.). The second thesis Spohn submits is that AGM belief revision theory does not have a dynamics that is even moderately complete (65ff.), although it has a notion of belief that is capable of being true. In a nutshell, the reason is that AGM belief revision theory does not have a notion of *conditional belief* and thus lacks the corresponding notions of (conditional) relevance and (conditional) independence. Bayesians and belief revision theorists alike do well to study this discussion.

In order to have a notion of belief that is capable of being true as well as a complete dynamics for this notion that can handle indefinite iterations of belief changes, one has to represent the agent's doxastic state by a *ranking function*. The agent's conditional beliefs can be adequately captured in this way only. Here is a slightly revised version of Spohn's definition 5.5 with notation adapted (70): let **A** be a complete algebra of propositions over the set of possibilities or possible worlds *W*. Then κ is a ranking function on

A if and only if κ is a function from W into the extended natural numbers $\mathbf{N}^+ = \mathbf{N} \cup \{\infty\}$ such that at least one possibility w from W is assigned rank 0. Function κ is extended to propositions A from **A** by defining $\kappa(\emptyset) = \infty$ and $\kappa(A) = \min\{\kappa(w) : w \in A\}$ for each nonempty A from **A**. The rank of a proposition A, $\kappa(A)$, is interpreted as the agent's grade of disbelief for A. The agent believes a proposition A just in case she assigns a positive degree of disbelief to the negation of A, $W \backslash A : \kappa(W \backslash A) > 0$. The defining clause that at least one possibility be assigned rank 0 requires the agent to not disbelieve every possibility and, thus, to not disbelieve every proposition. In other words, the static law of belief encapsulated in the definition of a ranking function is consistency.

The dynamic law of belief is stated in terms of conditional ranks, which definition 5.15 defines as differences of unconditional ranks (78): $\kappa(w|A) = \kappa(w) - \kappa(A)$, if $w \in A$, and ∞ otherwise, provided $\kappa(A) < \infty$. The agent believes A conditional on C, just in case she assigns a positive degree of disbelief to the negation of A, $W \backslash A$, conditional on $C : \kappa(W \backslash A|C) > 0$. Definition 5.24 then states what is often called Spohn conditionalization, the rank-theoretic analogue to Jeffrey conditionalization in probability theory (83; in slightly revised form and with notation adapted): let κ be a ranking function on **A** and let A be a proposition from **A** such that $\kappa(A)$, $\kappa(W \backslash A) < \infty$, and let n be a number from $\mathbf{N}^+$. Then the $A \rightarrow n$-conditionalization $\kappa_{A \rightarrow n}$ of κ is defined by $\kappa_{A \rightarrow n}(w) = \kappa(w|A)$ if $w \in A$, and $\kappa_{A \rightarrow n}(w) = \kappa(w|W \backslash A) + n$ if $w \in W \backslash A$. Spohn conditionalization preserves and generalizes AGM belief revision theory (88ff.) and provides a complete dynamics. It can be shown to adequately handle indefinite iterations of belief changes (171ff., based on Hild and Spohn 2008), which at the same time answers the question of where the numbers come from by providing a theory of measurement for ranks in terms of what belief revision theorists call "contractions" (belief changes in which the old beliefs are contracted to make them consistent with some new belief). I personally prefer a different argument for the thesis that an agent should update her beliefs by Spohn conditionalization: the disadvantage is that I have to presuppose a way of measuring ranks; the advantage is that we can formulate the dynamic law of belief as diachronic consistency so that the "laws of belief" from the title of the book, as formulated in ranking theory, require nothing but consistency in a synchronic as well as a diachronic sense (Huber 2007).

The Philosophical Applications of Ranking Theory. Space does not permit to discuss more than one philosophical application of ranking theory. This application, though, is so important that it would have deserved a book on its own. It is *causation*.

What I consider to be Spohn's most radical philosophical thesis could have played a more prominent role in his book. In its general form the view

is only hinted at in a few places (12ff., 271ff., 442ff., 469ff.). In its general content the view is not new, as it has been held by David Hume on at least one interpretation. However, in the particular form in which it is spelled out for the particular problem at hand, it is genuinely revolutionary yet, I fear, not well received.

Spohn's view is that all there is are nonmodal facts and doxastic modalities, a view he shares with Lewis. Lewis's program of Humean supervenience was to show that all alethic modalities, such as counterfactuality, causation, lawhood, chance, and so on, supervene on nonmodal "local matters of particular fact" (Lewis 1986). Spohn thinks this does not work (272; Spohn 2010). Rather, what he calls "natural" modalities (causation, lawhood, dispositions, chances) are the results of projecting our doxastic modalities (belief and degrees of belief) onto the nonmodal facts: we are beefing up a world that is much more meager than we believe it to be. Spohn represents the doxastic modalities by ranking functions and subjective probability measures. The question is what it means that we project our thus represented doxastic modalities onto the nonmodal facts. Spohn spells out the projectivist metaphor for his theory of causation (chap. 14; Spohn 2005) in constructive detail and with formal precision. The main thought is the following one (chap. 15; Spohn 1993): causation is defined relative to a subjectively interpreted ranking function, but some features of this subjective ranking function can be brought into one-to-one correspondence with certain nonmodal propositions that are objectively true or false. For instance, the unconditional beliefs of a subjective ranking function can be brought into one-to-one correspondence with certain nonmodal propositions that are objectively true or false, namely, the contents of those unconditional beliefs. Therefore, unconditional belief, as defined in rank-theoretic terms, can be objectified (445ff.). Each feature of a ranking function that can be uniquely associated with a nonmodal proposition can itself be said to be objectively true or false, depending on whether the associated nonmodal proposition is true or false. Thus, the question is which features of a ranking function can be objectified in this sense and, in particular, whether causation can be so objectified. Spohn defines causation as the transitive closure (422ff.) of direct causation (354ff.) and shows that these two notions can be conditionally objectified (chap. 15).

While I remain skeptical whether the "natural" or alethic modalities that can be objectified in this sense are rich enough for causation (Huber 2011) and metaphysics in general, I admire the general spirit of the project: to work out in constructive detail and formal precision the positive claims and see how far one gets. Doing so is difficult, and working through it is not easy either. But that is, mainly, because philosophy is difficult.

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