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REVIEWS

The Association for Symbolic Logic publishes analytical reviews of selected books and articles in the field of symbolic logic. The reviews were published in ***The Journal of Symbolic Logic*** from the founding of the Journal in 1936 until the end of 1999. The Association moved the reviews to this Bulletin, beginning in 2000.

The Reviews Section is edited by Clinton Conley (Managing Editor), Mark van Atten, Benno van den Berg, Thomas Colcombet, Samuel Coskey, Bradd Hart, Bernard Linsky, Antonio Montalbán, Rahim Moosa, Christian Retoré, and Nam Trang. Authors and publishers are requested to send, for review, copies of books to *ASL, Department of Mathematics, University of Connecticut, 341 Mansfield Road, U-1009, Storrs, CT 06269-1009, USA*.

N. Francez, ***Proof-theoretic semantics***, Studies in Logic, vol. 57, College Publications, London, 2015, xx + 415 pp.

This book, addressed to an audience of mathematicians, computer scientists, philosophers, and linguists, presents in a comprehensive way the viewpoint of proof theoretic semantics. According to proof theoretic semantics,[1] the meaning of a sentence lies in its proof-conditions rather than in its truth-conditions. The first part of the book is devoted to the development of this idea for logical formulas, constituting an impressive and timely work of synthesis. The second part focuses instead on proof-conditions for sentences belonging to fragments of natural language and—to our knowledge—represents one of the few attempts to specify the meaning of natural language sentences in a proof-theoretic way. The mainstream formal semantics for natural language is in fact model-theoretical, so that Francez's contribution is more than welcome. This review will follow the structure of the book.

§1. First part of the book. The first and the second chapters present a succinct and convincing account of the motivations behind the idea that meaning should be explained in terms of proof-conditions. They also offer an introduction to various systems of natural deduction. Systems of natural deduction are the main formal tools used to characterise the semantics of a sentence throughout the book; the semantics of a sentence A is defined in terms of *canonical derivations* of the sentences. Canonical derivations of A are—roughly speaking—detour-free derivations of A having a particular structure, e.g., whose last rule is an introduction rule for the main connective of A. Various systems of natural deduction are introduced together with their meta-theoretical properties. However, proofs of these properties are omitted. In our opinion the exposition would benefit from their inclusion. Some of those proofs are classic, and punctual references are given; nonetheless, both experienced logicians and neophytes would benefit from further exposition.

The third chapter exposes some criticisms that are raised against the idea that the meaning of the logical constants lies in their deduction rules. Francez illuminates how these criticisms are overcome by the partisans of proof theoretic semantics. In particular we find a detailed discussion of the *tonk* connective introduced by N. Prior.[2] This connective is defined in terms of deduction rules and has the unwelcome property of trivialising the deductive system. Francez exposes how we could prevent the definition of this kind of connective by imposing

[1]Some classical references are D. Prawitz, ***Towards A Foundation of A General Proof Theory***, Studies in Logic and the Foundations of Mathematics, vol. 74 (1973), pp. 225–250 and M. A. E. Dummett, ***The Logical Basis of Metaphysics***, The William James lectures, Harvard University Press, 1993.

[2]N. Prior, *The runabout inference-ticket*. ***Analysis***, vol. 21, no. 2, pp. 38–39.

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certain conditions on the rules that are regarded as meaning-conferring. In particular the chapter contains a detailed discussion of the concept of *harmony* between introduction and elimination rules and a comprehensive comparison between the various definitions of harmony that one can find in the proof theoretic semantics literature.

The fourth chapter exposes some—rather exotic—material even for a partisan of proof-theoretic semantics. Proof-theoretic semantics has its roots in constructive mathematics and the aim of the material presented in this chapter is to give a proof-theoretical semantics of classical logic. Francez presents two main alternatives: systems of multi-conclusion natural deduction that remind one of sequent calculi systems and natural deduction enriched by a primitive operation of denial *distinguished from negation*. This chapter contains also a detailed discussion of the role of *structural rules* in the proof-theoretic semantics enterprise. This kind of discussion is quite uncommon in the proof-theoretic semantics literature and deserves appreciation. In our opinion this is one of the chapters that would have benefited from the inclusion of proofs concerning meta-theoretical properties of the system presented. The book insists meaning lies in proofs so, to better understand these quite exotic systems, proofs of their properties would be more than welcome.

In chapter five, which is called proof semantic values, N. Francez' personal viewpoint starts to emerge more conspicuously. Usually the partisans of proof-theoretic semantics states that the meaning of the logical constants are given *implicitly* by their canonical proof conditions and the focus of people working in proof-theoretic semantics has been on finding a suitable definition of logical consequence cast in terms of canonical proof conditions. The author however does the exact opposite. The semantic values of formulas are defined explicitly in terms of set of canonical proofs constructed in some formal system of natural deduction and the notion of logical consequence is not studied in detail. This chapter is surely one of the most interesting in the book, though unfortunately it is also one of the hardest to follow. The notation is quite heavy and some conceptual clarification of the definitions would be welcome. The sixth chapter details some alternative views on proof-theoretic semantics that use sequent calculus rather than natural deduction as the meaning-conferring proof system. An interesting view heavily inspired by logic programming is also presented.

§2. Second part of the book. The second part of the book is concerned with developing Proof Theoretic Semantics for small fragments of natural language. After a short introduction in chapter seven, chapter eight defines natural deduction proof systems for fragments of natural language containing sentences with a simple structure[3] e.g., subject, verb object, quantifiers and relative clauses. In particular the author gives natural deduction systems for natural language generalised quantifiers like "all" and "some." Generalised quantifiers are usually defined in terms of relations between subsets of an arbitrary universe U. For instance $U \models All(A, B)$ iff $[|A|] \subseteq [|B|]$ in U. Francez instead designs natural deduction rules for the two quantifiers. He also states that one can prove the normalisation theorem for a natural deduction system including these rules. Again an explicit proofs of this claim would have been warmly welcomed.

Chapter nine is an interesting approach to the meaning of subsentential units and could be read as an attempt to formalize Frege's *context principle*. Concretely the author shows how to specify the meaning of subsentetial expressions up to words from the meaning of the sentences in which they occur by using a syntactic analysis of the sentence carried out in type theoretical grammar.

Chapter ten presents a detailed study of the proof-theoretic semantics of determiners encompassing generalised quantifiers. Francez takes a particularly interesting approach to the phenomena of monotonicity, one which does not rely on an external—to the proof system—partial ordering on the expressions. We think that a comparison of this approach to the one of natural logic would be fruitful.

Chapter eleven is devoted to the semantics of intensional transitive verbs. These are verbs like *seeks*, which occur in sentences like *John seeks a killer*. Such sentences are semantically

[3]Like it is common in natural logic see L. Moss, ***Natural Logic***, second ed., Chapter 18, John Wiley and Sons, 2015, pp. 561–592.

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ambiguous between a specific reading and a general reading. The treatment of intensional transitive verbs is notoriously difficult in model theoretic semantics and the solution proposed by Francez is elegant and intuitive.

Chapter twelve explains how one can handle a natural language sentence's *implicit* context of utterance in proof-theoretical terms. The truth conditions of a natural language sentences usually depend upon a certain context of utterance. This context restricts the quantificational scope of the generalised quantifier to a certain given situation. For instance the sentence "all the bottles are empty" usually means something like "all the bottles in the fridge are empty." First Francez explain why it is hard to deal with context dependence in model theoretic terms. Then he propose to model context as assumptions $H_1, \ldots, H_n$ in a natural deduction derivation. The context dependence of a sentence A is modelled by restricting the introduction rule of the main symbol of the first order logic formula representing A to the context $\Gamma, H_1 \ldots H_n$ where Γ is arbitrary. This chapter also contains natural deduction rules for the ι operator of Russel, which is typically used to express denotational uniqueness e.g., used to capture the logical form of a noun phrase like "The president of France." The book ends with a brief afterword summarising the main proposal while pointing the way to open problems in proof-theoretic semantics.

To sum up, this book offers a good introduction to proof-theoretic semantics. An introduction which has the virtue of not lingering on endless philosophical discussions and also the remarkable virtue of being the only comprehensive introduction on the topic. We would have preferred this book to be more self contained, e.g., by detailing the proof of the propositions but the synthesis provided by Francez and the biography on which he relies are quite impressive. We have particularly appreciated the second part of the book: it is a breath of fresh air in formal semantics, an area which is heavily "dominated" by the model-theoretic approach to meaning.

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