

# MARKET POWER AND THE AGGREGATE SAVING RATE

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Can increasing market power cause a decrease in the aggregate savings? We answer this question by using a heterogeneous agents model that features both idiosyncratic labor and capital income risk. Under complete markets, the saving rate does not depend on the degree of market power, but when markets are incomplete, higher markups substantially reduce the aggregate saving rate. This is due to endogenous changes in the distribution of income and wealth. A calibration of the model using the observed changes in market power in the United States since the 1970s closely matches the decline in the US saving rate. Furthermore, when market power increases, the model generates distributional changes that are consistent with the data.

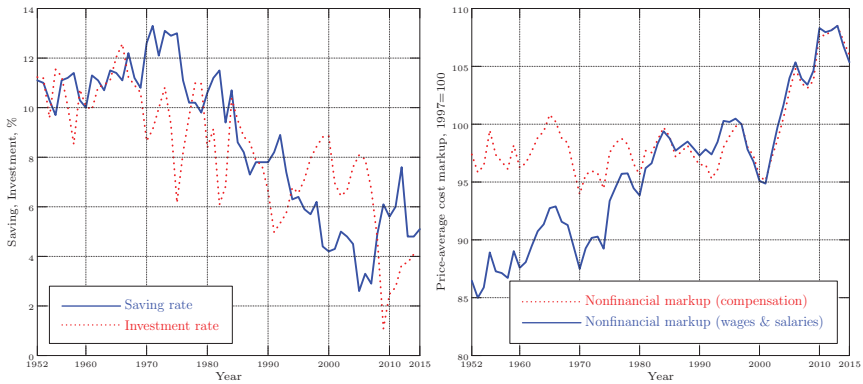
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## 1. INTRODUCTION

Since the mid-1970s, there has been a considerable decline in the US personal saving rate accompanied by an equally worrying collapse in business investment.<sup>1</sup> There is considerable controversy surrounding the causes of these trends, and most explanations focus either on savings or investment. For instance, as explanations for the declining saving rate, the literature has highlighted rising precautionary savings in the rest of the world [Bernanke (2005)]: an aging population with higher propensity to consume [Gokhale et al. (1996)], increasing access to credit and accumulation of capital gains especially in housing [Mian and Sufi (2014a,b)], and rising inequality and escalating consumption norms [Alvarez-Cuadrado and El-Attar (2012)].<sup>2</sup> On the investment side, the literature has singled out uncertainty as the main driver of weak investment [see, e.g., Barkbu et al. (2015), Banerjee et al. (2015)]. İşcan (2011) also considers the effect of expectations on investment, as well as on consumption-saving decisions in a general equilibrium model.

In this paper, we show that an increase in market power is capable of explaining much of the relative changes in savings and investment observed in the US data (Figure 1). We argue that market power has a strong impact on precautionary

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**FIGURE 1.** Aggregate saving, net investment, and price-average cost markup, US 1952–2015. The estimates of the price-average cost markup follow Nekarda and Ramey (2013). They construct several measures of markup as the inverse of the labor share, and report it as an index number (1997=100). Sources: National Income and Product Accounts (NIPA) by U.S. Department of Commerce, Bureau of Economic Analysis (2016).

savings and capital accumulation in an economy with incomplete financial markets. We use a model with a perfectly and a monopolistically competitive production sector, in which entry to the monopolistic sector is idiosyncratic. All agents face idiosyncratic labor endowments, including those that are in the monopolistic sector (entrepreneurs). Entrepreneurs use physical capital to produce intermediate goods, which are an input to the competitive sector. On the one hand, the risk-facing entrepreneurs tend to reduce investment in physical capital (the portfolio effect). On the other hand, since these agents have market power, they benefit from partial insurance against fluctuations in demand. As market power increases, the riskiness of capital income decreases, leading to a substitution toward capital due to precautionary motives. At the same time, market power also determines aggregate demand: the purchasing power of all agents declines when market power increases, and this reduces aggregate demand and capital accumulation (the wealth effect). We find that, under realistic calibrations, an increase in market power causes the aggregate saving rate to decline, suggesting that the wealth and portfolio effects dominate precautionary motives. Our findings complement the macroeconomic literature on the importance of market incompleteness for aggregate savings [Carroll (1992, 1997)]. Under complete financial markets, the saving rate is independent of the degree of market power in our model. This identifies an original and distinct channel for agent heterogeneity as a determinant of the aggregate savings. We also find that an increase in market power that matches the US data has considerable effects on the distribution of income and wealth by benefiting primarily the wealthiest 10% of the agents. The top 1% of agents more than triple their share of total wealth, while the top 10% ends up owning almost all wealth. Since our model abstracts from a range of factors that are likely to affect

the long-run distributional dynamics, we interpret these findings as the marginal contribution of higher market power to changes in income and wealth inequality.

Markups in the United States show a pronounced upward trend over the past four decades (Figure 1, right panel); see also Nekarda and Ramey (2013) and Council of Economic Advisors (2016). We treat this change as exogenous in this paper, by taking the point of view that market power has increased mostly because of political economy factors. This interpretation builds on micro-level evidence that mergers and acquisitions have significantly increased market power in the United States [Blonigen and Pierce (2016)] as well as accumulating evidence from the literature on industrial concentration and consolidation.

- Pryor (2001, 2002) estimates that industrial concentration decreased in the United States from 1960 until the early 1980s, but increased gradually thereafter at least until 1997. The factors behind the concentration ratios in the United States may include international trade, for instance, which could increase average concentration through selective destruction of those industries that are less protected by barriers to entry. Consistent with this hypothesis, Wang and Whalley (2014) find that concentration ratios in Chinese manufacturing industries are substantially lower than those in the United States.
- Swan (2005) studies the airline industry, which was deregulated in the early 1980s. Despite rising competition in the aftermath of deregulation, he finds that, since the late 1980s competition declined relative to its prederegulation levels. Koenig and Mayerowitz (2015) argue that consolidation is ongoing in the industry.
- Carroll et al. (2000) find a similar industry response to deregulation in the telecommunications industry. The breakup of AT&T in 1984, and the Telecom Act of 1996 were all legislated with the stated objectives of achieving a more competitive market structure. Instead, the Telecom Act even in its immediate aftermath culminated in consolidation in the industry.
- The health insurance industry in the United States has also gone through considerable consolidation over the last decade with the five largest insurers increasing their market share from an estimated 74% in 2006 to 83% in 2014 [Commonwealth Fund (2015)]. Dafny et al. (2012) find that consolidation in this industry has been conducive to more monopsonistic power by the insurers, reducing employment, and earnings of healthcare workers in the industry.<sup>3</sup>
- Agriculture, once among the most fiercely competitive sectors, has also experienced considerable consolidation. According to MacDonald et al. (2013), from 1982 to 2007 average acreage held by farms with more cropland than the midpoint acreage had nearly doubled (from 589 acres to 1,105). They attribute this increase in farm size to industry consolidation, and find that consolidation is associated with increased profitability.
- The impressive pace of consolidation of the banking industry in the United States is perhaps the most well documented among all other industries.

Starting with Berger et al. (1995), there has been extensive research on both the causes and consequences of this trend, with considerable agreement that consolidation has resulted in larger profit margins; see Jones and Critchfield (2005) for a survey of the literature up to the Great Recession.

- The case of the retail giant Walmart and consolidation in the retail industry is perhaps the best studied in the economics literature [Basker (2007)].

Of course, it is possible that consolidation may be caused by business dynamism rather than deregulation. If this were the case, we should observe a sustained and high rate of entry and exit from a sector. However, according to Decker et al. (2016), at least since the early 2000's, this dynamism has been on the decline uniformly in all industries. Hathaway and Litan (2014) find that the dynamism has started declining as early as mid-1980's, with lower entry rates accounting for much of the trend. Finally, Gutiérrez and Philippon (2016) find that higher concentration reduces investment at the industry level.

*Related literature.* Aside from few exceptions, heterogeneity and imperfect competition have been treated separately, leaving a gap between those models that have desirable empirical features linking aggregate demand to output (non-competitive models) and those that have desirable quantitative features linking distributional issues to macroeconomic outcomes, such as saving. While macroeconomic models of Blanchard and Kiyotaki (1987) and Rotemberg and Woodford (1992) consider noncompetitive markets, they are representative agent models. More recently, Opp et al. (2014) develop a tractable model of monopolistic competition with heterogeneous industries, owned by a representative individual. By contrast, while incomplete-markets models with uninsured idiosyncratic income risk [surveyed in Heathcote et al. (2009)] exhibit heterogeneous agents, they have competitive product markets. In this paper, we bridge these two strands in the literature. McKay and Reis (2016) also model both monopolistic competition and uninsurable idiosyncratic income risk. In their model, the interest rate is determined exogenously by the discount rate of a perfectly diversified representative saver, whereas here it is determined endogenously. Moreover, in our case, prices depend on market power, as well as the distributions of agents across capital and bond holdings.

To put our results into sharper perspective, consider two important strands in the incomplete markets literature where all agents are price takers. In the first strand of these models, income uncertainty originates from labor earnings alone, as in Aiyagari (1994), and uninsured idiosyncratic risk generates demand for precautionary saving. In the second strand of these models, idiosyncratic income uncertainty originates from capital income—as in Covas (2006), Angeletos (2007), and Evans (2014)—and holding private equity entails idiosyncratic risk. This produces two effects: on the one hand, the associated uncertainty in income induces a precautionary saving motive. On the other hand, risky returns on capital induce agents to limit their exposure to risks by reducing capital accumulation. In realistic calibrations of the US economy, Angeletos (2007) finds that the second

effect dominates the first. Here, as in Nirei and Aoki (2016), we use a framework featuring both capital and labor income uncertainty and allow for a risk-free asset in zero net supply.

The outline of the paper is follows: Section 2 describes the model. Section 3 presents our calibration and establishes robustness. Section 4 presents the main results. Section 5 concludes. Derivations, proofs, and the numerical solution algorithm are contained in the appendices.

**2. THE MODEL**

Consider an infinite horizon economy in discrete time (indexed by  $t$ ), and populated by a continuum of agents indexed by  $i \in \mathcal{I} = [0, 1]$ . The economy produces a final good, and a variety of intermediate inputs (materials). Below, we denote the agent-specific variables in lowercase Latin letters, and the corresponding aggregate variables in uppercase letters.

**2.1. Production**

There are two sectors: a final-good sector and an intermediate-good sector. The final-good sector is perfectly competitive and it produces a single commodity. The production function of this sector is

$$Y_t = Z_A M_t^\alpha L_t^{1-\alpha}, \tag{1}$$

where  $Z_A$  is total factor productivity (TFP) in the final-good sector,  $L_t$  is labor employed in the final-good sector,  $M_t$  is a composite of intermediate goods, and  $0 < \alpha < 1$  is the elasticity of output with respect to this composite good. The final-good producer combines these intermediate goods using the following production function:

$$M_t = \left[ \int_{\mathcal{J}_t} (x_{i,t})^{\frac{\nu-1}{\nu}} di \right]^{\nu/(\nu-1)}, \tag{2}$$

where  $\mathcal{J}_t \subset \mathcal{I}$  denotes the set of intermediate inputs,  $x_{i,t}$  ( $i \in \mathcal{J}_t$ ) denotes the demand for a typical intermediate input, and  $\nu > 1$  is the elasticity of substitution across varieties. As is standard in the models of monopolistic competition with constant elasticity of substitution (CES) aggregators, this parameter determines the extent of market power in the economy, with larger values of  $\nu$  corresponding to lower degrees of pricing power.

Each variety of intermediate inputs is produced by an entrepreneur: agent  $i$  is the owner-operator of firm  $i$ , which produces the unique variety  $y_i$  and sets the price of this variety as a monopoly producer. The production function of each variety is

$$y_{i,t} = z_{i,t} k_{i,t}^\alpha n_{i,t}^{1-\alpha}, \tag{3}$$

where  $z_{i,t}$  is an idiosyncratic TFP shock faced by agent (or firm)  $i$ ,  $k_{i,t}$  is the capital stock owned by agent  $i$ , and  $n_{i,t}$  is the labor endowment of agent  $i$ , all at time  $t$ .

*Idiosyncratic shocks.* Each agent faces two exogenous idiosyncratic shocks, both independently distributed across agents: a labor endowment shock  $n_{i,t}$  and a shock to entrepreneurial activity  $z_{i,t}$ . The labor endowment process is as in Aiyagari (1994):

$$\log n_{i,t} = \rho \log n_{i,t-1} + \epsilon_{i,t}, \quad \epsilon_{i,t} \sim \mathcal{N}(0, \sigma^2(1 - \rho^2)), \tag{4}$$

where  $\mathcal{N}$  denotes the standard normal distribution, and  $\rho$  and  $\sigma$  are the persistence and variance parameters.

The entrepreneurial activity, on the other hand, follows a two-state process with values  $z_{i,t} \in \{0, z\}$ ,  $z > 0$ . We refer to an agent with  $z_{i,t} = z$  as an “entrepreneur,” and with  $z_{i,t} = 0$  as a “worker.”<sup>4</sup> The transition matrix for this two-state process is

$$\begin{array}{cc} & \begin{array}{cc} \text{entrepreneur} & \text{worker} \end{array} \\ \begin{array}{c} \text{entrepreneur} \\ \text{worker} \end{array} & \begin{bmatrix} q_1 & 1 - q_1 \\ 1 - q_2 & q_2 \end{bmatrix}, \end{array} \tag{5}$$

where  $q_1$  is the probability of an entrepreneur in the current period to remain an entrepreneur in the next period, and  $1 - q_1$  is the probability of an entrepreneur in the current period to go out of business and hence become a worker in the next period, and similarly for  $q_2$  and  $1 - q_2$ .

*Earnings.* Labor endowment is indivisible. Thus, an agent is either an entrepreneur and produces its intermediate variety, or works in the final-good sector.<sup>5</sup> In either case, the labor endowment of the agent is  $n_{i,t}$ . Given these idiosyncratic shock processes, agents have the following (nonfinancial) earnings per time period:

$$\begin{aligned} \text{entrepreneur} &: p_{i,t} y_{i,t} \\ \text{worker} &: n_{i,t} W_t, \end{aligned}$$

where  $p_{i,t}$  is the price set by the entrepreneur  $i \in \mathcal{J}_t$  producing  $y_{i,t}$  units of intermediate variety at time  $t$ , and  $W_t$  is the market wage rate, which is equal to the marginal value product of labor in the final-good sector. We assume that  $\mathcal{J}_t$  is Lebesgue measurable for all  $t$  and by the law of large numbers, its measure is constant. Furthermore, since the elasticity of substitution across intermediate goods is constant, its exact composition is irrelevant, and therefore we simply denote this set by  $\mathcal{J}$  in what follows. Capital stock owned by workers earns no (gross) return in the current period. Individuals can change their capital stock through investment, and this can be undertaken by either lending or borrowing at the risk-free interest rate.

**2.2. Preferences and Budget Constraints**

Each agent has a time-separable, expected discounted utility function defined over a consumption sequence  $\mathbb{E}_0 \sum_t \beta^t u(c_{i,t})$ , where  $\mathbb{E}_0$  is the conditional expectation operator,  $0 < \beta < 1$  is the subjective discount factor,  $c_{i,t} > 0$  is the consumption

of agent  $i$  in period  $t$ , and the instantaneous utility function  $u$  is given by Epstein and Zin (1989) recursive preferences. Agents make contingent consumption and portfolio investment decisions to maximize

$$u_{i,t} = U(c_{i,t}) + \beta U(\Upsilon^{-1}[\mathbb{E}_t \Upsilon(U^{-1}(u_{i,t+1}))]), \tag{6}$$

where time and risk preferences are captured by

$$U(c) = \frac{c^{1-(1/\theta)}}{1 - (1/\theta)}, \quad \Upsilon(c) = \frac{c^{1-\gamma}}{1 - \gamma},$$

where  $\theta > 0$  is the elasticity of intertemporal substitution, and  $\gamma > 0$  is the coefficient relative risk aversion. For each agent, the portfolio decision consists of allocating wealth between physical capital  $k$  and a risk-free financial asset  $b$  (bond). The bond is in zero net supply and its return,  $r$ , is denominated in terms of the final good. The bond market indirectly serves as a market for physical capital: a worker can use their idle physical capital to purchase bonds and the relatively more productive entrepreneurs can issue bonds to increase their scale of production. The sequence of budget constraints is

$$c_{i,t} + k_{i,t+1} + b_{i,t+1} = \mathbb{I}_{i,t} p_{i,t} y_{i,t} + (1 - \mathbb{I}_{i,t}) n_{i,t} W_t + (1 - \delta) k_{i,t} + (1 + r) b_{i,t}, \tag{7}$$

where  $\mathbb{I}_{i,t} = 1$  if  $z_{i,t} > 0$  and zero otherwise, and  $0 < \delta < 1$  is the depreciation rate.<sup>6</sup>

### 2.3. Recursive Equilibrium

The noncompetitive sector in this paper is closely related to monopolistically competitive models with Dixit and Stiglitz (1977) preferences [e.g., Blanchard and Kiyotaki (1987)]: firms face a downward-sloping demand curve for their products, take other firms' prices as given, and ignore the effect of their own price changes on the aggregate price level. These models typically introduce firm heterogeneity through either demand shifters or random differences across firms' direct marginal costs. Here, heterogeneity is driven by exogenous stochastic processes with no free entry; as such, the monopolistically competitive sector requires neither a fixed cost to production nor increasing returns to scale at the firm level. This modeling choice is consistent with the heterogeneous-agents literature as it retains the main sources of income risks present in these models. In Aiyagari (1994), only  $n$  is stochastic, the wage rate  $W$  and the rate of return to physical capital are *common* across agents, and the income risk is due to idiosyncratic labor income  $W \times n_i$ . By contrast, in Angeletos (2007),  $n$  is fixed and identical across all agents, and labor income  $W \times n$  is common across agents. There are firm-specific productivity shocks, and the income risk is due to idiosyncratic capital income  $r_i$ . In our setup, each agent faces both capital and labor income risk.<sup>7</sup>

At the beginning of period  $t$ , each agent's capital stock  $k_{i,t}$  and bond holdings  $b_{i,t}$  are given. Then, agents observe the realization of their idiosyncratic shocks,

$n_{i,t}$  and  $z_{i,t}$ . Entrepreneurs set the price  $p_{i,t}$  for the variety of intermediate input they produce, decide how much to consume, and invest (physical capital and bonds). Workers decide how much to consume and invest. The succession of these events determine endogenously the distribution of capital and bonds across agents, which is part of the definition of an equilibrium. (From now on, we drop the time subscripts, and let a prime denote the next period.) The state of an agent  $i$  in the current period consists of the capital stock, bonds, labor endowment, and idiosyncratic productivity:  $s_i = (k_i, b_i, n_i, z_i) \in S \subset \mathbb{R}^4$ , for a Lebesgue measurable set  $S$ . The aggregate state of the economy is described by  $\Gamma : S \rightarrow [0, 1]$ , which is the measure of agents over  $S$  in the current period. In general, this distribution is time dependent. For all  $S \in \mathcal{S}$ ,  $\Gamma(S)$  is also the probability that an agent's state belongs to  $S$  in a given time period. Let  $\Gamma_0$  and  $\Gamma_1$  denote the marginal distributions of  $\Gamma$  over the levels of  $z_i$ . Due to the laws of large numbers,  $\Gamma$  is time invariant; hence, the marginal distributions of workers  $\Gamma_0$  and entrepreneurs  $\Gamma_1$  are also time invariant.

We now state the recursive formulation of the optimization problem faced by agent  $i \in \mathcal{I}$  in state  $s_i$ :

$$\begin{aligned}
 v(k_i, b_i, n_i, z_i) &= \max_{k'_i, b'_i} [u(c_i) + \beta \mathbb{E}_i v(k'_i, b'_i, n'_i, z'_i)], & (8) \\
 \text{s.t. } c_i + k'_i + b'_i &= \mathbb{I}_i p_i z_i k_i^\alpha n_i^{1-\alpha} + (1 - \mathbb{I}_i) n_i W + (1 - \delta)k_i + (1 + r)b_i, \\
 c_i \geq 0, \quad k'_i \geq 0, \quad p_i &> 0,
 \end{aligned}$$

where  $v(k_i, b_i, z_i, n_i)$  is the indirect utility of agent  $i$ .

**DEFINITION 1.** *A stationary recursive equilibrium is a pair of distributions of workers  $\Gamma_0$  and entrepreneurs  $\Gamma_1$  over capital, bonds, and labor endowments; a risk-free rate  $r$  for bonds; a wage rate  $W$  in the final-good sector; a price index  $P$  for the composite intermediate input  $M$ ; employment  $L$  in the final-good sector; such that*

1. *the supply of each intermediate good equals its demand:  $y_i = x_i$ ;*
2. *agents choose their capital stock and bond holdings to maximize (8);*
3. *entrepreneurs choose price  $p(k, b, n)$  and supply  $x(k, b, n)$  to maximize profits, and the price index is consistent with the individual pricing function:  $P M = \int p_i x_i \Gamma_1(dk, db, dn)$ ;*
4. *the labor market clears:  $L = \int n_i \Gamma_0(dk, db, dn)$ ; and*
5. *the bond market clears:  $\int b_i \Gamma_0(dk, db, dn) + \int b_i \Gamma_1(dk, db, dn) = 0$ .*

As in standard monopolistic competition models, the equilibrium price index for the composite intermediate input (materials) is given by

$$P = \left[ \int_{i \in \mathcal{J}} p_i^{1-\nu} di \right]^{1/(1-\nu)}, \tag{9}$$



where  $p_i$  is the price chosen by the producer of variety  $i \in \mathcal{J}$ . A key nonstandard feature of our equilibrium is that the heterogeneity of entrepreneurs causes prices to be *asymmetric* in equilibrium as determined by the optimal choices of entrepreneurs. However, the fundamental source of the pricing power (the degree of substitutability across intermediate goods) is identical across entrepreneurs and the profit maximizing price level is given by

$$p_i = \left[ \frac{P^{\frac{\nu(1-\alpha)-1}{1-\alpha}} \alpha^{1/(1-\alpha)} L}{z_i k_i^\alpha n_i^{1-\alpha}} \right]^{1/\nu} \tag{10}$$

for  $i \in \mathcal{J}$  and indeterminate (and irrelevant) for  $i \notin \mathcal{J}$ . As a consequence of these different individual prices, entrepreneurs need to compute  $P$  to determine their own prices. They do so by using the distribution of agents across the state space, which is part of the equilibrium definition. Furthermore, because marginal costs are convex and prices are effectively chosen one period in advance,<sup>8</sup> prices are biased upward and, for a given  $\nu$ , markups will be higher than in standard monopolistic competitions models [Kimball (1989)]. Fernández-Villaverde et al. (2015) show that the upward bias in prices is negatively related to the elasticity of substitution across varieties, since the curvature of the profit function declines with lower values of  $\nu$ . Consequently, this decreasing bias attenuates the magnitude of the main mechanism studied in this paper, which is to show that increasing market power may increase the aggregate price level by increasing markups.

### 2.4. Complete-Markets Steady-State Equilibrium

The complete markets steady-state equilibrium is a benchmark in which agents trade a complete set of Arrow–Debreu contracts to fully insure against the idiosyncratic risks. As such, agents will be indifferent to the sector in which they are employed. As there is no aggregate risk, the deterministic complete-markets equilibrium can be characterized analytically. Below, we will be comparing the stationary recursive equilibrium allocations with those from the complete markets counterpart of this model in a steady state. This comparison is not central to our main focus, but it is useful to better understand our results under incomplete markets. This equilibrium is a solution to the following problem:

$$v(k, n) = \max_{k', p} U(c) + \beta v(k', n'), \tag{11}$$

$$\text{s.t. } c + k' = p(1 - L)x + L W + (1 - \delta)k,$$

where all variables refer to a representative agent. In this case (see Appendix A for details), the steady-state price of each intermediate variety is  $p_{ss} = \frac{(1-L)^{\frac{\nu(1-\alpha)-1}{1-\nu}} \alpha L^{1-\alpha}}{(z k^\alpha)^{1-\alpha}}$ . Our first two propositions show that when market power increases, the intermediate-goods sector responds by accumulating less capital and decreasing production. This results in lower aggregate output (derivations

in Appendix B). At the same time, the increases in capital stock and output are proportional, so that, as the elasticity of substitution parameter changes, the saving rate remains constant, as Proposition 2 shows.

**PROPOSITION 1.** *In the complete-markets economy, the equilibrium levels of output and capital are increasing functions of the elasticity of substitution parameter  $\nu$ .*

**PROPOSITION 2.** *In the complete-markets economy, the steady-state aggregate saving rate is independent of the elasticity of substitution parameter  $\nu$ .*

The following proposition shows that, in the complete market economy, the productivity parameters in the final-good and intermediate-goods sectors have opposite effects on the market power held by entrepreneurs.

**PROPOSITION 3.** *The complete-markets steady-state optimal price for each intermediate good  $p$  is an increasing function of  $Z_A$  and a decreasing function of  $z$ .*

High productivity in the final-good sector ( $Z_A$ ) increases aggregate consumption expenditures and the demand faced by each intermediate-good producer. Consequently, the price of materials rises. Conversely, when productivity in the intermediate-goods sector ( $z$ ) increases, the price of materials falls and this enables the intermediate-goods sector to sell the higher quantity of materials at a lower cost. Since the aggregate price level is increasing in  $p$ , the implications of changes in these productivity parameters for  $P$  are immediate.

Finally, we consider the effects of changing the share of entrepreneurs in the economy, as this directly affects their market power. The following proposition shows that, in a complete-markets economy, the mass of entrepreneurs has a nonmonotonic effect on aggregate output but has no effect on the aggregate saving rate.

**PROPOSITION 4.** *In the complete-markets economy, aggregate output is non-monotonic in the share of employment in the intermediate-goods sector. The saving rate is independent of the share of entrepreneurs.*

### 3. CALIBRATION

In this section, we discuss our calibration strategy and solve the model numerically. A detailed description of the algorithm used to compute the equilibrium of the model is contained in Appendix C.

#### 3.1. Baseline Calibration

Our functional forms and calibrations draw on Aiyagari (1994) and Angeletos (2007), although there are several parameters that are unique to our setup. For those parameters that are common in our model and those of Aiyagari (1994)

**TABLE 1.** Baseline parameter values

| Description                                     | Label    | Value | Source                                 |
|---|----------|-------|--|
| Elasticity of output w.r.t. nonlabor inputs     | $\alpha$ | 0.36  | Aiyagari (1994); Angeletos (2007)      |
| Standard deviation of shocks to labor endowment | $\sigma$ | 0.20  | Aiyagari (1994)                        |
| Persistence of shocks to labor endowment        | $\rho$   | 0.3   | Aiyagari (1994)                        |
| Subjective time discount factor                 | $\beta$  | 0.96  | Angeletos (2007)                       |
| Depreciation rate                               | $\delta$ | 0.08  | Aiyagari (1994), Angeletos (2007)      |
| Elasticity of intertemporal substitution        | $\theta$ | 1.00  | Aiyagari (1994), Angeletos (2007)      |
| Coefficient of relative risk aversion           | $\gamma$ | 3.00  | Angeletos (2007)                       |
| Probability of staying entrepreneur             | $q_1$    | 0.90  | Moskowitz and Vissing-Jorgensen (2002) |
| Probability of staying worker                   | $q_2$    | 0.98  | Wolff (2012)                           |
| Elasticity of substitution across goods         | $\nu$    | 10    | See the text                           |

and Angeletos (2007), we match their parametrizations closely. The duration of each period is one year. Table 1 lists the baseline parameter values. The subjective time discount factor is directly from Angeletos (2007), whereas Aiyagari (1994) pins down this parameter by targeting the real interest rate. For the elasticity of intertemporal substitution, our parameter value is identical to that of Angeletos (2007). Aiyagari (1994) uses isoelastic instantaneous utility function, so in his analysis the elasticity of intertemporal substitution is the inverse of the coefficient of relative risk aversion. Otherwise, the coefficient of relative risk aversion we use is common to both Aiyagari (1994) and Angeletos (2007), and they both use two additional values for sensitivity analysis:  $\gamma \in \{1, 3, 5\}$ . We set  $\nu = 10$  in the baseline to bring the model close to the perfectly competitive limit, thus allowing a meaningful comparison with Aiyagari (1994) and Angeletos (2007). The baseline equilibrium reported below is quantitatively very similar to equilibria obtained by setting  $\nu = 15, 20,$  and  $50$ . Similarly, in order to facilitate comparisons with the existing models in this literature, we set  $Z_A = z = 1$ .

The probability that an entrepreneur stays in business ( $q_1$ ) is based on Moskowitz and Vissing-Jorgensen (2002), who report that the survival rate of private firms is around 34% over the first 10 years of a firm’s life. We set  $q_2 = 0.98$ , so that one-sixth of the agents are entrepreneurs. Wolff (2012, Table 6) reports that about 12% of households in the United States were owners of unincorporated businesses, and, depending on the inclusiveness of the definition of an entrepreneur, they can account for more than 20% of all households.

The labor endowment shocks are as in Aiyagari (1994). In our framework, these shocks also determine the idiosyncratic history of entrepreneurs, and thus

we need to choose a value of  $\rho$  that is also consistent with Angeletos (2007). At the baseline, we use  $\rho = 0.3$ , which implies a standard deviation of 0.169 for the average entrepreneur's income. If the average entrepreneur is exposed to the total factor productivity shocks of Angeletos (2007), the standard deviation of their income would be 0.165, at the baseline. In the sensitivity analysis below, we consider different persistences for workers and entrepreneurs.

### 3.2. Baseline Calibration Results

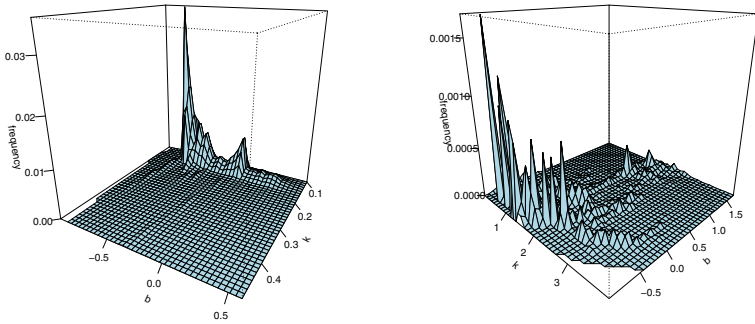
The baseline calibration yields an equilibrium interest rate of 2.53% and aggregate savings in excess of the complete markets benchmark by 1.77%. For all labor endowments and bond and capital holdings, the indirect utility of entrepreneurs is higher than that of a worker. Consequently, an agent with  $z_{i,t} > 0$  will never chose to be a worker. In Figure 2, we report the equilibrium distribution of wealth. The distributions feature considerable dispersion in capital and bond holdings. The dispersion in the distribution of workers (Figure 2a and 2c) is largely driven by the idiosyncratic labor endowment shocks, and it is mainly in the bond holding dimension. This is intuitive, as workers draw no immediate utility from capital except for its option value, in case they become entrepreneurs in the future—which is why some find it optimal to hold capital nonetheless. A small fraction of workers hold atypically large amounts of capital, possibly due to recently switching from being entrepreneurs to workers.

The dispersion in the distribution of entrepreneurs involves both capital and bond holding dimensions (Figure 2b and 2d). The idiosyncratic labor endowment shocks interact here with the duration of the entrepreneurial activity. Those agents that have just become entrepreneurs are likely to sell or issue bonds to finance investment, while long-term entrepreneurs, experiencing diminishing marginal returns to their own capital, likely choose to buy bonds to finance smaller and, at the margin, more productive firms. This illustrates how, in this model, capital is allocated across agents. The supply of bonds is met by the demand and it is worth emphasizing that these distributional differences across workers and entrepreneurs are endogenously generated in our model. The societal picture in Figure 2 involves four main groups of agents: workers and entrepreneurs that have had been relatively lucky and unlucky in their labor endowments.

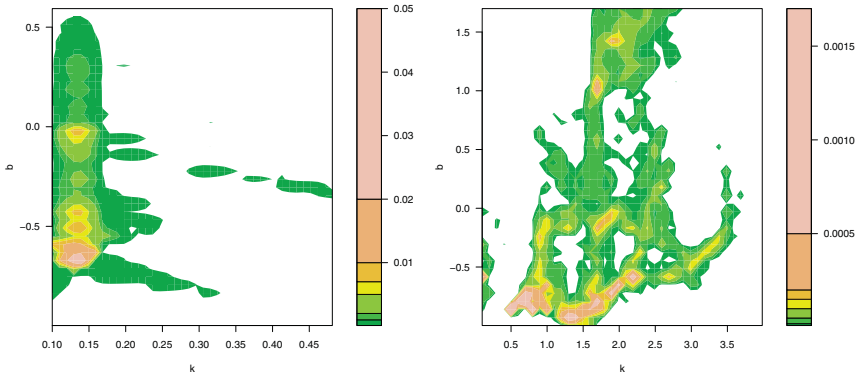
### 3.3. Sensitivity Analysis

We now conduct a sensitivity analysis of the baseline equilibrium to the values of several key parameters:  $\theta$ ,  $\rho$ ,  $\sigma$ ,  $q_2$ ,  $Z_A$ , and  $z$ . All of our comparisons are relative to the saving rate in the corresponding complete market situation.

*Elasticity of intertemporal substitution* ( $\theta$ ). Following Angeletos (2007), we consider  $\theta \in \{1/3, 1, 2\}$ . Table 2 shows that for  $\sigma = 0.2$  the saving rate decreases with  $\theta$ . However, this response is not independent of the value of  $\sigma$ . Similarly, increasing  $\theta$  does not have monotonic effects on the risk-free rate in the baseline



(a) Distribution of  $b$  and  $k$  for workers. (b) Distribution of  $b$  and  $k$  for entrepreneurs.



(c) Levels of the distribution of workers. (d) Levels of the distribution of entrepreneurs.

**FIGURE 2.** Ergodic distributions (baseline). The frequencies are scaled by the relative sizes of the corresponding populations. (a) Distribution of  $b$  and  $k$  for workers. (b) Distribution of  $b$  and  $k$  for entrepreneurs. (c) Levels of the distribution of workers. (d) Levels of the distribution of entrepreneurs.

calibration. Intuitively, both the demand for physical capital and precautionary savings are affected as the  $\theta$  increases, and this causes the saving rate to change nonmonotonically.

*Persistence of labor endowments ( $\rho$ ).* Following Aiyagari (1994), we consider  $\rho \in \{0, 0.3, 0.6\}$ . Table 2 shows that the saving rate increases with  $\rho$ : this parameter determines the strength of the precautionary savings motive. Consequently, depending on the parameter values, our model economy is capable of generating both a higher or lower saving rate relative to its complete-markets steady-state counterpart. Cooper and Haltiwanger (2006) estimate that the persistence of firm’s profitability is 0.885; using this persistence parameter for the labor endowment process of entrepreneurs—keeping the workers’ at the baseline—reduces the aggregate saving rate to 0.8% below the complete-markets level. Equation (4) shows

**TABLE 2.** Income uncertainty, the saving rate, and the risk-free rate

|  | $\Delta$ Saving rate, % |                | Interest rate, % |                |
|--|-------------------------|----------------|------------------|----------------|
|  | $\sigma = 0.2$          | $\sigma = 0.4$ | $\sigma = 0.2$   | $\sigma = 0.4$ |
| Baseline                                 |                         |                |                  |                |
| $\theta = 1, \rho = 0.3, \sigma = 0.2$   | +1.77                   | +3.60          | 2.53             | 2.23           |
| Elasticity of intertemporal substitution |                         |                |                  |                |
| $\theta = 1/3$                           | +3.09                   | +3.40          | 3.35             | 3.30           |
| $\theta = 2$                             | +0.88                   | +3.42          | 4.00             | 4.85           |
| Persistence of shocks to labor endowment |                         |                |                  |                |
| $\rho = 0$                               | -0.29                   | -0.12          | 2.50             | 2.58           |
| $\rho = 0.6$                             | +3.04                   | +4.93          | 2.50             | 2.20           |

*Notes:* The saving rate is the saving rate under incomplete (stationary equilibrium) minus the saving rate under complete markets (steady state). In all of the cases considered, the complete-markets steady-state saving rate does not change. The reported parameters are as follows:  $\theta$  is the elasticity of intertemporal substitution (EIS),  $\sigma$  is the standard deviation of shocks to labor endowment, and  $\rho$  is the persistence of shocks to labor endowment. The baseline parameter values are  $\theta = 1, \sigma = 0.2,$  and  $\rho = 0.3.$  All data are in percent.

that the variance of the labor endowment shocks decreases when  $\rho$  increases. Intuitively, this makes capital relatively more risky for entrepreneurs, and the magnitude of this portfolio effect is consistent with Angeletos (2007).

*Standard deviation of labor shocks ( $\sigma$ ).* As in Aiyagari (1994), we consider  $\sigma \in \{0.2, 0.4\}.$  Table 2 shows that the saving rate increases with  $\sigma$ : an increase of  $\sigma$  strengthens the precautionary saving motive.

*Productivities. ( $Z_A, z_i$ ).* In Table 3, we report the changes in the saving rate when the two sectors of the model experience differential productivity growth. Relatively faster productivity growth in the final-good sector has economically little impact on the aggregate saving rate. An increase in  $Z_A$  raises all agents’ incomes, which leads to an increase in the supply of loanable funds, and a decrease in the risk-free interest rate. By contrast, when entrepreneurs have higher productivity, the saving rate decreases monotonically, as entrepreneurs are able to reduce their optimal investment in risky capital. While, in principle, differential productivity growth could explain the declining saving rates in this model, the empirical evidence points to converging trends across sectors [Nordhaus (2008)].

*Share of entrepreneurs ( $\mathcal{J}$ ).* The transition probabilities between workers and entrepreneurs determine their ergodic proportion. Table 4 shows that as the share of entrepreneurs increases, the aggregate saving rate decreases monotonically, while the interest rate displays a U-shaped pattern.<sup>9</sup> The transition probabilities have substantial impact on the degree of uncertainty each agent faces. It is intuitive that labor income uncertainty should mostly matter for workers, while capital income uncertainty should mostly matter for entrepreneurs. As the share of entrepreneurs in the economy increases, the significance of the portfolio effect becomes more

**TABLE 3.** Productivity, saving rates, and interest rates

| $Z_A$ | $z_i$ | $\Delta$ Saving rate, % | Interest rate, % |
|-------|-------|-------------------------|------------------|
| 1.02  | 1.00  | +1.53                   | 2.39             |
| 1.05  | 1.00  | +1.64                   | 2.40             |
| 1.10  | 1.00  | +1.71                   | 1.60             |
| 1.00  | 1.02  | +1.59                   | 2.21             |
| 1.00  | 1.05  | +1.45                   | 2.19             |
| 1.00  | 1.10  | +0.84                   | 2.17             |

*Notes:* The saving rate is reported relative to its complete-markets steady-state value; In all of the cases considered, the complete-markets steady-state saving rate does not change as  $Z_A$  and  $z_i$  vary (Proposition 3);  $Z_A$  is TFP in the final-good sector, and  $z_i$  is the TFP of entrepreneurs.

**TABLE 4.** Share of entrepreneurs, saving rate, and the interest rate

| $q_1 - q_2$   | Entrepreneurs, % | $\Delta$ Saving rate, % | Interest rate, % |
|---------------|------------------|-------------------------|------------------|
| 0.900 – 0.999 | 1.0              | +16.62                  | 8.00             |
| 0.900 – 0.990 | 9.1              | +5.67                   | 2.50             |
| 0.900 – 0.980 | 16.6             | +1.77                   | 2.53             |
| 0.900 – 0.970 | 23.1             | –0.51                   | 3.20             |
| 0.900 – 0.950 | 33.3             | –0.93                   | 4.19             |
| 0.900 – 0.900 | 50.0             | –1.95                   | 4.80             |

*Notes:* The saving rate is reported relative to its complete-markets steady-state value; in all of the cases considered, the complete-markets steady-state saving rate does not change as  $Z_A$  and  $z_i$  vary (Proposition 4);  $q_1$  is the probability that an entrepreneur in the current period remains an entrepreneur in the next period, and  $q_2$  is the probability that a worker in the current period remains a worker in the next period.

prominent and the aggregate saving rate eventually decreases below the complete-markets benchmark.

*Entrepreneurial dynamism.* Decker et al. (2016, Figure 2a) report that new businesses as a share of existing ones went from a little less than 14% in the late 1970s to slightly more than 8% in the 2010s, while Wolff (2012, Table 8) reports that entrepreneurs as a share of the population has not changed significantly. In the model, the measure of entrepreneurs depends only on the transition probabilities and it is equal to

$$\pi_1 \equiv \frac{1 - q_2}{2 - q_1 - q_2},$$

while the share of new entrepreneurs is equal to  $1 - q_1$ .<sup>10</sup> In our baseline calibration, this share is equal to 10%, and here we consider a value of 6% to investigate the effect of falling business dynamism on the saving rate. To keep the share of entrepreneurs equal to the baseline, we set  $q_1 = 0.94$  and  $q_2 = 0.988$ . The excess of savings over the complete-market benchmark declines from 1.77% in the baseline to 0.52% in this scenario. Intuitively, this is driven by the concavity of

the policy function: each entrepreneur has a target steady-state capital stock. An agent that has just become an active entrepreneur is expected to be farther away from their target—and thus accumulate capital faster—than existing entrepreneurs, which explains why lower dynamism decreases the saving rate in our model. Our conjecture is that this effect is due to the dynamics of the capital accumulation process and not to the portfolio effect. Finally, the average markup in the economy remains equal to the baseline (3%), showing that in our calibrated model market power is not affected by business dynamism.

*Summary.* We conclude that the aggregate saving rate responds only modestly—as compared to the data in Figure 1—to changes in most of the parameters considered here. There are only three notable exceptions to this conclusion. First, if the labor endowment shocks had become smaller (from  $\sigma = 0.4$  to  $\sigma = 0.2$ ) and less persistent (from  $\rho = 0.6$  to  $\rho = 0$ ), then the calibrations point to a decline of 5.31 percentage points in the saving rate; Gottschalk and Moffitt (2009) find that the observed changes in the US data are in the opposite direction, and that labor income has become significantly less stable. The second notable exception involves the share of entrepreneurs. However, to match the changes in the US saving rate through this channel, the calibrated model would need to postulate unrealistic changes to the share of entrepreneurs in the population and that entrepreneurs *increased* dramatically over the past three decades; for example, that their share increased from one-tenth to half of the population. Finally, a decline in the share of new entrepreneurs can decrease the aggregate saving rate, but it has no effect on market power.

#### 4. INCREASING MARKET POWER

In the model, there are, in principle, three sets of parameters that increase market power. The first is the difference between TFP in the aggregate and individual production functions. When  $z_i$  increases relative to  $Z_A$ , the entrepreneurs are able to extract larger rents. The first two columns of Table 5 show that this does not substantially change market power in our model; this is consistent with the sensitivity analysis of Section 3.3, whereby the saving rate is shown to be rather insensitive to these parameters.

The second set of parameters that may affect market power consist of the switching probabilities of entrepreneurs and workers. While decreasing the share of entrepreneurs endows them with more market power, it is not obvious that this would increase it at the macroeconomic level. In the model, a decrease in the share of entrepreneurs simultaneously increases the size of the perfectly competitive sector, which tends to decrease the average markup in the economy. Table 5 shows that the aggregate average markup (i) is nonmonotonic in the size of  $\mathcal{J}$  (share of entrepreneurs), and (ii) does not vary much as  $\mathcal{J}$  varies. Therefore, in this framework, altering the transition probabilities does not affect market power significantly.

These first two sets of parameters are potentially relevant for a full account of the declining saving rate in the United States since the 1970s. Simply put,



**TABLE 5.** Average markups

| $z_i$ (%) | Average markup (%) | $J$ (%) | Average markup (%) | $\nu$ | Average markup (%) |
|-----------|--------------------|---------|--------------------|-------|--------------------|
| 1.00      | 2.93               | 1.0     | 3.08               | 50    | 0.56               |
| 1.02      | 2.96               | 9.1     | 3.12               | 20    | 1.40               |
| 1.05      | 2.95               | 16.6    | 3.00               | 15    | 1.89               |
| 1.10      | 2.98               | 23.1    | 2.78               | 10    | 3.00               |
|           |                    | 33.3    | 2.87               | 8     | 3.72               |
|           |                    | 50.0    | 3.17               | 6     | 5.26               |
|           |                    |         |                    | 4     | 7.92               |
|           |                    |         |                    | 2     | 19.14              |

*Notes:* Average markups are reported as  $100(1 - \frac{p_i}{MC})$ , where MC is marginal cost.

**TABLE 6.** Markups and the aggregate saving rate

| $\nu$ | Average markup | Markup index     |              | Saving rate, (%) |
|-------|----------------|------------------|--------------|------------------|
|       |                | Wages & salaries | Compensation |                  |
| 50    | 1.0056         | 87.50            | 93.95        | 10.93            |
| 20    | 1.0140         | 88.23            | 94.73        | 10.47            |
| 15    | 1.0189         | 88.66            | 95.19        | 10.11            |
| 10    | 1.0300         | 89.62            | 96.23        | 9.56             |
| 8     | 1.0372         | 90.25            | 96.90        | 8.98             |
| 6     | 1.0526         | 91.59            | 98.34        | 8.84             |
| 4     | 1.0792         | 93.90            | 100.82       | 8.04             |
| 2     | 1.1914         | 103.66           | 111.30       | 5.24             |

*Notes:* In this table, the average markup is reported as a factor. The markup indexes are normalized to the 1970 value of the indexes shown in Figure 1.

the calibrated model is unable to account for increasing market power through changes in  $\mathcal{J}$  or  $z_i$ , and it narrows the focus on changes in the substitutability across intermediate goods—the third parameter that determines market power in the model. Table 5 shows that changes in  $\nu$  have a predictable and large effect on markups and, as such, our empirical strategy is to model increases in market power by decreasing  $\nu$ . We think of these exogenous changes in  $\nu$  as driven either by technology or by the regulatory environment. The technological changes we capture through a decreased substitutability across goods include the large network externalities benefiting internet search engines, social media businesses, as well as electronic commerce, and auction websites. The regulatory changes that we capture include patenting and licensing [Boldrin and Levine (2013)] as well rent-seeking regulations [Council of Economic Advisors (2016)].

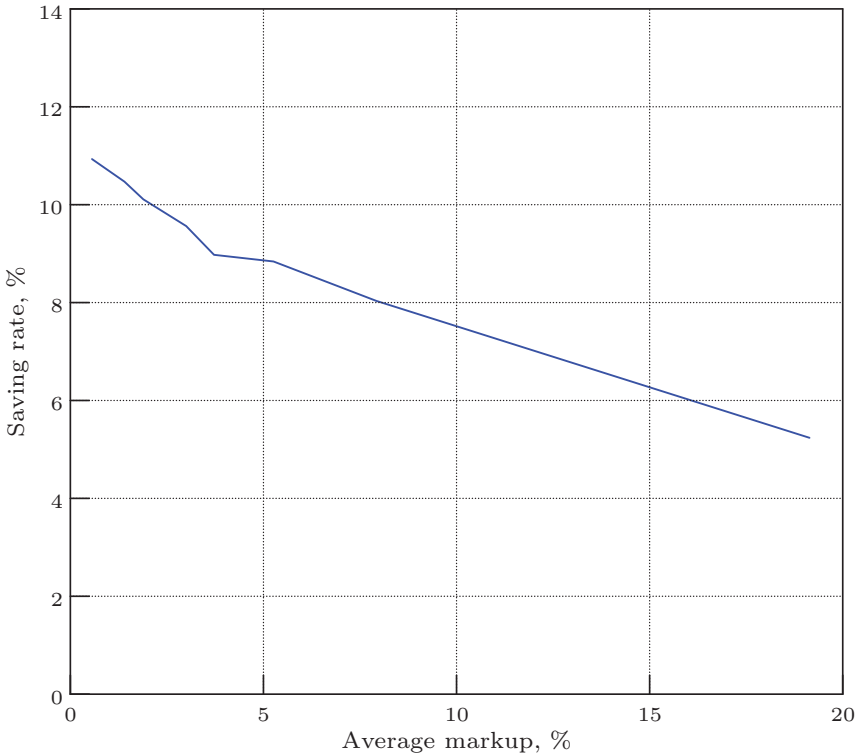


FIGURE 3. Market power and the aggregate saving rate.

#### 4.1. The Aggregate Saving Rate

Table 6 shows the main findings of the paper. We construct two indexes of markups drawn from the calibrations that are normalized to match the 1970 values of the indexes reported in Nekarda and Ramey (2013) (columns 2, 3, and 4) for  $\nu = 50$ . As  $\nu$  declines from 50 to 2, the indexes increase to 103.66 and 111.30, which matches closely the values attained by these indexed in 2015 (105.95 and 105.35, respectively). This is evidence that the values of  $\nu$  we consider here, are a realistic progression that tracks the dynamics of market power in the US economy. Furthermore, the value of  $\nu = 50$  delivers an aggregate markup of 0.56%. Based on data from 1949 to 1986, Norrbin (1993) reports an average value of 1.2% for aggregate markups in the United States. The calibrated saving rate declines by 5.69 percentage points from 10.93% to 5.24%, which compares favorably to the US data, where the aggregate saving rate declined by 7.5 percentage points, from 12.6% in 1970 to 5.1% in 2015. The sensitivity analysis of the preceding section indicates that these quantitative results are robust. Figure 3 shows the relationship between markups and saving rates, and it provides evidence that higher market power has caused most of the decline in the US saving rate.

**TABLE 7.** Income and wealth inequality: Model

| Share of top (%) | Markup: 0.56% |         |        | Markup: 3% |         |        | Markup: 19.14% |         |        |
|------------------|---------------|---------|--------|------------|---------|--------|----------------|---------|--------|
|                  | Wealth        | Capital | Income | Wealth     | Capital | Income | Wealth         | Capital | Income |
| 0.1              | 1.53          | 0.99    | 0.47   | 1.62       | 0.99    | 0.46   | 4.37           | 1.53    | 0.73   |
| 1                | 13.75         | 8.22    | 4.10   | 14.31      | 7.94    | 4.02   | 41.70          | 7.38    | 5.96   |
| 5                | 52.37         | 30.37   | 16.81  | 53.56      | 29.28   | 16.52  | 77.66          | 99.81   | 16.82  |
| 10               | 83.32         | 49.50   | 28.99  | 86.15      | 47.98   | 28.57  | 100.00         | 99.81   | 28.69  |

*Notes:* This table reports the income and wealth shares earned or owned by the top 10% , 5% , 1% , and 0.1% of the agents in our models. Wealth includes physical capital and net bond holdings.

## 4.2. Income and Wealth Distributions

Income and wealth inequality change significantly and realistically with our calibrated increases in  $\nu$ . We do not claim that this factor alone can explain the trends in income and wealth inequality in the United States. Rather, we interpret the results shown here as an estimate of the marginal effect on inequality of increased market power.

Table 7 reports the shares of income, capital, and wealth (capital plus net bond holdings) earned or owned by the top 10%, 5%, 1%, and 0.1% of the agents in our models. Increasing market power skews the distributions toward the top decile. Much of this compression involves physical capital. The reason is that with higher markups, the real purchasing power of workers decreases and they tend to save less. Entrepreneurs, on the other hand, face less compelling motives for physical capital accumulation, resulting in a larger concentration of capital in the hands of continuing entrepreneurs. Within the top decile, the group that benefits the most from increasing market power is the top 1%. Table 7 also shows that increased market power mostly affects the share of income of the top 1% and 0.1%.

## 5. CONCLUSION

This paper identifies an overlooked determinant for the decline of the US saving rate. We use an incomplete-market model with monopolistic competition and heterogeneous agents in which both the rate of return to entrepreneurial investment and the risk-free interest rate are determined endogenously. We calibrate this model to match the increasing degree of market power observed in US data and show that it can account for the majority of the decline in the aggregate saving rate. We also find that market power has considerable influence on income and wealth inequality, but fails to fully explain the increased inequality in the United States [Piketty (2014)].

### NOTES

1. This is especially true after accounting for adjustments in investment due to spending on software (1990s) and residential structures (2000s).

2. For an earlier survey, see Parker (2000).

3. Consolidation in the health insurance industry has been accompanied by rising concentration in the provision of hospital services. According to the American Hospital Association, the number of hospitals per capita nearly halved in the United States from 1975 to 2013.

4. In the baseline model parametrization, we use  $Z_{A,t} = 1$  and  $z = 1$ , and later discuss the sensitivity of the results to alternative values.

5. Entrepreneurs may choose to be workers in the final-good sector instead. However, in all of the calibrations we consider, all entrepreneurs optimally choose to operate their firms.

6. Capital depreciates even when not in use, which is consistent with the data published in the national income accounts.

7. Angeletos (2007) and Nirei and Aoki (2016) allow for trading of a risk-free bond, as we do here. Nirei and Aoki (2016) allow for capital and labor income risk. In these papers, there is no public equity, markets are perfectly competitive, and capital and labor income risk are not correlated.

8. From equation (10), by choosing next period's capital stock, an entrepreneur also targets an expected profit maximizing price for the next period

9. It is possible to also consider changes in  $q_1$  while keeping  $q_2$  constant, with consistent results.

10. This share is derived as new entrants,  $(1 - q_2)(1 - \pi_1)$ , over existing entrepreneurs,  $\pi_1$ .

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## APPENDIX A: DERIVATIONS

### A.1. THE COMPLETE-MARKETS STEADY-STATE EQUILIBRIUM

Consider the following planning problem:

$$v(k, n) = \max_{k', p} U(c) + \beta v(k', n'), \tag{A.1}$$

$$\text{s.t. } c + k' = p(1 - L)x + L W + (1 - \delta)k.$$

The first-order conditions are

$$-U'(c) + \beta v_k = 0, \tag{A.2}$$

$$U'(c) = \beta U'(c')(1 + r), \tag{A.3}$$

where  $v_k = U'(c) \left(1 - \delta + \frac{\partial Y}{\partial M} \frac{\partial M}{\partial k}\right)$ . The relevant partial derivatives are

$$\frac{\partial Y}{\partial M} = \alpha Z_A \left[ \int_J x^{\frac{v-1}{v}} di \right]^{\frac{v(\alpha-1)}{v-1}} L^{1-\alpha},$$

$$\frac{\partial M}{\partial k} = \left[ \int_J x^{\frac{v-1}{v}} di \right]^{\frac{1}{v-1}} x^{-\frac{1}{v}} z \alpha k^{\alpha-1},$$

$$\frac{\partial Y}{\partial k} = \alpha^2 z Z_A k^{\frac{1-\alpha}{v}} \left[ \int_J x^{\frac{v-1}{v}} di \right]^{\frac{v(\alpha-1)+1}{v-1}} L^{1-\alpha}.$$

Define  $\gamma = \alpha^2 Z_A z^\alpha L^{1-\alpha} (1 - L)^{\frac{v(\alpha-1)+1}{v-1}}$ . Thus, the intertemporal optimality conditions become

$$U'(c) = \beta U'(c') \left(1 - \delta + \gamma k_{ss}^{\alpha^2-1}\right), \tag{A.4}$$

which, in the steady-state equilibrium, yields

$$k_{ss} = \left[ \frac{(\beta^{-1} - 1 + \delta)}{\gamma} \right]^{1/(\alpha^2-1)}. \tag{A.5}$$

Consequently, the marginal value product of capital is

$$\text{MPK}_{ss} = \alpha z k_{ss}^{\alpha-1}.$$

In equilibrium, materials prices are all equal. Hence,  $P = (1 - L)^{1/(1-\nu)} p_{ss}$ , where  $p_{ss}$  is the price of any variety of intermediate goods (with  $k_{ss}$  and  $n = 1$ ). Substituting this into equation (10) gives

$$\begin{aligned} p_{ss} &= \left[ \frac{\left[ (1 - L)^{\frac{1}{1-\nu}} p_{ss} \right]^{\frac{\nu(1-\alpha)-1}{1-\alpha}} (\alpha Z_A)^{\frac{1}{1-\alpha}} L}{z k^\alpha n^{1-\alpha}} \right]^{1/\nu} \\ &= \left[ \frac{\left[ (1 - L)^{\frac{1}{1-\nu}} \right]^{\frac{\nu(1-\alpha)-1}{1-\alpha}} (\alpha Z_A)^{\frac{1}{1-\alpha}} L}{z k^\alpha n^{1-\alpha}} \right]^{\frac{1}{\nu}} p_{ss}^{\frac{\nu(1-\alpha)-1}{\nu(1-\alpha)}}. \end{aligned}$$

Solving for the equilibrium price gives

$$p_{ss} = \frac{(1 - L)^{\frac{\nu(1-\alpha)-1}{1-\nu}} \alpha Z_A L^{1-\alpha}}{(z k^\alpha)^{1-\alpha}}. \tag{A.6}$$

The steady-state level of output is given by

$$\begin{aligned} Y_{ss} &= Z_A M_{ss}^\alpha L_{ss}^\alpha \\ &= Z_A \left[ \int_J x_{ss}^{\frac{\nu-1}{\nu}} dj \right]^{\frac{\alpha\nu}{\nu-1}} L^{1-\alpha} \\ &= Z_A J^{\frac{\alpha\nu}{\nu-1}} (x_{ss})^\alpha L^{1-\alpha} \\ &= Z_A (1 - L)^{\frac{\alpha\nu}{\nu-1}} (z k_{ss}^\alpha)^\alpha L^{1-\alpha} \\ &= Z_A z^\alpha (1 - L)^{\frac{\alpha\nu}{\nu-1}} \left( \left[ \frac{(\beta^{-1} - 1 + \delta)}{\alpha^2 Z_A z^\alpha L^{1-\alpha} (1 - L)^{\frac{\nu(\alpha-1)+1}{\nu-1}}} \right]^{\frac{1}{\alpha^2-1}} \right)^{\alpha^2} \\ &= Z_A z^\alpha (1 - L)^{\frac{\alpha\nu}{\nu-1}} \left[ \frac{\overbrace{\alpha^2 Z_A z^\alpha L^{1-\alpha} (1 - L)^{\frac{\nu(\alpha-1)+1}{\nu-1}}}^{\xi_2}}{\underbrace{\beta^{-1} - 1 + \delta}_{\xi_1}} \right]^{-\frac{\alpha^2}{\alpha^2-1}} \\ &= Z_A z^\alpha \left( \frac{\xi_2}{\xi_1} \right)^{-\frac{\alpha^2}{\alpha^2-1}} (1 - L)^{\frac{\alpha\nu}{\nu-1} - \frac{\nu\alpha^2(\alpha-1)+\alpha^2}{(\nu-1)(\alpha^2-1)}} L^{1-\alpha - \frac{\alpha^2(1-\alpha)}{\alpha^2-1}} \\ &= \underbrace{\zeta}_{\zeta} (1 - L)^{\frac{\alpha[\alpha(\nu-1)-\nu]}{(\nu-1)(\alpha^2-1)}} L^{\frac{1}{1+\alpha}}. \end{aligned}$$



## APPENDIX B: PROOFS OF PROPOSITIONS

### B.1. PROOF OF PROPOSITION 1

From equation (A.5), we have

$$\begin{aligned}
 k_{ss} &= \left[ \frac{\alpha^2 Z_A z^\alpha L^{1-\alpha} (1-L)^{\frac{v(\alpha-1)+1}{v-1}}}{(\beta^{-1} - 1 + \delta)} \right]^{-\frac{1}{\alpha^2-1}} \\
 &= \left[ \frac{\alpha^2 Z_A z^\alpha L^{1-\alpha}}{(\beta^{-1} - 1 + \delta)} \right]^{-\frac{1}{\alpha^2-1}} (1-L)^{\frac{v(\alpha-1)+1}{(1-\alpha^2)(v-1)}}.
 \end{aligned}
 \tag{B.1}$$

Thus, we have

$$\frac{\partial k_{ss}}{\partial v} = k_{ss} \log(1-L) \frac{-\alpha}{(1-\alpha^2)(v-1)^2} > 0,$$

where the inequality follows from  $\log(1-L) < 0$  since  $L \in (0, 1)$ .

Similarly, we have

$$\frac{\partial Y_{ss}}{\partial v} = Y_{ss} \log(1-L) \frac{-\alpha}{(1-\alpha^2)(v-1)^2} > 0.$$

### B.2. PROOF OF PROPOSITION 2.

The saving rate in the steady state is

$$s = \frac{\delta K_{ss}}{Y_{ss}} = \frac{\delta(1-L)k_{ss}}{M_{ss}^\alpha L^{1-\alpha}}.
 \tag{B.2}$$

Using  $M_{ss} = [\int_J y_{ss}^{v/(v-1)} dj]^{v/(v-1)} = (1-L)^{v/(v-1)} k_{ss}^\alpha$  gives

$$\begin{aligned}
 s &= \delta(1-L)^{1-\frac{\alpha v}{v-1}} L^{\alpha-1} k_{ss}^{1-\alpha^2} \\
 &= \delta(1-L)^{1-\frac{\alpha v}{v-1}} L^{\alpha-1} \left[ \left( \frac{1/\beta - 1 + \delta}{\Gamma} \right)^{\frac{1}{\alpha^2-1}} \right]^{1-\alpha^2} \\
 &= \delta(1-L)^{1-\frac{\alpha v}{v-1}} L^{\alpha-1} \left( \frac{\Gamma}{1/\beta - 1 + \delta} \right) \\
 &= \left( \frac{\alpha^2 \delta}{1/\beta - 1 + \delta} \right) L^{\alpha-1+1-\alpha} (1-L)^{1-\frac{\alpha v}{v-1} + \frac{v\alpha-v+1}{v-1}} \\
 &= \frac{\alpha^2 \delta}{1/\beta - 1 + \delta}.
 \end{aligned}$$

**B.3. PROOF OF PROPOSITION 3.**

This follows immediately from the pricing equation for each intermediate good:

$$p_{ss} = \frac{(1 - L)^{\frac{\nu(1-\alpha)-1}{1-\nu}} \alpha Z_A L^{1-\alpha}}{(zk^\alpha)^{1-\alpha}}.$$

**B.4. PROOF OF PROPOSITION 4.**

The steady-state output is equal to  $Y_{ss} = \zeta(1 - L)^{\frac{\alpha(\nu-1)-\nu}{(\nu-1)(\alpha^2-1)}} L^{\frac{1}{1+\alpha}}$ . The first exponent is positive as it is the ratio of two negatives: the numerator is negative because  $\nu > \nu - 1$ , and furthermore since  $\alpha$  is between 0 and 1,  $\nu > \alpha(\nu - 1)$ . The denominator is negative because  $\alpha^2 < 1$ . The second exponent is positive. Thus, the steady-state level of output is a product of two factors involving  $L$ , one of which is increasing and the other is decreasing in  $L$ . Hence, output is nonmonotonic in  $L$ .

Taking the first-order condition of  $Y_{ss}$  with respect to  $L$ , we obtain

$$\phi_1(1 - L)^{\phi_1-1} L^{\phi_2} = (1 - L)^{\phi_1} \phi_2 L^{\phi_2-1},$$

where

$$\phi_1 = \frac{\alpha[\alpha(\nu - 1) - \nu]}{(\nu - 1)(\alpha^2 - 1)}, \quad \phi_2 = \frac{1}{1 + \alpha}.$$

This expression simplifies to

$$\phi_1(1 - L)^{\phi_1-1-\phi_1} = \phi_2 L^{\phi_2-1-\phi_2},$$

which can be solved for  $L = \frac{\phi_2/\phi_1}{1+\phi_2/\phi_1}$ . To prove the second statement of this proposition, see the proof of Proposition 2.

## APPENDIX C: THE NUMERICAL SOLUTION ALGORITHM

We use a value function iteration method to numerically solve for optimal consumption and saving decisions by workers and entrepreneurs. To solve for the equilibrium numerically, we begin with a guess of the aggregate price level  $P$  and of the risk-free rate  $r$ . We then solve for optimal decisions and aggregate the agents' choices. If the resulting aggregate price level differs from the initial guess, or that the net supply of bonds differs from zero, we update the guesses, and repeat. A summary of these steps follows.

1. Guess the aggregate price level  $P$  and the interest rate on bonds  $r$ . Iterate on the value function to obtain the policy functions for the agents (see below for details).
2. Define grids for physical capital  $\mathcal{K}$ , bonds  $\mathcal{B}$ , and labor endowment levels  $\mathcal{E}$ . The number of points must be greater than the grids used in step 1.
3. Set  $i = 0$ . Define  $f_0(\mathcal{K}, \mathcal{B}, \mathcal{E})$  to be a tabulated uniform distribution.
4. Set  $f_{i+1} = 0$ , then for all  $ik \in \mathcal{K}$ , all  $ib \in \mathcal{B}$  and all  $ie \in \mathcal{E}$  repeat the following calculations:
  - (a) Calculate the optimal levels of capital and bonds for the next period ( $k'$  and  $b'$ ) from the policy functions obtained in step 1.
  - (b) Find indexes  $j$  and  $\ell$  such that  $\mathcal{K}[j - 1] \leq k' \leq \mathcal{K}[j]$  and such that  $\mathcal{B}[\ell - 1] \leq b' \leq \mathcal{B}[\ell]$ . If  $k' > \max(\mathcal{K})$  or  $b' > \max(\mathcal{B})$ , then set  $j$  or  $\ell$  equal to the number of points in the respective grid.
  - (c) If  $k' \leq \max(\mathcal{K})$  and  $b' \leq \max(\mathcal{B})$ , calculate for all  $ie' \in \mathcal{E}$

$$f_{i+1}(j - 1, \ell - 1, ie') + \\ = P(ie, ie') \frac{(k' - \mathcal{K}[j - 1])(b' - \mathcal{B}[\ell - 1])}{(\mathcal{K}[j] - \mathcal{K}[j - 1])(\mathcal{B}[\ell] - \mathcal{B}[\ell - 1])} f_i(ik, ib, ie),$$

$$f_{i+1}(j - 1, \ell, ie') + \\ = P(ie, ie') \frac{(k' - \mathcal{K}[j - 1])(\mathcal{B}[\ell] - b')}{(\mathcal{K}[j] - \mathcal{K}[j - 1])(\mathcal{B}[\ell] - \mathcal{B}[\ell - 1])} f_i(ik, ib, ie),$$

$$f_{i+1}(j, \ell, ie') + = P(ie, ie') \frac{(\mathcal{K}[j] - k')(\mathcal{B}[\ell] - b')}{(\mathcal{K}[j] - \mathcal{K}[j - 1])(\mathcal{B}[\ell] - \mathcal{B}[\ell - 1])} f_i(ik, ib, ie),$$

$$f_{i+1}(j, \ell - 1, ie') + \\ = P(ie, ie') \frac{(\mathcal{K}[j] - k')(b' - \mathcal{B}[\ell - 1])}{(\mathcal{K}[j] - \mathcal{K}[j - 1])(\mathcal{B}[\ell] - \mathcal{B}[\ell - 1])} f_i(ik, ib, ie).$$

- (d) Else, if  $k' > \max(\mathcal{K})$  and  $b' \leq \max(\mathcal{B})$ , calculate for all  $ie' \in \mathcal{E}$

$$f_{i+1}(j, \ell - 1, ie') + = P(ie, ie') \frac{(k' - \max(\mathcal{K}))(b' - \mathcal{B}[\ell - 1])}{(k' - \max(\mathcal{K}))(\mathcal{B}[\ell] - \mathcal{B}[\ell - 1])} f_i(ik, ib, ie),$$

$$f_{i+1}(j, \ell, ie') + = P(ie, ie') \frac{(k' - \max(\mathcal{K}))(\mathcal{B}[\ell] - b')}{(k' - \max(\mathcal{K}))(\mathcal{B}[\ell] - \mathcal{B}[\ell - 1])} f_i(ik, ib, ie),$$

(e) Else, if  $k' \leq \max(\mathcal{K})$  and  $b' > \max(\mathcal{B})$ , calculate for all  $ie' \in \mathcal{E}$

$$f_{i+1}(j-1, \ell, ie') += P(ie, ie') \frac{(k' - \mathcal{K}[j-1])(b' - \max(\mathcal{B}))}{(\mathcal{K}[j] - \mathcal{K}[j-1])(b' - \max(\mathcal{B}))} f_i(ik, ib, ie),$$

$$f_{i+1}(j, \ell, ie') += P(ie, ie') \frac{(\mathcal{K}[j+1] - k')(b' - \max(\mathcal{B}))}{(\mathcal{K}[j] - \mathcal{K}[j-1])(b' - \max(\mathcal{B}))} f_i(ik, ib, ie).$$

(f) Else, calculate for all  $ie' \in \mathcal{E}$

$$f_{i+1}(j, \ell, ie') += P(ie, ie') f_i(ik, ib, ie).$$

5. Calculate  $\sum_{ik, ib, ie} |f_{i+1} - f_i|$ . If this sum is above a set tolerance level, set  $f_i = f_{i+1}$  and repeat from step 4.
6. Integrate the policy functions to obtain the excess demand for bonds and the supply of materials. Given the supply of materials, calculate the aggregate price level from equation (9).
7. If the aggregate price level calculated in step 6 is different from the guess made in step 1, or if the absolute value of the excess demand for bonds is not below a set tolerance level, adjust the guesses and repeat from step 1.

**C.1. VALUE-FUNCTION ITERATION STEP.**

The value-function iteration step employs a standard algorithm employing a bilinear approximation of the value function. The only nonstandard feature of this step is related to the optimization routine used to solve the Bellman equation. Each agent has a binding borrowing constraint (this could be set equal to the natural borrowing limit or to a more stringent level), and, because of this, the maximization of the utility function is subject to a constraint. There is also a nonnegativity constraint on the level of consumption. To enforce these constraints, we employ a trust-region optimization algorithm that performs the following calculations.

1. Choose a point in  $(k_0, b_0) \equiv x_0 \in \mathbb{R}^2$  that does not violate the borrowing constraint and the zero-lower bound on consumption.
2. Choose a maximum and initial trust-region radius  $\hat{\Delta}, \Delta_0 \in (0, \hat{\Delta})$ . Set  $k = 0$ .
3. Update  $k = k + 1$  and perform one iteration of the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm to obtain a candidate step  $p_k \in \mathbb{R}^2$ .
4. If  $\|p_k\| > \Delta_k$ , set  $p_k = p_k \times \frac{\Delta_k}{\|p_k\|}$ . If the point  $x_k + p_k$  violates any of the constraints, set  $\Delta_{k+1} = 0.5\Delta_k$  and repeat from step 3.
5. Calculate

$$\rho_k = - \frac{v(x_k) - v(x_k + p_k)}{\nabla v + \frac{1}{2} p_k^T H p_k},$$

where  $\nabla v$  is the gradient of the value function and  $H$  is an estimate of its Hessian (obtained from the BFGS algorithm).

6. If  $\rho_k > 0.5$ , set  $x_{k+1} = x_k + p_k$ , otherwise set  $x_{k+1} = x_k$ .
7. If  $\rho_k > 0.75$  and  $\|p_k\| > 0.8\Delta_k$ , set  $\Delta_{k+1} = \min(2\Delta_k, \hat{\Delta})$ , else set  $\Delta_{k+1} = \Delta_k$ .
8. If  $\rho_k < 0.1$ , set  $x_{k+1} = x_k$ , and  $\Delta_{k+1} = 0.5\Delta_k$ .
9. Stop if either  $\|\nabla v\|$  or  $\Delta_k$  are small enough, otherwise repeat from step 3.

This algorithm requires some explanations, since to the best of our knowledge, the use of trust-region methods for constrained optimization is novel in the economics literature. Many iterative optimization methods (e.g., BFGS) minimize a second-order Taylor expansion of the objective function. In such cases, the function  $v$  is replaced by a quadratic “model” of it around the current operating point, as a function of the displacement step  $p$

$$m(p) = v + \nabla v^T p + \frac{1}{2} p^T H p.$$

The trust-region approach seeks to find a neighborhood in which the quadratic approximation is a “good enough” representation of the objective function. To this end, calculate the ratio of the *actual* change to the *predicted* change

$$\rho = \frac{v(x) - v(x + p)}{m(0) - m(p)},$$

as a measure of the reliability of the quadratic approximation. If the function is well approximated by the quadratic model, and the step does not violate any constraints, then it is accepted. Furthermore, if the step length is close to the boundary of the trust region, then the trust region is enlarged to allow for a potentially bigger step in the next iteration. If, instead, the constraints are violated, then the trust-region is shrunk, so that the next iteration is set up to take a step that, being smaller, has a better chance to not violate the constraints. If the optimal point is a corner solution, then the trust-region shrinks to a very small radius and the algorithm terminates.