THE JOURNAL OF NAVIGATION (2004), **57**, 245–259. © The Royal Institute of Navigation
DOI: 10.1017/S0373463304002693 Printed in the United Kingdom

Localizability Analysis for GPS/Galileo Receiver Autonomous Integrity Monitoring

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With the European Commission (EC) and European Space Agency's (ESA) plans to develop a new satellite navigation system, Galileo and the modernisation of GPS well underway the integrity of such systems is as much, if not more, of a concern as ever. Receiver Autonomous Integrity Monitoring (RAIM) refers to the integrity monitoring of the GPS/Galileo navigation signals autonomously performed by the receiver independent of any external reference systems, apart from the navigation signals themselves. Quality measures need to be used to evaluate the RAIM performance at different locations and under various navigation modes, such as GPS only and GPS/Galileo integration, etc. The quality measures should include both the *reliability* and *localizability* measures. Reliability is used to assess the capability of GPS/Galileo receivers to detect the outliers while localizability is used to determine the capability of GPS/Galileo receivers to correctly identify the detected outlier from the measurements processed.

Within this paper, the fundamental equations required for effective outlier detection and identification algorithms are described together with the measures of reliability and localizability. Detailed simulations and analyses have been performed to assess the performances of GPS only and integrated GPS/Galileo navigation solutions with respect to reliability and localizability. Simulation results show that, in comparison with the GPS-only solution, the localizability of the integrated GPS/Galileo solution can be improved by up to 270%. The results also indicate an expectation of a considerable increase in the sensitivity to outliers and accuracy of their estimation with the augmentation of the Galileo system with the existing GPS system.

KEY WORDS

1. GPS. 2. Galileo. 3. Integrity. 4. Localizability.

1. INTRODUCTION. Over the past decade, Global Navigation Satellite Systems (GNSS) have become increasingly familiar entities in the fields of surveying, geodesy and other position-sensitive disciplines, such as transportation, personal location and telecommunications. With the process of modernisation well underway for the existing US developed GPS and Russian GLONASS, the European Commission (EC) in a joint project with the European Space Agency (ESA) have also established plans for the development of a new satellite navigation system in

response to the ever increasing demands from civilian and commercial sectors around the globe. However, for all these satellite-based navigation systems, the integrity of navigation solutions is still one of the major concerns.

Integrity, as defined in the International Civil Aviation Organisation's GNSS Standards and Recommended Practices (SARPS) is: *a measure of the trust which can be placed in the correctness of the information supplied by the total system. Integrity includes the ability of a system to provide timely and valid warnings to users* (ESA, 2002). Monitoring the integrity of a navigation system is essential to ensure that the navigation solution is within tolerable constraints. Ideal integrity monitoring involves the detection, isolation and removal of faulty measurement sources from the navigation solution.

Integrity monitoring of the GNSS navigation signals can be achieved at the user end autonomously performed by the receiver independent of any external reference systems, excluding the navigation signals themselves. Such monitoring is referred to as Receiver Autonomous Integrity Monitoring (RAIM). The RAIM performance is measured in terms of the *maximum allowable alarm rate* and the *minimum detection probability* and is dependent on the failure rate of measurement sources, range accuracies and measurement geometry. Optimal RAIM algorithms should exhibit high detection rates and low false alarm rates.

Significant efforts have been made to develop and analyse RAIM methods and algorithms over the past decade. Among others, Gao (1993) has investigated a GPS integrity test procedure with reliability assurance to offer real-time precision and reliability checks on navigation solutions. Walter and Enge (1995) presented a versatile weighted form of RAIM where measurement sources are weighted based on *a priori* information or broadcast weighting information. A probabilistic approach to the determination of geometrical criteria for the evaluation of GPS RAIM availability has been discussed in Sang and Kubik (1997). More recently, Romay *et al.* (2001) investigates the availability of RAIM computed for GPS, Galileo and combined GPS/Galileo constellations through simulations. The RAIM capability of the Galileo system, when used alone and when combined with GPS, is assessed by Ochieng *et al.* (2002).

Statistical testing procedures, focussed on reliability of detecting fault measurements or outliers, have traditionally been the basis for current RAIM techniques. The reliability of GNSS systems is essentially dependant on the redundancy and geometry of the measurement system. Reliability assesses the capability of GNSS receivers to detect the outliers. A minimum of five satellites with sufficiently strong measurement geometry must be available to provide the redundancy essential to permit measurement consistency checks necessary to evaluate the reliability measure. However, with only five satellites available, it is possible to detect one outlier but impossible to identify which measurement in the solution is the outlier. It should also be noted that greater redundancy and geometric strength of the measurement system significantly improves the capability of GNSS RAIM procedures in both detecting and identifying the outliers. Therefore, some quality measures need to be used to evaluate the GNSS RAIM performance at different locations and under various navigation modes, such as GPS only and GPS/Galileo integration, etc. The quality measures should include both the *reliability* and *localizability*. Localizability is used to assess the capability of GNSS receivers to correctly identify the detected outlier from the measurements processed.

In this paper, the mathematical models for RAIM and its measures of reliability and localizability are described. Detailed simulations have also been carried out to evaluate the reliability and separability performance of GPS only and integrated GPS/Galileo navigation solutions.

2. RAIM ALGORITHMS.

2.1. *Outlier detection and identification.* The GNSS data acquisition process is highly autonomous once the user has initialised the receiver in the desired manner. While this mitigates the potential for human observation errors it also eliminates the user's faith in the quality of individual measurements that he/she may obtain when taking the measurements manually. For this reason, and due to the abundance of data acquired by many GNSS positioning and navigation techniques, it is absolutely essential that statistical methods of quality control are implemented. Through the proper implementation of quality control measures the user can have much more faith in the quality of the measurements. Measurements containing gross errors cannot be described or accounted for by the stochastic model and will therefore undesirably affect parameter estimations and related variances. As a result there is an extremely high risk that an outlier will go undetected unless statistical testing methods are employed. One widely accepted procedure is called Detection, Identification and Adaptation (DIA), which is discussed below.

The linearised functional model of measurements can be expressed as:

$$Ax = l + v \tag{1}$$

where A is the $n \times m$ design matrix, x is the vector of m parameters, l is the vector of n measurements and v is the vector of n residuals. Complementing the above functional model is the stochastic variance covariance (VCV) matrix of measurements Q_l. This matrix, which is assumed to be known, describes the 'noise' characteristics related to the measurements. Define

$$P = Q_l^{-1} \tag{2}$$

as the $n \times n$ weight matrix of measurements. For the estimation of the unknown parameters and measurement n residuals the following least squares formulae are used:

$$\hat{x} = (A^T PA)^{-1} A^T Pl \tag{3}$$

$$\hat{v} = A\hat{x} - l \tag{4}$$

where $\hat{x}$ is the vector of m least squares estimates of x, $\hat{v}$ is the vector of n least squares estimates of v.

The *a posteriora* variance factor (or unit variance) test is used to indicate whether or not the adjustment model is satisfactory and capable of detecting the presence of any gross anomalies. The variance factor is determined as:

$$\hat{\sigma}_0^2 = \frac{v^T Pv}{n-m} \tag{5}$$

where v is the vector of n residuals, P is the $n \times n$ weight matrix of measurements and m is the number of unknown parameters.

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The variance factor can be tested against the two-tailed test limits derived from the Chi-squared distribution:

$$\frac{\chi^2_{1-\alpha/2,\,n-m}}{n-m} \leq \hat{\sigma}_0^2 \leq \frac{\chi^2_{\alpha/2,\,n-m}}{n-m} \tag{6}$$

where $n-m$ is the degrees of freedom in the solution and α is the significance level of the test.

If the variance factor exceeds the test limit (critical value), the adjustment model is considered to be invalid. The expected value of the variance factor is equal to one and is often referred to as the unit variance statistic. The variance factor test may fail as a result of input or programming errors, the presence of outliers in measurements, model errors or poor estimation of the *a priori* covariance matrix. In this study, we assume an accurate model and covariance matrix and that only the outliers may cause the failure of the variance factor test.

Assuming that an outlier ∇S_i exists, the linearised adjustment model may be defined as:

$$A\hat{x} + e_i \nabla S_i = l + \hat{v} \tag{7}$$

where A is the $n \times m$ design matrix, $\hat{x}$ is the vector of m least squares parameter estimates, l is the vector of n measurements and $\hat{v}$ is the vector of the n least squares residual estimates and e_i is a unit vector in which the ith component has a value equal to one and dictates the measurement to be tested:

$$e_i = [0 \quad 0 \quad \ldots \quad 1 \quad 0]^T$$

A least squares estimation of the magnitude of the outlier ∇S_i can be determined by:

$$\nabla \hat{S}_i = -(e_i^T P Q_{\hat{v}} P e_i)^{-1} e_i^T P \hat{v} \tag{8}$$

with a variance of:

$$\sigma^2_{\nabla \hat{S}_i} = (e_i^T P Q_{\hat{v}} P e_i)^{-1} \tag{9}$$

Statistical testing for the *identification* of an outlier ∇S_i relies on the null hypothesis, which shows that the measurements are outlier free, and the alternative hypothesis which, when proved true, indicates the existence of an outlier of magnitude ∇S:

Null Hypothesis:

$$H_0\colon\ E(\nabla \hat{S}_i) = 0 \tag{10}$$

Alternative Hypothesis:

$$H_a\colon\ E(\nabla \hat{S}_i) = \nabla S_i \neq 0 \tag{11}$$

Provided that the adjustment model is correct, the w-test can be used to identify an outlier. The test statistic is (Baarda, 1968; Cross *et al.*, 1994; Teunissen, 1998)

$$w_i = \frac{-e_i^T P \hat{v}}{\sqrt{e_i^T P Q_{\hat{v}} P e_i}} \tag{12}$$

which has a standard normal distribution under the null hypothesis. However, under the alternative hypothesis, the distribution of the statistic will have the following

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non-centrality:

$$\delta_i = \nabla S_i \sqrt{e_i^T P Q_{\hat{v}} P e_i} \tag{13}$$

The critical value for w_i to be tested against is $N_{1-\alpha/2}(0, 1)$. For the situation where:

$$|w_i| > N_{1-\alpha/2}(0, 1) \tag{14}$$

the *i*th measurement is a 'supposed' outlier. The test is carried out with respect to each measurement and the largest value that exceeds the critical value is deemed an outlier and is removed from the model. The *w*-test is performed again to see if any more outliers exist. If another outlier is found it should be removed from the model and the measurement that was first regarded as an outlier should be reinstated and the model retested. This procedure should be repeated until no more outliers are detected.

There are five possible outcomes of the hypothesis testing procedure and only two of these are correct. The correct outcomes are achieved if the null hypothesis is accepted when it is in fact true or the null hypothesis is rejected when it is indeed false. The incorrect outcomes are rejecting the null hypothesis when it is true (Type I error), accepting the null hypothesis when it is actually false (Type II error) and removing the wrong measurement once an outlier has been detected (Type III error) (Hawkins, 1980). The probabilities of committing Type I and Type II errors are α and β, respectively. α is the probability of a false alarm and is referred to as the test level of significance. β is the probability that an outlier is undetected and the power of the test is the probability of detecting an outlier, $\gamma = 1 - \beta$.

The *Adaptation* phase refers to the effective handling of the outlier which, permits the null hypothesis to be legitimately accepted. The measurement regarded as an outlier may be eliminated from the adjustment computation or the resulting bias may be included as a parameter within the model to be estimated and accounted for. If the latter approach is chosen then a new null hypothesis must be selected in consideration of the new error parameter.

2.2. *Reliability and localizability*. Reliability refers to the consistency of the results provided by a system, dictating the extent to which they can be trusted or relied upon. More specifically, in terms of GNSS RAIM, reliability comprises the ability of the system to detect outliers, referred to as internal reliability, and measures of the influence of undetectable outliers on the parameter estimations, referred to as external reliability (Baarda, 1968).

Localizability refers to the ability to distinguish or localise a measurement from the other measurements. This ability is of the upmost importance as poorly separated measurements adversely affect the reliability of a navigation solution by manifesting a high risk of incorrectly flagging a 'good' measurement as an outlier (Type III error).

2.2.1. *Internal reliability*. The measure of internal reliability is quantified as the Minimal Detectable Bias (MDB) and is indicated by the lower bound for detectable outliers. The MDB is the magnitude of the smallest error that can be detected for a specific level of confidence and is determined, for correlated measurements (Baarda, 1968; Cross *et al.*, 1994) by:

$$\nabla_0 S_i = \frac{\delta_0}{\sqrt{e_i^T P Q_{\hat{v}} P e_i}} \tag{15}$$

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where δ_0 is the noncentrality parameter which depends on the given Type I and Type II errors (α_0 and β_0). It is usual practice to hold the power of the test fixed at a value of, for example, 80% and the given confidence level (99%) for the test for the determination of MDBs (Baarda, 1968; Cross *et al.*, 1994).

2.2.2. *External reliability*. External reliability of the system is characterised by the extent to which the MDB affects the estimated parameters. External reliability measures are evaluated as (Baarda, 1968; Cross *et al.*, 1994):

$$\nabla_0 \hat{x} = Q_x A^T P e_i \nabla_0 S_i \tag{16}$$

As external reliability is solely concerned with the effect of undetected outliers upon the final solution it is only possible to quantify the probability of detecting them if they do occur, and the effect they will have if they are not detected.

2.2.3. *Localizability*. For the case where a blunder is large enough to cause many w-test failures, resulting in many alternatives, it is essential to insure that any two alternatives can be separated. A measure of the separability of H_{ai} and H_{aj} is given by $(1-r)$, where r is the probability of committing a Type III error and depends on the correlation of the test statistics w_i and w_j. The separability is calculated for a given significance level α_0 and non-centrality parameter δ_0. The degree of correlation of the two test statistics is determined through derivation of the correlation coefficient (Förstner, 1983; Tiberius, 1998):

$$\rho_{ij} = \frac{\sigma_{ij}}{\sqrt{\sigma_i^2 \sigma_j^2}} = \frac{e_i^T P Q_{\hat{v}} P e_j}{\sqrt{e_i^T P Q_{\hat{v}} P e_i} \cdot \sqrt{e_j^T P Q_{\hat{v}} P e_j}} \tag{17}$$

where $|\rho_{ij}| \leqslant 1$. A correlation coefficient value of one and zero respectively indicates that the two test statistics are fully correlated and uncorrelated, respectively. The greater the correlation between two test statistics, the more difficult it is to separate the corresponding measurements. In such a situation where an outlier has been detected and the corresponding w-test statistic is highly correlated with other measurements, there is a strong probability that the wrong measurement will be identified as the outlier. The degree of correlation of the w-test statistics is dependant on the strength of the geometry. A strong geometry will deliver weakly correlated w statistics. For circumstances where only five satellites are available, all measurements are fully correlated to one another.

The measure of localizability is essentially a product of the internal reliability and separability $(1-r)$ between two alternative hypotheses H_{ai} and H_{aj}. The internal reliability is multiplied by the separability multiplying factor $K_{\rho_{ij}}$ (e.g., Li, 1986; Wang and Chen, 1994; Moore *et al.*, 2002):

$$K_{\rho_{ij}} = \frac{\delta_{0,\rho_{ij}}}{\delta_0} = K(\alpha_0, \gamma_0', r_0', \rho_{ij}) \tag{18}$$

where $\delta_{0,\rho_{ij}}$ is the critical value of the non-centrality parameter δ_0 satisfying the conditions of the power of the combined test γ_0' of w_i and w_j and the separability of $(1-r_0)$ for the rejection of the null hypothesis. In other words, the non-centrality parameter δ_0 must be large enough to permit the rejection of the null hypothesis for the assumed power and separability of the combined test. $\delta_{0,\rho_{ij}}$ will always be greater than or equal to δ_0 due to the correlation between the two test statistics. Localizability is therefore, expressed as:

$$\nabla_{\mathbf{0}} S_{ij} = K_{\rho_{ij}} \nabla_{\mathbf{0}} S_i \tag{19}$$

For all practical purposes it is sufficient to consider only the maximum correlation coefficient $\rho_{ij\,\max}$ ($\forall j \neq i$) for any measurement as this will obtain the lower bound value, in the worst case scenario, for an MDB which, can just be separated by the combined test. In such a case Equation (19) becomes:

$$\nabla_{\mathbf{0}} S_{ij\,\max} = K_{\rho_{ij\,\max}} \nabla_{\mathbf{0}} S_i \tag{20}$$

3. GPS/GALILEO RAIM PERFORMANCE STUDIES.

In order to analyse GPS/Galileo RAIM performance, a series of intensive simulation tests were conducted. UNSW GNSS (GPS and Pseudolite) measurement simulation and analysis software (Lee *et al.*, 2002), originally designed for GPS and Pseudolite simulation, was modified by the author to enable the following simulations involving the Galileo system. The analyses are actually based on the GPS and Galileo satellite and receiver coordinates. The GPS satellite coordinates were calculated by ephemeris (converted from the almanac files). The constellation that was implemented for Galileo was compiled from the following papers: Lucas and Ludwig (1999), Tytgat and Owen (1999), Ryan and Lachapelle (2000), Verhagen (2002) as it is still in the defining phase. The pseudo-constellation consists of 30 medium Earth orbit satellites in three perfect circular orbital planes with an inclination of 54 degrees and an altitude of 23 000 km. The simulations also assume that the satellites are equally spaced with 10 satellites on each orbital plane.

3.1. *RAIM performance analysis for a specific location and time.* Internal and external reliability, as well as localizability values may be determined for a specific location and time using simulated range data. The determined values provide a good indication of the capability of the measurement system to detect and correctly identify an outlier and the effect of an undetectable outlier on the measurement solution. An example of such a simulation is given here considering two different positioning scenarios. The first scenario uses only GPS measurements where 6 satellites are available and the second uses both GPS and Galileo systems satellite availability is increased to 14 (see Figure 1). Single frequency code measurements were simulated for both scenarios with the standard deviations set to $\sigma = 2{\cdot}8$ m. The measurements are assumed to be spatially and temporally uncorrelated. Note that satellite numbers from 1 to 31 represent GPS satellites, while numbers 40 to 70 refer to Galileo satellites.

As discussed previously, the Minimal Detectable Bias (MDB) gives the magnitude of the outlier which can be detected with γ probability using the *w*-test. The MDB values determined here are computed with $\gamma = 80\%$ probability at a significance level of $\alpha = 5\%$ and a non-centrality parameter $\delta_{\mathbf{0}} = 2{\cdot}8$. The results are given in Table 1. The internal reliabilities corresponding to the measurements of satellites 4, 10 and 30 in the case of GPS-only and those of satellites 10, 45, 63, and 70 in the GPS/Galileo case are slightly poorer than of the other satellites. The values are all based on the redundancy of least square estimation. This dependence can be expressed in terms of correlations between test statistics. Table 2 gives the correlation coefficients for

Table 1. MDBs for GPS and GPS/Galileo systems ($\gamma = 80\%$, $\alpha = 5\%$, $\delta_0 = 2{\cdot}8$).

GPS only		GPS and Galileo			
Satellite	MDB(m)	Satellite	MDB(m)	Satellite	MDB(m)
04	15·258	04	9·637	45	10·987
05	12·243	05	9·924	46	9·843
09	14·586	09	9·759	53	9·363
10	17·558	10	10·326	54	9·561
24	12·119	24	9·637	55	9·460
30	19·517	30	9·399	61	9·571
				63	11·977
				70	10·648

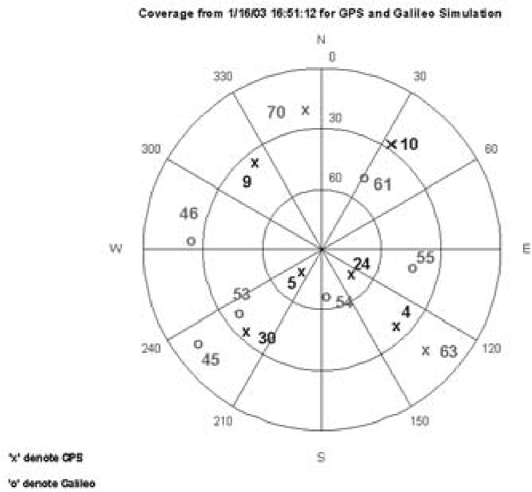

Figure 1. Skyplot of GPS and Galileo satellites for the simulation.

GPS-only and GPS/Galileo simulation scenarios. Upon comparison of the MDBs in Table 1 with the correlation coefficients in Table 2 it can be seen that the measurements of poorer internal reliability are generally more strongly correlated to other measurement w-test statistics. In addition, by comparing the correlations of the GPS-only system with the combine GPS/Galileo system it is revealed that greater redundancy delivers less correlation among the test statistics. Hence, the inclusion of the Galileo satellites enhances the internal reliabilities. In general, it is more difficult to localise highly correlated measurements than those with low correlation. This difficulty can be measured by localizability, and will be discussed in more detail.

For the case of PRN04 and PRN30 in the GPS-only system in Table 2(a), where the measurements are fully correlated and an outlier was detected corresponding to either of these measurements, it would be impossible to identify which measurement contains the gross error. Hence, in practice both measurements would have to be removed, leaving only 5 measurements which is the absolute minimum required for RAIM availability. Furthermore, if another outlier was detected it would not be possible to

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Table 2. Correlation matrices for *w*-test statistics, $\rho \in [0, 1]$.

(a) GPS-only system with 7 satellites.

PRN	04	05	09	10	24	30
04	1	0·119	0·869	0·862	0·419	1·000
05		1	0·387	0·399	0·952	0·141
09			1	1·000	0·085	0·859
10				1	0·099	0·851
24					1	0·438
30						1

(b) GPS/Galileo system with 15 satellites.

PRN	04	05	09	10	24	30	45	46	53	54	55	61	63	70
04	1	0·038	0·133	0·096	0·133	0·081	0·095	0·088	0·039	0·127	0·252	0·035	0·498	0·076
05		1	0·131	0·094	0·322	0·135	0·108	0·054	0·174	0·322	0·081	0·161	0·231	0·098
09			1	0·191	0·049	0·051	0·063	0·259	0·096	0·033	0·043	0·202	0·261	0·358
10				1	0·002	0·131	0·056	0·019	0·129	0·082	0·202	0·285	0·249	0·472
24					1	0·068	0·167	0·055	0·090	0·293	0·176	0·182	0·039	0·092
30						1	0·329	0·209	0·241	0·132	0·011	0·068	0·097	0·062
45							1	0·387	0·283	0·064	0·084	0·203	0·368	0·104
46								1	0·222	0·002	0·121	0·001	0·087	0·234
53									1	0·148	0·029	0·035	0·003	0·039
54										1	0·132	0·113	0·034	0·145
55											1	0·160	0·342	0·034
61												1	0·066	0·254
63													1	0·015
70														1

identify which measurement it corresponds to, for when only 5 satellites are available all the measurements are fully correlated, and all the measurements must be discarded.

Figure 2 depicts the computed external reliability values for the two measurement scenarios simulated here. The external reliability is a more practical measure than the internal reliability as it indicates the effect an undetectable outlier may have on the position fix. This is extremely useful as a large undetected outlier may have little effect on the fix if it relates to a very low weighted, low elevation satellite, or vice versa (Cross *et al.*, 1994). It can be seen in Figure 2 that a large error corresponding to satellite 5 measurements is induced in the vertical component even though it has relatively small MDB. Measurements of the signals from satellites 24 and 30 deliver different external reliabilities even though the internal reliability values are very similar. The results in Figure 2 reveal significant improvement in the external reliability values due to the augmentation of the GPS with the Galileo system.

In Table 3 the maximum correlation coefficients and separability multiplying factors are presented for the measurements corresponding to each satellite. Based on the separability multiplying factors in Table 3 and the MDBs in Table 1, Table 4 gives the localizability values. Localizability represents the lower bound for a MDB which can be separated. From Table 4 it is obvious to see the marked improvements in the localizability values of the combined GPS/Galileo system when compared to the GPS alone.

It is also revealed in Table 4 that the localizability values are increased from the MDB values more significantly for the GPS-only system than for the combined system

Table 3. Separability multiplying factor ($K_{\rho_{ij\,\max}}$) for GPS and GPS/Galileo systems.

GPS only			GPS and Galileo					
SVs	$\rho_{ij\,\max}$	$K_{\rho_{ij\,\max}}$	SVs	$\rho_{ij\,\max}$	$K_{\rho_{ij\,\max}}$	SVs	$\rho_{ij\,\max}$	$K_{\rho_{ij\,\max}}$
04	0·999	4·383	04	0·499	1·015	45	0·387	1·010
05	0·952	1·964	05	0·322	1·007	46	0·387	1·010
09	0·999	4·383	09	0·358	1·010	53	0·283	1·007
10	0·999	4·383	10	0·472	1·015	54	0·322	1·007
24	0·952	1·964	24	0·322	1·007	55	0·342	1·007
30	0·999	4·383	30	0·329	1·007	61	0·285	1·007
						63	0·499	1·015
						70	0·472	1·015

Table 4. Localizability of Outliers for GPS and GPS/Galileo systems.

GPS only			GPS and Galileo					
SVs	$\nabla_0 S_i$	$\nabla_0 S_{ij\,\max}$	SVs	$\nabla_0 S_i$	$\nabla_0 S_{ij\,\max}$	SVs	$\nabla_0 S_i$	$\nabla_0 S_{ij\,\max}$
04	15·258	66·877	04	9·637	9·782	45	10·987	11·097
05	12·243	24·045	05	9·924	9·993	46	9·843	9·941
09	14·586	63·931	09	9·760	9·857	53	9·363	9·428
10	17·558	76·958	10	10·326	10·481	54	9·561	9·628
24	12·119	23·802	24	9·637	9·704	55	9·460	9·526
30	19·517	85·544	30	9·399	9·465	61	9·571	9·638
						63	11·977	12·156
						70	10·648	10·808

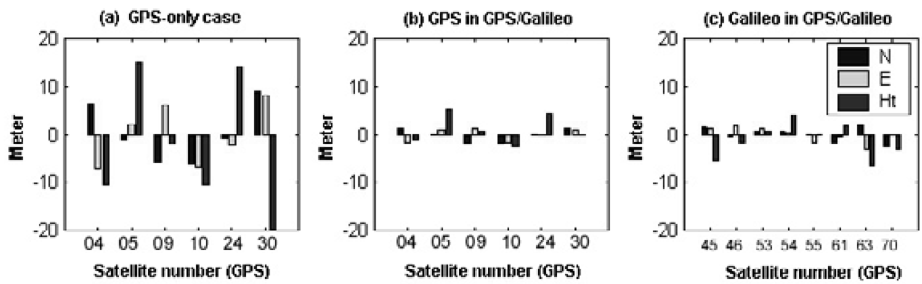

Figure 2. External reliability for GPS and GPS/Galileo systems.

due to the higher degree of correlation between the measurements of the GPS-only system. The average increase in the localizability values from the internal reliability values is over 270% for the GPS only system and approximately 1% for the combined system. The combined system, in this case, offers an improvement in localizability of up to 270 times that achieved by GPS alone.

The results in Table 5 show the estimation values for the outliers and resulting test statistic values corresponding to each satellite for the GPS only constellation where outliers of +20 m and +30 m have been introduced into the measurements associated with satellite 5. For both scenarios, a reasonable estimate is made of the

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Table 5. Simulation results for detecting outliers in the GPS only system.

	Scenario 1 SV 05+20 m		Scenario 2 SV 05+30 m	
SV	∇S	$\lvert w_i \rvert$	∇S	$\lvert w_i \rvert$
04	8·279	0·506	9·767	0·597
05	**24·717**	**1·884**	**34·717**	**2·647**
09	−7·311	0·468	−11·922	0·763
10	9·273	0·493	15·002	0·797
24	−24·415	1·880	−33·835	2·606
30	−11·408	0·546	−13·650	0·653

Table 6. Simulation results for detecting outliers in GPS and GPS/Galileo systems.

	Scenario 1 SV 05+20 m		Scenario 2 SV 05+30 m			Scenario 1 SV 05+20 m		Scenario 2 SV 05+30 m	
SV	∇S	$\lvert w_i \rvert$	∇S	$\lvert w_i \rvert$	SV	∇S	$\lvert w_i \rvert$	∇S	$\lvert w_i \rvert$
04	1·355	0·131	0·985	0·095	45	−0·448	0·038	0·751	0·064
05	**22·349**	**2·102**	**32·349**	**3·042**	46	−5·518	0·523	−6·056	0·574
09	2·219	0·212	0·936	0·089	53	0·680	0·068	0·959	0·096
10	2·006	0·181	2·983	0·270	54	−11·387	1·112	14·492	1·415
24	−12·078	1·170	−15·203	1·472	55	−0·007	0·001	0·775	0·076
30	0·753	0·075	−0·528	0·052	61	−0·595	0·058	2·145	0·209
					63	5·403	0·421	8·193	0·638
					70	−1·782	0·156	0·737	0·065

induced outliers. However, due to the high correlation of the measurements the procedure indicates the possibility of outliers in all of the measurements. The procedure also indicates that an equally sized outlier exists in the measurement corresponding to satellite 24 due to its high correlation with satellite 5 (Table 2(a)). Furthermore, the test statistic only indicates that an outlier exists for the +30 m induced outlier as it is greater than the critical value of 1·96 for the 5% significance level. Also due to the high correlation between satellite measurements 5 and 24, the test statistic corresponding to satellite 24 suggests the presence of an outlier.

For the combined GPS/Galileo system results, in Table 6, more accurate estimates for both of the induced outliers are achieved. Even though outliers of reasonable sizes are indicated for the measurements of satellites 24 and 54 (due to correlations with satellite 5, see Table 2(b)), it is clear, from the test statistics, that the outlier corresponds to satellite 5. Such clarity is due to decorrelation that is achieved through the greater availability and geometric strength of the combined system. These results reveal the marked improvement in the sensitivity of combined GPS/Galileo system upon the GPS only system.

3.2. *Worldwide RAIM localizability analysis.* The global results were obtained from snapshot simulations for 0:00 h on the 16th January, 2003 at 1 degree intervals of latitude and longitude and an altitude of 50 m. Snapshot results permit analysis based on spatial variations as time is held constant. Simulated measures include availability, maximum internal reliability, minimum and maximum localizability and

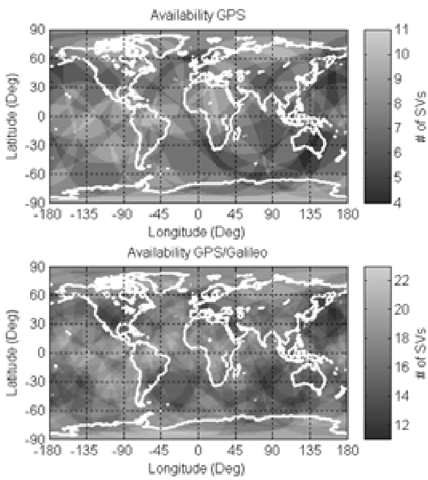

Figure 3. Availability for GPS (top) and combined GPS/Galileo (bottom).

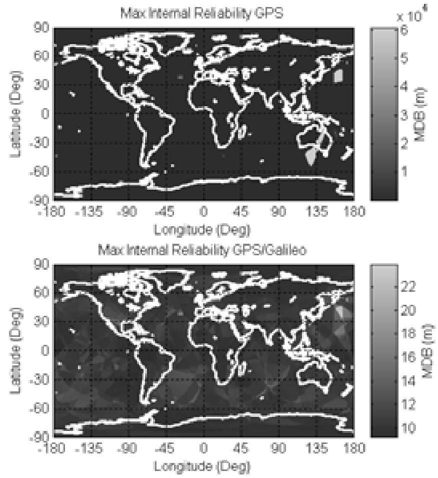

Figure 4. Maximum internal reliability for GPS (top) and combined GPS/Galileo (bottom).

correlation coefficients. The results from the global snapshot scenario are presented as orthographic global colour maps.

Figures 3 and 4 respectively exhibit the availabilities and maximum internal reliabilities for both the GPS only and the combined GPS/Galileo systems. It can be seen in Figure 4, upon comparison with Figure 3, that the maximum internal reliability computations fail to deliver sensible results where there are only 4 satellites available (see around 30°S, 135°E and 40°N, 165°E).

Figures 5 and 6 are representations of the spatial variations of the minimum and maximum correlation coefficients between two satellites for both constellations, GPS and GPS/Galileo. The results for the GPS/Galileo system show that the *w*-test statistics are far less correlated than those of the GPS system are. For the combined case, the maximum correlation coefficient is below 0·5 for the majority of the earth's surface. The maximum correlation coefficient results for the GPS system cover the majority of the earth with values of 0·7 or greater. The more correlated the *w*-test statistics are, the more poorly separated they are. Poor separation means that it may be impossible to distinguish one outlier in the measurements from another. If an outlier cannot be distinguished with a reasonable level of certainty, there is a large risk that, for a case where an outlier has been detected, the wrong measurement could be identified as the outlier. If this occurred then a good measurement would be removed and the outlier would remain and bias the navigation solution. It is also undesirable to remove more measurements than is absolutely necessary as this would weaken the measurement geometry and dilute the precision of the solution. Regions in Figure 5, for the GPS only system, where the minimum correlation coefficient is equal to one correspond to the areas where only 5 or less satellites are available (see Figure 3). It should be noted that there are no such regions in the combined GPS/Galileo system as the minimum availability across the globe is 11 satellites in this instance.

The global minimum and maximum localizability values are presented in Figures 7 and 8 respectively. A vast improvement is apparent with the inclusion of the Galileo

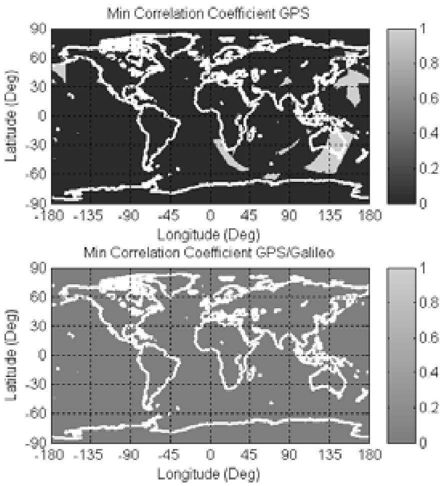

Figure 5. Minimum correlation coefficient for GPS (top) and combined GPS/Galileo (bottom).

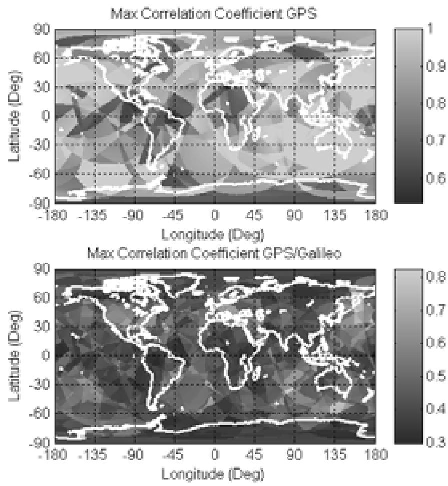

Figure 6. Maximum correlation coefficient for GPS (top) and combined GPS/Galileo (bottom).

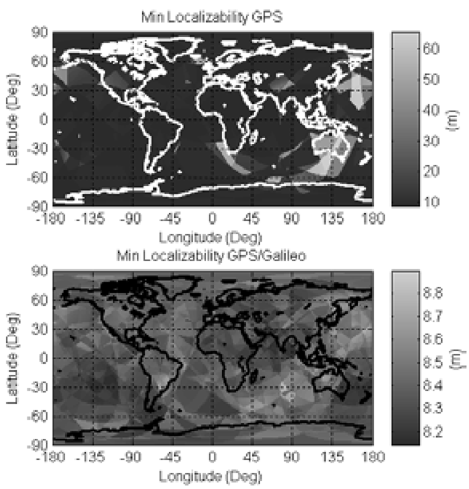

Figure 7. Minimum localizability for GPS (top) and combined GPS/Galileo (bottom).

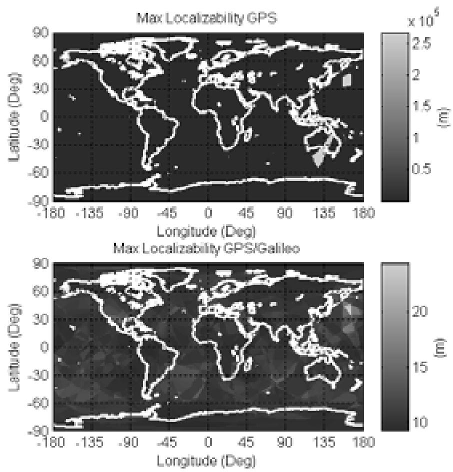

Figure 8. Maximum localizability for GPS (top) and combined GPS/Galileo (bottom).

system. From the minimum localizability values (see Figure 7) an improvement better than 500% can be seen for some locations due to decorrelation of the measurements as a result of the increase in satellite availability. The GPS/Galileo system not only gives improved results but it also offers greater consistency among the results. Figure 8 exposes the computational failure as a result of poor availability for the GPS only system.

As these results are snapshots in time it should be noted that the areas affected by insufficient availability to permit RAIM and separability functions (i.e. Satellites less

https://doi.org/10.1017/S0373463304002693 Published online by Cambridge University Press

than five), move and vary in size and number. The results also illustrate the benefits of the advent of Galileo, where marked improvements in satellite availability and subsequently, greater separability of outliers in measurements will occur.

4. CONCLUDING REMARKS.

A first effort for the development of a RAIM algorithm including both reliability and separability assurance measures has been presented along with an investigation and comparison of the abilities of the current and existing GPS and the anticipated combined GPS/Galileo system to distinguish and separate measurements.

Through simulations it has been shown that the advent of Galileo will markedly lower correlations between the statistics for the outlier detection through the provision of greater satellite availability. This, in turn, will improve RAIM performance, as it will enable another parameter, namely localizability, to be monitored. Localizability monitoring will provide users with a level of assurance that good measurements will not be flagged as outliers, and more importantly, that true outliers can easily be excluded. Such assurance will improve the robustness and integrity of the GNSS navigation solutions as it will enable gross errors to be removed and thus provide a reliable solution. Based on the results presented in this paper, localizability of the integrated GPS/Galileo solution is expected to improve by up to and in some instances (when satellite availability is less than 5) well over 500% on the GPS-only. It is also expected that the GPS/Galileo system will be markedly more sensitive to outliers and deliver better estimations of their values.

ACKNOWLEDGEMENTS

Hewitson, S. is supported in his PhD research by a scholarship funded by the Faculty of Engineering, UNSW. Lee, H.K. is supported in his PhD research by a scholarship funded by the Kwanjeong Educational Foundation of Korea.

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