J. Plasma Phys. (2021), *vol.* 87, 755870401
Published by Cambridge University Press
doi:10.1017/S0022377821000738

Extraordinary and upper-hybrid waves in spin quantum magnetoplasmas with vacuum polarization effect

Jun Zhu[1],†, Xiaoshan Liu[1] and Yuee Luo[2]

[1]School of Physics and Electronic Engineering, Shanxi University, Taiyuan 030006, PR China

[2]Department of Mechanical and Electronic Engineering, Jingdezhen University, Jingdezhen 333000, PR China

(Received 25 January 2021; revised 22 June 2021; accepted 23 June 2021)

The propagation of extraordinary and upper-hybrid waves in spin quantum magnetoplasmas with vacuum polarization effect is investigated. Based on the quantum magnetohydrodynamics model including Bohm potential, arbitrary relativistic degeneracy pressure and spin force, and Maxwell's equations modified by the spin current and vacuum polarization current, the dispersion relations of extraordinary and upper-hybrid waves are derived. The analytical and numerical results show that quantum effects (Bohm potential, degeneracy pressure and spin magnetization energy) and the vacuum polarization effect modify the propagation of the extraordinary wave. Under the action of a strong magnetic field, the plasma frequency is obviously increased by the vacuum polarization effect.

Key words: quantum plasma, astrophysical plasmas, plasma waves

1. Introduction

As an emerging research field of plasma physics, the quantum plasma, composed of ions and degenerate electrons, has attracted much attention and research interest (Markowich, Ringhofer & Schmeiser 1990; Haas, Manfredi & Feix 2000; Harding & Lai 2006; Brodin *et al.* 2007). An example of the real physical environments in which a quantum plasma exists is the dense astrophysical objects, such as white dwarfs and neutron stars. White dwarfs resist gravitational collapse by producing electron degeneracy pressures with extremely high number density, typically as high as 10^{30} cm^{-3}, whereas neutron stars have a higher density. Electrons in the quantum plasma obey the Fermi–Dirac distribution. According to the Pauli exclusion principle, the thermal pressure is replaced by the degenerate pressure between electrons (Haas 2011). In dense astrophysical plasmas, the conditions of high-electron-number density and low temperature make the thermal de Broglie wavelength of electrons become equal to or even larger than the characteristic scale of the plasma system, and the quantum tunnelling effect represented by the Bohm potential will appear (Bohm 1952; Manfredi 2005; Shukla 2006; Shukla & Eliasson 2006).

Since electrons are fermions (spin-1/2 quantum particles), under the action of a strong magnetic field there will appear an electron-spin current and a spin force acting on

† Email address for correspondence: zhujun@sxu.edu.cn

electrons due to the Bohr magnetization. In highly magnetized or cold plasmas, the spin effect is significant (Shukla & Eliasson 2011). The spin magnetohydrodynamic model was proposed by Marklund & Brodin (2007), in which the electrons were treated as a single fluid. Andreev then gave a generalized form of the quantum hydrodynamics (QHD) model for spin-1/2 particles (Andreev 2015), in which the electrons of spin-up and spin-down were treated as two different fluids. This model is called the separate spin evolution quantum hydrodynamics (SSE-QHD) model. Since then, a lot of research on the electron spin-1/2 effect has been carried out. Iqbal investigated the spin magnetoacoustic wave and hybrid wave instabilities (Iqbal, Khan & Murtaza 2018*a*; Iqbal, Khanum & Murtaza 2018*b*; Iqbal *et al.* 2019*a*,*b*), which indicated that the dispersion of an upper-hybrid wave is affected by spin effects. The extraordinary wave in a spin-1/2 quantum plasma was studied by Andreev (2017). A magnetohydrodynamic wave with relativistic electrons and positrons in degenerate spin-1/2 astrophysical plasmas was investigated by Maroof *et al.* (2015). A magnetohydrodynamic spin wave in degenerate electron–positron plasmas was analysed by Mushtaq *et al.* (2012).

The quantum electrodynamic (QED) effect in a strong field is a very large research area. The QED effect has been experimentally confirmed under many different conditions, but there is still one that has not been verified, called the Schwinger mechanism. QED theory points out that a vacuum will exhibit some special properties in the strong field. For example, when the field intensity reaches Schwinger's critical strength, the vacuum will break down, and a virtual electron–positron pair can be spontaneously excited into a real electron–positron pair. When the field strength is lower than the Schwinger's critical strength, the vacuum will still show a weak nonlinear dielectric effect due to the quantum fluctuation of the virtual electron–positron pair, which is the so-called QED vacuum polarization effect (Goldreich & Julian 1969; Gedalin, Merose & Gruman 1998; Marklund & Shukla 2006). The vacuum polarization effect can induce many new physical phenomena, such as photon–photon scattering, electron–positron pair generation, vacuum birefringence and photon acceleration in a vacuum. Shukla & Stenflo (2008) investigated the dispersion relations for elliptically polarized extraordinary waves and linearly polarized ordinary waves propagating across an external magnetic field in a dense magnetoplasma. Lundin *et al.* (2007) investigated circularly polarized waves propagating along an external magnetic field with a vacuum polarization effect. Stenflo *et al.* (2005) investigated a new low-frequency circularly polarized electromagnetic waves in an electron–positron plasma, taking into account the QED effect involving photon–photon scattering.

In this paper, we investigate the propagation of extraordinary and upper-hybrid waves in dense magnetoplasmas composed of immobile ions and electrons, taking into account QED vacuum polarization, as well as the Bohm potential, arbitrary relativistic degeneracy pressure and spin magnetization energy due to the electron-1/2 spin effect. As far as we know, only Shukla & Stenflo (2008) have previously studied the dispersion relationship of electromagnetic wave propagation in a dense magnetized plasma, in which the spin effect and the QED vacuum polarization effect were considered. However, in their theoretical model, the spin effect is only considered to modify the electron momentum equation, and it is not considered to modify Maxwell's equations. The theoretical model used here is composed of the electron momentum equation, which includes Bohm potential, arbitrary relativistic degeneracy pressure and spin force, and Maxwell's equations modified by the spin current and vacuum polarization current. This paper is organized as follows. In § 2, the quantum magnetohydrodynamics model composed of the continuity equation and the momentum equation, including Bohm potential, arbitrary relativistic degeneracy pressure and spin force, is presented, and Maxwell's equations modified by the spin current and vacuum polarization current are also provided. In § 3, starting from the quantum magnetohydrodynamics model and the Poisson equation, the dispersion relation of the upper-hybrid wave is deduced. In § 4, based on the spin quantum magnetohydrodynamics model and the Maxwell's equations, the dispersion relation of an extraordinary wave is derived. In § 5, the contributions of quantum effects and the vacuum polarization effect are quantitatively calculated and discussed with the real parameters of dense astrophysical plasmas.

2. Basic equations

In this paper, we consider a zero-temperature plasma composed of ions and electrons. Since the mass of ions is much more than that of electrons, ions are treated as a stationary neutralizing background, and only the motion of electrons is considered. The quantum magnetohydrodynamics model for spin-1/2 electrons is composed of the continuity equation:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\boldsymbol{u}) = 0, \tag{2.1}$$

and the momentum equation (Brodin & Marklund 2007; Marklund & Brodin 2007):

$$mn\left(\frac{\partial}{\partial t} + \boldsymbol{u}\cdot\nabla\right)\boldsymbol{u} = -en\left(\boldsymbol{E} + \frac{\boldsymbol{u}}{c}\times\boldsymbol{B}\right) - \nabla P + \frac{\hbar^2 n}{2m}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) + \frac{2n\mu_B}{\hbar}\nabla(\boldsymbol{S}\cdot\boldsymbol{B}), \tag{2.2}$$

where n is the number density of electrons, $\boldsymbol{u}$ is the fluid velocity of electrons, e is the charge of the electron, m is the electron mass, $\mu_B = e\hbar/2mc$ is the Bohr magneton and $\hbar$ is Planck's constant divided by 2π. Here, P denotes the relativistic electron degeneracy pressure in dense plasmas, which can be written as (Shukla & Eliasson 2011)

$$P = \frac{\pi m_e^4 c^5}{3h^3} f(\xi), \tag{2.3}$$

where $f(\xi) = \xi(2\xi^2 - 3)(1+\xi^2)^{1/2} + 3\sinh^{-1}(\xi)$, $\xi = p/mc$ and $p = (3\pi^2 n)^{1/3}\hbar$ is the momentum of an electron on the Fermi surface. Expanding (2.3) around the unperturbed density of electrons n_0 by the Taylor series expansion and neglecting the higher order terms, we have (Maroof *et al.* 2015; El-Shamy 2015)

$$P = P_0 + \frac{m v_{Fe}^2}{3\gamma_0} n_1, \tag{2.4}$$

where n_1 denotes the perturbed electron number density, $v_{Fe} = (3\pi^2 n_0)^{1/3}\hbar/m$ is the Fermi velocity of electrons, and $\gamma_0 = 1/\sqrt{1-\xi_0^2}$ with $\xi_0 = p_0/mc$, where $p_0 = (3\pi^2 n_0)^{1/3}\hbar$ is the Fermi momentum of electrons.

Neglecting the spin–thermal coupling terms and the nonlinear spin fluid contribution, the spin vector $\boldsymbol{S}$ in (2.2) satisfies the evolution equation

$$\frac{\mathrm{d}\boldsymbol{S}}{\mathrm{d}t} = \frac{2\mu_B}{\hbar}\boldsymbol{B}\times\boldsymbol{S}. \tag{2.5}$$

Neglecting the spin inertia, the spin vector is determined from $\boldsymbol{B}\times\boldsymbol{S} = 0$, which has a solution

$$\boldsymbol{S} = -\frac{\hbar}{2}\eta\left(\frac{\mu_B B}{k_B T_{Fe}}\right)\hat{\boldsymbol{B}}, \tag{2.6}$$

where B denotes the magnitude of the magnetic field, and $\hat{\boldsymbol{B}}$ is a unit vector in the direction of the magnetic field. Here, $\eta(\alpha) = \tanh\alpha$ is the Brillouin function, where $\alpha = \mu_B B_0/(k_B T_F)$.

The QED vacuum polarization effect is formulated by the Heisenberg–Euler Lagrangian density of electromagnetic fields, which is expressed as (Heisenberg & Euler 1936; Marklund & Shukla 2006)

$$\mathcal{L} = \frac{1}{8\pi}(E^2 - B^2) + \frac{\xi}{8\pi}[(E^2 - B^2) + (\boldsymbol{E}\cdot\boldsymbol{B})^2], \tag{2.7}$$

where $\xi = \hbar e^4/(45\pi m^4 c^7)$, and c is the speed of light in vacuum. The first term of (2.7) is the classical Lagrangian density, and the second term is the correction term originating from the vacuum polarization effect. The effective polarization and magnetization vectors of the vacuum derived from Lagrangian density can be expressed as (Shen, Yu & Wang 2003; Lundin *et al.* 2007)

$$\boldsymbol{P} = \frac{\xi}{4\pi}[2(E^2 - B^2)\boldsymbol{E} + 7(\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{B}], \tag{2.8}$$

and

$$\boldsymbol{M} = \frac{\xi}{4\pi}[-2(E^2 - B^2)\boldsymbol{B} + 7(\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{E}]. \tag{2.9}$$

Assuming that the amplitude of oscillation is small, we can solve the system by using linearized equations. The plasma equilibrium is assumed as $\boldsymbol{E}_0 = 0$, $\boldsymbol{u}_0 = 0$, therefore, the linearized continuity equation is derived as

$$\frac{\partial n_1}{\partial t} + n_0\nabla\cdot\boldsymbol{u}_1 = 0, \tag{2.10}$$

and the linearized momentum equation is obtained as

$$\frac{\partial \boldsymbol{u}_1}{\partial t} = -\frac{e}{m}\left(\boldsymbol{E}_1 + \frac{\boldsymbol{u}_1}{c}\times\boldsymbol{B}_0\right) - \frac{v_{Fe}^2}{3n_0\gamma_0}\nabla n_1 + \frac{\hbar^2}{4m^2 n_0}\nabla\nabla^2 n_1 + \frac{2\mu_B}{m\hbar}\nabla(\boldsymbol{S}\cdot\boldsymbol{B}_1). \tag{2.11}$$

The linearized Maxwell equations modified by the vacuum polarization effect and spin effect can be presented as

$$\nabla\times\boldsymbol{E}_1 = -\frac{1}{c}\frac{\partial\boldsymbol{B}_1}{\partial t}, \tag{2.12}$$

$$\nabla\times\boldsymbol{B}_1 = \frac{1}{c}\frac{\partial\boldsymbol{E}_1}{\partial t} + \frac{4\pi}{c}(\boldsymbol{J}_e + \boldsymbol{J}_M + \boldsymbol{J}_{\text{vac}}), \tag{2.13}$$

$$\nabla\cdot\boldsymbol{E}_1 = 4\pi(\rho_e + \rho_{\text{vac}}), \tag{2.14}$$

where $\rho_e = -en_1$ and $\boldsymbol{J}_e = -en_0\boldsymbol{u}_1$ are the charge and current density of electrons, respectively. Then $\boldsymbol{J}_M = -c\nabla\times(2n_0\mu_B\boldsymbol{S}/\hbar)$ is the magnetization spin current density, $\rho_{\text{vac}} = -\nabla\cdot\boldsymbol{P}$ and $\boldsymbol{J}_{\text{vac}} = \partial\boldsymbol{P}/\partial t + c\nabla\times\boldsymbol{M}$ are the effective vacuum charge and current density, respectively.

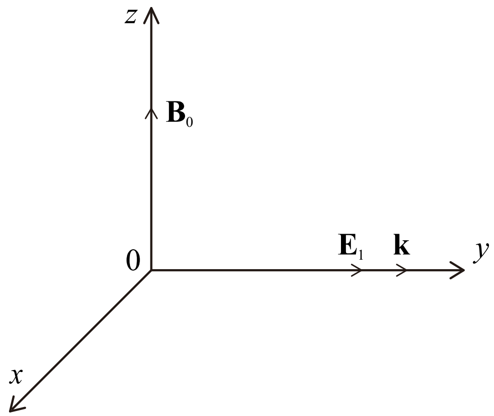

FIGURE 1. Cartesian coordinate system, chosen such that $\boldsymbol{B}_0$ is along $\hat{z}$ and $\boldsymbol{E}_1$ is along $\hat{y}$.

3. Dispersion relation of upper-hybrid wave

We choose the external magnetic field as $\boldsymbol{B}_0 = B_0\hat{z}$ with respect to the propagation direction determined by the wavenumber $\boldsymbol{k} = k\hat{y}$ of the wave. Since the upper-hybrid wave is an electrostatic wave, the first-order electromagnetic fields are chosen as $\boldsymbol{E}_1 = E_1\hat{y}$ and $\boldsymbol{B}_1 = 0$, as shown in figure 1. It should be noted that E^2 as well as B^2 in (2.8) are constant and that $\boldsymbol{E}\cdot\boldsymbol{B} = 0$. The effective vacuum charge density in (2.14) can be written as

$$\rho_{vac} = -\frac{\xi}{2\pi}(E^2 - B^2)\nabla\cdot\boldsymbol{E}_1. \tag{3.1}$$

Inserting (3.1) into (2.14), we derive a new Poisson equation modified by the vacuum polarization correction as

$$\nabla\cdot\boldsymbol{E}_1 = 4\pi\rho_e(1-\beta)^{-1}, \tag{3.2}$$

where

$$\beta = 2\xi(B_0^2 - E_1^2) \sim \frac{2\alpha B_0^2}{45\pi B_c^2}. \tag{3.3}$$

Here $E_1 \ll B_0$ is the amplitude of the firs-order electric field, $\alpha = e^2/\hbar c$ is the fine structure constant, and $B_c = m^2c^3/\hbar e \sim 4.44\times 10^{13}$Gs is the Schwinger critical magnetic field.

Supposing the perturbations are proportional to $\exp[\mathrm{i}(ky-\omega t)]$, (2.10) and (2.11) become

$$-\mathrm{i}\omega\boldsymbol{u}_1 = -\frac{e}{m}\left(\boldsymbol{E}_1 + \frac{\boldsymbol{u}_1}{c}\times\boldsymbol{B}_0\right) - \frac{\mathrm{i}kv_{Fe}^2 n_1}{3\gamma_0 n_0}\hat{y} - \frac{\mathrm{i}\hbar^2k^3n_1}{4m^2n_0}\hat{y} \tag{3.4}$$

and

$$n_1 = \frac{ku_{1y}}{\omega}n_0. \tag{3.5}$$

By solving (3.4) and (3.5), we have the component u_{1y} of the fluid velocity as

$$u_{1y} = -\frac{\mathrm{i}eE_1}{\omega m}\left(1 - \frac{\omega_c^2}{\omega^2} - \varDelta\right)^{-1}, \tag{3.6}$$

where $\omega_c = eB_0/mc$ is the electron cyclotron frequency and $\varDelta = k^2v_{Fe}^2/3\gamma_0\omega^2 + \hbar^2k^4/4m^2\omega^2$ is the quantum correction.

https://doi.org/10.1017/S0022377821000738 Published online by Cambridge University Press

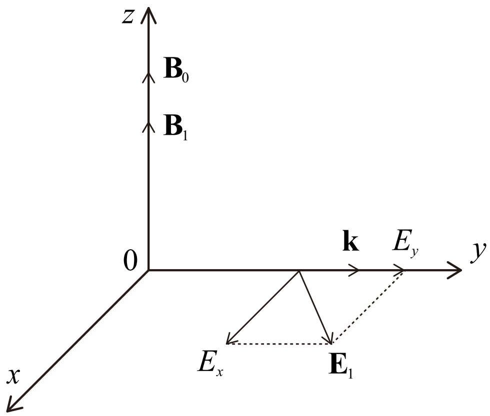

FIGURE 2. Cartesian coordinate system, chosen such that $\boldsymbol{B}_0$ and $\boldsymbol{B}_1$ are along $\hat{z}$, and $\boldsymbol{E}_1$ is in the plane of xoy.

Inserting (3.6) into (3.2), we derive the dispersion relation of upper-hybrid wave as

$$\omega_h^2 = \frac{\omega_p^2}{1-\beta} + \omega_c^2 + \frac{k^2 v_{Fe}^2}{3\gamma_0} + \frac{\hbar^2 k^4}{4m^2}. \tag{3.7}$$

When setting $\hbar \to 0$ and $\beta \to 0$, (3.7) can be degenerated to the frequency of upper-hybrid oscillation in the classical cold plasmas, and it indicates that an upper-hybrid oscillation can propagate in cold plasmas due to quantum effects.

4. Dispersion relation of an extraordinary wave

We choose the external magnetic field as $\boldsymbol{B}_0 = B_0\hat{z}$ with respect to the propagation direction determined by the wavenumber $\boldsymbol{k} = k\hat{y}$ of the wave. Since we investigate the propagation of an extraordinary wave, the first-order electromagnetic fields are particularly chosen as $\boldsymbol{E}_1 = E_{1x}\hat{x} + E_{1y}\hat{y}$ and $\boldsymbol{B}_1 = B_1\hat{z} = -kcE_{1x}/\omega\hat{z}$, as shown in figure 2. It should be noted that E^2 as well as B^2 are constant and that $\boldsymbol{E}\cdot\boldsymbol{B} = 0$. This means that

$$\boldsymbol{J}_{\text{vac}} = -\frac{\xi}{2\pi}(E^2 - B^2)\left(\nabla \times \boldsymbol{B}_1 - \frac{1}{c}\frac{\partial \boldsymbol{E}_1}{\partial t}\right). \tag{4.1}$$

Inserting (4.1) into (2.13) we derive a new equation

$$\nabla \times \boldsymbol{B}_1 = \frac{1}{c}\frac{\partial \boldsymbol{E}_1}{\partial t} + \frac{4\pi}{c}(\boldsymbol{J}_e + \boldsymbol{J}_M)(1-\chi)^{-1}, \tag{4.2}$$

where

$$\chi = \frac{2\alpha}{45\pi B_c^2}[(n^2-1)E_1^2 + B_0^2]. \tag{4.3}$$

Here $n = kc/\omega$ is the index of refraction and E_1 is the amplitude of the first-order electric field.

The spin magnetization current density in (2.13) is calculated as

$$\boldsymbol{J}_M = \frac{\mathrm{i}\mu_B\eta(\alpha)n_0k^2v_{1y}}{\omega}\hat{x}. \tag{4.4}$$

Supposing the perturbations are proportional to $\exp[\mathrm{i}(ky-\omega t)]$, (2.10) and (2.11) become

$$-\mathrm{i}\omega\boldsymbol{u}_1 = -\frac{e}{m}\left(\boldsymbol{E}_1 + \frac{\boldsymbol{u}_1}{c}\times\boldsymbol{B}_0\right) - \frac{\mathrm{i}kv_{Fe}^2n_1}{3\gamma_0n_0}\hat{y} - \frac{\mathrm{i}\hbar^2k^3n_1}{4m^2n_0}\hat{y} + \frac{\mathrm{i}\mu_B\eta(\alpha)k^2cE_{1x}}{m\omega}\hat{y} \tag{4.5}$$

and

$$n_1 = \frac{ku_{1y}}{\omega}n_0. \tag{4.6}$$

By solving (4.5) and (4.6), we have the two components u_{1x} and u_{1y} of the fluid velocity as

$$\left.\begin{aligned} u_{1x} &= \frac{e}{\omega m}\left\{-iE_{1x} - \left[\frac{i\omega_c^2}{\omega^2}\left(1-\frac{\omega}{\omega_c}S\right)E_{1x} + \frac{\omega_c}{\omega}E_{1y}\right]\left(1-\frac{\omega_c^2}{\omega^2}-\Delta\right)^{-1}\right\},\\ u_{1y} &= \frac{e}{\omega m}\left[\frac{\omega_c}{\omega}\left(1-\frac{\omega}{\omega_c}S\right)E_{1x} - iE_{1y}\right]\left(1-\frac{\omega_c^2}{\omega^2}-\Delta\right)^{-1}, \end{aligned}\right\} \tag{4.7}$$

where $S = (\mu_B\eta(\alpha)k^2c)/e\omega$ is the spin correction term.

From the linearized Maxwell's equations (2.12) and (4.2), we have

$$(\omega^2 - k^2c^2)E_{1x} = -4\pi\mathrm{i}\omega(J_{ey} + J_M)(1-\chi)^{-1}, \tag{4.8}$$

and

$$\omega^2E_{1y} = -4\pi\mathrm{i}\omega J_{ex}(1-\chi)^{-1}, \tag{4.9}$$

where $J_{ex} = -en_0u_{1x}$ and $J_{ey} = -en_0u_{1y}$ are the two components of electron current density $\boldsymbol{J}_e$, and $J_M = (\mathrm{i}\mu_B\eta(\alpha)n_0k^2v_{1y})/\omega$ is the magnitude of the spin magnetization current density.

According to (4.8) and (4.9), the dispersion equation can be obtained as

$$\begin{vmatrix} -\mathrm{i}\left(\frac{\omega_c}{\omega}-S\right)\omega_p^2 & \omega^2\left(1-\frac{\omega_c^2}{\omega^2}-\varDelta\right)(1-\chi)-\omega_p^2 \\ (\omega^2-k^2c^2)\left(1-\frac{\omega_c^2}{\omega^2}-\varDelta\right)(1-\chi)-\omega_p^2\left(1-\varDelta-S^2\right) & \mathrm{i}\left(\frac{\omega_c}{\omega}+S\right)\omega_p^2 \end{vmatrix} = 0, \tag{4.10}$$

where $\omega_p^2 = 4\pi n_0e^2/m$ is the plasma frequency.

Solving (4.10), the dispersion relation of the extraordinary wave in spin quantum magnetoplasmas with the vacuum polarization effect is derived as

$$\frac{k^2c^2}{\omega^2} = 1 - \frac{\varOmega_p^2}{\omega^2}\frac{\omega^2(1-\varDelta-S^2)-\varOmega_p^2}{\omega^2-\tilde{\omega}_h^2}, \tag{4.11}$$

where

$$\tilde{\omega}_h^2 = \varOmega_p^2 + \omega_c^2 + \frac{k^2v_{Fe}^2}{3\gamma_0} + \frac{\hbar^2k^4}{4m^2} \tag{4.12}$$

is the dispersion relation of an upper-hybrid wave, and $\varOmega_p^2 = \omega_p^2/(1-\chi)$.

https://doi.org/10.1017/S0022377821000738 Published online by Cambridge University Press

In the absence of quantum effects and the vacuum polarization effect we have $\Delta = 0$, $S = 0$ and $\chi = 0$, so (4.11) and (4.12) will reproduce to the dispersion relation of an extraordinary wave and the frequency of upper-hybrid oscillation in the classical cold plasmas.

5. Discussion and conclusion

Here, we adopt the typical parameters of a dense astrophysical object, such as a pulsar magnetosphere, for quantitative calculation, where the plasma parameters are chosen as $n_0 = 10^{29}\ \mathrm{cm}^{-3}$, $B_0 = 10^{13\sim 14}$ Gs and $T \sim 10^9$ K (Harding & Lai 2006).

It is well known that when the de Broglie wavelength λ_B of electrons becomes comparable to, or even larger than, the average interparticle distance of electrons (*viz.*, $\lambda_B^3 n_0 \geq 1$), the quantum effects will play a crucial role in plasma dynamics. From the expression $\lambda_B^3 n_0 \geq 1$, we have

$$\frac{n_0}{T^{3/2}} \geq 10^{16}\ \mathrm{cm}^{-3}/\mathrm{K}^{3/2}. \tag{5.1}$$

Obviously, the parameters of a pulsar magnetosphere satisfy the above quantum condition. Therefore, the Bohm potential and arbitrary relativistic degeneracy pressure should be considered.

Due to the complexity of the spin dynamics, it is difficult to give simple conditions when the spin effect is important. However, a few simple rules of thumb can be given: the spin effect is important if the energy difference between the two spin states is larger than the thermal energy or Fermi energy (*viz.*, $\mu_B B_0/K_B T \geq 1$ or $\mu_B B_0/K_B T_{Fe} \geq 1$). Calculating with the parameters of a pulsar magnetosphere we have

$$\mu_B B_0 > K_B T_{Fe} > K_B T. \tag{5.2}$$

Obviously, the energy difference between the two spin states is larger than the Fermi energy and thermal energy in the pulsar magnetosphere, and the spin effect should also be considered.

Equations (3.7) and (4.11) indicate that the contribution of vacuum polarization to the dispersion relation of linear waves is mainly reflected in the correction of the plasma frequency with the factor of β, and figure 3 shows that β can reach 10^{-3} with the super strong magnetic field $B_0 = 1.4 \times 10^{14}$ Gs.

Figure 4 shows the dispersion relation curves of extraordinary waves in classical plasmas, quantum plasmas and spin quantum plasmas, where the plasma parameters are chosen as $n_0 = 10^{29}\ \mathrm{cm}^{-3}$ and $B_0 = 10^{13}$Gs. In figure 4(*a*), the three dispersion relation curves almost coincide and are indistinguishable, because the value range of the wave vector is relatively large ($k = 10^2 \sim 10^{10}\ \mathrm{cm}^{-1}$). If the wave vector is restricted to the relatively high range $k \sim 10^{10}\ \mathrm{cm}^{-1}$, as shown in figure 4(*b*), it shows that the dispersion curve of extraordinary waves is significantly modified by the quantum effects and the spin effects. Meanwhile, if the wave vector is restricted to the relatively low range $k \sim 10^5\ \mathrm{cm}^{-1}$, as shown in figure 4(*c*), the dispersion curve of extraordinary waves is significantly modified by the quantum effects (Bohm potential and arbitrary relativistic degeneracy pressure), but the contribution of the spin effects is not obvious. Therefore, it can be concluded that in the low-frequency range, the correction to the dispersion relation of extraordinary waves produced by spin effects can be ignored, but in the high-frequency range, the correction is more obvious.

In summary, we present a theoretical investigation on the propagation of extraordinary and upper-hybrid waves in spin quantum magnetoplasmas with vacuum polarization effect.

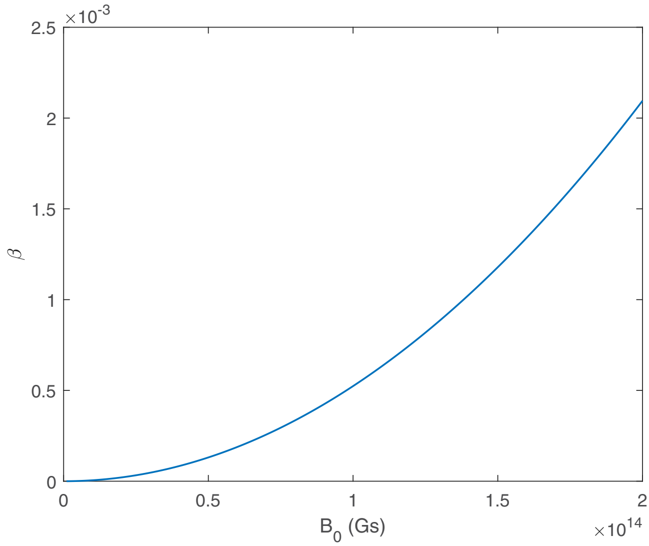

FIGURE 3. The modification of vacuum polarization effects to the electron plasma frequency.

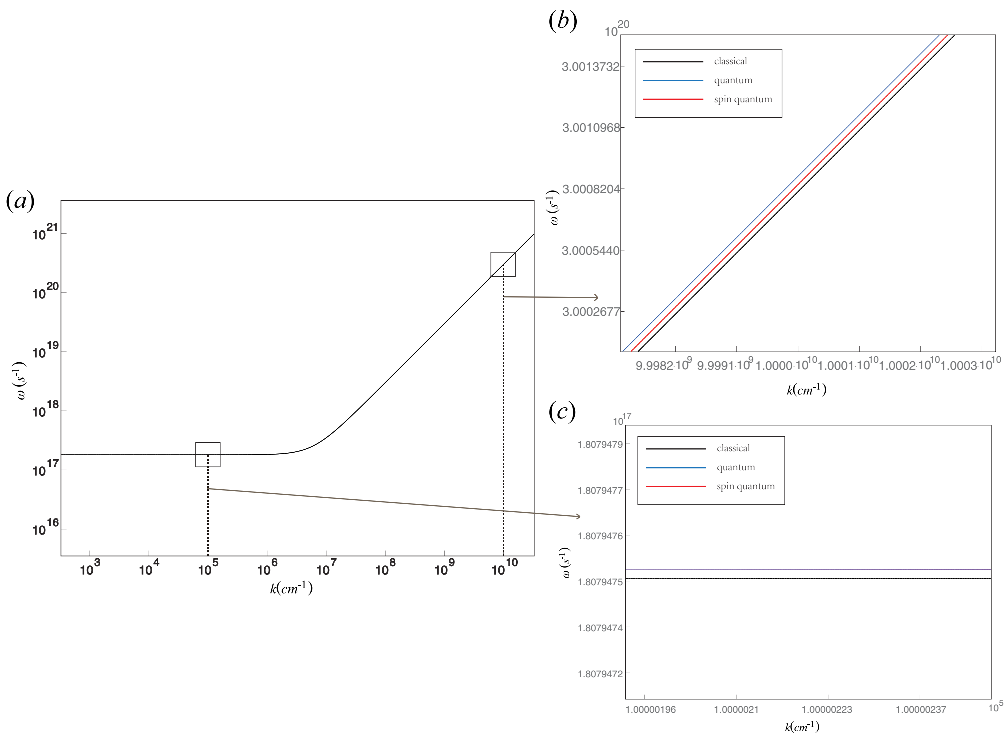

FIGURE 4. (*a*) The dispersion relation curves of extraordinary waves in classical plasmas, quantum plasmas and spin quantum plasmas, where the value of k ranges from 10^2 to 10^{11}. (*b*) The dispersion relation curves with a k value of approximately 10^{10}. (*c*) The dispersion relation curves with k value of approximately 10^5. The plasma parameters: $n_0 = 10^{29}\ \text{cm}^{-3}$, $B_0 = 10^{13}$ Gs.

https://doi.org/10.1017/S0022377821000738 Published online by Cambridge University Press

Based on the quantum magnetohydrodynamics model including Bohm potential, arbitrary relativistic degeneracy pressure and spin force, and Maxwell's equations modified by the spin current and vacuum polarization current, the dispersion relations of an extraordinary wave and upper hybrid wave are derived. The analytical and numerical results show that quantum effects (Bohm potential, arbitrary relativistic degeneracy pressure and spin magnetization energy) and the vacuum polarization effect should be considered in the dense astrophysical objects, such as a pulsar magnetosphere, as they significantly modify the propagation of an extraordinary wave under certain circumstances. Under the action of a strong magnetic field, the plasma frequency is obviously increased by the vacuum polarization effect. This theoretical research may be useful for understanding the propagation properties of the high-frequency waves in dense astrophysical objects, and also provides important reference for the experimental study on the intense laser–solid density plasma interaction.

Funding

This work is supported by the National Natural Science Foundation of China (Grant No 11705110 and Grant No 11665009) and the Scientific and Technological Innovation Programs of Higher Education Institutions (Grant No 2016103).

Editor Roger Blandford thanks the referees for their advice in evaluating this article.

Declaration of interests

The authors report no conflict of interest.

REFERENCES

ANDREEV, P. A. 2015 Separated spin-up and spin-down quantum hydrodynamics of degenerated electrons: spin-electron acoustic wave appearance. *Phys. Rev.* E **91** (3), 033111.

ANDREEV, P. A. 2017 Extraordinary spin-electron acoustic wave. *Phys. Plasmas* **24** (2), 022123.

BOHM, D. 1952 A suggested interpretation of the quantum theory in term of hidden variables. *Phys. Rev.* **85** (2), 166–193.

BRODIN, G. & MARKLUND, M. 2007 Spin magnetohydrodynamics. *New J. Phys.* **9**, 277.

BRODIN, G., MARKLUND, M., ELIASSON, B. & SHUKLA, P. K. 2007 Quantum-electrodynamical photon splitting in magnetized nonlinear pair plasmas. *Phys. Rev. Lett.* **98** (12), 125001.

EL-SHAMY, E. F. 2015 Nonlinear ion-acoustic cnoidal waves in a dense relativistic degenerate magnetoplasma. *Phys. Rev.* E **91** (3), 033105.

GEDALIN, M., MEROSE, D. B. & GRUMAN, E. 1998 Long waves in a relativistic pair in a strong magnetic field. *Phys. Rev.* E **57** (3), 3399–3410.

GOLDREICH, P. & JULIAN, W. H. 1969 Pulsar electrodynamics. *Astrophys. J.* **157**, 868–880.

HAAS, F. 2011 An introduction to quantum plasmas. *Braz. J. Phys.* **41**, 349–363.

HAAS, F., MANFREDI, G. & FEIX, M. 2000 Multistream model for quantum plasmas. *Phys. Rev.* E **62** (2), 2763–2772.

HARDING, A. K. & LAI, D. 2006 Physics of strongly magnetized neutron stars. *Rep. Prog. Phys.* **69**, 2631–2708.

HEISENBERG, W. & EULER, H. 1936 Folgerungen aus der Diracschen Theorie des Positrons. *Z. Phys.* **98** (11), 714–732.

IQBAL, Z., AYUB, M., SHAH, H. A. & MURTAZA, G. 2019*a* Energy behavior of spin electron cyclotron wave in a spin polarized plasma. *Phys. Lett.* A **383**, 2903–2907.

IQBAL, Z., KHAN, I. A. & MURTAZA, G. 2018*a* On the upper hybrid wave instability in a spin polarized degenerate plasma. *Phys. Plasmas* **25**, 062121.

IQBAL, Z., KHANUM, U. & MURTAZA, G. 2018*b* Lower hybrid wave instability in a spin–polarized degenerate plasma. *Contrib. Plasma Phys.* **59** (3), 284–291.

IQBAL, Z., YOUNAS, M., KHAN, I. A. & MURTAZA, G. 2019*b* Spin magnetoacoustic wave. *Phys. Plasmas* **26**, 112101.

LUNDIN, J., STENFLO, L., BRODIN, G., MARKLUND, M. & SHUKLA, P. K. 2007 Circularly polarized waves in a plasma with vacuum polarization effects. *Phys. Plasmas* **14** (6), 064503.

MANFREDI, G. 2005 How to model quantum plasmas. Fields Institute Communications 46.

MARKLUND, M. & BRODIN, G. 2007 Dynamics of spin-$\frac{1}{2}$ quantum plasmas. *Phys. Rev. Lett*. **98** (2), 025001.

MARKLUND, M. & SHUKLA, P. K. 2006 Nonlinear collective effects in photon-photon and photon-plasma interactions. *Rev. Mod. Phys*. **78** (2), 591–640.

MARKOWICH, P. A., RINGHOFER, C. & SCHMEISER, C. 1990 *Semiconductor Equations*. Springer.

MAROOF, R., ALI, S., MUSHTAQ, A. & QAMAR, A. 2015 Magnetohydrodynamic waves with relativistic electrons and positrons in degenerate spin-1/2 astrophysical plasmas. *Phys. Plasmas* **22** (11), 112102.

MUSHTAQ, A., MAROOF, R., AHMAD, Z. & QAMAR, A. 2012 Magnetohydrodynamic spin waves in degenerate electron-positron-ion plasmas. *Phys. Plasmas* **19** (5), 052101.

SHEN, B., YU, M. Y. & WANG, X. 2003 Photon–photon scattering in a plasma channel. *Phys. Plasmas* **10** (11), 4570–4571.

SHUKLA, P. K. 2006 A new dust mode in quantum plasmas. *Phys. Lett.* A **352**, 242–243.

SHUKLA, P. K. & ELIASSON, B. 2006 Formation and dynamics of dark solitons and vortices in quantum electron plasmas. *Phys. Rev. Lett*. **96** (24), 245001.

SHUKLA, P. K. & ELIASSON, B. 2011 Colloquium: nonlinear collective interactions in quantum plasmas with degenerate electron fluids. *Rev. Mod. Phys*. **83** (3), 885–906.

SHUKLA, P. K. & STENFLO, L. 2008 Dispersion relations for electromagnetic waves in a dense magnetized plasma. *J. Plasma Phys*. **74** (6), 719–723.

STENFLO, L., BRODIN, G., MARKLUND, M. & SHUKLA, P. K. 2005 A new electromagnetic wave in a pair plasma. *J. Plasma Phys*. **71** (5), 709–713.