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Nonlinear propagation of ion acoustic waves in quantum plasma in the presence of an ion beam

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Research Article

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Abstract

Nonlinear propagation of ion acoustic waves has been studied in unmagnetized quantum (degenerate) plasma in the presence of an ion beam using the one-dimensional quantum hydrodynamic model. The Korteweg–de Vries (K–dV) equation has been derived by using the reductive perturbation technique. The solution of ion acoustic solitary waves is obtained from the K–dV equation. The theoretical results have been analyzed numerically for different values of plasma parameters and the results are presented graphically. It is seen that the formation and structure of solitary waves are significantly affected by the ion beam in quantum plasma. The solitary waves will be compressive or rarefactive depending upon the values of velocity, concentration, and temperature of the ion beam. The critical value of ion beam density for the nonexistence of solitary wave has been numerically estimated, and its variation with velocity and temperature of ion beam has been discussed graphically. The results are new and would be very useful for understanding the beam–plasma interactions and the formation of nonlinear wave structures in dense quantum plasma.

Introduction

The most general class of nonlinear wave structures is solitary waves or soliton having approximately self-consistent wave amplitudes and phases. Solitons are important and extensively studied because a whole class of nonlinear differential equations encountered in plasma physics, solid-state physics, particle physics, hydrodynamics, nonlinear optics, and biology are seen to support the solutions of solitary waves. In the last few years, solitary waves in plasma have been studied theoretically and experimentally by various authors. Washimi and Taniuti (1966) first theoretically studied ion acoustic solitary waves (IASWs) in cold collisionless unmagnetized plasma deriving the K–dV equation using the reductive perturbation technique. The solitary wave in plasma was observed experimentally by Ikezi (1973), Lonngren (1983) and others. A lot of theoretical works on the nonlinear propagation of ion acoustic (IA) waves have been done by various authors incorporating different parameters in the plasma example, ion temperature (Tagare, 1973), two-temperature electrons (Ghosh *et al*., 2008), resonant electrons (Schamel, 1973), gravitation (Paul *et al*., 2017), negative ion (Chattopadhyaya and Paul, 2012), positrons (Paul *et al*., 2012), nonthermal electrons (Gill *et al*., 2004), kappa-distributed electron (Baluku and Hellberg, 2012), Tsallis-distributed electrons (Bala *et al*., 2017), etc. and have shown that these plasma parameters have considerable impact on the excitation of solitary waves. However, in recent years, there is considerable interest in the nonlinear propagation of waves in a plasma consisting of electron/ion beams. The studies of the propagation of waves in beam plasma are important in magnetospheric and solar physics (Goldman, 1983; Hoffmann and Evans, 1968). The nonlinear structure of a plasma may change considerably in the presence of electron/ion beams. An important property of an electron beam plasma is that it can change the propagation characteristic of the Trivelpiece–Gould (TG) solitons (Krivoruchko *et al*., 1975). The effects of an ion beam on the motion of solitons in an ion beam-plasma system has been studied by Gell and Roth (1977). The effects of ion beams are more important on the excitation of ion acoustic solitary waves in plasma. Abrol and Tagare (1979) obtained a modified K–dV equation for an IASW in an ion–beam–plasma system with cold ions, beam ions, and nonisothermal electrons. They investigated the effect of the resonant electrons, both trapped and free electrons, of the plasma–ion to beam–ion mass ratio and of the beam–ion concentration on the amplitude and width of the solitary wave. Later, many authors (Karmakar *et al*., 1988; Das and Singh, 1991; El-Labany, 1995; Huibin and Kelin, 2009; Zank and McKenzie, 1998; Das *et al*. 2011; Misra and Adhikary, 2011; Das, 2012; Das and Deka, 2015) have studied solitary waves in ion beam plasma and have obtained some fascinating results which are important in different situations in laboratory and space plasma. Recently, Kaur *et al*. (2017) have investigated the nonlinear propagation of ion acoustic solitary waves (IASWs) in an unmagnetized plasma composed of a positive warm ion fluid, two-temperature electrons obeying kappa-type distribution, and penetrated by a positive ion beam. They have used the reductive perturbation method to derive the nonlinear equations, namely K–dV, modified K–dV (mK–dV), and Gardner equations. The characteristic features of both compressive and rarefactive nonlinear excitations from the solution of these equations are studied and compared in the context with the observation of the He^+ beam in the polar cap region near solar maximum by the Dynamics Explorer 1 satellite. It is observed that the superthermality and density of cold electrons, number density, and temperature of the positive ion beam crucially modify the basic properties of compressive and rarefactive IASWs in the K–dV and mK–dV regimes. It is further analyzed that the amplitude and width of Gardner solitons are appreciably affected by different plasma parameters. The characteristics of double layers (DLs) are also studied in detail below the critical density of cold electrons. The theoretical results may be useful for the observation of nonlinear excitations in laboratory and ion beam-driven plasmas in the polar cap region near solar maximum and polar ionosphere as well in Saturn's magnetosphere, solar wind, pulsar magnetosphere, etc., where the population of two-temperature superthermal electrons is present. More recently, Kaur *et al.* (2018) have investigated the propagation characteristics of dust acoustic solitary and rogue waves in an unmagnetized ion beam plasma with electrons and ions following the kappa-type distribution in nonplanar geometry. The reductive perturbation method is employed to derive the cylindrical/spherical K–dV equation, which is further transformed into a standard K–dV equation by neglecting the geometrical effects. Using new stretching coordinates, the nonlinear Schrödinger equation has also been derived from the standard K–dV equation to study the different order rational solutions of dust acoustic rogue waves. The impact of various physical parameters on the characteristics of dust acoustic solitary waves is elaborated specifically in nonplanar geometry. Further, the effects of ion beam and superthermality of electrons/ions on the characteristics of rogue waves are studied. It is predicted that the results obtained may be useful in comprehending a variety of phenomena in Earth's magnetosphere polar cap region where the presence of positive ion beam has been detected and also in other regions of space/astrophysical environments where dust along with superthermal electrons and ions exists.

However, in recent years, researchers are very much interested to study the propagation of waves in a quantum plasma because quantum effects become important in a variety of environments, such as in intense laser–solid density plasma interaction experiments, dense astrophysical and cosmological environments, ultrasmall electronic devices, and metal nanostructures. The recent developments in nanoscience (Manfredi *et al.*, 2009), biophotonics (Barnes *et al.*, 2003), fiber optics (Agrawal, 1989), quantum confinement (Ang *et al.*, 2003), ultracold plasmas (Killian, 2007), etc., the study has gained momentum in a totally new direction. The quantum effects are also important in laboratory-produced ultra-dense plasmas (Kremp *et al.*, 1999) and laser-based inertial fusion experiments (Azechi, 2006) as well as the interior of Jovian planets, neutron stars (Shapiro and Teukolsky, 1983), and white dwarfs (Madelung, 1927), etc., where the density is very high. Quantum plasmas show numerous nonlinearities which are absent in classical plasmas. This has led to the investigation of waves in quantum plasma more important than ever. There have been many modes of waves in plasmas like ion acoustic waves (IAWs), electron acoustic waves, dust acoustic waves, dust ion acoustic waves, electron plasma waves, and shock waves. Using the Quantum Hydrodynamic (QHD) model, Haas *et al.* (2003) has studied the importance of the role of quantum diffraction in a linear and nonlinear regime. They adopted an equation of state pertaining to a zero-temperature Fermi gas for the electrons by disregarding pressure effects for the ions. By an appropriate rescaling of the variables, they identified a nondimensional parameter H, proportional to quantum diffraction effects. The system is shown to support linear waves, which, in the limit of small H, resemble the classical IAWs. In the weakly nonlinear limit, the quantum plasma is shown to support waves described by a deformed K–dV equation which depends in a nontrivial way on the quantum diffraction parameter H. In the fully nonlinear regime, the system also admits traveling waves which can exhibit periodic patterns. The works of Haas *et al.* (2003) have encouraged a large number of authors to study the nonlinear propagation of acoustic waves (e.g., solitary waves and modulation instability) in quantum plasma. We have here mentioned some of the important works on solitary waves in quantum plasma. Ali and Shukla (2006) have considered a one-dimensional QHD model for a three-species quantum plasma to derive the K–dV equation incorporating quantum corrections and studied the nonlinear properties of dust acoustic solitary waves. They examined the quantum mechanical effects numerically both on the profiles of the amplitude and the width of dust acoustic solitary waves. It is found that the amplitude remains constant but the width shrinks for different values of a dimensionless electron quantum diffraction parameter H. Subsequently, Ali *et al.* (2007) have investigated the linear and nonlinear properties of the IAWs by using the QHD equations together with the Poisson equation in a three-component quantum electron–positron–ion plasma. They have derived the linear dispersion relation, the K–dV equation, and an energy equation containing quantum corrections. They have also performed the computational investigations to examine the quantum mechanical effects on the linear and nonlinear waves. It is found that both the linear and nonlinear properties of the IAWs are significantly affected by the inclusion of the quantum corrections. Later, Misra and Bhowmik (2007) have considered a nonplanar spherical geometry to study the nonlinear properties of ion acoustic (IA) waves in an electron–ion quantum plasma with the effects of quantum corrections. Using the QHD model and standard reductive perturbation method, they derived the K–P equation having a variable coefficient and examined numerically the importance of quantum mechanical effects on the compressive and rarefactive solitons. It is found that H plays a significant role in the formation of compressive and rarefactive solitons. A critical value of H is also found which depends on the phase velocity of the wave and the ion to electron Fermi temperature ratio, for which the soliton formation ceases to exist. Subsequently, Mushtaq and Khan (2007) have studied ion acoustic solitary wave with weakly transverse perturbations in quantum electron–positron–ion plasma using the QHD model. They have derived the linear dispersion relation in the linear regime and the K–P equation in the nonlinear regime. It is found that compressive solitary wave can propagate in this system. The quantum effects are also studied graphically for both the linear and nonlinear profiles of ion acoustic wave. Using the energy consideration method, conditions for the existence of stable solitary waves are obtained. It is found that stable solitary waves depend on quantum corrections, positron concentration, and direction cosine of the wave vector.

Later, Sah and Manta (2009) have investigated the nonlinear wave structure of electro-acoustic waves (EAWs) in a three-component unmagnetized dense quantum plasma consisting of two distinct groups of electrons (one inertial cold electron and other inertialess hot electrons) and immobile ions. Using one-dimensional QHD and the standard reductive perturbation technique, they derived the K–dV equation governing the dynamics of EAWs. Both compressive and rarefactive solitons along with periodical potential structures are found to exist for various ranges of dimensionless quantum parameter H. The quantum mechanical effects are also examined numerically on the profiles of the amplitude and the width of electro-acoustic solitary waves. It is observed that both the amplitude and the width of electro-acoustic solitary waves are significantly affected by the parameter H. Akbari-Moghanjoughi (2010) has investigated large-amplitude IASW in a degenerate dense electron–positron–ion plasma considering the ion temperature as well as electron/positron degeneracy effects. It is shown that the ion temperature effects play an important role in the existence criteria and allowed a Mach-number range in such plasmas. He has also pointed out the fundamental difference in the existence of supersonic IASW propagations between degenerate plasmas with nonrelativistic and ultra-relativistic electrons and positrons. Chandra *et al.* (2012) have theoretically studied the linear and nonlinear propagation of electron plasma wave in a two-component unmagnetized dense quantum plasma with streaming of ions both analytically and numerically using one-dimensional QHD. It has been shown that the quantum effect modifies the linear dispersion character of the electron plasma waves in the presence of streaming motion and makes the possibility of two distinct modes. They have also derived the K–dV equation and have shown that both compressive and rarefactive solitary waves would be excited in the model plasma on some critical values of the quantum diffraction parameter. Dip *et al.* (2017) have investigated higher-order nonlinearity of the EAWs, specifically IA waves in an unmagnetized, collisionless, quantum electron–positron–ion plasma. They derived the mK–dV equation. The plasma system is supposed to be formed of positively charged inertial heavy ions, inertialess electrons and positrons to analyze the solitary waves (SWs), and the standard Gardner (SG) equation to analyze the higher-order SWs as well as DLs. The basic features (namely, amplitude, width, phase speed, etc.) of the IA SWs and DLs are examined. The comparison between the mK–dV SWs and SG SWs is also made. It is found that the amplitude, width, and phase speed of the IA SWs and DLs are significantly modified by the effects of both Fermi temperatures as well as pressures and Bohm potentials of electrons and positrons.

However, the propagation of ion acoustic and EAWs in quantum (degenerate) plasma having electron beam or ion beam would give fascinating results which are not studied yet critically, though many authors studied the wave propagation in nondegenerate/classical plasma (Yajima and Wadati, 1990; Mondal *et al.*, 1998; El-Labany, 1995; Zank and McKenzie, 1998; Bailung *et al.*, 2010; Saberian *et al.*, 2013; Danehkar, 2018). So, in the present paper, we are interested to study the nonlinear propagation of ion acoustic waves in a quantum plasma consisting of warm ion beams. Using the reductive perturbation method, we have derived the K–dV equation and have obtained the solution of ion acoustic solitary in the quantum plasma having nonrelativistic ion beam. The effects of ion beam parameters, that is ion beam density, ion beam temperature, and ion beam velocity, on the solitary waves are critically discussed with graphical representation. The quantum diffraction parameter H has a significant role in the width of solitary waves in the quantum plasma. For $H < 2$ and $H > 2$, the width may be increased or decreased depending upon the values of ion beam parameters.

Basic equations

We consider that the quantum plasma is unmagnetized and collissionless and it consists of positive ions, beam ions, and inertialess electrons. All species of the plasma are assumed to follow the Fermi–Dirac statistics. The positive ions and positive beam ions are considered to be singly ionized. Ion acoustic wave is a low-frequency wave, in which the restoring force comes from the pressure of inertialess electrons and the ion masses provide the driving force to maintain the wave. The electrons are considered inertialess by assuming that the phase velocity of ion acoustic wave is much less than the Fermi velocity of electron and much greater than the Fermi velocities of positive and beam ions. We assume that the plasma particles behave as a one-dimensional Fermi gas at zero temperature and therefore the pressure law is:

$$p_{\mathrm{j}} = \frac{m_{\mathrm{j}} V_{\mathrm{Fj}}^2}{3 n_{\mathrm{j0}}^2} n_{\mathrm{j}}^3 \tag{1}$$

where j = e for electrons, j = i for ions, and j = b for beam ions; m_{j} is the mass; $V_{\mathrm{Fj}} = \sqrt{2k_{\mathrm{B}} T_{\mathrm{Fj}}/m_{\mathrm{j}}}$ is the Fermi speed, T_{Fj} is the Fermi temperature, and k_{B} is the Boltzmann constant; n_{j} is the number density with the equilibrium value n_{j0}. $\omega_{\mathrm{pe}} = \sqrt{4\pi n_0 e^2/m_{\mathrm{e}}}$ is the electron plasma oscillation frequency and V_{Fe} is the Fermi thermal speed of electrons.

The normalized equations governing the plasma dynamics are:

$$0 = \frac{\partial \phi}{\partial x} - n_{\mathrm{e}} \frac{\partial n_{\mathrm{e}}}{\partial x} + \frac{H^2}{2} \frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_{\mathrm{e}}}} \frac{\partial^2 \sqrt{n_{\mathrm{e}}}}{\partial x^2}\right] \tag{2}$$

$$\frac{\partial n_{\mathrm{i}}}{\partial t} + \frac{\partial (n_{\mathrm{i}} u_{\mathrm{i}})}{\partial x} = 0 \tag{3}$$

$$\left(\frac{\partial}{\partial t} + u_{\mathrm{i}} \frac{\partial}{\partial x}\right) u_{\mathrm{i}} = -\frac{\partial \phi}{\partial x} - \sigma_{\mathrm{i}} n_{\mathrm{i}} \frac{\partial n_{\mathrm{i}}}{\partial x} \tag{4}$$

$$\frac{\partial n_{\mathrm{b}}}{\partial t} + \frac{\partial (n_{\mathrm{b}} u_{\mathrm{b}})}{\partial x} = 0 \tag{5}$$

$$\left(\frac{\partial}{\partial t} + u_{\mathrm{b}} \frac{\partial}{\partial x}\right) u_{\mathrm{b}} = -\mu \frac{\partial \phi}{\partial x} - \mu \sigma_{\mathrm{b}} n_{\mathrm{b}} \frac{\partial n_{\mathrm{b}}}{\partial x} \tag{6}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \chi n_{\mathrm{e}} - n_{\mathrm{i}} - (1-\chi) n_{\mathrm{b}} \tag{7}$$

It is to be mentioned that the following normalization has been used in the above equations:

$$x \to \frac{x \omega_{\mathrm{pi}}}{V_{\mathrm{Fi}}},\ t \to t\omega_{\mathrm{pi}},\ \varphi \to \frac{e\varphi}{2k_{\mathrm{B}} T_{\mathrm{Fj}}},\ n_{\mathrm{j}} \to \frac{n_{\mathrm{j}}}{n_0} \text{ and } u_{\mathrm{j}} \to \frac{u_{\mathrm{j}}}{V_{\mathrm{Fe}}}$$

and $H = \hbar\omega_{pe}/2k_B T_{Fe}$ is a nondimensional quantum parameter proportional to the quantum diffraction and is equal to the ratio between the plasma energy $\hbar\omega_{pe}$ (energy of an elementary excitation associated with an electron plasma wave) and the Fermi energy $k_B T_{Fe}$; $\chi = n_{e0}/n_{i0}$ is the ratio of unperturbed electron density and ion density; $\mu = m_i/m_b$ is the ratio of positive ion and beam ion masses and $\sigma_{i,b} = T_{Fi,b}/T_{Fe}$ is the ratio of ion (beam) Fermi temperature to electron Fermi temperature.

K–dV equation

In order to investigate the behavior of IA, we make the following perturbation expansions for the field quantities n_e, n_i, n_b, u_e, u_i, u_b, and φ about their equilibrium values:

$$\begin{bmatrix} n_e \\ u_e \\ n_i \\ u_i \\ n_b \\ u_b \\ \phi \end{bmatrix} = \begin{bmatrix} n_{e0} \\ 0 \\ n_{i0} \\ 0 \\ n_{b0} \\ u_{b0} \\ 0 \end{bmatrix} + \varepsilon \begin{bmatrix} n_{e1} \\ u_{e1} \\ n_{i1} \\ u_{i1} \\ n_{b1} \\ u_{b1} \\ \phi_1 \end{bmatrix} + \varepsilon^2 \begin{bmatrix} n_{e2} \\ u_{e2} \\ n_{i2} \\ u_{i2} \\ n_{b2} \\ u_{b2} \\ \phi_2 \end{bmatrix} + \ldots \tag{8}$$

where $n_{e0} = \chi$, $n_{b0} = 1 - \chi$, and $n_{i0} = 1$ which are obtained from the equilibrium condition of densities after normalization.

To get the desired K–dV equation describing the nonlinear behavior of ion acoustic waves, we use the standard reductive perturbation technique (Washimi and Taniuti, 1966). We introduce the usual stretching of the space and time variables:

$$\xi = \varepsilon^{1/2}(x - V_p t) \quad \text{and} \quad \tau = \varepsilon^{3/2} t \tag{9}$$

where V_p is the phase speed normalized by electron Fermi speed V_{Fi} and ε is a smallness parameter measuring the dispersion and nonlinear effects.

Using Eq. (9), Eqs (2)–(7) are written in terms of the stretched coordinates ξ and τ and then the perturbation expansions (8) are substituted. Solving the lowest-order equations in ε (i.e., $\varepsilon^{3/2}$), we obtain the linear long-wave phase speed V_p in terms of plasma parameters after using the boundary conditions:

$$\chi - \frac{1}{V_p^2 - \sigma_i} - \frac{(1-\chi)\mu}{[(V_p - u_{b0})^2 - \sigma_b(1-\chi)^2]} = 0 \tag{10}$$

Simplifying Eq. (10), we obtain the following equation:

$$c_4 V_p^4 - 2c_3 V_p^3 + c_2 V_p^2 + 2c_1 V_p + c_0 = 0 \tag{11}$$

where

$$\begin{aligned} c_4 &= 1 \\ c_3 &= b_2/2 \\ c_2 &= -[b_1 - b_2 - b_3 + \mu(1-\chi)]/\chi \\ c_1 &= [b_2(b_1 - 1)]/2\chi \\ c_0 &= -[b_3(b_1 + 1)]/\chi \\ b_3 &= [u_{b0}^2 - \sigma_b(1-\chi)^2] \\ b_2 &= 2u_{b0} \\ b_1 &= \sigma_i \end{aligned}$$

Equation (11) is a bi-quadratic equation in V_p and it gives four values of phase speed of the ion acoustic wave in quantum plasma (real or imaginary). The four values of V_p from the above bi-quadratic equation are obtained as follows:

$$\begin{aligned} V_p &= \frac{1}{2}[(c_3 + d_1) + \sqrt{(d_1 + c_3) - 4(d_2 + d_3)} \\ V_p &= \frac{1}{2}[(c_3 + d_1) - \sqrt{(d_1 + c_3) - 4(d_2 + d_3)} \\ V_p &= \frac{1}{2}[(c_3 - d_1) + \sqrt{(d_1 - c_3) - 4(d_2 - d_3)} \\ V_p &= \frac{1}{2}[(c_3 - d_1) - \sqrt{(d_1 - c_3) - 4(d_2 - d_3)} \end{aligned} \tag{12}$$

where

$$\begin{aligned} d_1^2 &= c_3^2 - c_2 + 2d_2 \\ d_1 d_3 &= c_1 + c_3 d_2 \\ d_3^2 &= d_2^2 - c_0 \end{aligned}$$

$$d_2 = \left[\frac{1}{2}(d_4 + \sqrt{(d_4^2 - d_5)}\right]^{1/3} + \left[\frac{1}{2}(d_4 - \sqrt{(d_4^2 - d_5)}\right]^{1/3} + \frac{1}{6}c_2$$

$$\begin{aligned} d_4 &= \frac{1}{6}c_2(c_0 + c_1 c_3) + \frac{1}{108}c_2^3 - \frac{1}{2}(c_0 c_2 - c_0 c_3^2 - c_1^2) \\ d_5 &= \frac{4}{729}\left[3(c_0 + c_1 c_3) + \frac{1}{4}c_2^2\right]^2 \end{aligned}$$

From Eq. (12), the phase speed of IAW in quantum plasma in the presence of ion beam can be studied analytically and numerically, and the effects of plasma parameters on the phase speed would be understood.

Now, to derive the K–dV equation, we use the stretching coordinates given by Eq. (9). Taking the terms of $\epsilon^{5/2}$, we obtain from Eqs (2)–(7),

$$0 = \frac{\partial \varphi_2}{\partial \xi} - n_{e0}\frac{\partial n_{e2}}{\partial \xi} - n_{e1}\frac{\partial n_{e1}}{\partial \xi} + \frac{H^2}{4}\frac{\partial^3 n_e^{(1)}}{\partial \xi^3} \tag{13}$$

$$\frac{\partial n_{i1}}{\partial \tau} - V_p\frac{\partial n_{i2}}{\partial \xi} + n_{i0}\frac{\partial u_{i2}}{\partial \xi} + \frac{\partial}{\partial \xi}(n_{i1}u_{i1}) = 0 \tag{14}$$

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$$\frac{\partial u_{i1}}{\partial \tau} - V_p\frac{\partial u_{i2}}{\partial \xi} + u_{i1}\frac{\partial u_{i1}}{\partial \xi} = -\frac{\partial \varphi_2}{\partial \xi} - \sigma_i n_{i0}\frac{\partial n_{i2}}{\partial \xi} - \sigma_i n_{i1}\frac{\partial n_{i1}}{\partial \xi} \tag{15}$$

$$\frac{\partial n_{b1}}{\partial \tau} - (V_p - u_{b0})\frac{\partial n_{i2}}{\partial \xi} + n_{b0}\frac{\partial u_{i2}}{\partial \xi} + \frac{\partial}{\partial \xi}(n_{b1}u_{b1}) = 0 \tag{16}$$

$$\begin{aligned}\frac{\partial u_{b1}}{\partial \tau} - (V_p - u_{b0})\frac{\partial u_{b2}}{\partial \xi} + u_{b1}\frac{\partial u_{b1}}{\partial \xi} \\ = -\mu\frac{\partial \varphi_2}{\partial \xi} - \sigma_b\mu n_{b0}\frac{\partial n_{b2}}{\partial \xi} - \sigma_b\mu n_{b1}\frac{\partial n_{b1}}{\partial \xi}\end{aligned} \tag{17}$$

$$\frac{\partial^2 \varphi_1}{\partial \xi^2} = \chi n_{e2} - n_{i2} - (1-\chi)n_{b2} \tag{18}$$

Differentiating Eq. (18) with respect to ξ once more, we get

$$\frac{\partial^3 \varphi_1}{\partial \xi^3} = \chi\frac{\partial n_{e2}}{\partial \xi} - \frac{\partial n_{i2}}{\partial \xi} - (1-\chi)\frac{\partial n_{b2}}{\partial \xi} \tag{19}$$

Putting the first-order values of n_{e1}, n_{i1}, n_{b1}, u_{i1}, u_{b1} in terms of ϕ_1 in Eqs (13)–(18) and then eliminating n_{e2}, n_{i2}, n_{b2}, u_{e2}, u_{i2}, u_{b2}, ϕ_2, the following K–dV equation in quantum plasma having ion beams has been obtained from (19):

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1\frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \tag{20}$$

where

$$\begin{aligned} A = {} & \frac{\chi\mu(\sigma_i - V_p^2)(V_p^2-\sigma_i)^2(V_b^2-\mu\sigma_b)^2}{(V_p^2-\sigma_i)(V_b^2-\sigma_b)[2\chi\mu V_p(\sigma_i - V_p^2 - (1/\chi))(V_p^2-\sigma_i) - 2\mu V_p(V_b^2-\mu\sigma_b)]} \\ & - \frac{\mu(\sigma_i + 3V_p^2)(V_b^2-\mu^2\sigma_b) + \chi\mu^2(\mu\sigma_b + 3V_b^2)(\sigma_i - V_p^2 - (1/\chi))(V_p^2-\sigma_i)^2}{(V_p^2-\sigma_i)(V_b^2-\mu\sigma_b)[2\chi\mu V_p(\sigma_i - V_p^2 - (1/\chi))(V_p^2-\sigma_i) - 2\mu V_p(V_b^2-\mu\sigma_b)]}\end{aligned} \tag{20a}$$

$$B = \frac{(H^2-4)(V_p^2-\sigma_i)^2(V_b^2-\mu\sigma_b)}{4[2\chi V_p(\sigma_i - V_p^2 - (1/\chi))(V_p^2-\sigma_i) - 2V_p(V_b^2-\mu\sigma_b)} \tag{20b}$$

where $V_b = V_p - u_{b0}$ and the parameters μ, σ_i, σ_b, χ, u_{b0}, H, and V_p are defined earlier.

It is known that the solitary wave structure is formed due to the balance between the dispersive effect and the nonlinear effect. The relative strength of these two effects determines the characteristic of such solitary wave structure. The coefficients A of the nonlinear term and coefficient B of the dispersion term thus play a crucial role in determining the solitary wave structure.

To find the steady-state solution of the K–dV equation [Eq. (20)], the independent variables ξ and τ are transformed into one variable $\eta = \xi - U\tau$ where U is the normalized constant speed of the wave frame. Applying the boundary conditions: as $\eta \to \pm\infty$; (i) $\varphi_1 \to 0$, (ii) $(d\varphi_1/d\eta) \to 0$, (iii) $(d^2\varphi_1/d\eta^2) \to 0$, the solution is obtained as

$$\varphi_1 = \varphi_0 \sec h^2\left(\frac{\eta}{\Delta}\right) \tag{21}$$

where the amplitude φ_0 and width Δ of the solitary structure are given by:

$$\varphi_0 = \frac{3U}{A}, \quad \Delta = \sqrt{\frac{4B}{U}} \tag{22}$$

For the positive value of A, the solitary wave will be compressive and the negative value of A will generate the rarefactive solitary wave. Since A is independent of H, the amplitude of the solitary wave will remain the same for any value of H. On the other hand, the width of the solitary wave depends on H and other plasma parameters σ_i, σ_b, n_{b0}, u_{b0}, μ.

Results and discussions

We have numerically analyzed the behavior of ion acoustic solitary waves in quantum plasma in the presence of an ion beam from the K–dV equation [Eq. (20)] and its solution is given by Eq. (21). It is seen that the structure of solitary waves depends on the nonlinear coefficient (A) and the width depends on dispersion coefficient (B). The characteristics of solitary waves depend on the various plasma parameters, namely, velocity (u_{b0}), concentration (n_{b0}), and temperature (σ_b) of the ion beam. For the numerical estimation we have considered a model quantum plasma and have used the value of quantum diffraction diffraction (H) from Hass *et al.* (2003) and Chandra *et al.* (2013); and for ion beam velocity from Kaur *et al.* (2017). The results obtained from numerical estimation for the variations of A and B, the structure and width of solitary waves are given below.

Variation of the coefficients of the nonlinear term and the dispersive term

It is seen from the K–dV equation [Eq. (20)] that the coefficient A of the nonlinear term depends on the plasma

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parameters, that is μ, σ_i, σ_b, χ, u_{b0} but diffraction parameter H has no effect on A; whereas the coefficient B of dispersive term depends on the plasma parameters including quantum diffraction parameter H. Thus, the nonlinear and dispersion coefficients get modified by the ion beam, quantum diffraction effect, and the quantum statistical effect through electron–ion Fermi temperatures. The nonlinear coefficient A has been numerically estimated for a model quantum plasma for plasma parameters related to ion beam and is graphically shown in Figure 1a–1c.

Figure 1a shows the variation in the nonlinear coefficient (A) with ion beam temperature (σ_b) for different values of ion beam density (n_{b0}) in a quantum plasma having fixed values of other parameters as $\sigma_i = 0.01$, $u_{b0} = 0.5$, $H = 0.1$, $V_p = 1.6$, $\mu = 1$ (H^+ ion beam).

It is seen that the nonlinear coefficient (A) becomes negative or positive depending on the values of ion beam temperature. The negative values of A indicate that the solitary waves are rarefactive and, for positive values of A, the solitary waves will be compressive. The nonlinear coefficient A decreases up to the certain value of ion beam temperature ($\sigma_b \approx 1.2$) and then it is suddenly increased to a maximum positive value and finally it decreases slowly with the increase of ion beam temperature keeping a positive value.

Similarly, the variation of A with ion beam velocity (u_{b0}) for different values of velocity ion beam density (n_{b0}) is shown in Figure 1b in a quantum plasma with fixed values of other parameters as $H = 0.1$. $V_p = 1.6$, $\sigma_i = 0.1$, $\sigma_b = 0.1$, $\mu = 1$.

It is seen from Figure 1b that the nonlinear coefficient (A) becomes negative or positive depending on the values of ion beam velocity. Therefore, the solitary waves will be rarefactive (compressive) when $A < 0$ ($A > 0$). It is interesting to see that the nonlinear coefficient A decreases up to the certain value of ion beam velocity ($u_{b0} \approx 1.3$) and then it is suddenly increased to a maximum positive value and finally it starts to decrease with the increase of the ion beam velocity keeping a positive value. Moreover, the nonlinear term is seen to increase with the increase of the ion beam density (n_{b0}).

The variation in the nonlinear coefficient (A) with ion beam velocity (u_{b0}) for different ion beam temperatures (σ_b) can be understood from Figure 1c for a quantum plasma having fixed values of $\sigma_i = 0.01$, $n_{b0} = 0.5$, $H = 0.1$, $V_p = 1.6$, $\mu = 1$.

It is observed that the nonlinear coefficient A becomes positive or negative depending upon the values of ion beam velocity and ion beam temperature. The positive values of A indicate that the solitary wave will be compressive and the negative value of A means the rarefactive solitary will be excited.

Similarly, the dispersion coefficient (B) is numerically estimated for a quantum plasma and its variation with the quantum diffraction parameter H and the parameters related to the ion beam are graphically shown in Figure 2a–2c.

Figure 2(a) shows the variation of B with ion beam velocity (u_{b0}) and H in a quantum plasma having fixed values of $\sigma_i = 0.1$, $\sigma_b = 0.1$, $u_{b0} = 0.5$, $V_p = 1.6$, $\mu = 1$.

It is seen that B is significantly affected by H. The coefficient B is negative and it increases with ion beam velocity for $H < 2$ and $B = 0$ for $H = 2$. But, B is positive and it decreases with ion beam velocity for $H > 2$.

In Figure 2b, the variation of B with ion beam temperature for different values of H is shown for a quantum plasma having fixed values of $\sigma_i = 0.01$, $u_{b0} = 0.5$, $n_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$.

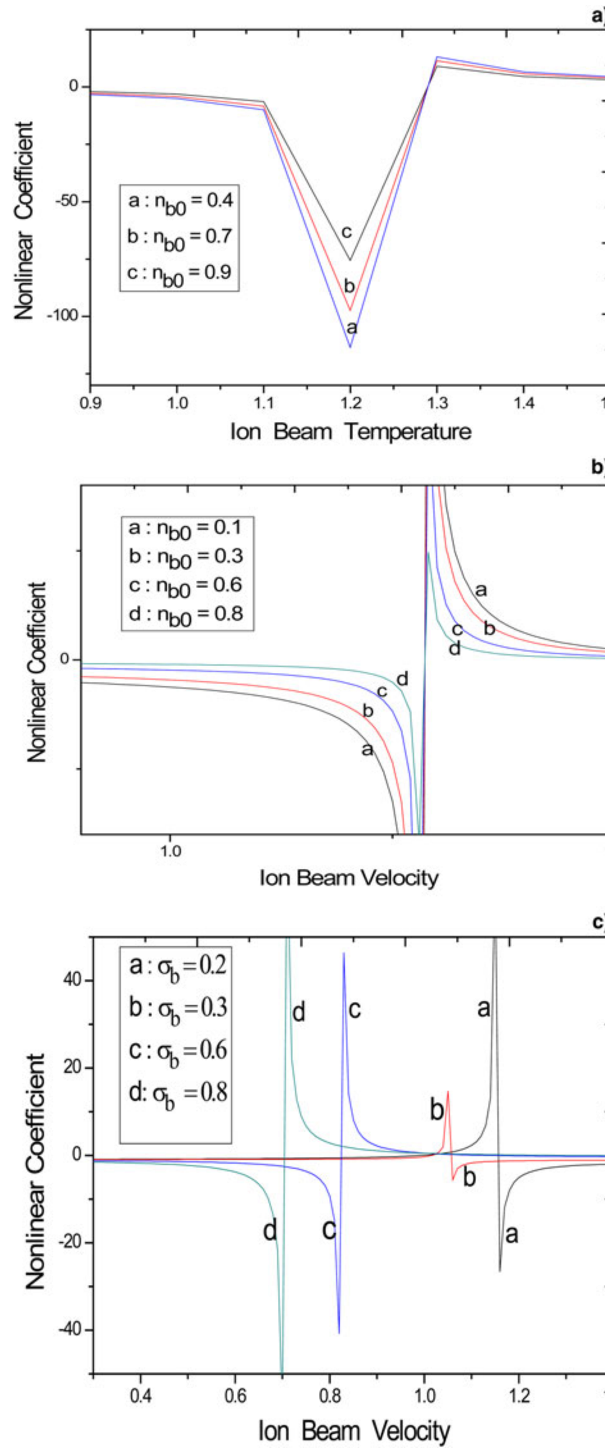

Fig. 1. (a) Variation of Nonlinear coefficient with ion beam temperature (σ_b) for different value of ion beam density (n_{b0}) in quantum plasma with fixed values of plasma parameters $\sigma_i = 0.01$, $u_{b0} = 0.5$, $V_p = 1.6$, $\mu = 1$ (H^+ ion beam). (b) Variation of nonlinear coefficient with ion beam velocity (u_{b0}) for different value of ion beam density (n_{b0}) in quantum plasma with fixed values of plasma parameters $V_p = 1.6$, $\sigma_i = 0.1$, $\sigma_b = 0.1$, $\mu = 1$ (H^+ ion beam). (c) Variation of nonlinear coefficient with ion beam velocity (u_{b0}) for different value of ion beam temperature (σ_b) in quantum plasma with fixed values of plasma parameters $\sigma_i = 0.01$, $n_{b0} = 0.5$, $V_p = 1.6$, $\mu = 1$ (H^+ ion beam).

In this case also, B is negative and it increases with ion beam temperature for $H < 2$ and $B = 0$ for $H = 2$. But, B becomes positive and it decreases with ion beam temperature for $H > 2$.

The variation of B with n_{b0} and H is shown in Figure 2c in a quantum plasma having fixed values of $\sigma_i = 0.01$, $\sigma_b = 0.1$, $n_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$.

It is seen that B is negative for $H < 2$ and it increases with the increase of n_{b0}. The coefficient $B = 0$ when $H = 2$. For $H > 2$, the diffraction coefficient B is positive and it decreases with the increase of n_{b0}.

Ion acoustic solitary waves

To understand the nature of ion acoustic solitary waves, the profiles of solitary waves are drawn in the quantum plasma in the presence of an ion beam for different values of velocity, density, and temperature of the beam. In Figure 3a, the structure of solitary waves is shown with a variation of u_{b0} in the quantum plasma having parameters (dimensionless) $\sigma_i = \sigma_b = 0.1$, $n_{b0} = 0.5$, $V_p = 1.6$, $\mu = 1$.

It is observed that both compressive and rarefactive solitary waves may be excited in the quantum plasma depending upon the values of u_{b0}. For $u_{b0} = 0.1 - 1.2$, the solitary wave will be rarefactive and the amplitude will decrease with the increase of u_{b0}, but for $u_{b0} = 1.3$ and 1.4, the compressive solitary wave will be excited and the amplitude will increase with the increase of u_{b0}.

The effect of ion beam density on solitary waves is shown in Figure 3b in the quantum plasma with fixed values of $\sigma_i = 0.01$, $\sigma_b = 0.1$, $u_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$.

It is seen that the values of n_{b0} decide to excite the compressive or rarefactive solitary waves. For low values of n_{b0} (=0.1, 0.2, and 0.3), the solitary waves are compressive and the amplitudes are increased with the increase of n_{b0}, but for large values of n_{b0} (0.7, 0.8), the solitary wave will be rarefactive in nature and the amplitudes are decreased with the increase of n_{b0}.

In Figure 3c, the structure of solitary waves for different values of ion beam temperature (σ_b) is shown for a quantum plasma having fixed values of $\sigma_i = 0.01$, $n_{b0} = 0.5$, $u_{b0} = 0.5$, $V_p = 1.6$, $\mu = 1$.

It is seen that both compressive and rarefactive solitary waves may be excited in the quantum plasma and these are dependent on the values of σ_b. For $\sigma_b = 0.0$, 0.2, 0.25, the solitary waves are compressive and the amplitude increases with the increase of σ_b. But when $\sigma_b = 0.3$ and 0.35, the rarefactive solitary waves are excited and the amplitudes are decreased with the increase of σ_b.

Since the width is very important for the study of solitary waves in quantum plasma, we have drawn the profiles of width for different values of ion beam parameters. From Eq. (22), we see the width of solitary depends on H and other plasma parameters σ_i, σ_b, n_{b0}, u_{b0}.

In Figure 4a, the variation in width with σ_b for different values of H is plotted for a quantum plasma having fixed values of $\sigma_i = 0.1$, $\sigma_b = 0.1$, $n_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$.

It is seen from Figure 4a that the width of solitary waves decreases with the increase of ion beam temperature and it becomes zero at $\sigma_b \approx 0.58$. Moreover, the widths are decreased with the increase of H when $H < 2$. But the widths are increased with the increase of H when $H > 2$.

In Figure 4b, the variation of widths with ion beam velocity (u_{b0}) for different values of H are depicted for a quantum plasma with fixed values of $\sigma_i = 0.1$, $\sigma_b = 0.1$, $u_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$.

It is observed from Figure 4b that the width decreases with the increase of ion beam velocity and it becomes zero at $u_{b0} \approx 1.3$. When u_{b0} <1.3, the width decreases with the increase of H when $H < 2$ and it increases with the increase of H when $H > 2$. But, when u_{b0} >1.3, the width increases with the increase of H.

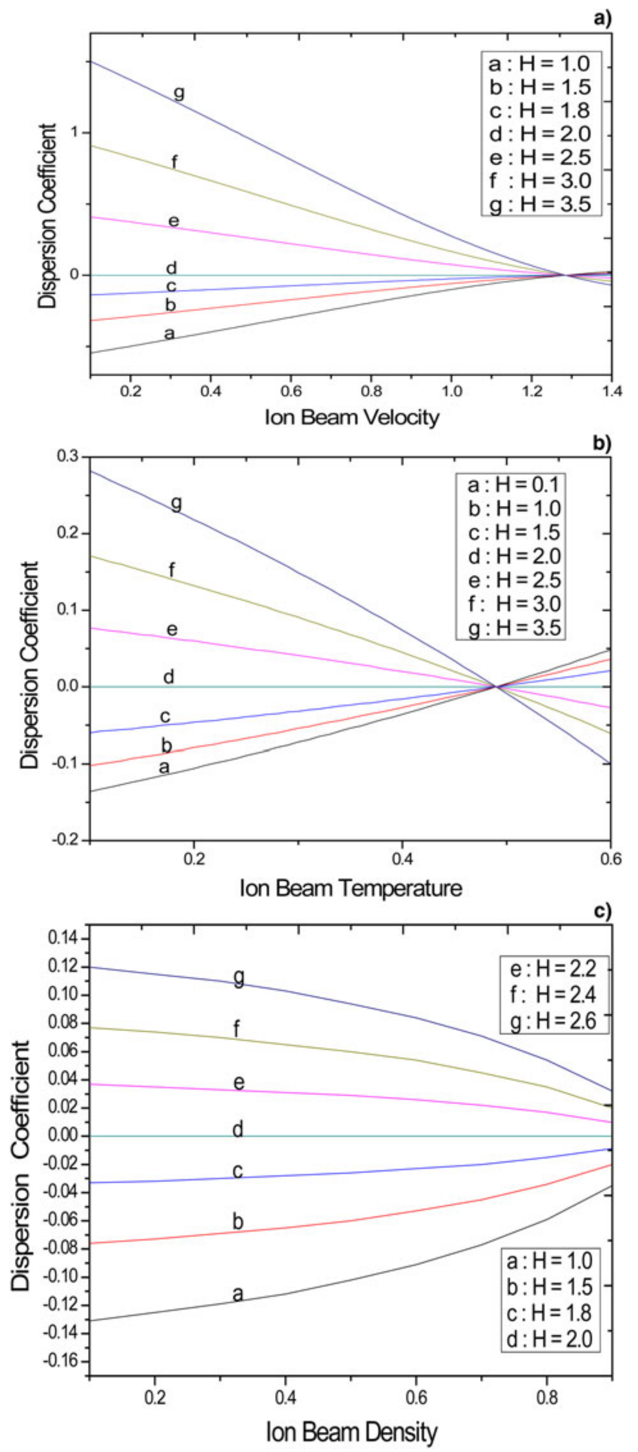

Fig. 2. (a) Variation of dispersion coefficient with ion beam velocity (u_{b0}) for different value of quantum diffraction parameter (H) in quantum plasma with fixed values of plasma parameters $\sigma_i = 0.1$, $\sigma_b = 0.1$, $n_{b0} = 0.5$, $V_p = 1.6$, $\mu = 1$ (H^+ ion beam). (b) Variation of the dispersion coefficient with ion beam temperature (σ_b) and quantum diffraction parameter (H) in quantum plasma with fixed values of plasma parameters $\sigma_i = 0.01$, $u_{b0} = 0.5$, $n_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$ (H^+ ion beam). (c) Variation of the dispersion coefficient with ion beam density (n_{b0}) and quantum diffraction parameter (H) in quantum plasma with fixed values of plasma parameters having $\sigma_i = 0.01$, $\sigma_b = 0.1$, $u_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1.4$ (H^+ ion beam).

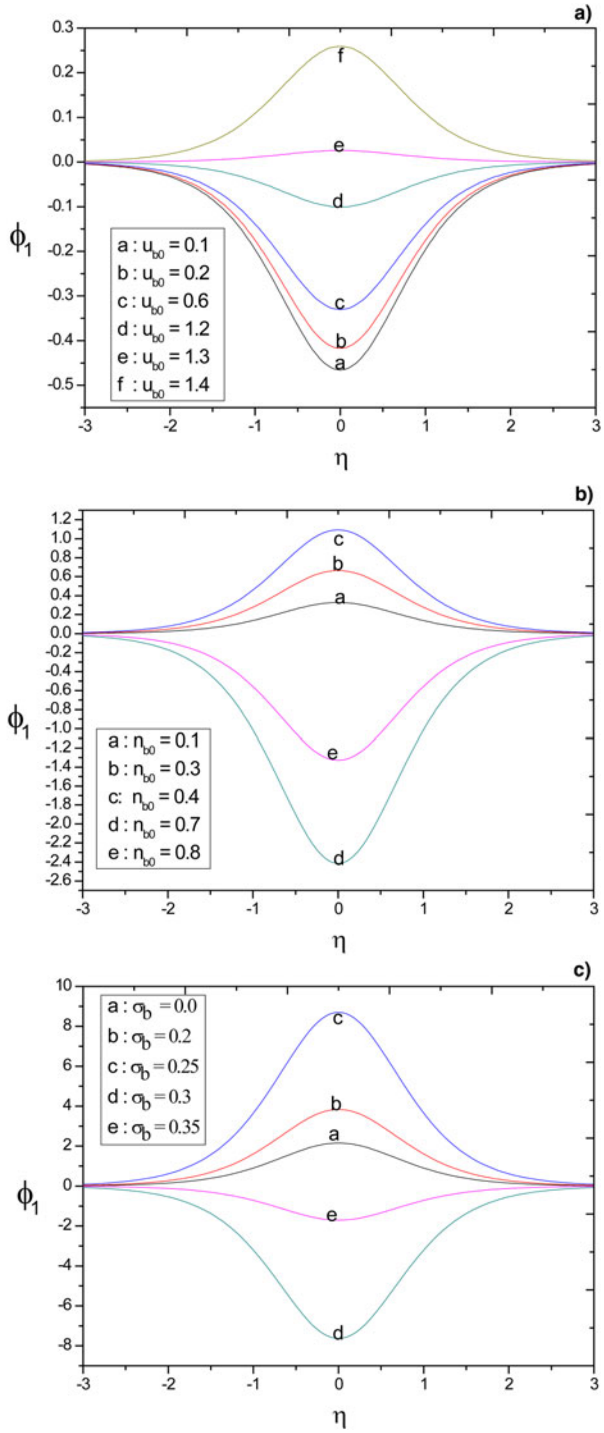

Fig. 3. (a) The profiles of compressive and rarefactive solitary waves in quantum plasma with variation of ion-beam velocity (u_{b0}) with fixed values of plasma parameters $\sigma_i = \sigma_b = 0.1$, $n_{b0} = 0.5$, $V_p = 1.6$, $\mu = 1$ (H^+ ion beam), $U = 0.1$. (b) The profiles of compressive and rarefactive solitary waves in quantum plasma with variation of ion-beam density (n_{b0}) with fixed values of plasma parameters $\sigma_i = 0.01$, $\sigma_b = 0.1$, $u_{b0} = 0.5$, $n_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$ (H^+ ion beam), $U = 0.1$. (c) The profiles of compressive and rarefactive solitary waves in quantum plasma with variation of ion-beam temperature (σ_b) with fixed values of plasma parameters $\sigma_i = 0.01$, $u_{b0} = 0.5$, $n_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$ (H^+ ion beam), $U = 0.1$.

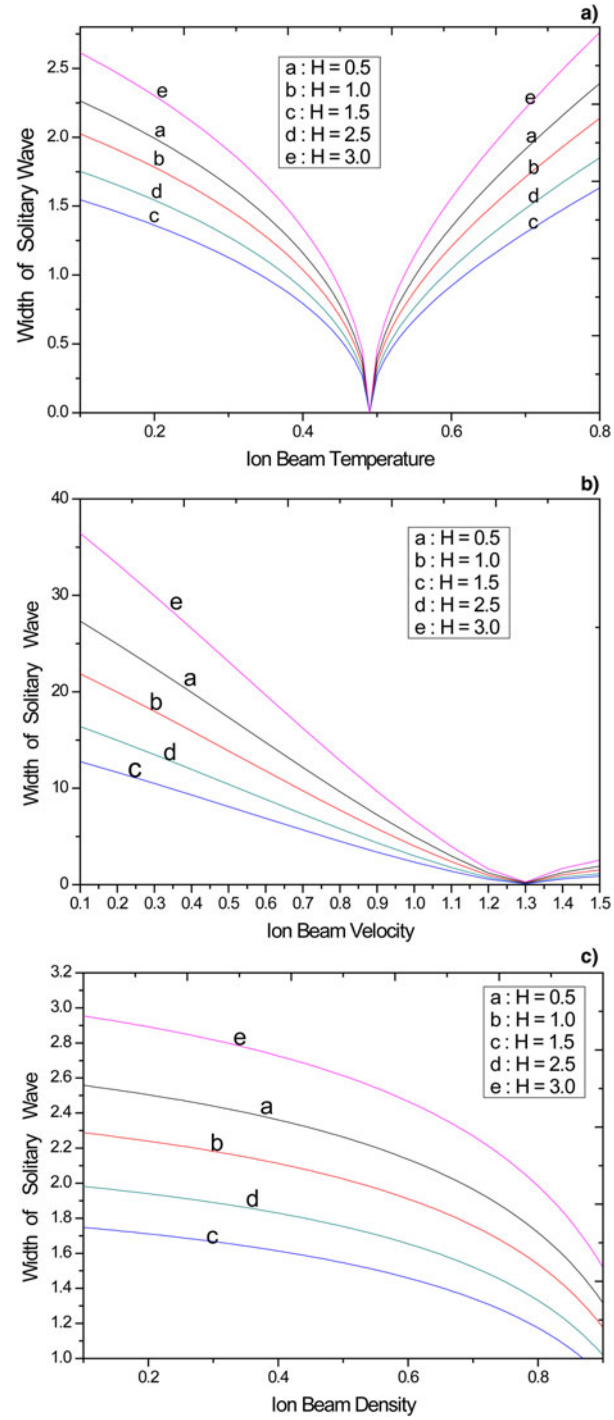

Fig. 4. (a) Variation of width of solitary wave with ion beam temperature (σ_b) and quantum diffraction (H) in quantum plasma with fixed values of plasma parameters $\sigma_i = 0.01$, $n_{b0} = 0.5$, $u_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$ (H^+ ion beam). (b) Variation of width of solitary waves with ion beam velocity (u_{b0}) for different value of quantum diffraction parameter (H) in quantum plasma with fixed values of plasma parameters $\sigma_i = 0.1$, $\sigma_b = 0.1$, $n_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$ (H^+ ion beam). (c) Variation of width of solitary waves with ion beam density (n_{b0}) for different value of quantum diffraction parameter (H) in quantum plasma with fixed values of plasma parameters $\sigma_i = 0.01$, $\sigma_b = 0.1$, $u_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$ (H^+ ion beam).

The variation in the width of solitary waves with ion beam density (n_{b0}) for different values of quantum diffraction parameter (H) is shown in Figure 4c with fixed values of plasma parameters $\sigma_i = 0.01$, $\sigma_b = 0.1$, $u_{b0} = 0.5$, $V_p = 1.2$, $\mu = 1$.

Figure 4c shows that the width of solitary waves decreases with the increase of ion beam density. Moreover, the width decreases with increase of H when $H < 2$ and it increases with the increase of H when $H > 2$.

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It is important to point out that the solitary wave will not exist when the coefficient of the nonlinear term of the K–dV equation [Eq. (20)] vanishes for some critical values of the plasma parameters μ, σ_i, σ_b, χ, u_{b0}, and H. Assuming $A \approx 0$, we obtain

$$\chi[(\sigma_i - V_p^2)(\sigma_b^2 - \mu V_b)^2 - \mu(\mu\sigma_b - 3V_b^2)(\sigma_i - V_p^2)] = (\sigma_i - 3V_p^2)(\sigma_b^2 - \mu V_b^2) - \mu(\mu\sigma_b - V_b^2) \tag{23}$$

From Eq. (23), the critical values of the plasma parameters can be obtained for the nonexistence of the solitary waves in the quantum plasma. The critical value of ion beam density (N_{bc}) is

$$N_{bc} = 1 - \frac{[(\sigma_i - 3V_p^2)(\sigma_b^2 - \mu V_b^2) - \mu(\mu\sigma_b - V_b^2)]}{[(\sigma_i - V_p^2)(\sigma_b^2 - \mu V_b)^2 - \mu(\mu\sigma_b - 3V_b^2)(\sigma_i - V_p^2)]} \tag{24}$$

Again, for some critical values H, V_p, and V_b, the solitary wave will be non-dispersive if the coefficient $B \approx 0$ in K–dV equation [Eq. (20)]. In this case, $H = 2$, or $V_p = \sqrt{\sigma_i}$, or $V_b = \sqrt{\mu\sigma_b}$.

The variation of critical ion beam density (N_{bc}) with ion beam velocity and ion beam temperature is shown in Figure 5.

It is observed that N_{bc} does not depend on H and increases slowly with the increase of u_{b0} and after a certain value of u_{b0}, it is suddenly increased to attain a maximum value. Then, N_{bc} decreases rapidly with the increase of u_{b0}. Moreover, N_{bc} is small for large values of u_{b0}.

But, the solitary waves near the critical density of ion beam in the quantum plasma can be obtained through the derivation of a modified K–dV equation using different stretching coordinates

$$\xi = (x - V_p t) \text{ and } \tau = \varepsilon^3 t \tag{25}$$

The solution of the modified K–dV equation near critical ion beam density will be represented by Paul *et al.* (1996) and Mondal *et al.* (1998)

$$\varphi_{1c} = \varphi_{0c} \sec h(\eta/\Delta') \tag{26}$$

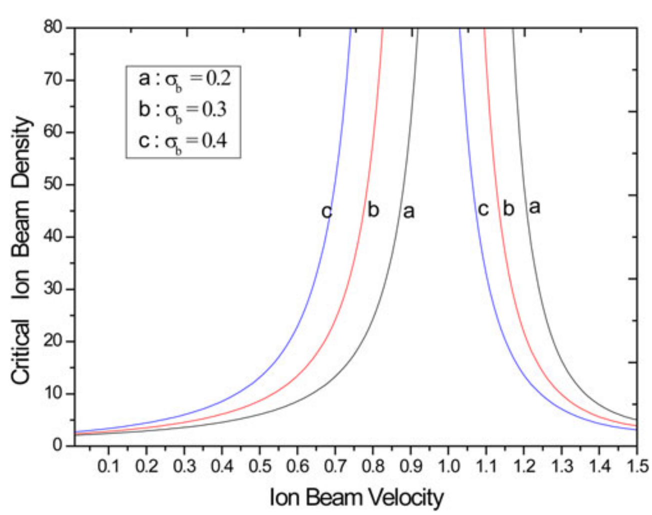

Fig. 5. Variation of critical value of ion beam density with ion velocity (n_{b0}) for different ion beam temperature (σ_b) in quantum plasma with fixed values of plasma parameters $V_p = 1.6$, $\sigma_i = 0.1$, $\mu = 1$ (H^+ ion beam).

where φ_{0c} and Δ' are the amplitude and width of solitary wave near critical ion beam density. Moreover, double-layer (shock-like) solution of ion acoustic wave in the quantum plasma near critical ion beam density may be obtained from the nonlinear equation combining the K–dV equation and the modified K–dV equation (Paul *et al.*, 1996; Mondal *et al.*, 1998; Kaur *et al.*, 2017),

$$\varphi_d = \frac{\varphi_{0d}}{2}[1 - \tanh(\eta/\Delta'')] \tag{27}$$

where φ_{0d} and Δ'' are the amplitude and the width of DLs.

Summary and conclusions

In this paper, we have used the one-dimensional QHD model to investigate ion acoustic solitary waves in a dense quantum plasma in the presence of ion beam. For the study of the nonlinear behavior of ion acoustic waves, the K–dV equation has been derived by using the reductive perturbation technique. It is seen that the formation and structure of solitary waves are significantly affected by the presence of ion beam in quantum plasma. Ion acoustic solitary waves in the quantum plasma may be compressive or rarefactive depending upon the values of ion beam parameters, that is velocity, density, and temperature of the ion beam. Our results may be useful for understanding the beam–plasma interactions and the formation of nonlinear wave structures in dense quantum plasma. The model includes a quantum statistical effect through the equation of state and quantum diffraction effect through the parameter H. It is known that quantum effects in plasma become important when thermal de Broglie wavelength becomes much larger than the average interparticle distance. In most practical situations, quantum effects for the ions may be neglected because of their heavier mass than the electrons.

To the best of our knowledge, no work on the ion acoustic solitary waves in degenerate plasma in the presence of ion beam has been reported till now. In our analysis, the explicit form of solitary waves in the quantum plasma is obtained from the K–dV equation and the structure of solitary waves is graphically discussed for different values of density, velocity, and temperature of the ion beam.

Our main findings are:

i) Both compressive and rarefactive solitary waves may be excited in the quantum plasma depending upon the values of ion beam parameters but the quantum diffraction has no role in it.
ii) Compressive solitary wave will be excited for ion beam velocity $u_{b0} = 1.3$ and 1.4, and amplitude will increase with the increase of u_{b0}; but the solitary wave will be rarefactive for ion beam velocity $u_{b0} = 0.1$–1.2 and the amplitude will decrease with the increase of u_{b0}.
iii) For low values of ion beam density ($n_{b0} = 0.1$, 0.2, and 0.3), the solitary waves will be compressive and the amplitudes are increased with the increase of beam density; but for large values of n_{b0} (0.7, 0.8), the solitary wave will be rarefactive and amplitude decreases with the increase of beam density.
iv) Compressive solitary waves will be excited for ion beam temperature $\sigma_b = 0.0$, 0.2, 0.25 and the amplitude increases with the increase of σ_b; but for $\sigma_b = 0.3$ and 0.35, the solitary waves will be rarefactive in nature and amplitude decreases with the increase of σ_b.

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v) For the numerical estimation of A, B, and the electrostatic potential of solitary waves, we have considered H^+ ion beam and the value of $\mu = 1(\mu = m_i/m_b)$. For He^+ beam ($\mu = 1/4$) and Ar^+ beam ($\mu = 1/40$), numerical estimation can be made and in these cases, the structure of solitary will be different. From numerical estimation and graphical representation of solitary waves, it is seen that the solitary waves will be always rarefactive.

vi) The quantum diffraction parameter H plays important roles in the width of solitary waves in the quantum plasma. For $H < 2$ and $H > 2$, the width may be increased or decreased depending upon the values of ion beam parameters.

However, with reference to laboratory and space plasma, interesting results would be obtained on the propagation of electrostatic waves in a bounded system like plasma-filled waveguides. The boundaries in such a system may add additional effects. The wave with no dispersion in an unbounded system may become dispersive in the bounded system; and the stable wave of an unbounded system may become unstable in bounded geometry (Ghosh *et al*., 2014; Paul *et al*., 2016). Moreover, relativistic effect in plasma is important in many practical situations, for example, in space–plasma phenomena (Vette, 1970), the plasma sheet boundary of Earth's magnetosphere (Grabbe, 1989), Van Allen radiation belts (Shprits *et al*., 2013), and laser–plasma interaction experiments (Marklund and Shukla, 2006). Following the works of Das and Paul (1985) in nondegenerate plasma, ion acoustic solitary waves in degenerate (quantum) plasma have been studied by Sahu (2011). Assuming the relativistically degenerate plasma, the nonlinear propagation of electrostatic solitary has been studied by Mamun and Shukla (2010) and others (Akbari-Moghanjoughi, 2011; Masood and Eliasson, 2011). Later, Chandra *et al*. (2013) have considered two-temperature electrons in a relativistically degenerate plasma and have theoretically investigated the electro-acoustic solitary waves. It has been shown that the relativistic degeneracy parameter significantly influences the conditions of the formation and properties of solitary waves. Recently, Hossen and Mamun (2014) have studied theoretically and numerically the nonlinear propagation of modified EAWs in an unmagnetized, collisionless, relativistic degenerate quantum plasma through the derivation of the nonplanar K–dV equation which admits a wave solution for the solitary wave profile. More recently, Paul *et al*. (2019) have considered an ultra-relativistic degenerate dense electron–ion–positron plasma for the study of envelope solitary waves and rogue waves and have obtained significant results which can be further studied in the presence of an electron beam or ion beam following the concept of the present paper.

Some works on the nonlinear propagation of ion and electron acoustic waves in a fully relativistic quantum plasma with ion and electron beams are in progress and these will be reported later.

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