Relativistic and ponderomotive effects on evolution of dark hollow Gaussian electromagnetic beams in a plasma

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Abstract

Nonlinear parabolic partial differential equation governing the evolution of complex envelope in slowly varying envelope approximation is solved using variational approach. The basic nonlinear phenomena of relativistic and ponderomotive self-focusing in a plasma are taken into account. Self-focusing, self-phase modulation as well as self-trapping of dark hollow Gaussian beam is studied for higher orders of hollow Gaussian beam (n).

Keywords: Hollow Gaussian beam; Self-focusing and self-trapping; Self-phase modulation; Relativistic and ponderomotive

1. INTRODUCTION

Theoretical and experimental study of the evolution of high power laser beams as they propagate through plasmas is an active area of research due to their potential relevance in plasma based accelerators (Sarkisov *et al.*, 1999), inertial confinement fusion (Tabak *et al.*, 1994; Regan *et al.*, 1999), ionospheric modification (Perkins & Goldman, 1981; Guzdar *et al.*, 1998; Gondarenko *et al.*, 2005), and new radiation sources (Suckewer & Skinner, 1990, 1995; Benware *et al.*, 1998; Foldes *et al.*, 1999; Fedotov *et al.*, 2000). Specifically, in the laser plasma interaction, the generic process of the self-focusing of the laser beams (Chiao *et al.*, 1964; Kelley, 1965; Sodha *et al.*, 1974, 1976; Milchberg *et al.*, 1995; Saini & Gill, 2006; Gill & Saini, 2007; Yu *et al.*, 2007; Gill *et al.*, 2010; Kaur *et al.*, 2010) has been focus of attention as it affects many other nonlinear processes. It plays crucial role in beam propagation and arises due to increase of the on-axis index of refraction relative to edge of the laser beam. For example, for ponderomotive force type nonlinearity, electrons are expelled from the region of high intensity laser field. On the other hand, relativistic self-focusing results from the effect of quiver motion leading to reduced local plasma frequency. The self-focusing is counterbalanced by the tendency of the beam to spread because of diffraction. In the absence of nonlinearities, the beam will spread substantially in a Rayleigh length, $R_d(\sim ka_0^2)$, where k is the wavenumber and a_0 is the initial spot size of the laser beam.

Self-focusing and filamentation are among the most dangerous nonlinear phenomena that destroy the uniformity of overall irradiation required for direct-drive fusion experiment as well as leads to seeding and growth of hydrodynamic instabilities. Experimental as well as theoretical observations of relativistic self-focusing and ponderomotive self-channeling have been reported in a number of investigations (Faenov *et al.*, 2007; Chessa *et al.*, 1998; Kurki-Suonio *et al.*, 1989; Abramyan *et al.*, 1992; Tzeng & Mori, 1998; Monot *et al.*, 1995; Krushelnick *et al.*, 1997; Konar & Manoj, 2005; Wagner *et al.*, 1997; Chen *et al.*, 1998; Borisov *et al.*, 1998). The dynamics of ponderomotive channeling in underdense plasma has recently been studied experimentally (Borghesi *et al.*, 2007). Relativistic laser-plasma interaction physics has also been focus of attention as many nonlinear processes playing key roles in the generation of new ion sources as reported recently (Laska *et al.*, 2007; Strangio *et al.*, 2007; Torrisi *et al.*, 2008). Further, there have been a series of novel experiments to study rich physics issues in nuclear and particle physics, atomic physics (Stoehlker *et al.*, 2003), plasma physics (Hoffmann *et al.*, 2005; Schaumann *et al.*, 2005) and applied sciences (Kuehl *et al.*, 2007; Kasperczuk *et al.*, 2008). Such experiments have opened new vistas in this field and are supposed to pay rich dividents to multidisciplinary character of laser produced plasma.

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Some of the approximate theories used to study self-focusing process are a paraxial ray approximation (PRA) (Akhmanov *et al.*, 1968; Sodha *et al.*, 1974, 1976), moment theory approach (Firth, 1977; Lam *et al.*, 1977), variational approach (Firth, 1977; Anderson & Bonnedal, 1979), and source-dependent expansion method (Sprangle *et al.*, 2000). Each of these theories has limitations in describing completely the experimental/computer simulation results. However, the most used theory based on Wentzel-Krammers-Brillouin approximation and paraxial ray approach (PRA) is given by Akhmanov *et al.* (1968) and developed by Sodha *et al.* (1976). Due to its mathematical simplicity, it has been extensively used to account for general features of self-focusing as witnessed by the increasing number of publications. However, this theory being local in character overemphasizes the field closest to beam axis and lacks global pulse dynamics. Furthermore, it also predicts unphysical phase relationship (Karlsson *et al.*, 1991). It has also been pointed out that PRA is not applicable when high power laser beams are used (Subbarao *et al.*, 1998). Another global approach is variational approach, though crude to describe the singularity formation and collapse dynamics, is fairly general in nature to describe the propagation and correctly predicts the phase.

Most of the research work on self-focusing of the laser beam has been confined to the cylindrically symmetric Gaussian beams symmetry (Kumar *et al.*, 2006; Hora, 1975; Esarey *et al.*, 1997; Sharma *et al.*, 2004; Akhmanov *et al.*, 1968). Only a few investigations have been reported on the self-focusing of super-Gaussian beams (Grow *et al.*, 2006; Fibich, 2007), self-trapping of degenerate modes of laser beams (Karlsson, 1992), self-trapping of Bessel beams (Johannisson *et al.* 2003), elliptical Gaussian beam (Cornolti *et al.* 1990), and hollow elliptical Gaussian beam (Cai & Lin, 2004). These types of beams have different type of irradiance distribution. Recently, the optical beams with central shadow, generally known as dark hollow beams (DHB) have attracted the attention of the physics community because of their wide potential applications in the field of modern optics, atomic optics, and plasmas (Cai *et al.*, 2003; Yin *et al.*, 2003; York *et al.*, 2008). A DHB can be expressed as a finite sum of Laguerre Gaussian beams or Gaussian beams. DHBs have been widely studied both from experimental and theoretical aspect (Arlt & Dholakia, 2000; Zhang *et al.*, 2004). For the explanation of dynamics and other propagation characteristics, several theoretical models for dark hollow Gaussian Beam (DHGB) like the *TEM*$_{01}$ mode doughnut beam, some higher order Bessel beams, superposition of off-axis Gaussian beams, and DHGB have been reported in several recent investigations (Arlt & Dholakia, 2000; Zhu *et al.*, 2002; Deng *et al.*, 2005; Mei & Zhao, 2005). However, the propagation characteristics of coherent DHBs in a plasma or any other nonlinear media have received little attention. In recent investigations, Sodha *et al.* (2009*a*, 2009*b*) have presented a modified paraxial like approach to study self-focusing of a HGB in plasma.

In the present investigation, authors have studied evolution of DHGB in a plasma when relativistic and ponderomotive nonlinearities are considered.

The organization of the paper is as follows: In Section 2, a model is setup in a weakly relativistic regime in underdense plasma starting from Ampere's and Faraday's laws. These equations under approximate conditions leads to an evolution equation. Lagrangian for the problem is setup and variational approach is used. In Section 3, authors have studied the self-trapped mode, and Section 4 is devoted to discussion. In Section 5, conclusions of present investigation are presented.

2. BASIC FORMULATION

The present model is setup in a weakly relativistic regime in underdense plasma. Starting from Ampere's and Faraday's laws and in the absence of the external charge and current, we have the following set of equations:

$$\vec{\nabla} \times \vec{B} = \frac{1}{c}\frac{\partial \vec{D}}{\partial t}, \tag{1}$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}. \tag{2}$$

where $\vec{E}$ and $\vec{B}$ are the electric and magnetic field vectors, respectively. $\vec{D} = \epsilon\vec{E}$ is the displacement vector. Also, the induced current density due to laser-plasma interaction exists in the dielectric permittivity, ϵ.

Combining the above two equations gives:

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} + \frac{1}{c^2}\frac{\partial^2 \vec{D}}{\partial t^2} = 0. \tag{3}$$

Assuming wave propagation in *z*-direction and the electric and magnetic fields in the *xy*-plane, Eq. (3) is reduced to:

$$\nabla^2\vec{E} + \frac{\omega^2}{c^2}\epsilon\vec{E} = 0. \tag{4}$$

Further the relativistic ponderomotive force on the electrons modifies the electron density. The relativistic ponderomotive force on electrons is given by:

$$\vec{F}_{pe} = -m_{e0}c^2\nabla(\gamma - 1), \tag{5}$$

where m_{e0} is the electron mass in the absence of the external field and γ is called the relativistic factor defined as follows:

$$\gamma = \sqrt{1 + \left(\frac{eE}{m_{0e}c\omega}\right)^2}. \tag{6}$$

Following Niknam *et al.* (2009), the dielectric permittivity

for the cold plasma is:

$$\epsilon = 1 - \frac{\omega_{pe}^2}{\gamma\omega^2}, \tag{7}$$

$$\epsilon = 1 - \frac{\omega_{pe}^2}{\gamma\omega^2} exp\left(-\frac{m_{0e}c^2}{T_e}(\gamma-1)\right), \tag{8}$$

where

$$\omega_{pe0}^2 = \frac{4\pi n_{e0}e^2}{m_{e0}}, \tag{9}$$

$$n_e = n_{e0}exp\left(-\frac{m_{0e}c^2}{T_e}(\gamma-1)\right), \tag{10}$$

where n_{e0} is the maximum electron density at the place in which the laser electric field is zero. We obtain the following evolution equation governing the electric field envelope in collisionless plasmas as follows:

$$\left[\nabla_\perp^2 - 2\iota k\frac{\partial}{\partial z} + \frac{\omega^2}{c^2}\left(1 - \frac{\omega_{pe}^2}{\gamma\omega^2}\right.\right.$$
$$\left.\left.\times exp\left(-\frac{m_{0e}c^2}{T_e}(\gamma-1)\right)\right)\right]\psi(r, z) = 0. \tag{11}$$

Eq. (11) is a nonlinear parabolic partial differential equation in which the first term has its origin in diffractional divergence, in third term within parenthesis, "1" is the free space propagation term and rest corresponds to relativistic and ponderomotive self-focusing terms. Since relativistic and ponderomotive channeling occur together, we investigate their combined effects on the evolution of intense laser beam in a plasma. Variational approach that have rigorous basis, as applied in other fields, is used here to investigate nonlinear wave propagation. We use procedure of Anderson and Bonnedal (1979) to reformulate Eq. (11) into a variational problem corresponding to a Lagrangian L, so as to make $\frac{\delta L}{\delta z} = 0$. Thus, Lagrangian L corresponding to Eq. (11) is given by:

$$L = \iota kr\left(\psi\frac{\partial\psi^*}{\partial z} - \psi^*\frac{\partial\psi}{\partial z}\right) - r\left|\frac{\partial\psi}{\partial r}\right|^2$$
$$+ r\left[\frac{\omega^2}{c^2}\alpha^2|\psi|^2 - \frac{\omega_{pe0}^2}{52c^2}\left(1 - e^{-52\alpha^2|\psi|^2}\right) + \frac{\omega_{pe0}^2}{2704c^2}\right. \tag{12}$$
$$\times\left(1 - e^{-52\alpha^2|\psi|^2}\right) - \frac{\omega_{pe0}^2}{52c^2}\alpha^2|\psi|^2 e^{-52\alpha^2|\psi|^2}\right],$$

where $\alpha = \frac{e}{\sqrt{2}m_{e0}c\omega}$

Thus, the solution to the variational problem

$$\delta\iiint Ldxdydz = 0, \tag{13}$$

also solves the nonlinear Schrödinger Eq. (11). Using the

trial function as Gaussian beam of the form as follows:

$$\psi(r, z) = \psi_0(z)\left(\frac{r^2}{2a^2(z)}\right)^n exp$$
$$\times\left(-\frac{r^2}{2a^2(z)} + \iota b(z)r^2 + \iota\phi(z)\right), \tag{14}$$

where $a(z)$ is the beam width, $b(z)$ is the spatial chirp, and $\phi(z)$ is the phase of the laser beam. Using the ansatz, with expression for ψ as a trial function, we can perform the integration to write:

$$<L> = <L_0> + <L_1>, \tag{15}$$

where

$$<L_0> = \frac{\iota ka^2}{2^{2n+1}}\left(\psi_0\frac{\partial\psi_0^*}{\partial z} - \psi_0^*\frac{\partial\psi_0}{\partial z}\right)(2n)!$$
$$+ k|\psi_0|^2\frac{a^4}{2^{2n}}\frac{db}{dz}(2n+1)! + k|\psi_0|^2\frac{a^2}{2^{2n}}\frac{d\phi}{dz}(2n)!$$
$$- \frac{n^2|\psi_0|^2}{2^{2n-1}}(2n-1)! + \frac{n|\psi_0|^2}{2^{2n-1}}(2n)! \tag{16}$$
$$- \frac{|\psi_0|^2}{2^{2n+1}}(2n+1)! - \frac{b^2|\psi_0|^2a^4}{2^{2n-1}}(2n+1)!$$

$$<L_1> = \frac{\omega^2a^2}{c^2}\frac{\alpha^2|\psi_0|^2}{2^{2n+1}}(2n)! + \frac{\omega_{pe0}^2a^2}{104c^2}$$
$$\times\left[c' - 2n(1-n)\left(c' - \frac{2}{e}\right)Log(52\alpha^2|\psi_0|^2)\right] -$$
$$\frac{\omega_{pe0}^2a^2}{5408c^2}\left[c' - 2n(1-n)\left(c' - \frac{2}{e}\right)Log \tag{17}$$
$$(52\alpha^2|\psi_0|^2)\right] + \frac{\omega_{pe0}^2a^2}{5408c^2}[1 - (2n - 4n^2)$$
$$\times(c' + Log(52\alpha^2|\psi_0|^2)],$$

where $c' = 0.57721$ is Euler's constant. Using the procedure of (Anderson & Bonnedal, 1979; Saini & Gill, 2006), we arrive at the following equation for a:

$$\frac{d^2a}{dz^2} = \frac{2}{k^2a^3} - \frac{4n}{k^2a^3(2n+1)} + \frac{2^{2n+2}}{a(2n+1)!}$$
$$\times\left[-\frac{(2n)!}{2^{2n+2}} + \frac{51\omega_{pe0}^2}{10816\omega^2\alpha^2|\psi_0|^2}(c' - 2n(1-n)\right.$$
$$\times\left(c' - \frac{2}{e}\right)Log(52\alpha^2|\psi_0|^2)$$
$$+ \frac{\omega_{pe0}^2}{10816\omega^2\alpha^2|\psi_0|^2}(1 - (2n - 4n^2) \tag{18}$$
$$\times(c' + Log(52\alpha^2|\psi_0|^2)) + \frac{51\omega_{pe0}^2}{5408\omega^2\alpha^2|\psi_0|^2}$$
$$\times 2n(1-n)\left(c' - \frac{2}{e}\right) + \frac{\omega_{pe0}^2}{5408\omega^2\alpha^2|\psi_0|^2}$$
$$\times(2n - 4n^2)].$$

After normalization using $\eta = \frac{cz}{\omega a_0^2}$, we arrive at the following equations for a_n, ϕ:

$$\frac{d^2 a_n}{d\eta^2} = \frac{2}{a_n^3} - \frac{4n}{(2n+1)a_n^3} + \frac{2^{2n+2}k^2 a_0^2}{a_n(2n+1)!}$$

$$\times \left[-\frac{(2n)!}{2^{2n+2}} + \frac{51\omega_{pe0}^2}{10816\omega^2\alpha^2|\psi_0|^2}(c' - 2n(1-n) \right.$$

$$\left(c' - \frac{2}{e} \right) Log(52\alpha^2|\psi_0|^2) + \frac{\omega_{pe0}^2}{10816\omega^2\alpha^2|\psi_0|^2}$$

$$\times (1 - (2n - 4n^2)(c' + Log(52\alpha^2|\psi_0|^2)))$$

$$+ \frac{51\omega_{pe0}^2}{5408\omega^2\alpha^2|\psi_0|^2} 2n(1-n)\left(c' - \frac{2}{e} \right) + \frac{\omega_{pe0}^2}{5408\omega^2\alpha^2|\psi_0|^2}$$

$$\left. (2n - 4n^2) \right] \tag{19}$$

$$\frac{d\phi}{d\eta} = \frac{n}{2a_n^2} + \frac{2^{2n-1}k^2 a_0^2}{(2n)!}\left[-\frac{(2n)!}{2^{2n+2}} + \frac{51\omega_{pe0}^2}{10816\omega^2\alpha^2|\psi_0|^2} \right.$$

$$\times \left(c' - 2n(1-n)\left(c' - \frac{2}{e} \right) Log(52\alpha^2|\psi_0|^2) \right)$$

$$+ \frac{\omega_{pe0}^2}{10816\omega^2\alpha^2|\psi_0|^2}(1 - (2n - 4n^2)$$

$$\left. \times (c' + Log(52\alpha^2|\psi_0|^2)) \right) + \frac{51\omega_{pe0}^2}{5408\omega^2\alpha^2|\psi_0|^2} 2n(1-n) \tag{20}$$

$$(c' - \frac{2}{e}) + \frac{\omega_{pe0}^2}{5408\omega^2\alpha^2|\psi_0|^2}(2n - 4n^2) \right]$$

$$+ \frac{2^{2n-1}k^2 a_0^2}{(2n)!}\left[-\frac{(2n)!}{2^{2n+1}}\alpha^2|\psi_0|^2 + \frac{51}{5408} \right.$$

$$\omega_{pe0}^2\omega^2 2n(1-n)\left(c' - \frac{2}{e} \right) + \left. \frac{\omega_{pe0}^2(2n - 4n^2)}{5408\omega^2} \right].$$

3. SELF-TRAPPED MODE

For an initially plane wave front, $\frac{da}{dz} = 0$ and $a = 1$ at $z = 0$, the condition $\frac{d^2a}{dz^2} = 0$ leads to the propagation of DHGB in the uniform waveguide/self-trapped mode. The conditions under which this occurs are termed as critical conditions and their graphical representation is known as the critical curve.

By putting $\frac{d^2a}{dz^2} = 0$ in Eq. (18), we obtain a relation between dimensionless initial beam width parameter (ρ_0) and critical values of power of the beam $\Pi(=\alpha^2 |\psi_0|^2)$ taking into account relativistic-ponderomotive type nonlinearity. The critical condition leads to general expression for determination of critical threshold for various orders of n. The expression when simplified is given as follow:

$$1 = \frac{2n}{(2n+1)} - \frac{2^{2n+1}}{(2n+1)!}\frac{\omega^2 a_0^2}{c^2}$$

$$\times \left[\frac{-(2n)!}{2^{2n+2}} + \frac{51\omega_{pe0}^2}{10816\omega^2\alpha^2|\psi_0|^2}(c' - 2n(1-n) \right.$$

$$\left(c' - \frac{2}{e} \right) Log(52\alpha^2|\psi_0|^2).$$

$$+ \frac{\omega_{pe0}^2}{10816\omega^2\alpha^2|\psi_0|^2}(1 - (2n - 4n^2)$$

$$\times (c' + Log(52\alpha^2|\psi_0|^2)) + \frac{51\omega_{pe0}^2}{5408\omega^2\alpha^2|\psi_0|^2} 2n(1-n)$$

$$\times \left(c' - \frac{2}{e} \right) + \frac{\omega_{pe0}^2}{5408\omega^2\alpha^2|\psi_0|^2}(2n - 4n^2) \right], \tag{21}$$

where $\rho_0 = \frac{a_0\omega}{c}$, is the initial dimensionless beam width parameter.

The nature of self-focusing is further highlighted through the critical curves. DHB undergoes self-focusing when the condition $\frac{d^2a}{dz^2} < 0$ is satisfied whereas for $\frac{d^2a}{dz^2} > 0$, HGB displays either oscillatory or steady state self-focusing.

4. DISCUSSION

Eq. (19) describes the beam dynamics in plasma with relativistic and ponderomotive nonlinearities taken into account. Eqs. (19) and (20) are nonlinearly coupled ordinary second order differential equations governing the normalized beam width parameter a_n and phase ϕ. There are several terms appearing on right hand side (R.H.S.) of Eqs. (19) and (20). Analytical solutions to these equations are not possible. We therefore seek numerical computational techniques to study beam dynamics. Before that, it is worthnoting to understand the physical mechanisms and origin of various terms on R.H.S. of Eq. (19). The first and second terms on R.H.S of Eq. (19) are responsible for diffractional divergence of the laser beam which has its origin in the Laplacian ($\nabla_\perp^2$) appearing in the evolution Eq. (11). It is noteworthy to point out that the second term in Eq. (19) weakens the divergence of laser beam solely because of its sign. Further, within parenthesis, the first term comes from free space propagation and the rest of the terms represent the combined effects of ponderomotive self-channeling and relativistic self-focusing. Self-focusing/defocusing of laser beam in plasma is determined by the competing mechanisms on the R.H.S. of Eq. (19). The normalized beam width parameter, $a_n < 1$ corresponds to self-focusing and $a_n > 1$ is the result of diffractional dominance over all other terms leading to defocusing of laser beam. However, as mentioned in the Introduction, long distance of several Rayleigh lengths (R_d) are prerequisite for novel applications of laser produced plasma. In Figure 1, the plot of normalized beam width parameter, a_n with dimensionless distance of propagation, η is shown for various orders of n. In this case, free space propagation term on R.H.S of Eq. (19) within parenthesis is not taken into account. Further, it is observed that defocusing of the laser beam occurs for all n values but decrease with increase in n. However, situation changes drastically when the combined effects of both relativistic and
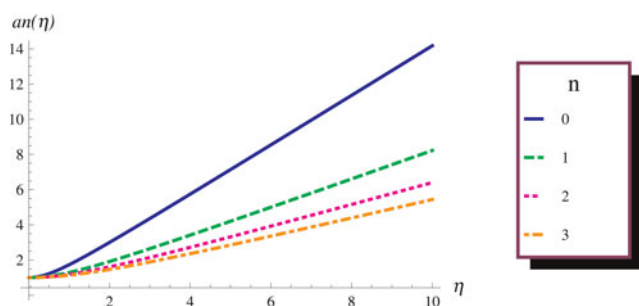
Fig. 1. (Color online) Variation of normalized beam width $a_n(\eta)$ with dimensionless distance of propagation η for the case when free space propagation term is not taken into account with the following set of parameters for the various orders of n: $a_0 = 0.002$ cm, ω_{pe0} $0.01 \times \omega$, $\omega = 1.778 \times 10^{14}$ $\frac{\text{rad}}{\text{sec}}$, $k = 1.25 \times 10^3$ cm^{-1}, intensity parameter, $\alpha^2 |\psi_0|^2 = 0.8$. Solid curve corresponds to $n = 0$, dashed curve to $n = 1$, dotted curve to $n = 2$ and dotdashed curve to $n = 3$.

Fig. 3. (Color online) Plot of longitudinal phase $\phi(\eta)$ versus dimensionless distance of propagation η for the case when both the mechanisms (relativistic self-focusing and ponderomotive self-channeling) are considered for the same set of parameters as mentioned in the caption of Figure 1. Solid curve corresponds to $n = 0$, dashed curve to $n = 1$, dotted curve to $n = 2$ and dotdashed curve to $n = 3$.

ponderomotive nonlinearities along with free space propagation term are taken into account. It is observed from Figure 2 that oscillatory self-focusing takes place for all higher orders of n. It is also important to notice that beam propagates oscillatory and infinitely over several number of Rayleigh lengths. Inspite of higher order, defocusing is not observed for all powers. It is in contrast to the results of Sodha *et al.* (2009a) where defocusing is observed with increase in n. This is apparently a consequence of variational approach where in averaging process the contribution of whole wave front is considered. On the other hand, PRA takes into account only rays that are very close to the beam-axis. Another aspect of this phenomenon observed here is that oscillatory character becomes slow with increase in the value of n. We have numerically analyzed the evolution of normalized beam width as a function of distance of propagation, η for four values of n with other parameters chosen as follows:

$$a_0 = 0.002 \, \text{cm}, \, k = 1.25 \times 10^3 \, \text{cm}^{-1},$$

$$\omega_{pe0} = 0.01 \times \omega, \, \omega = 1.778 \times 10^{14} \, \frac{\text{rad}}{\text{sec}}.$$

Eq. (20) describes the evolution of longitudinal phase, ϕ with the dimensionless distance of propagation (η). As is apparent from the R.H.S of Eq. (20), which is a complicated function of n, normalized beam width a_n and intensity parameter, $\alpha^2 |\psi_0|^2$ besides other parameters, determines the nature of phase. It is observed from Figure 3, the phase is negative for lower values of n ($=0,1$), but becomes positive with further increase in n. However, for lowest order of n, observed phase is negative and does not show oscillatory character. This is due to the fact that the R.H.S. of Eq. (20) for $n = 0$ depends only on $\alpha^2 |\psi_0|^2$ maintaining a linear relationship. However, oscillatory character is obtained for $n \neq 1$, as self-focusing of the main beam enters into the picture. Wiggles observed in ϕ *versus* η graph are more pronounced for higher values of n.

To further elucidate the results for deleanating the underlying physics, we numerically analyze the dependence of dimensionless initial beam width parameter (ρ_0) as a function of critical values of beam power Π ($=\alpha^2 |\psi_0|^2$) for various
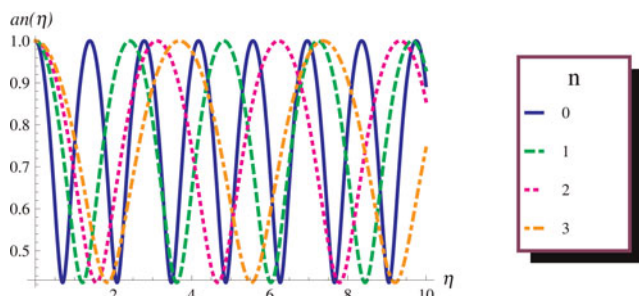

Fig. 2. (Color online) Variation of normalized beam width $a_n(\eta)$ with dimensionless distance of propagation η for the case when both relativistic and ponderomotive nonlinearities alongwith free space propagation term are considered with the same set of parameters as in the caption of Figure 1 for the various orders of n. Solid curve corresponds to $n = 0$, dashed curve to $n = 1$, dotted curve to $n = 2$ and dotdashed curve to $n = 3$.
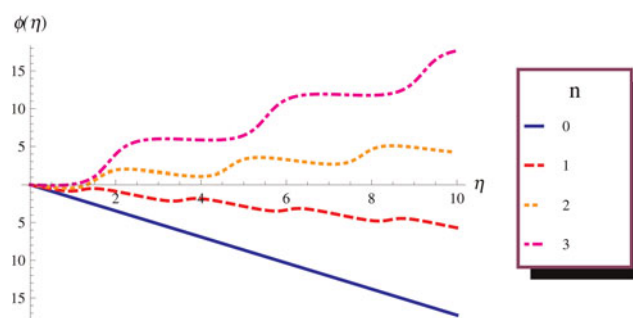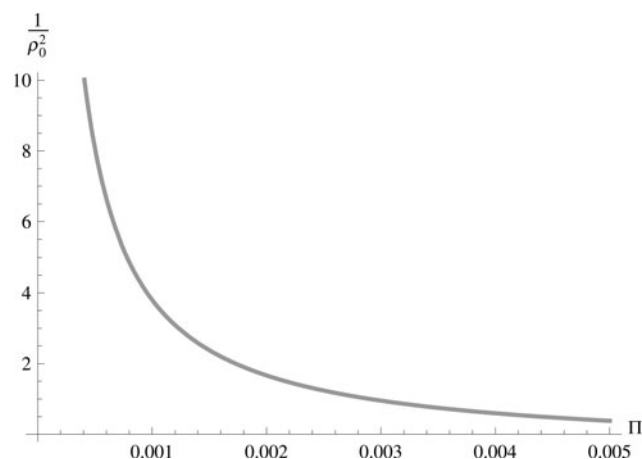

Fig. 4. Variation of the dimensionless initial beam width (ρ_0) as a function of measure of critical beam power (Π) for the case when both the mechanisms are present with $n = 0$.
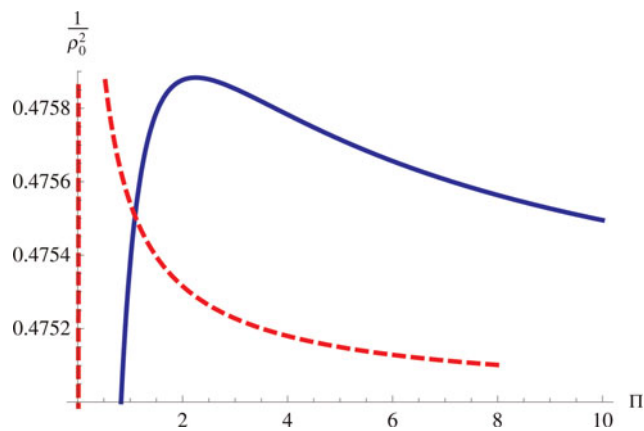
Fig. 5. (Color online) Variation of the dimensionless initial beam width (ρ_0) as a function of Π with $n = 2$ and $n = 4$. Solid curve corresponds to $n = 2$ and dashed to $n = 4$.

Fig. 7. Dependence of the dimensionless initial beam width (ρ_0) as a function of Π with $n = 3$.

orders of n when both relativistic and ponderomotive nonlinearities are taken into account. The results are depicted in the form of graphs. The critical curves for the propagation of the HGB in a plasma have been computed for a chosen set of parameters, n, c' and $\Omega(= \frac{\omega_{pe0}}{\omega})$. The critical curve for the DHGB characterizes the self-focusing region in $1/\rho_0^2 - \Pi$ space. Those points that lie below the critical power curve correspond to the propagation of the HGB with self-focusing whereas the points lying above the critical curve result in oscillatory or steady defocusing of HGB. Points on the critical curve lead to the propagation of the HGB in self-trapped mode. Figure 4 depicts such graph showing that the dependence of $1/\rho_0^2$ for $n = 0$ is much weaker on the high intensity, a result consistent with earlier calculation based on variational approach (Anderson, 1978). However, initial beam width is much higher than the earlier investigations. Substantial decrease in dimensionless beam width as a function of Π is observed for higher order of n. Although the results corresponding to $n = 4$ in
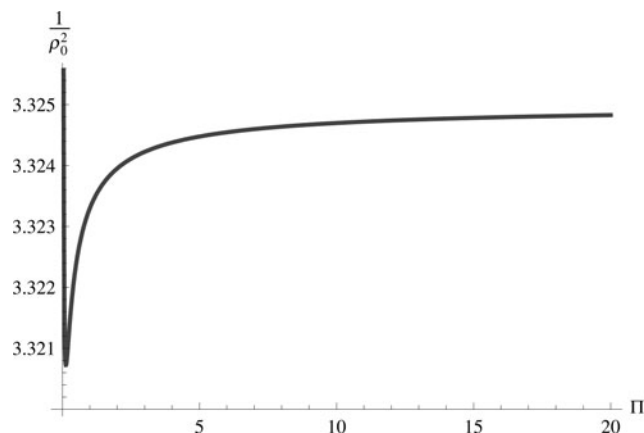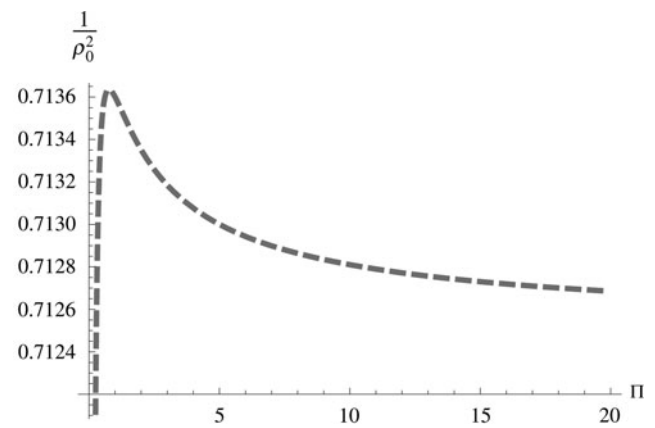
Figure 5 are quite similar to those obtained for $n = 0$, however, the following departure is observed:

Dependence of $1/\rho_0^2$ as a function of Π is initially fast which becomes slow for large Π. However, contrary results are observed for $n = 2$ in Figure 5, where $1/\rho_0^2$ reaches maximum followed by steady decrease with Π. The result is quite similar to those of Misra and Mishra (2009).

It is worthnoting that critical power curves exhibit some distinct features for different values of n, particularly for odd values of n. Both qualitatively as well as quantitatively different characteristics are obtained in critical curves. There is a sharp contrast witnessed for even and odd values of n shown in Figures 4–8 where $1/\rho_0^2$ as a function of Π is displayed. Critical curves shown in Figures 7 and 8 are quite similar but with different trapped powers. However, behavior for $n = 1$ in Figure 6 is quite astonishing and interesting when significant departure is observed. There is a significant decrease in $1/\rho_0^2$ with Π till minimum value is obtained. This is followed by increase in Π and finally exhibit a saturation behavior with Π (independent of Π).

Fig. 6. Dependence of the dimensionless initial beam width (ρ_0) as a function of Π with $n = 1$.

Fig. 8. Dependence of the dimensionless initial beam width (ρ_0) as a function of Π with $n = 5$.

5. CONCLUSIONS
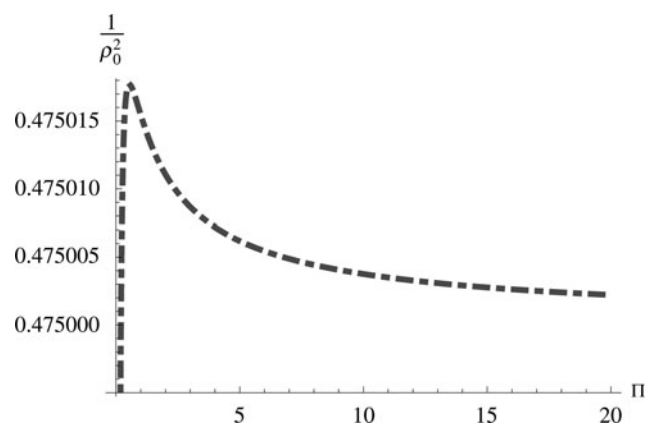
In the present investigation, authors have studied the self-focusing and self-phase modulation of laser beam with relativistic and ponderomotive nonlinearities. Equation for normalized beam width and phase is derived using variational approach. When free space propagation term is not considered, the beam defocuses for all values of n. However, when relativistic as well as ponderomotive nonlinearities alongwith free space term are taken into account, oscillatory self-focusing is observed over several Rayleigh lengths. This result is in contrast to those of Sodha *et al.* (2009*a*). Phase exhibit peculiar behavior for different values of n as it becomes positive as well as negative. Lastly, we have studied some distinct features of critical power curves for even and odd values of n. Substantial decrease in dimensionless beam width (ρ_0) as a function of Π is also observed for higher orders of n.

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