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The distribution of urine deposited on a pasture from grazing animals

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SUMMARY

Intensive agricultural production practices are known to cause far-reaching effects on water quality. The current paper addresses and quantifies these effects caused by high stocking rates.

A set of stochastic difference equations describing the development of the proportion of a grazed field either unaffected by urine deposition, or affected by multiple (1, 2, …) urine depositions is described. A solution to this set of equations is found for the expected value of multiple (0, 1, 2, …) urine depositions, and the variances of these depositions. It is assumed that an animal voids urine with a Poisson probability distribution, and that each urine deposition covers a random area with a Gaussian probability density. Given these reasonable assumptions, the probability distributions for each multiplicity of patch distribution can be found numerically.

The utility of the results obtained is illustrated for a problem in assessing the nitrogen (N) pollution of ground water from different grazing strategies. It is demonstrated quantitatively that mob stocking (typical of winter management regimes in New Zealand) is often caused by rotational grazing. The latter is often used to optimize grass growth and intake, especially in winter. This increases (more than linearly) the level of N pollution in ground water. This is because of the increased frequency of multiple urine depositions, i.e. more than one urine deposition on the same patch of land in a short time.

INTRODUCTION

Animals grazing on pasture excrete urine back onto the pasture in patches. The areas of these patches depend on the volume of urine excreted, and the number of urine patches depends on the frequency of urination and on the number of animals present (the stocking rate); see, for example, Tilman *et al.* (2002).

There is interest in the frequency distribution of urine patches over a field for two reasons. The organic matter from animals forms an important component of plant nutrition, and the concentration of N in a urine patch may be sufficient to travel through the soil to pollute the water table. When overlap of urine patches occurs the increase in N concentration may be a factor in increasing pollution, depending on the other factors operating such as plant N uptake.

Clearly, the distribution of N over a field resulting from urine patches will depend on factors such as the type of animal and the stocking rate. Each of these factors is under the control of the farmer and may be manipulated in an effort to control the distribution of N and the consequent pollution. However, this evaluation requires a quantitative description of the relationship between these variables and the frequency of N concentration over the field. This problem has been considered by a number of authors who have measured the distribution of dung and urine patches in a grazed field (Hirata *et al.* 1991; Saunders 1984). Petersen *et al.* (1956) found a negative binomial distribution to be a suitable approximation to the frequency distribution of the area of a field covered by excreta over a long period of time.

New Zealand farmers typically adopt one of two grazing systems, particularly over the winter period. Animals may be set stocked, which refers to the practice of grazing animals at low stocking rates (two cows/ha) for an extended period of time (3 months).

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Alternatively, animals may be mob stocked (also called rotational grazing), where animals are grazed at very high stocking rates (up to 900 cows/ha) for a short time (1 day). A farmer may adopt different grazing strategies in different seasons. Mob stocking is generally employed in the winter, so as to optimize feed intake.

A derivation of the frequency distribution of the area of urine patches in a grazed field must deal with the fact that each urine deposition will have a randomly distributed area, and with the problem of overlaps whereby a second urine event is deposited over some fraction of an earlier deposit. Clearly, the frequency of overlap will depend on the stocking rate of the animals.

In the current paper, a direct relationship between the type of animal grazed, the characteristics of urine deposition and the stocking rate with the frequency of urine deposition in a grazed field is derived. Cattle or sheep are included, with appropriate parameter differences, but data and parameters appropriate for cattle are normally used in the examples below. The derivation deals with urine patch overlaps in a natural way. The current paper deals only with the spatial distribution problem of urine concentration. There is also a temporal effect due to climatic factors, which is not considered in the current paper.

N deposited as urine on pasture may be evaporated, taken up by plants or leached. The amount leached will be subject to variation in temperature, which drives evaporation and plant growth, and rainfall which affects plant growth and leaching. Thus, the effect of a urine overlap will depend on the time difference between the first deposition and the second. For example, in a urine deposition at time zero the grass will be able to absorb a certain amount of N before a second urine deposition overlaps at a later time. Further, there will be some evaporation between depositions, this being at a rate proportional to the amount of urine present. Hence, the results in the current paper provide an upper bound on the actual effect of depositions in practice.

THE EVOLUTION OF A PROBABILITY DENSITY FOR MULTIPLE URINE DEPOSITIONS

It is supposed that an animal urinates randomly over time, and each of these urinations covers a proportion of the area of grazing z_t. It is assumed that the proportion of the area of the field not covered by urine at time t is $y_t^{(0)}$, the proportion of the area of the field covered by a single urine patch at time t is $y_t^{(1)}$, and the proportion of the area of the field covered by m overlapping urine patches at time t is $y_t^{(m)}$. Then, assuming that at time $t=0$ there are no urine patches present ($y^{(0)}_{t=0}=1$; $y^{(m)}_{t=0}=0$, $m>0$), the proportion of the area covered by urine patches with different degrees of overlap is given by the following set of difference equations. This set of equations is constructed by noting that the frequency of overlap (from zero to m) depends on the area of urine in each deposition and on the relative areas of the field already covered by urine patches with different degrees of overlap. Accordingly, letting z_t be the random variable describing the proportion of the field covered by one urine event at time t gives:

$$\begin{aligned}
&y_{t+1}^{(0)}=(1-z_t)y_t^{(0)}\\
&y_{t+1}^{(m+1)}=(1-z_t)y_t^{(m+1)}+z_t y_t^{(m)}\\
&0\leqslant z_t\leqslant 1\\
&y_0^{(0)}=1\\
&y_0^{(m)}=0,\quad m\geqslant 1
\end{aligned}\tag{1}$$

Equation (1) is a system of stochastic difference equations that does not have a simple solution. However, taking expected values of Eqn (1):

$$\begin{aligned}
&E[y_{t+1}^{(0)}]=E[(1-z_t)y_t^{(0)}]\\
&E[y_{t+1}^{(m+1)}]=E[(1-z_t)y_t^{(m+1)}+z_t y_t^{(m)}]\\
&y_0^{(0)}=1{\cdot}0\\
&y_0^{(m)}=0,\quad m\geqslant 1
\end{aligned}$$

Expanding the expectation of the function about the mean gives:

$$E[y_{t+1}^{(0)}]=(1-E[z_t])E[y_t^{(0)}]+\text{cov}(z_t,y_t^{(0)})$$

$$\begin{aligned}
E[y_{t+1}^{(m+1)}]=&(1-E[z_t])E[y_t^{(m+1)}]+E[z_t]E[y_t^{(m)}]\\
&+\text{cov}(z_t,y_t^{(m+1)})+\text{cov}(z_t,y_t^{(m)})
\end{aligned}$$

Since the area covered by one urination event at time t and the area already covered or not covered by urine events with various degrees of overlap are independent the covariance terms above are zero. This gives the original system (1) with z_t replaced by $E[z_t]$ and with $y_t^{(m)}$ replaced by $E[y_t^{(m)}]$.

Letting $\alpha=E[z_t]$ for notational convenience this set of equations has the solution:

$$E[y_t^{(0)}]=(1-\alpha)^t$$

$$E[y_t^{(m)}]=\begin{cases}\dfrac{\alpha^m(1-\alpha)^{t-m}}{m!}\displaystyle\prod_{k=0}^{m-1}(t-k), & 1\leqslant m\leqslant t\\[2ex] 0, & m>t\end{cases}\tag{2}$$

These results hold for any probability density for the area of urine deposited on pasture in time t (z_t) that has an expected value.

The variance of the proportion of the field covered by urine deposition of different degrees of overlap is similarly expressed by using a Taylor expansion about the mean and dropping the covariance terms.

Thus:

$$\left(\sigma_{t+1}^{(0)}\right)^2 = \sigma_z^2(1-\alpha)^{2t} + \left(\sigma_t^{(0)}\right)^2(1-\alpha)^2 + \sigma_z^2\left(\sigma_t^{(0)}\right)^2$$

$$\begin{aligned}\left(\sigma_{t+1}^{(m+1)}\right)^2 &= \left((1-\alpha)^2+\sigma_z^2\right)\left(\sigma_t^{(m+1)}\right)^2 + (\alpha^2+\sigma_z^2)\left(\sigma_t^{(m)}\right)^2 \\ &\quad + \left[(1-\alpha)^2\binom{t}{m}^2 + \alpha^2\binom{t}{m+1}^2\right] \\ &\quad \times \sigma_z^2\alpha^{2m}(1-\alpha)^{2(t-m-1)}, \\ \left(\sigma_t^{(m)}\right)^2 &= 0, \quad m>t \\ \left(\sigma_{t=0}^{(m)}\right)^2 &= 0, \quad m>0.\end{aligned} \tag{3}$$

where for example

$$E[y_t^{(1)}y_t^{(0)}] = \frac{(\alpha(1-\alpha)-\sigma_z^2)}{\sigma_z^2}\left[((1-\alpha)^2+\sigma_z^2)^t - (1-\alpha)^{2t}\right]$$

The first equation in (3) has the solution:

$$\left(\sigma_t^{(0)}\right)^2 = [(1-\alpha)^2+\sigma_z^2]^t - (1-\alpha)^{2t} \tag{4}$$

The second set of equations in Eqn (3) has no closed form solution, but a solution can be found iteratively. However, Eqn (4), which is the variance of the proportion of the area not covered by urine patches is also by symmetry the variance of the area covered by urine patches of all degrees of overlap $0 \ldots m$.

The variance of the proportion of the area not urinated on increases through time to a maximum before falling. This time of maximum variance t^* is given by:

$$t^* = \frac{\ln\left\{\dfrac{2\ln(1-\alpha)}{\ln[(1-\alpha)^2+\sigma_z^2]}\right\}}{\ln\left\{\dfrac{(1-\alpha)^2+\sigma_z^2}{(1-\alpha)^2}\right\}} \tag{5}$$

For the range of stocking rates and parameters considered here, the time of maximum variance occurs between 0 and 1 h of grazing. After this time the variance declines steadily.

Measurements of the frequency distribution of the area covered by single urine depositions have not been reported. If this area is assumed to be Gaussian distributed and the frequency of the number of urination events (n) through time by one animal is Poisson with parameter λ, the area covered by urine from a single animal will be described by a distribution given by

$$P[z_t] = \sum P[z_t|n,\lambda]P[n,\lambda] = \sum_n \frac{\lambda^n e^{-\frac{\lambda(z_t-\alpha)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}\,n!} \tag{6}$$

In this case, the expected value of z_t above is $\lambda\mu$ and the variance is $\lambda\sigma^2$ where λ is the Poisson parameter, μ is the average area of a urine patch and σ^2 is the variance of the area of a urine patch. Where there are s animals being grazed, the expected value of z_t is $s\lambda\mu$ with variance $\lambda\sigma^2s^2$.

RESULTS

To illustrate the issues, consider two grazing strategies involving 360 cows grazed over 100 ha for a typical 150-day winter grazing period. Strategy (A) makes one rotation of the grazing area, shifting the cows to fresh grazing every day. To fit into the area available and the time of 150 days means grazing the cows on 0·66 ha each day, a stocking rate of 450 cows/ha. Strategy (B) involves grazing the same animals at a lower stocking rate of 180 cows/ha. Thus strategy (B) grazes 1·67 ha/day. Over the 150-day grazing period these cows will return to graze the same area every 2 months. Thus, there will be 2·5 rotations over 150 days for strategy (B). A time of 60 days between each grazing permits the assumption that there is no practical effect from urine deposition in the previous grazing. This is justified by it being the lowest time between two overlaps for which the residual urine-N left from the first deposition is sufficiently small to be ignored.

Applying the result of Eqn (2) shows that, in each 24 h period under strategy (B), the animals will deposit one urine patch on an average of $0·0825 \times 1·67$ ha or 1378 m^2, two overlapping urine patches on an average of 60 m^2, and three overlapping urine patches on 1·6 m^2. This will deposit an average of 27·1 kg N/day into the ground water, or 4065 kg N over the 150-day grazing period.

The same calculations for strategy (A) show that each 24 h period will deposit $0·1812 \times 0·66$ hectares or 1200 m^2 of single urine patch, 130 m^2 of double urine patches and 9 m^2 of triple urine patches. This equates to 28·6 kg of N/day into the ground water, or 4290 kg N over the 150-day grazing period.

Thus, the grazing strategy that increases the frequency of overlapping urine patches over a short period of time increases the expected deposition of N pollution into the ground water, in this case by a maximum of 225 kg (5%) over the 150-day grazing period. It should be noted that it is not uncommon for farmers to graze animals at twice the rate of the 450 cows/ha used in the current example.

Williams & Haynes (1994) found that urine deposited by cattle covered an area of between 0·38–0·42 m^2, and that deposited by sheep covered an area of 0·04–0·06 m^2. Afzal & Adams (1992) measured an urination rate of $10·9 \pm 1·8$/day for cattle and urine patch areas at $0·49 \pm 0·035$ m^2. Petersen *et al.* (1956) reported that dairy cows averaged eight urinations/day and that the average area of each urine patch was 0·3 m^2.

Adopting an urination rate of $10/24 = 0·42$ events/h with an average urination patch area of 0·5 m^2

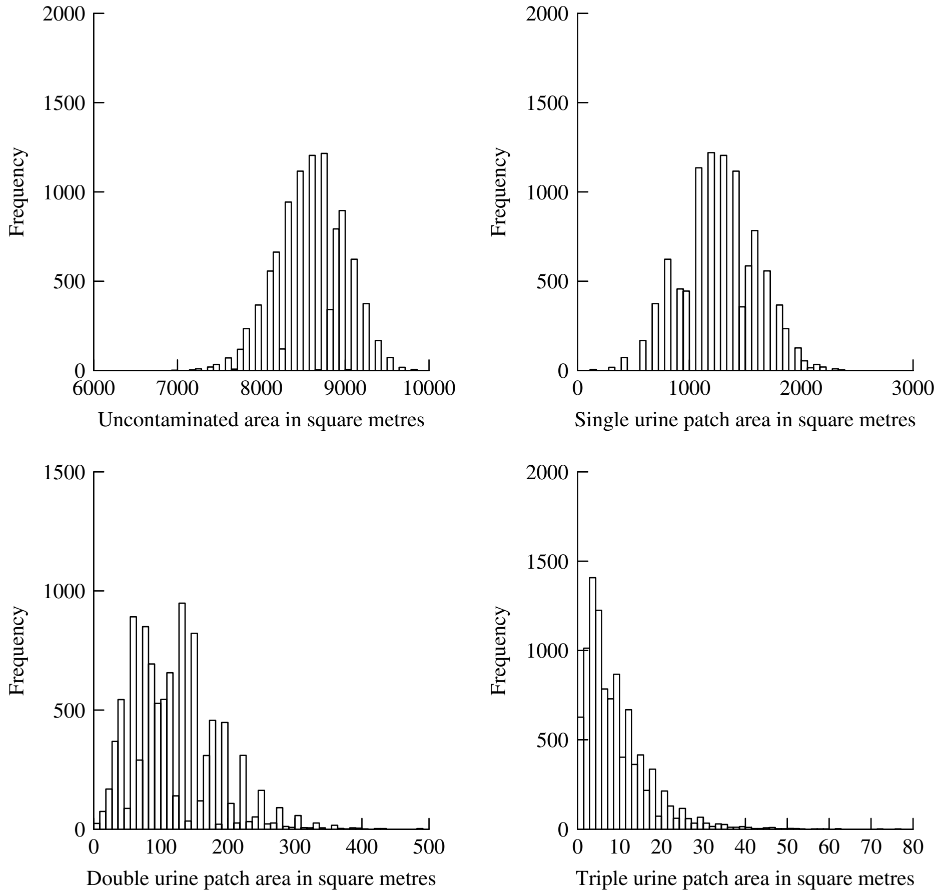

Fig. 1. Histograms for the areas (metres squared) of a 1 ha field covered by uncontaminated pasture, and pasture contaminated by a single urine patch or by double or triple overlapping urine patches when grazing at a stocking rate of 300 cows/ha for 24 h.

with standard deviation of $0{\cdot}035\ \mathrm{m}^2$, the frequency distributions for the proportion of the area uncontaminated, or contaminated by one urine patch or two overlapping urine patches, is shown in Fig. 1 for a grazing time $t = 24$ h.

None of these frequency distributions is Gaussian. The frequency distribution for uncontaminated pasture is significantly ($P < 0{\cdot}01$, using the approximate test given by Snedecor & Cochran 1967) negatively skewed, so that there are more frequent higher values encountered than would be expected if the distribution were Gaussian. The frequency distribution for pasture contaminated by one urine patch has significant ($P < 0{\cdot}05$, using the approximate test given by Snedecor & Cochran 1967) negative kurtosis, meaning that there are more values closer to the mean than would be the case if the distribution were Gaussian. The skewness for the distributions describing the proportion of two and three urine overlaps is obvious.

The expected values for the area of a field contaminated by urine are easily calculated from Eqn (2) for any given animal type or stocking rate. The expected value of the area of a 1 ha field not contaminated by urine, or contaminated by one or two overlapping urinations over a 24 h period is given in Fig. 2 for a series of stocking rates for dairy cows. Mob grazing at the high stocking rates (100 to 900 cows per hectare) considered is typical of winter grazing management practices in New Zealand. The effect can be seen in the increase of the proportion of overlapping urine patches.

The maximum variance of uncontaminated pasture calculated from Eqn (5) shows $t^* < 1$ h for all feasible stocking rates. Since the variance at $t = 0$ is zero by definition this means that the maximum variance occurs at time $t = 1$, or at the first time step (1 h in these examples).

A second application of the above equations lies in assessing the transfer of nutrients within a grazed

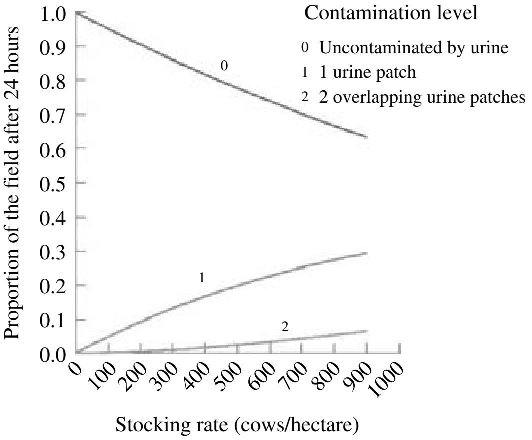

Fig. 2. The proportion of a field having one, two or three urine patches deposited on the surface over a 24-h period for a range of cattle stocking rates. Results calculated from Eqn (3) using $\lambda = 0{\cdot}42$ urinations per h and $\mu = 0{\cdot}5$ m^2.

field. Ideally, soil nutrients should be spread evenly over the field, since this will promote the best grass growth. However, in the short term, animal excreta will concentrate nutrients into patches. An important question is to decide what period of time and what stocking rate of animals is required to, for example, cover proportion x of the area of the field. Manipulating Eqn (2) for the expected value for the proportion of the field covered in urine (of various degrees of overlap) by time t gives:

$$t = \frac{\ln(1-x)}{\ln(1-s\lambda\mu)} \tag{7}$$

For example, Eqn (7) gives the expected time to cover 0·9 of a 1 ha field in urine patches, grazing at 500 cows/ha, to be 218 h, or 9 days.

A similar calculation shows that if a farmer operates a 30-day grazing rotation over 150 days of winter then animals will spend 5 days grazing the same ground. If the farmer has a stocking rate of 508 cows/ha then there will be an average of 0·75 of the area covered in urine (at least one patch).

DISCUSSION

The effect of N pollution of ground water due to different grazing strategies can be evaluated using the current results. Differences may arise because overlapping urine patches introduce more N to the soil than can be taken up by the plant. The N not taken up by the plant is available to leach into the ground water. A typical urination event deposits the equivalent of 500 kg N/ha on 0·5 m^2 (Afzal & Adams 1992). Nitrogen leaching in the winter is significant if it is assumed that 0·35 of the N in a single urination event reaches the ground water after plant uptake and evaporation (Barraclough *et al.* 1984). This equates to 175 kg/ha. A double urination event will deposit twice the amount of N on the soil surface, i.e. 1000 kg/ha. If the plant uptake of N is saturated by the first urination event, most of the extra N in the second urination event will be volatilized or leached to enter the ground water (Whitehead 1995). If it is assumed 0·70 of the additional urine added in a double urine patch reaches the ground water, then the amount of N entering the ground water will then be 525 kg/ha when two urine patches overlap. For a triple overlap this figure will be 825 kg/ha.

These calculations are dependent on the assumptions regarding the amount of extra N in multiple urine depositions reaching the ground water, and on the rate of urination by the animal and on the average area covered by these urinations.

Nutrient transfer through stock camps is also of concern. The degree of this can be calculated by knowing the area of the stock camp, the amount of time the animals spend on and off the stock camp and the frequency of urination at each time. With this information, the above equations can be used in the appropriate way to estimate the urine deposition on the surface. Any circumstances which result in animals spending different proportions of time in different parts of a field can be treated in the same manner.

The current example shows the relative importance of grazing strategy in the management of N pollution. By adopting a grazing strategy that increased the stocking rate from 180 cows/ha to 450 cows/ha, the N pollution in the ground water increased on average by 5% due to the increase in multiple overlapping urine patches. However, a farmer opting for a grazing policy with a lower stocking rate may also opt for a lower level of pasture utilization, and a lower income. Furthermore, awareness of stock pollution can assist with determining the upper level of fertilizer application, noting that there is some uptake by pasture of elements from urine depositions; see for example Williams *et al.* (1989). Nevertheless, the savings in N pollution of the ground water appear to be worth consideration if implemented on a large scale across a number of farms. It has been shown that, if the stock is held in high concentrations, albeit with the overall average stocking rate being unchanged, this will lead to increased N leaching with consequences for ground-water quality. The cumulative effect of neighbouring farms adopting similar practices could lead to significant deterioration in water quality. A balance should be sought between low overlap in deposition and good grass utilization so as to optimize the overall outcomes. The current work suggests that policy should be considered that addresses this issue.

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