Macroeconomic Dynamics, **19**, 2015, 1045–1073. Printed in the United States of America.
doi:10.1017/S1365100513000692

THE TAYLOR PRINCIPLE AND (IN-) DETERMINACY WITH HIRING FRICTIONS AND SKILL LOSS

ANSGAR RANNENBERG
Macroeconomic Policy Institute, Hans-Böckler Foundation

We introduce skill decay during unemployment into a New Keynesian model with hiring frictions and real-wage rigidity. Plausible values of quarterly skill decay and real-wage rigidity turn the long-run marginal cost–unemployment relationship positive in a "European" labor market with little hiring but not in a fluid "American" one. If the marginal cost–unemployment relationship is positive, determinacy requires a passive response to inflation in the central bank's interest feedback rule if the rule features only inflation. Targeting steady-state output or unemployment helps to restore determinacy.

Keywords: Monetary Policy Rules, Taylor Principle, Determinacy, Skill Decay

1. INTRODUCTION

The Taylor principle states that, in response to an increase in inflation, the central bank should eventually increase the nominal interest rate more than one for one. The conventional wisdom in monetary economics says that to ensure a unique and stable equilibrium, monetary policy should follow the Taylor principle [Taylor (1993)]. We show that an active monetary policy may instead induce indeterminacy if unemployed workers lose a fraction of their skills per quarter of unemployment, the real wage responds only imperfectly to changes in a worker's skill level, and labor market flows are low.

Specifically, we add skill decay during unemployment along the lines of Pissarides (1992) to the New Keynesian model with hiring frictions and real-wage rigidity of Blanchard and Gali (2010). With skill decay, a persistent increase in unemployment lowers the average productivity of previously unemployed, newly hired workers by increasing average unemployment duration. With real-wage rigidity, the lower productivity of the newly hired is only partly reflected in a reduction of their real wage. Hence skill decay and real-wage rigidity create a channel via which an unemployment increase tends to increase the unit labor cost of the newly hired and thus inflation. For some calibrations, this channel is sufficiently

I would like to thank Alan Sutherland, Andrew Hughes Hallett, Charles Nolan, George Evans, Gregory de Walque, Jens Iversen, Michael Krause, Stephane Moyen, Raf Wouters, and two anonymous referees for valuable comments and suggestions. The usual disclaimer applies. Address correspondence to: Ansgar Rannenberg, International Business-Cycle Analysis and Economic Policy Unit, Hans-Böckler-Stiftung Macroeconomic Policy Institute (IMK), Hans-Böckler-Str. 39, 40476 Düsseldorf, Germany; e-mail: Ansgar-Rannenberg@boeckler.de; www.imk-boeckler.de.

© 2014 Cambridge University Press 1365-1005/14

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

strong to change the overall long-run effect of an increase in unemployment on inflation from negative to positive. In this case, an expectation-driven, sufficiently persistent increase in unemployment ultimately increases inflation. If the central bank responds more than one for one to inflation, the real interest rate will increase, thus lowering demand and validating the increase in unemployment: hence there is a self-fulfilling prophecy. Correspondingly, we find that a reversal of the inflation–unemployment relationship almost always requires a passive response to inflation if the central bank responds only to current, expected future, or lagged inflation.

Plausible values of quarterly skill decay and real-wage rigidity suffice to generate a positive long-run inflation–unemployment relationship and thus a failure of the Taylor principle to ensure determinacy if the job-finding probability takes a low "European" value. By contrast, for an "American" calibration with a high job-finding probability, the long-run marginal cost–unemployment relationship never becomes positive for plausible values of skill decay, even if the real wage is perfectly rigid. Correspondingly, a coefficient on inflation larger than one guarantees determinacy.

Furthermore, adding a negative response to unemployment or a positive response to the deviation of output from its steady state to the policy rule generally helps to deliver determinacy. Such a policy helps to ensure that an increase in unemployment will eventually cause a real interest rate decline, regardless of whether the long-run inflation–unemployment relationship is positive or negative.

Our results extend an evolving literature that argues that an active monetary policy may induce indeterminacy if the interest rate set by the central bank has some indirect or direct effect on marginal cost. Such a channel may arise because the interest rate affects capital accumulation, job creation in models with matching frictions, or the cost of working capital needed to fund the wage bill. Most of these contributions find that an active monetary policy may induce indeterminacy if the nominal interest rate responds only to expected *future* inflation, whereas responding to current inflation remains stabilizing.[1] In contrast, Sveen and Weinke (2005, 2007), who consider a model with firm-specific capital, Christiano et al. (2010), who consider a model with a working capital constraint applied to labor and raw material inputs, and Kienzler and Schmid (2013), who add an ad hoc feedback mechanism from actual output to natural output to the basic New Keynesian model, find that even an active response to current inflation may induce indeterminacy. Kurozumi and Van Zandweghe (2010) consider a model with hiring frictions equivalent to Blanchard and Gali (2010) but with Nash-bargained wages, and find that for some calibrations, an active response to current inflation induces indeterminacy, whereas for most calibrations they consider, only an active response to expected inflation creates such problems. Unlike the case in the model developed in the following, however, in these contributions the specific mechanism rendering the Taylor principle ineffective does not imply a reversal of the long-run marginal cost–unemployment relationship.

Furthermore, Bilbiie (2008) finds that if a sufficiently large fraction of households do not participate in asset markets, the central bank has to respond passively

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

to (current or expected future) inflation, whereas Zubairy (in press) finds that a response to output in addition to inflation becomes necessary for determinacy in the presence of deep habits. However, the mechanism driving these result relies on the "aggregate demand" side of the respective models and is thus very different from ours. Esteban-Pretel and Faraglia (2010) introduce skill decay during unemployment into a monetary model but do not examine the implications for determinacy, and also follow a modeling strategy different from ours. Finally, Tesfaselasie and Schaling (2009) investigate how determinacy and E-stability in the original Blanchard and Gali (2010) model depend on the cost of hiring.

The remainder of the paper is structured as follows: Section 2 derives the model. Section 3 shows how the long-run effect of a permanent increase in unemployment on marginal cost is affected by the introduction of skill decay. Section 4 explores the conditions for determinacy and how they are affected by skill decay. Section 5 concludes.

2. THE MODEL

This section introduces skill decay during unemployment into Blanchard and Gali's (2010) sticky price model with hiring frictions.

2.1. Households

The economy is populated by a continuum of infinitely lived households. A household consists of a continuum of members who may be employed or unemployed but who are all allocated the same level of consumption. The household's period-t income derives from total wage payments $W_t^A N_t$ earned by employed household members N_t, with W_t^A denoting their average real wage; nominal interest payments i_{t-1} on holdings of a nominal risk-less bond; and firms' profits F_t. The household allocates its income to buying a CES basket of consumption goods C_t and the riskless bond B_t to maximize

$$E_t\left[\sum_{i=0}^{\infty}\beta^i U_{t+i}^h\left(C_{t+i}-hC_{t+i-1}\right)\right],\ \text{with } U_t^h\left(C_t-hC_{t-1}\right)=\log\left(C_t-hC_{t-1}\right),$$

subject to the budget constraint

$$W_t^A N_t+\frac{B_{t-1}}{P_t}(1+i_{t-1})+F_t\geq C_t+\frac{B_t}{P_t},$$

where β, $U_t^h\left(C_t-hC_{t-1}\right)$, h, and P_t denote the utility discount factor, the per-period utility function of the representative household, the degree of internal habit formation, and the price level of the CES basket, respectively.

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

2.2. Firms

There are two types of firms. Final-goods firms indexed by i produce the varieties in the CES basket of goods consumed by households. They use the intermediate good $X_t\,(i)$ in the linear technology:

$$Y_t\,(i) = X_t\,(i)\,.$$

The demand curve for variety i resulting from the household spreading its expenditure across varieties in a cost-minimizing way is given by $c_t(i) = C_t(p_t(i)/P_t)^{-\theta}$, where $c_t(i)$ and $p_t(i)$ denote the consumption and price of variety i, respectively, whereas $\theta > 1$ denotes the elasticity of substitution between varieties. Final-goods firms face nominal rigidities in the form of Calvo (1983) contracts; i.e., only a randomly chosen fraction $1 - \omega$ of firms can reoptimize their price in a given period. They accordingly maximize

$$E_t\left\{\sum_{k=0}^{\infty}(\omega\beta)^k\,\frac{U^h_{c,t+k}}{U^h_{c,t}}\left[\left(\frac{p_t\,(i)}{P_{t+k}}\right)^{1-\theta} - mc_{t+k}\left(\frac{p_t\,(i)}{P_{t+k}}\right)^{-\theta}\right]C_{t+k}\right\},$$

where $U_{c,t}$ denotes the marginal utility of consumption and mc_t denotes real marginal costs. The price level evolves according to $P_t^{1-\theta} = (1-\omega)(p_t^*(i))^{1-\theta} + \omega(P_{t-1})^{1-\theta}$, where $p_t^*(i)$ denotes the price set by those firms allowed to reset their price in period t.

The intermediate-goods firms employ labor to produce intermediate goods $X_t(j)$. Intermediate-goods firms operate under perfect competition and are owned by households. A fixed fraction δ of jobs are destroyed each period. Thus, employment of firm j evolves according to $N_t(j) = (1-\delta)N_{t-1}(j) + H_t(j)$, where $H_t(j)$ denotes the amount of hiring in firm j. Aggregate hiring is accordingly given by

$$H_t = N_t - (1-\delta)N_{t-1}. \tag{1}$$

Note that the lower δ is, the more H_t will depend on the change of employment as opposed to the level. The number of job seekers at the beginning of the period is defined as U_t. U_t consists of those workers who had not found a job at the end of period $t-1$ and those whose jobs were destroyed at the beginning of t:

$$U_t = 1 - N_{t-1} + \delta N_{t-1} = 1 - (1-\delta)N_{t-1}. \tag{2}$$

As in Blanchard and Gali (2010), we assume that every hire generates a cost G_t that is proportional to the productivity of a newly hired worker,

$$G_t = A_t B' x_t^{\alpha}, \tag{3}$$

where A_t denotes the average productivity of newly hired workers, to be defined later, B' is a constant, and x_t denotes labor market tightness, defined as the ratio

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

between aggregate hiring H_t and U_t:

$$x_t = \frac{H_t}{U_t}. \tag{4}$$

As shown in Blanchard and Gali (2010), assuming a hiring cost of (3) is a short-cut equivalent to assuming a constant-return-to-scale matching function and a flow cost of posting a vacancy as in the Diamond–Mortensen–Pissarides model [Mortensen and Pissarides (1994)], a route followed for instance by Kurozumi and Van Zandweghe (2010). G_t increases in x_t because if hiring is high relative to the number of job seekers, it takes longer on average to fill a vacancy, thus increasing the average cost of hiring a worker. We interpret labor-market tightness x_t as the probability of an unemployed person moving into employment in period t.

Following Pissarides (1992), we assume that the productivity of a newly hired worker is the product of exogenous technology A_t^P and his skill level. An unemployed worker loses a fraction $\delta_s \in [0, 1]$ of his skill per quarter of his unemployment spell. Hence the skill level of a worker with unemployment spell i is denoted by β_s^i, where $\beta_s = 1 - \delta_s$ and $\delta_s \in [0, 1]$. i equals zero if the newly hired worker lost his previous job in period t, one if he lost his job in period $t - 1$, and so on. Thus, the productivity of a worker with unemployment duration i is given by $A_t^P \beta_s^i$. The idea that a worker's skill may decay during unemployment is already present in Phelps (1972) and can be supported by two lines of evidence. First, there is evidence in favor of a negative effect of the length of the unemployment spell on the probability that an unemployed person moves into employment.[2] According to Jackman et al. (1991), an important reason might be the belief of employers that the long-term unemployed lack vital skills and work habits. They cite various studies finding that morale and motivation decline the longer a person remains unemployed.[3] Second, there is evidence saying that the difference between the wage a formerly unemployed worker earns upon reemployment and the wage he earned in his previous job is negatively related to the length of the unemployment spell.[4] These findings suggest that the unemployed become less attractive to a potential employer the longer their unemployment spell lasts and are thus indicative of skill decay during unemployment.

We assume further, following Pissarides (1992), that the unemployed regain all their skills after one quarter of employment, that when intermediate-goods firms make the decision whether to hire or not and thus pay the hiring cost G_t, they know the state of exogenous technology A_t^P but not the type of worker with whom they are going to be matched, and that they meet workers according to the share of these workers among job seekers. Furthermore, for simplicity, we assume that a firm does not hire individual workers, but only a group of workers sufficiently large for the distribution of skills in the group to match the distribution of skills in the job-seeking population U_t. This ensures that the average skill level of the group hired by the firm equals the average skill level in the job-seeking population.[5] This assumption implies that job seekers with different unemployment durations

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

have the same job-finding rate, which is in contrast to the evidence of negative duration dependence of the probability of leaving unemployment citedpreviously. However, during economic downturns, it does imply a low average job-finding rate, a high average unemployment duration, and thus a low average skill level of job seekers. The countercyclical unemployment duration would also arise in a more realistic model with negative duration dependence of the job-finding rate.[6]

We denote the average skill level in the job-seeking population as A_t^L, implying that the average productivity of newly hired workers is given by

$$A_t = A_t^P A_t^L, \tag{5}$$

whereas A_t^L is given by

$$A_t^L = \sum_{i=0}^{\infty} \beta_s^i s_t^i, \tag{6}$$

where s_t^i denotes the share of those unemployed for i periods among job seekers. Note that $A_t^L < 1$ if $\delta_s > 0$, whereas for $\delta_s = 0$, we have $\beta_s = A_t^L = 1$. The shares of the various types of workers among the total number of job seekers U_t is denoted as s_t^i, and is defined by

$$s_t^i = \frac{\delta N_{t-i-1} \prod_{j=1}^{i} \left(1 - x_{t-j}\right)}{U_t}. \tag{7}$$

Note that δN_{t-i-1} represents all the workers who had a job in period $t-i-1$ but lost it in period $t-i$, whereas $\Pi_{j=1}^{i}(1-x_{t-j})$ represents the fraction of the workers laid off in period $t-i$ who are still unemployed at the end of period $t-1$. Hence the numerator consists of all workers laid off in period $t-i$ and still unemployed in period $t-1$.

As in Blanchard and Gali (2010), who in turn follow the seminal contribution of Hall (2005), we assume that the real wage of a worker is rigid. The wage W_t^i of a worker who has been unemployed for i periods is given by

$$W_t^i = \Theta' \left(\beta_s^i\right)^{1-\gamma} \left(A_t^P\right)^{1-\gamma_P}, \tag{8}$$

with $0 \leq \gamma \leq 1$, $0 \leq \gamma_P \leq 1$, and $\Theta' > 0$. Hence, for $\gamma > 0$ or $\gamma_P > 0$, an increase in the worker's skill level or an increase in technology will cause a less-than-proportional increase in his real wage. Although the degree of real-wage rigidity with respect to technology γ_P does not actually matter for the determinacy results that are the subject of this paper, the degree of rigidity with respect to the worker's skill level γ does. By assumption, Hall's (2005) "fixed wage rule," as well as Blanchard and Gali's real wage schedule, always lies inside the bargaining set. This implies that it neither prevents the formation of matches with a positive surplus nor results in inefficient separations.[7] Under our assumption that firms are

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

restricted to hiring a representative sample of job seekers, this condition is also satisfied in our model.

Hall (2005) interprets his constant wage rule as a social norm preventing employers from lowering wages, and, what is more, from paying lower wages to newly hired employees than to their existing workforce, because doing so would hurt morale and thus productivity. A growing survey-based literature supports the existence of downward nominal and real-wage rigidity.[8] On the other hand, the econometric evidence regarding the flexibility of the wage of the newly hired is less clear.[9] It is worth stressing that the degree of real-wage rigidity with respect to technology γ_P, which is consistent with a wide range of wage volatilities for new hires, does not affect the results of this paper.[10] We will in any case allow γ to vary between 0 and 1 in Section 4.

The average real wage of the group the firm hires is given by

$$W_t = \Theta' \left(\sum_{i=0}^{\infty} \beta_s^{i(1-\gamma)} s_t^i \right) \left(A_t^P \right)^{1-\gamma_P}. \tag{9}$$

Θ' is calibrated to support a desired steady-state combination of x, δ, and N, as shown in Appendix A. For future reference, we denote the skill-dependent part of the average real wage as

$$W_t^L = \left(\sum_{i=0}^{\infty} \beta_s^{i(1-\gamma)} s_t^i \right). \tag{10}$$

The intermediate-goods firms will hire additional groups until the hiring cost of an additional group equals the present discounted value of the profits generated by this group. Thus we have

$$G_t = \frac{P_t^I}{P_t} A_t^P A_t^L - W_t + E_t \left[\sum_{i=1}^{\infty} (1-\delta)^i \beta^i \frac{U_{c,t+i}^h}{U_{c,t}^h} \left(\frac{P_{t+i}^I}{P_{t+i}} A_{t+i}^P - W_{t+i}^0 \right) \right],$$

where P_t^I / P_t denotes the real price of intermediate goods, whereas $\beta^i \frac{U_{c,t+i}^h}{U_{c,t}^h}$ denotes the stochastic discount factor of the representative household. The terms $\frac{P_t^I}{P_t} A_t^P A_t^L - W_t$ and

$$E_t \left[\sum_{i=1}^{\infty} (1-\delta)^i \beta^i \frac{U_{c,t+i}^h}{U_{c,t}^h} \left(\frac{P_{t+i}^I}{P_{t+i}} A_{t+i}^P - W_{t+i}^0 \right) \right]$$

represent the flow profit generated in period t (when the group has just been hired) and the present discounted value of profits generated in period $t+1$ and later, respectively. Note that because of our assumption that a worker regains all his skills after one period, the expression for the flow profit in period t is different from the expression for the flow profit in period $t+1$ and later. Rewriting this

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

equation as a difference equation and noting that the real price of intermediate-goods firms equals the marginal cost of final goods firms (hence $P_t^I/P_t = \mathrm{mc}_t$), we have

$$\begin{aligned}\mathrm{mc}_t A_t^P A_t^L &= W_t + G_t - \beta(1-\delta) \\ &\times E_t\left\{\frac{U_{c,t+i}^h}{U_{c,t}^h}\left[G_{t+1} + \mathrm{mc}_{t+1}A_{t+1}^P - W_{t+1}^0 - \left(\mathrm{mc}_{t+1}A_{t+1}^P A_{t+1}^L - W_{t+1}\right)\right]\right\}.\end{aligned} \tag{11}$$

The left-hand side represents the real marginal revenue product of a group of newly hired job seekers, which increases with the average skill level of the group A_t^L. The right-hand side comprises the net cost of adding this group to the work force. This cost increases with the real wage of the newly hired W_t and the cost associated with hiring them G_t, whereas it decreases with the present expected value of the benefit associated with hiring the group in t rather than $t+1$. This benefit consists of the future hiring costs saved (G_{t+1}) and the difference between the real profit generated by a fully skilled group (with productivity A_{t+1}^P and real wage W_{t+1}^0) and a group hired in period $t+1$ (with productivity $A_{t+1}^P A_{t+1}^L$ and real wage W_{t+1}). Hence although an increase in the skill level (the real wage) of the newly hired in period t A_t^L (W_t) will decrease (increase) the price of intermediate goods and thus the marginal cost of final goods firms mc_t, an increase in A_{t+1}^L (W_{t+1}) will increase (decrease) mc_t by reducing (increasing) the gain from hiring in period t rather than in period $t+1$.

The average productivity of the whole workforce after the newly hired A_t^A and the production functions of gross output Y_t (i.e., output including hiring costs) and consumption goods C_t are added is then given by

$$A_t^A = A_t^P\left[s_t^N A_t^L + \left(1 - s_t^N\right)\right],\ s_t^N = \frac{H_t}{N_t} = \frac{N_t - (1-\delta)N_{t-1}}{N_t}, \tag{12}$$

$$Y_t = A_t^A N_t, \tag{13}$$

$$\begin{aligned}C_t &= A_t^A N_t - B' x_t^\alpha A_t^P A_t^L H_t \\ &= A_t^A N_t - B' x_t^\alpha A_t^P A_t^L\left(N_t - (1-\delta)N_{t-1}\right),\end{aligned} \tag{14}$$

where s_t^N denotes the share of the newly hired in the workforce.

3. MARGINAL COST AND UNEMPLOYMENT IN THE PRESENCE OF SKILL LOSS

In this section we show how skill decay during unemployment may reverse the long-run relationship between unemployment and marginal cost and thus lay the ground for the discussion of the determinacy results in Section 4. Combining log-linear approximations of (1) to (14) allows us to express the percentage deviation of marginal cost from its steady state as a function of unemployment (we suppress

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

the state of technology from now on because it does not affect determinacy):

$$\widehat{\mathrm{mc}}_t = -a_1^L \widehat{a}_t^L + a_2^L E_t \widehat{a}_{t+1}^L + w_1^L \widehat{w}_t^L - w_2^L E_t \widehat{w}_{t+1}^L + h_0^{'} \widehat{n}_t + h_L^{'} \widehat{n}_{t-1}$$
$$+ h_F^{'} E_t \widehat{n}_{t+1} - h_c E_t \widehat{\mathrm{mc}}_{t+1} - \beta(1-\delta) X \left(E_t \widehat{U^h_{c,t+1}} - \widehat{U^h_{c,t}} \right), \qquad \textbf{(15)}$$

where $a_1^L,\ a_2^L,\ w_1^L,\ w_2^L,\ p_0,\ p_1,\ h_0^{'},\ h_c > 0,\ h_L^{'},\ h_F^{'} < 0,$

$$\widehat{a}_t^L = -\sum_{i=1}^{\infty} a_i^u \widehat{u}_{t-i},\ a_i^u > 0, \qquad \textbf{(16)}$$

$$\widehat{w}_t^L = -\sum_{i=1}^{\infty} w_i^u \widehat{u}_{t-i},\ w_i^u > 0. \qquad \textbf{(17)}$$

A lowercase variable with a circumflex denotes the percentage deviation of the respective uppercase variable from its steady state, with the exception of $\widehat{u}_t$, which denotes the percentage point deviation of unemployment from its steady state. The definitions of the various coefficients are displayed in Table 1. Note that although the average skill level among job seekers in period t, $\widehat{a}_t^L$, negatively affects marginal cost, the period-$(t+1)$ average skill level $E_t \widehat{a}_{t+1}^L$ enters with a positive sign because, as was discussed previously, a higher average period-$(t+1)$ skill level reduces the benefit of hiring today rather than tomorrow and thus increases marginal cost. Analogously, an increase in $E_t \widehat{w}_{t+1}^L$ lowers $\widehat{\mathrm{mc}}_t$.

With no habit formation ($h = 0$) and no skill decay (i.e., for $\delta_s = 0$), the marginal cost equation becomes

$$\widehat{\mathrm{mc}}_t = -\frac{h_0}{(1-u)} \widehat{u}_t - \frac{h_L}{(1-u)} \widehat{u}_{t-1} - \frac{h_F}{(1-u)} E_t \widehat{u}_{t+1}, \qquad \textbf{(18)}$$
$$\text{where} \qquad h_0 > 0,\ h_L, h_F < 0,$$

where the coefficients are exactly as in Blanchard and Gali (2010). Note that marginal cost depends negatively on current unemployment but positively on lagged and lead unemployment because of the effect of unemployment on the cost of hiring an additional worker. A decrease in $\widehat{u}_t$ increases period-t hiring and thus labor market tightness and the cost of hiring. A decrease in $\widehat{u}_{t-1}$ lowers period-t hiring for a given $\widehat{u}_t$ and thus period-t hiring cost. A decrease $E_t \widehat{u}_{t+1}$ increases period-$(t+1)$ hiring and hiring cost, thus increasing the benefit of creating jobs today and correspondingly reducing marginal cost today. The effects of lagged and lead unemployment increase in absolute value as the job destruction rate δ falls, because this enhances the effect of past employment on current hiring and of current employment on future hiring, respectively, as can be seen from (1). Therefore, the effect of a permanent increase in unemployment on marginal cost becomes less negative as δ declines. Nevertheless, it always remains negative, as $-\frac{h_0}{(1-u)} - \frac{h_L}{(1-u)} - \frac{h_F}{(1-u)} < 0.$[11]

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

As we will see shortly, this is not always true in the presence of skill decay and real-wage rigidity. Let us denote reductions of the skill level of the average job seeker and the average real wage caused by a one-percentage-point increase in unemployment as $a^u = \sum_{i=1}^{\infty} a_i^u$ and $w^u = \sum_{i=1}^{\infty} w_i^u$, respectively. The following proposition summarizes the properties of these reductions and their derivatives with respect to δ_s and γ:

PROPOSITION 1. *Let $a^u = \sum_{i=1}^{\infty} a_i^u$ and $w^u = \sum_{i=1}^{\infty} w_i^u$ denote the decline of the average skill level and the average real wage, respectively, caused by a permanent one-percentage-point increase in unemployment. Then it is possible to prove the following three results:*

(i) $a^u > w^u > 0$ *if and only if* $\gamma > 0$ *and* $\delta_s > 0$.
(ii) $\partial a^u/\partial \delta_s > \partial w^u/\partial \delta_s > 0$ *if* δ_s *is close to 0 and* $\gamma > 0$.
(iii) $\partial w^u/\partial \gamma < 0$ *if and only if* $\delta_s > 0$ *and* $\gamma < 1$.

Proof. See Appendix B. ■

Hence in the presence of real-wage rigidity ($\gamma > 0$) and skill decay ($\delta_s > 0$), a permanent rise in unemployment increases the ratio between the (average) wage of the newly hired and their average productivity (i). A rise in unemployment increases the share of workers with longer unemployment durations and—with $\delta_s > 0$—causes lower skill levels in the job-seeking population and thus lower real wages for the newly hired. If $\gamma > 0$, the average real wage of newly hired job seekers declines by a smaller percentage than the average skill level of newly hired workers; i.e., the unit labor cost of the newly hired increases. Furthermore, the size of the increase in the newly hired's unit labor cost increases in δ_s (ii). A higher δ_s lowers the skill level of unemployed workers with longer unemployment durations. Hence an increase in the share of this group among job seekers causes a faster decline in the average skill level and the average real wage if δ_s is higher. $\gamma > 0$ implies that the decline in the average skill level is accelerated by more than the decline in the average real wage. Finally, the size of the increase of the newly hired's unit labor cost increases with γ. The higher γ, the less a newly hired worker's real wage depends on his skill level, and the smaller the reduction in the average real wage associated with a decline of the average skill level of job seekers.

Hence skill decay in combination with real-wage rigidity creates a channel via which a permanent increase in unemployment pushes up marginal cost, and more so the higher δ_s and γ are. One can see this channel formally by writing the long-run marginal cost–unemployment relationship as

$$\lambda \widehat{\mathrm{mc}} = -\kappa \widehat{u}, \tag{19}$$

$$\kappa = \frac{\frac{h_0'+h_L'+h_F'}{(1-u)} - \left[a^u \left(a_1^L - a_2^L\right) - w^u \left(w_1^L - w_2^L\right)\right]}{(1+h_c)}\lambda.$$

A detailed derivation can be found in Appendix C. $-\kappa$ gives the effect of a permanent increase in unemployment on marginal cost. Conveniently, substituting

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

the definitions of $h_0^{'}$, $h_L^{'}$, and $h_F^{'}$ shows that $h_0^{'}+h_L^{'}+h_F^{'}$ exactly equals $h_0+h_L+h_F$ and is thus always positive and independent of δ_s. Hence only the term in the square brackets and h_c actually depend on skill loss.

The squared brackets encapsulate the "skill loss channel" from unemployment to marginal cost. It will be zero if $\delta_s = 0$, implying that $\kappa > 0$ and thus there is a negative effect of a permanent increase in unemployment on marginal cost. The first term represents the decline of the skill level of the average applicant caused by the increase in $\widehat{u}$ (a^u) times the net effect of a permanent skill level decline on marginal cost $\left(a_1^L - a_2^L\right)$. The second term represents the decline of the skill-dependent real wage caused by the increase in $\widehat{u}$ (w^u) times the net effect of a permanent decline in the skill-dependent real wage on marginal cost $(-\left(w_1^L - w_2^L\right))$. From Table 1 we obtain $a_1^L > a_2^L$ and $w_1^L > w_2^L$, because the gain from hiring today rather than tomorrow is uncertain ($\delta > 0$) and is discounted ($\beta > 0$). Furthermore, $a_1^L - a_2^L$ and $w_1^L - w_2^L$ will be quite close for reasonable calibrations. Proposition 1 would then imply that for positive δ_s and γ, the term in the square brackets is positive and increases with δ_s and γ. Thus skill decay and real-wage rigidity would indeed render the effect of unemployment on marginal cost less negative, and more so the higher δ_s and γ are. We confirm this by proving the following proposition:

PROPOSITION 2. *Let* κ, *as defined in* (19) , *be the decline in marginal cost caused by a permanent one percentage point increase in unemployment and let* δ_s *be close to zero. Then* $\partial\kappa/\partial\delta_s < 0$ *if* $\gamma > \frac{B'x^{\alpha}M\beta(1-\delta)}{1-B'x^{\alpha}M[1-\beta(1-\delta)]}$.[12] *Furthermore,* $\partial\kappa/\partial\gamma < 0$ *if and only if* $\delta_s > 0$, $\gamma < 1$ *and* $x(1-\delta)\beta + \frac{x[1-(1-\delta)\beta]}{\left[1-(1-x)\beta_s^{1-\gamma}\right]} > (1-x)\left(1-\beta_s^{1-\gamma}\right)$.

Proof. See Appendix C. ■

The conditions under which $\partial\kappa/\partial\delta_s < 0$ and $\partial\kappa/\partial\gamma < 0$ are easily fulfilled for the calibrations we will adopt later.

Hence the long-run effect of unemployment on marginal cost may turn positive if the skill decay channel [i.e., $a^n\left(a_1^L - a_2^L\right) - w^n\left(w_1^L - w_2^L\right)$], via which an increase in unemployment increases the unit-labor cost of the newly hired, starts to dominate the "hiring cost channel" of Blanchard and Gail (2010) [i.e., $\frac{h_0^{'}+h_L^{'}+h_F^{'}}{(1-u)}$]. The strength of the skill decay channel increases with δ_s if $\gamma > 0$ and with γ if $\delta_s > 0$. If the skill decay channel reverses the long-run effect of unemployment on marginal cost and thus inflation, this has consequences for the merits of the Taylor principle as a guide for monetary policy, as we show in the next section.

4. DETERMINACY

In this section we explore how the conditions for determinacy in the preceding model are shaped by skill decay and real-wage rigidity. After Section 4.1 discusses

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

the calibration of the nonpolicy parameters, Section 4.2 compares the determinacy properties of the model in the absence of skill decay with the results of Kurozumi and Van Zandweghe (2010). Section 4.3 presents conditions for determinacy if the central bank responds only to inflation, whereas Section 4.4 performs various robustness checks. In Section 4.5, we investigate the effect of adding other variables to the policy rule. The linearized model consists of the following equations:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \widehat{\mathrm{mc}}_t, \tag{M.1}$$

$$\lambda \widehat{\mathrm{mc}}_t = -a^* \widehat{a}_t^L + w^* \widehat{w}_t^L - \kappa_0^* \widehat{u}_t + \kappa_L^* \widehat{u}_{t-1} + \kappa_F^* E_t \widehat{u}_{t+1} - \lambda h_c E_t \widehat{\mathrm{mc}}_{t+1} - \beta(1-\delta) X \left(E_t \widehat{U_{c,t+1}^h} - \widehat{U_{c,t}^h} \right), \tag{M.2}$$

$$\widehat{a}_t^L = (1-x) \left[-(1-\beta_s) \frac{\widehat{u}_{t-1}}{u(1-u)} + \beta_s \widehat{a}_{t-1}^L \right], \tag{M.3}$$

$$\widehat{w}_t^L = (1-x) \left[-\left(1-\beta_s^{1-\gamma}\right) \frac{\widehat{u}_{t-1}}{u(1-u)} + \beta_s^{1-\gamma} \widehat{w}_{t-1}^L \right], \tag{M.4}$$

$$\widehat{c}_t = c_L \widehat{a}_t^L - c_0^* \widehat{u}_t - c_1^* \widehat{u}_{t-1}, \tag{M.5}$$

$$\widehat{U_{c,t}^h} = E_t \widehat{U_{c,t+1}^h} + \left(\widehat{i}_t - E_t \pi_{t+1} \right), \tag{M.6}$$

$$\widehat{U_{c,t}^h} = -\frac{1}{(1-h)(1-\beta h)} \left[\widehat{c}_t - h \widehat{c}_{t-1} - \beta h \left(E_t \widehat{c}_{t+1} - h \widehat{c}_t \right) \right], \tag{M.7}$$

$$\widehat{i}_t = \phi_\pi E_t \pi_{t+j}, \ \phi_\pi \geq 0, \ -1 \leq j \leq 1. \tag{M.8}$$

(M.1) is the New Keynesian Phillips curve, with π_t denoting the deviation of inflation from its steady state. (M.3) and (M.4) are merely quasi-differenced versions of the laws of motion of the average skill level and the average real wage [(16) and (17)]. (M.2) is the marginal cost equation and results from combining (M.3) and (M.4) with (15). (M.5) is derived by linearizing (12)–(14) and combining the resulting expressions.[13] The definitions of all reduced-form coefficients can be found in Table 1. (M.6) is the consumption Euler equation, whereas (M.7) is the marginal utility of consumption. (M.8) is the interest feedback rule of the central bank, which may be current or forward- or backward-looking. Unfortunately, we cannot establish the conditions for determinacy analytically.[14] Therefore, we solve the model numerically for a range of values of ϕ_π and other key parameters in order to check for which parameter combinations a unique and stable equilibrium exists. We use the Dynare software to solve the model.

4.1. Calibration

The calibration is displayed in Table 2. Wherever possible, the parameters are as in Blanchard and Gali (2010). We set $\beta = 0.99$ and $\lambda = 0.08$, implying that prices remain fixed on the average for about four quarters (i.e., $\omega = 0.76$), and $\theta = 6$, implying a steady-state mark-up M of 1.2. The degree of habit

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

TABLE 1. Reduced-form coefficients

$h_c = \beta(1-\delta)\frac{(1-A^L)}{A^L}$	$\Phi' = 1 - gM - (1-\gamma_P)\frac{M}{A^L}W$
$h'_F = -\beta(1-\delta)\left(\frac{\alpha g M}{\delta}\right)$	$a_i^u = \frac{1}{u(1-u)}(1-x)^i\left(\beta_s^{i-1}-\beta_s^i\right)$
$h'_0 = \left(\frac{\alpha g M}{\delta}\right)\left[1+\beta(1-\delta)^2(1-x)\right]$	
$h'_L = -\left(\frac{\alpha g M}{\delta}\right)(1-\delta)(1-x)$	
$a_1^L = 1-gM$	$w_i^u = \frac{1}{u(1-u)}(1-x)^i\left(\beta_s^{(1-\gamma)(i-1)} - \beta_s^{(1-\gamma)i}\right)$
$a_2^L = \beta(1-\delta)[1-gM]$	$\kappa_0^* = \frac{\lambda}{1-u}\left[h_0' + \frac{(1-x)}{u}\left[a_2^L(1-\beta_s) - w_2^L\left(1-\beta_s^{1-\gamma}\right)\right]\right]$
$w_1^L = \frac{M}{A^L}W,\ w_2^L = \beta(1-\delta)\frac{M}{A^L}W$	$\kappa_L^* = \frac{-\lambda h_L'}{1-u},\ \kappa_F^* = \frac{-\lambda h_F'}{1-u}$
$p_1 = \beta(1-\delta)\left[\left(\frac{1-A^L}{A^L}\right)+Mg\right]$	$a^* = \lambda\left[a_1^L - a_2^L(1-x)\beta_s\right]$
$p_0 = \beta(1-\delta)\left[\begin{array}{c}\left(\frac{1-A^L}{A^L}\right)+Mg+(1-\gamma_P)w_1^L\\ -\frac{M}{A^L}W^o(1-\gamma_P)\end{array}\right]$	$w^* = \lambda\left[w_1^L - w_2^L(1-x)\beta_s^{1-\gamma}\right]$
$X = gM + \frac{1-A^L-M\left(\Theta'-W\right)}{A^L}$	$c^L = \frac{A^L\delta(1-g)}{A^A - A^L g\delta}$
$g = B'x^\alpha$	$y^L = \frac{A^L\delta}{A^A}$
$\xi_0' = \frac{A^L[1-g(1+\alpha)]}{A^A-A^L g\delta}$	$y_0 = \frac{A^L}{A^A(1-u)}$
$\xi_1' = \frac{(1-\delta)\left[(1+\alpha(1-x))A^L g+\left(1-A^L\right)\right]}{A^A-A^L g\delta}$	$y_1 = \frac{\left(1-A^L\right)(1-\delta)}{A^A(1-u)}$

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

TABLE 2. Calibration

Parameter	"American"	"European"
β	0.99	0.99
h	0;0.8	0;0.8
λ	0.08	0.08
ω	0.76	0.76
θ	6	6
M	1.2	1.2
α	1	1
x	0.9	0.2
u	0.05	0.1
δ	0.47	0.03
B'	0.12; 0.024	0.12; 0.024
γ	[0,1]	[0,1]

formation h is allowed to take a value of 0 or 0.8, following Kurozumi and Van Zandweghe (2010). With respect to the labor market flows and the unemployment rate, Blanchard and Gali's calibration distinguishes between an American labor market, on one hand, characterized by a high job-finding probability of $x = 0.7$ with a low unemployment rate $u = 0.05$ and a high job destruction rate, and a European labor market with a high unemployment rate of $u = 0.1$ and low flows into and out of unemployment ($x = 0.25$), on the other hand. Note, however, that for a given value of x, whether u is set equal to 0.1 or 0.05 has only marginal effects on our results. Regarding the parameters pertaining to the hiring cost, α and B', $\alpha = 1$, as this is consistent with estimates of matching functions. Setting $B' = 0.12$ implies a share of hiring costs in GDP of about 1% of GDP under the American calibration. The share of hiring costs in GDP under the European calibration is always lower than under the American calibration, because x is lower. As mentioned previously, there is mixed evidence regarding the flexibility of the real wage of newly hired workers. Therefore, in every grid search conducted in the following, we set the degree of real-wage rigidity $\gamma = [0, 1]$. The interval for skill decay δ_s is $\delta_s = [0, 0.07]$. The interval for the coefficient on inflation in the policy rule is $\phi_\pi = [0, 3]$.

4.2. Comparison with Kurozumi and Van Zandweghe (2010)

We first investigate the determinacy properties of our model in the absence of skill decay. It turns out that for all calibrations we consider, $\phi_\pi > 1$ is sufficient for determinacy, regardless of whether policy targets lagged, current, or expected future inflation. This is in contrast to the results of Kurozumi and Van Zandweghe (2010), who assume Nash bargaining over wages and internal habit formation but otherwise use a model equivalent to that of Blanchard and Gali (2010).[15] Under most of the calibrations they consider in their sensitivity analysis, they find

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

that with monetary policy targeting only expected inflation, $\phi_\pi = (1, k)$, with k larger than but very close to 1. Under the remaining calibrations, $\phi_\pi = (0, 1)$ is a necessary and sufficient condition under forecast-based policies and a sufficient condition under outcome-based policies. The failure of $\phi_\pi > 1$ to guarantee determinacy is due to the fact that with costly hiring, an increase in the real interest rate associated with an increase in inflation expectations tends to increase future marginal cost (the so-called "vacancy channel" of monetary policy). As households substitute future for current consumption, current hiring and employment and thus employment carried over to the next period decline. The recovery of consumption in the next period therefore requires a costly increase in hiring above the steady state, which tends to increase marginal cost and inflation. This mechanism is captured by the positive coefficient on lagged unemployment in (18) .

However, the severely limited ability of the Taylor principle, especially for forecast-based policies, to deliver determinacy in the model of Kurozumi and Van Zandweghe (2010) appears to be partly due to the assumption of Nash-bargained wages. This assumption enhances the overall increase of marginal costs and inflation caused by an increase in hiring. Assuming an exogenous real wage, as in Blanchard and Gali (2010), strongly increases k. Furthermore, the calibration of Blanchard and Gali (2010) implies a much lower value of B' and thus a lower steady-state value of a filled job (i.e., a lower steady-state value of G) than in Kurozumi and Van Zandweghe (2010). According to Kurozumi and Van Zandweghe (2010), a lower match value enhances the success of the Taylor principle in ensuring determinacy by implying a lower elasticity of marginal cost with respect to labor market tightness. Correspondingly, when the calibration of Kurozumi and Van Zandweghe (2010) is modified so that the implied value of B' is as in Blanchard and Gali (2010), k generally increases and all but one of the cases where determinacy is consistent with $\phi_\pi = (0, 1)$ vanish.[16]

We conclude that the assumptions of Blanchard and Gali (2010) regarding calibration and wage-setting imply a substantial weakening of the "vacancy channel" as compared to the setup of Kurozumi and Van Zandweghe (2010), and thereby restore the ability of the Taylor principle to deliver determinacy for plausible values of ϕ_π. Furthermore, it can be shown that in the Phillips curve of the Kurozumi and Van Zandweghe (2010) model, a permanent increase in unemployment will lower marginal cost and inflation under all permissible calibrations, just as in the model of Blanchard and Gali (2010). Our contribution is to show that even with a weaker vacancy channel, the Taylor principle may fail to induce determinacy because skill decay generates a positive relationship between the unit labor cost of the newly hired and unemployment, which may reverse the long-run relationship between inflation and unemployment.

4.3. Pure Inflation Targeting

We now investigate the determinacy properties of the model in the presence of skill decay. For the American calibration, we find that a unique and stable equilibrium

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

TABLE 3. European calibration ($x = 0.25$) —critical values of skill decay δ_s

γ	Critical value of δ_s	Wage loss implied by a one-year unemployment spell
0.1	None	Not applicable
0.2	None	Not applicable
0.3	0.052	13.9%
0.4	0.031	7.3%
0.5	0.023	4.5%
0.6	0.018	2.9%
0.7	0.015	1.8%
0.8	0.013	1.0%
0.9	0.011	0.4%
1.0	0.01	0.0%

Note: The second column displays for each value of γ the value of δ_s for which the determinacy requirement switches from $\phi_\pi > 1$ to $\phi_\pi < 1$ (i.e., the "critical value" of δ_s in the policy rule $\widehat{i}_t = \phi_\pi E_t \pi_{t+j}$, $-1 \leq j \leq 1$. The third column displays the percentage difference between the wage a worker earned in his previous job and the wage he earns upon reemployment following a one-year unemployment spell if δ_s equals its respective critical value. The wage loss is calculated as $100\,[1 - (\beta_s^4)^{1-\gamma}]$, where $\beta_s = 1 - \delta_s$.

requires $\phi_\pi > 1$ for all values of skill decay δ_s and real-wage rigidity γ that we consider in the grid search; i.e., following the Taylor principle guarantees a unique and stable equilibrium. In contrast, $\phi_\pi > 1$ is not always sufficient to induce determinacy under the European calibration. Figure 1 reports the determinacy regions for the current looking rule and $h = 0$. Whereas for very flexible real wages ($\gamma \leq 0.1$), $\phi_\pi > 1$ is sufficient for determinacy, for $\gamma = 0.3$ and $\delta_s \geq 0.03$, the determinacy requirement switches to$\phi_\pi < 1$: The central bank now has to lower the real interest rate in response to an increase in inflation. Indeed, for every value of real-wage rigidity γ higher than or equal to 0.3, there is a *critical threshold value* of δ_s. If δ_s equals or exceeds this value, determinacy requires a passive response to inflation. As can be seen from Figure 1 and Table 3, the critical value of δ_s declines as γ increases. The determinacy regions for the backward- and forward-looking policy rules (not shown) are almost identical. In particular, the critical values of δ_s are identical across the three rules, which suggests that it is not the timing of the active response to inflation but the active response to inflation per se that induces indeterminacy. The critical values of δ_s are also unaffected by setting the degree of habit formation to $h = 0.8$.

The switch in the determinacy requirement to $\phi_\pi < 1$ as δ_s reaches its critical value is closely related to a change in the long-run relationship between marginal cost and unemployment $-\kappa$ as defined in (19) . As discussed in Section 3, this relationship is always negative for $\delta_s = 0$ (i.e., $\kappa > 0$) but in the presence of real-wage rigidity becomes less negative as δ_s increases (as then $\partial\kappa/\partial\delta_s < 0$) and may ultimately turn positive ($-\kappa > 0$). The reason is that increasing δ_s boosts the rise in

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

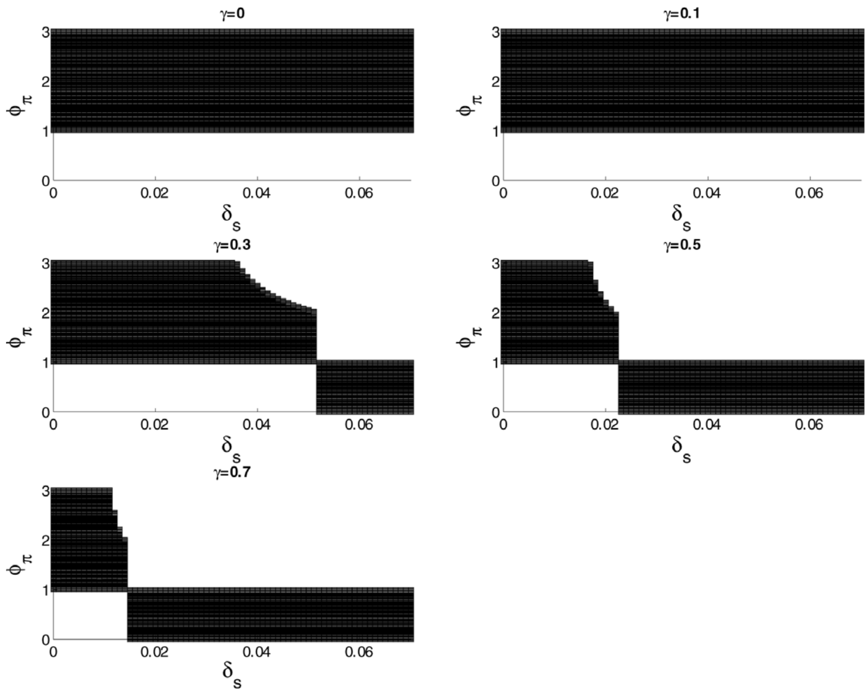

FIGURE 1. European calibration: determinacy regions for selected values of γ. The black and white area indicates regions of determinacy and indeterminacy, respectively. No explosive solutions were found. The figure reports results from a grid search over $\gamma = [0, 1]$, stepsize 0.1; $\delta_s = [0, 0.07]$, stepsize 0.001; and $\phi_\pi = [0, 3]$, stepsize 0.01, with $x = 0.25$ and $u = 0.1$. The figure changes only marginally for $u = 0.05$. The policy rule is $\widehat{i}_t = \phi_\pi E_t \pi_{t+j}$, with $j = 0$; the determinacy regions are almost identical for $j = -1$ or $j = 1$.

the newly hired's unit labor cost caused by an increase in unemployment. However, under the American calibration, κ never turns negative for the combinations of γ and δ_s in our grid. In contrast, under the European calibration, κ does turn negative for some combinations of γ and δ_s. Figure 2 plots κ against δ_s for the European calibration. Each line corresponds to a different degree of real-wage rigidity γ. For $\gamma = 0$, κ is essentially flat, whereas it decreases with skill decay δ_s for $\gamma \geq 0.1$. Furthermore, for any given $\delta_s > 0$, higher values of γ are associated with lower values of κ, in line with Proposition 2.

κ turns negative only for $\gamma \geq 0.3$. Hence, κ turns negative only for those degrees of real-wage rigidity for which a critical value of δ_s exists. Furthermore, the respective critical values are in most cases identical to and in the remaining calibrations only slightly higher than the value of δ_s turning κ negative (the difference never exceeds 0.009). Hence, we can broadly conclude that if marginal cost, and thus inflation, increases in response to a permanent increase in the unemployment

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

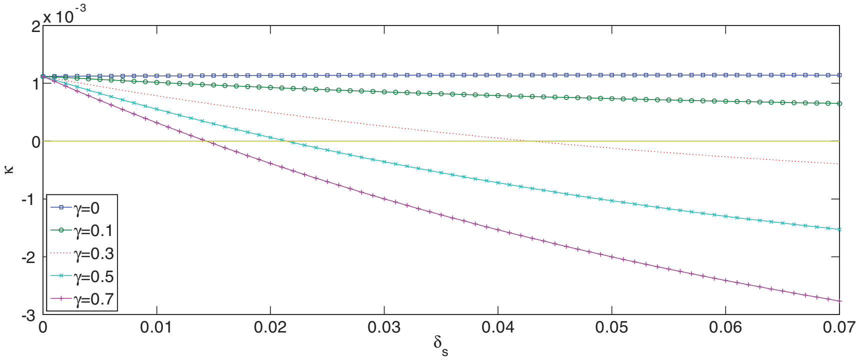

FIGURE 2. European calibration: κ against δ_s for selected values of γ.

rate—because δ_s exceeds the critical level and thus we have $\kappa < 0$—the central bank should lower the real interest rate to ensure a unique equilibrium.

This prescription should rule out self-fulfilling prophecies. In response to a sunspot-driven persistent decrease in demand and increase in unemployment, the central bank would boost demand and hence would not validate the rise in unemployment. In contrast, with $\phi_\pi \geq 1$, there is scope for sunspot equilibria if δ_s exceeds its corresponding critical value: A sufficiently persistent increase in unemployment will ultimately lead to an increase in inflation and (as $\phi_\pi \geq 1$) the real interest rate, irrespective of whether the central bank responds to lagged, current, or expected future inflation. This lowers demand and thus validates the increase in unemployment.

The reason that the critical value of δ_s sometimes exceeds the value that turns the long-run inflation–unemployment relationship positive may be that stationary sunspot-driven fluctuations might cause very persistent, but never permanent increases in unemployment. If the increase in unemployment is merely very persistent, for an increase in inflation and thus an increase in the real interest rate to ultimately occur, δ_s has to exceed the value that turns the long-run inflation–unemployment relationship positive.

The primary reason that there is no critical value of skill decay δ_s under the American calibration within the interval of δ_s used in the grid search is that, because of the more fluid labor market associated with the American calibration, for any combination of γ and δ_s, κ is significantly higher than under the continental European calibration. The higher job-finding rate x and thus the higher job-destruction probability δ under the American calibration imply that the positive effects of lagged and lead unemployment on period-t marginal cost arising from the presence of costly hiring ares lower than under the European calibration. We discussed the intuition for this after (18).

We now show that the duration-dependent wage loss implied by values of skill decay δ_s equal to or above its corresponding critical value is reasonable in light of empirical estimates of how the difference between the wage a formerly

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

unemployed worker earns upon reemployment and the wage he earned in his previous job relates to the length of his unemployment spell. Therefore, given each degree of real-wage rigidity γ and the corresponding critical value of δ_s, Table 3 displays the percentage difference implied by (8) between the wage a worker earned in his previous job and the wage he earns upon reemployment following a one-year unemployment spell. For$\gamma = 0.3$, the wage loss associated with values of δ_s equal to the critical value of 0.03 equals 13.9%. For $\gamma > 0.3$, the wage loss associated with the corresponding critical value is of course lower because both $1-\gamma$ and δ_s are lower, becoming zero for $\gamma = 1$. A range of 0–13.9% is not excessive when compared to what is found in the literature. Pichelmann and Riedel (1993), Gregory and Jukes (2001), Gregg and Tominey (2005), and Gangji and Plasman (2007) find that a one-year unemployment spell reduces the real wage by 24%, 11%, 10%, and 8%, respectively, whereas Nickell et al. (2002) find that a jobless period in excess of six months implies an additional permanent earnings loss between 6.8% and 10.6%.

4.4. Robustness

We now show that the interpretation of the results of Section 4.3 offered in the preceding is consistent with results based on a wider range of values for the job-finding probability x. We repeat the grid search for values of the job-finding probability x between 0.2 and 0.9, with a stepsize of 0.05.[17] The results reported here are based on a steady-state unemployment rate of 0.05, but the results differ only marginally if the unemployment rate $u = 0.1$. The critical values of skill decay δ_s are again the same across all three policy rules.

For each value of x, Figure 3 plots the critical values of δ_s against the degree of real-wage rigidity γ. Each line consists of a set of critical values of δ_s associated with a given value of x. Hence, the region equal to or above a given line consists of the combinations of γ and δ_s for which determinacy requires $\phi_\pi < 1$. The lowest line corresponds to $x = 0.2$, whereas the line in the upper right corner of the graph (which consists of only a single point) corresponds to $x = 0.65$. No critical values exist for $x \geq 0.7$. For each value of x, the critical value of δ_s declines with γ. Furthermore, as we would expect from our comparison of the American and the European calibrations, for each value of γ the critical value of δ_s increases with x.

Moreover, we again observe a strong correspondence between a determinacy requirement of $\phi_\pi < 1$ and a positive long-run relationship between marginal costs and unemployment, i.e., a negative value of κ. If a critical value of δ_s exists for a given combination of x and γ, we always have $\kappa < 0$ at the critical value. Furthermore, the critical value of δ_s is in most cases identical to, and in virtually all other cases only slightly larger than the value of δ_s turning κ negative.[18]

Finally, we repeat all of the previous experiments for a lower value of B', namely $B' = 0.024$. Our main result is strengthened, in the sense that the critical

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

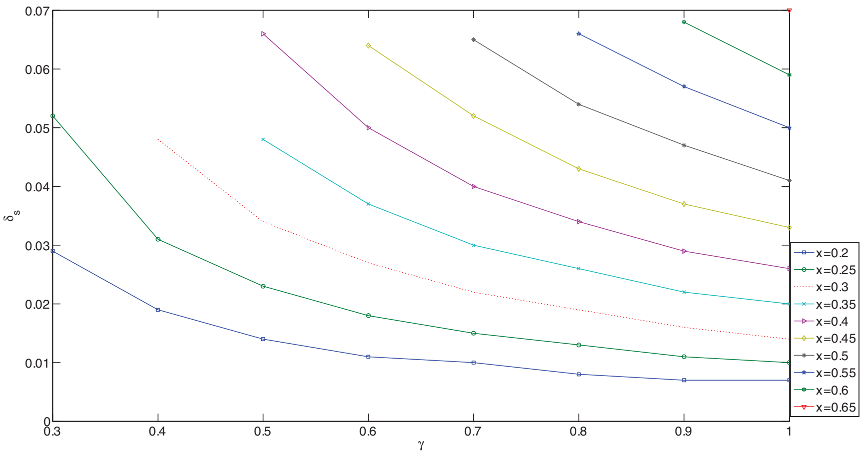

FIGURE 3. Critical values of δ_s. This figure reports results from a grid search over $x = [0.2, 0.9]$, stepsize 0.05; $\gamma = [0, 1]$, stepsize 0.1; $\delta_s = [0, 0.07]$, stepsize 0.001; and $\phi_\pi = [0, 3]$, stepsize 0.1. Each line corresponds to a different value of x and plots on the vertical axis the value of δ_s for which the determinacy requirement switches to $\phi_\pi < 1$ (i.e., the critical value of δ_s) against γ on the horizontal axis. The policy rule is $\widehat{i_t} = \phi_\pi E_t \pi_{t+j}$, $\phi_\pi \geq 0, -1 \leq j \leq 1$. The critical values of δ_s are reported for a steady-state unemployment rate of $u = 0.05$ but are virtually identical if $u = 0.1$.

values are lower than in the case of $B' = 0.12$ and the number of combinations of x and γ for which a critical value exists grows.

4.5. Flexible Inflation Targeting

We now investigate whether the determinacy issues caused by an aggressive response to inflation under the European calibration and some of the other calibrations of labor market flows considered in the preceding can be solved by adding variables other than inflation to the policy rule. We include in the grid search those values of x for which critical values of δ_s exist ($x = [0.2, 0.65]$). The results that follow are reported for $u = 0.05$ and the degree of habit formation $h = 0$, but results differ only marginally if $u = 0.1$ and/or $h = 0.8$.

To allow for interest rate smoothing, we replace (M.8) with $\widehat{i_t} = (1 - \rho_i)\,\phi_\pi E_t \pi_{t+j} + \rho_i \widehat{i}_{t-1}$, $j = 0, 1$ and $\widehat{i_t} = (1 - \rho_i)\,\phi_\pi \pi_{t-1} + \rho_i \widehat{i}_{t-1}$, and set $\rho_i = [0,\ 0.9]$. We find that the critical values of δ_s are exactly those plotted in Figure 3. This result is in line with the intuition given in Section 4.3, as interest rate smoothing does not change the long-run response of the real interest rate to an increase in unemployment.

To render the long-run real interest rate response to an increase in unemployment more negative, we add a negative response of the nominal interest rate to unemployment. We replace (M.8) with $\widehat{i_t} = \phi_\pi E_t \pi_{t+j} - \phi_u \widehat{u}_t$, $j = 0, 1$. We set$\phi_u = [0, 1]$. For the European calibration and the degree of real-wage rigidity

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

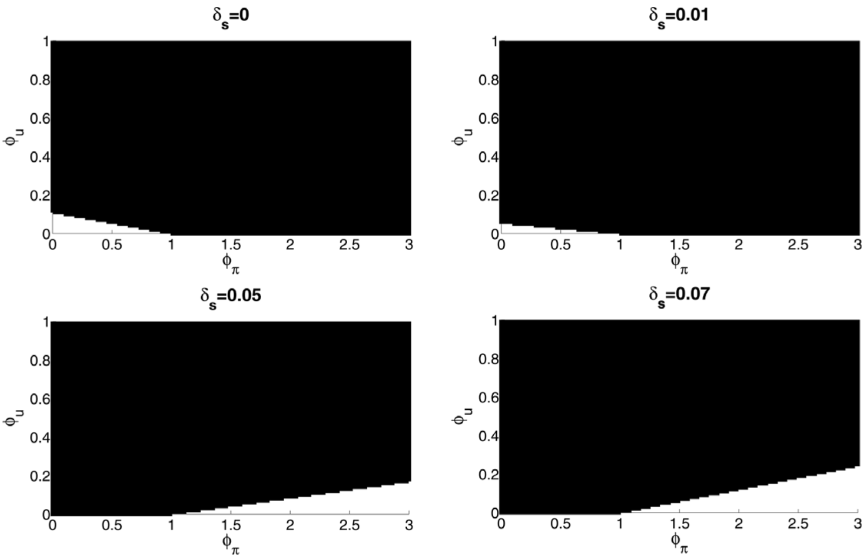

FIGURE 4. European calibration, flexible inflation targeting, and determinacy regions for $\gamma = 0.5$ and selected values of δ_s. The black and white areas indicate regions of determinacy and indeterminacy, respectively. No explosive solutions were found. The figure reports results from a grid search over $\gamma = 0.5$; $\delta_s = [0, 0.07]$, stepsize 0.001; $\phi_\pi = [0, 3]$, stepsize 0.01; and $\phi_u = [0, 1]$, stepsize 0.01, with $x = 0.25$ and $u = 0.1$. The figure changes only marginally for $u = 0.05$. The policy rule is $\widehat{i_t} = \phi_\pi E_t \pi_{t+j} - \phi_u \widehat{u}_t$, with $j = 0$, but results are virtually identically for $j = 1$.

$\gamma = 0.5$, Figure 4 plots the combinations of ϕ_π and ϕ_u that deliver determinacy for $\delta_s = \{0;\ 0.01;\ 0.05;\ 0.07\}$. Responding to unemployment clearly expands the determinacy region by allowing values of ϕ_π that would imply indeterminacy for $\phi_u = 0$. For values of δ_s smaller than the critical value, which equals 0.023 (see Table 3), and thus a negative long-run inflation–unemployment relationship, ϕ_u andϕ_π are substitutes, in that increasing ϕ_u allows one to reduce ϕ_π below one (upper two panels of Figure 4). In contrast, for $\delta_s > 0.023$ and thus a positive long-run inflation–unemployment relationship, the minimum value of ϕ_u sufficient for determinacy increases withϕ_π (lower two panels of Figure 4).

Furthermore, as we would expect from our discussion in Section 4.3, it can be shown that the minimum values of ϕ_u necessary for determinacy in our grid search are well proxied for by the requirement that the long-run response of the real interest rate to unemployment should be negative [i.e., $\partial\widehat{(i - \pi)}/\partial\widehat{u} < 0$], although it also has to be true that $\phi_u \geqq 0$, which implies that $\phi_u > -\frac{(\phi_\pi - 1)\kappa}{1-\beta}$ if $-\frac{(\phi_\pi - 1)\kappa}{1-\beta} > 0$ and $\phi_u = 0$ otherwise.[19] In over 90% of the cases where for a certain combination of $x, \gamma,\ \delta_s$, and ϕ_π, determinacy requires $\phi_u > 0$, we have $-\frac{(\phi_\pi - 1)\kappa}{1-\beta} > 0$ as well. Furthermore, for all parameterizations for which

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

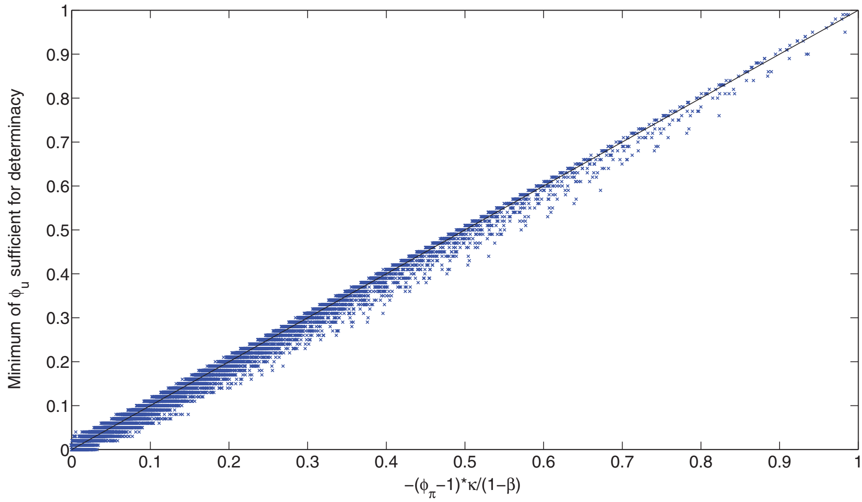

FIGURE 5. Minimum value of ϕ_u sufficient for a long-run negative response of the real interest rate to unemployment against the minimum value of ϕ_u sufficient for determinacy. This figure reports results from a grid search over $x = [0.2, 0.65]$, stepsize 0.05; $\gamma = [0, 1]$, stepsize 0.1; $\delta_s = [0, 0.07]$, stepsize 0.005; and $\phi_\pi = [0, 3]$, stepsize 0.1, and $\phi_u = [0, 1]$, stepsize 0.1. For each combination of x, γ, δ_s, and ϕ_π for which the minimum value of ϕ_u sufficient to ensure that $\partial(\widehat{i-\pi})/\partial\widehat{u} < 0$ is positive, the figure plots this value [calculated as $(-\frac{(\phi_\pi-1)\kappa}{1-\beta})$] on the horizontal axis, and the minimum value of ϕ_u sufficient for determinacy for the same combination of x, γ, δ_s, and ϕ_π on the vertical axis. Results are reported for a steady-state unemployment rate of $u = 0.05$, but change only marginally if $u = 0.1$. The policy rule is $\widehat{i}_t = \phi_\pi E_t\pi_{t+j} - \phi_u\widehat{u}_t$, with $j = 0$, but the figure is virtually identical for $j = 1$.

$-\frac{(\phi_\pi-1)\kappa}{1-\beta} > 0$, in Figure 5 we plot $-\frac{(\phi_\pi-1)\kappa}{1-\beta}$ on the horizontal axis against the minimum value of ϕ_u sufficient for determinacy. There is a strong correspondence, as can be observed from the proximity of the points to the 45° line. The difference between the two exceeds 0.05 only in a small fraction of parameter combinations, and never exceeds 0.09.[20] We obtain analogous results if we replace the negative response to unemployment with a positive response to the deviation of output from its steady state.

Regarding the merits of the Taylor principle, as in the case of pure inflation targeting, we find that it is an unreliable guide for monetary policy if skill decay and real-wage rigidity are present. $\partial(\widehat{i-\pi})/\partial\widehat{u} < 0$ implies the Taylor principle only if $\kappa > 0$ and thus the long-run inflation–unemployment relationship is negative. In cases where the long-run inflation–unemployment relationship is positive, it implies that the nominal interest rate will eventually increase less than one-for-one in response to a permanent increase in inflation. Hence, the introduction of skill decay strengthens the argument made by Blanchard and Gali (2010) that if there is little hiring and firing, the central bank should focus more on

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

stabilizing unemployment and less on stabilizing inflation than in case of a very fluid labor market. Their optimal simple rule puts a much smaller weight on inflation stabilization under the European than under the American calibration. In the presence of skill decay and real-wage rigidity, a sufficiently aggressive response to unemployment emerges as a robust way to deliver determinacy under the European calibration, as it succeeds both for a negative and for a positive long-run unemployment–inflation relationship.

5. CONCLUSION

This paper adds duration-dependent skill decay during unemployment to the sticky price model with hiring costs and real-wage rigidity developed by Blanchard and Gali (2010) as an additional labor-market friction and shows the implications of this modification for determinacy. For a low "European" value of the job-finding probability and very moderate real-wage rigidity, there is a critical threshold level of skill decay. If the quarterly skill-decay percentage equals or exceeds this level and the central bank responds only to inflation, determinacy requires a coefficient on inflation in the interest feedback rule smaller than one. This holds regardless of whether the central bank responds to current, lagged, or expected future inflation.

The switch in the determinacy requirement caused by some values of δ_s is always associated with or preceded by a reversal of the long-run relationship between marginal cost and unemployment from negative to positive as δ_s increases toward its critical value. In such a scenario, a persistent increase in unemployment will ultimately increase inflation. If the central bank responds more than one for one to inflation, the real interest rate increases, which lowers demand and thus validates the rise in unemployment. Correspondingly, adding a negative response to unemployment or a positive response to the deviation of output from its steady state to the policy rule generally helps to deliver determinacy.

NOTES

1. Examples are Carlstrom and Fuerst (2005), Duffy and Xiao (2008), and Kurozumi and van Zandweghe (2008) for models with capital, and Surico (2008) and Llosa and Tuesta (2009) for models with Ravenna and Walsh (2006)-type working capital.
2. Recent examples are Tatsiramos (2009) for Denmark, France, Germany, Greece, Ireland, Italy, Spain, and the United Kingdom, Carrol (2006) for Australia, Roed and Zhang (2005) for Norway, and van den Berg and van Ours (1999) van den Berg and van der Klaauw (2001) for France.
3. See Jackman et al. (1991, p. 259).
4. Evidence along these lines includes Gangji and Plasman (2007) for Belgian workers, Gregg and Tominey (2005) for British male youths, Gregory and Jukes (2001) and Nickell et al. (2002) for British male workers, and Pichelmann and Riedel (1993) for Austrian workers.
5. This assumption rules out the possibility that, after paying the hiring cost, a firm meets an individual worker that it might not want to hire because as a result of skill decay, his productivity is too low relative to the wage it has to pay him. The subsequent analysis will be substantially simplified by this assumption.
6. We thank an anonymous referee for raising these points.
7. See Hall (2005, p. 56).

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

8. See Bewley (1998, 1999), Agell and Lundborg (2003), Fabiani et al. (2010), Babecky et al. (2009), and Galuščák et al. (2010). Similar results are obtained by Falk and Fehr (1999) from an experiment.

9. For instance, Pissarides (2009) cites various studies finding that the wages of newly hired workers are in fact quite flexible, whereas Gertler and Trigari (2009) argue that his results might be driven by a failure to account for compositional effects.

10. Moreover, for any $\gamma < 1$, the real wages of the newly hired will be more procyclical than the real wages of continuing jobs, as unemployment duration is countercyclical and thus the skill level and the skill-dependent real wage are procyclical. We thank an anonymous referee for raising these points.

11. This is easily shown: We want to prove that $\frac{1}{1-u}(h_0 + h_L + h_F) = \frac{1}{1-u}\frac{\alpha g M}{\delta}[1 + \beta(1-\delta)^2(1-x) - (1-\delta)(1-x) - \beta(1-\delta)] > 0$. Using the fact that $1-\delta = \frac{N-x}{N(1-x)}$, this can be simplified to $(1-N)x^2 + (N-x)N(1-\beta) > 0$. This holds for all permissible values of x, β, and N, because the maximum value x can take without violating $\delta \leq 1$ is N.

12. A more general proof without restrictions on δ_s would have been desirable but was not feasible here because of the complexity of the expression resulting from $\partial\kappa/\partial\delta_s$.

13. Throughout we use $\widehat{n}_t = \frac{-\widehat{u}_t}{1-u}$.

14. In the absence of skill decay ($\delta_s = 0$) and habit formation ($h = 0$), it is possible to establish the conditions for determinacy for an interest feedback rule where the central bank responds only to inflation analytically, as we show in Rannenberg (2009), by reducing it to a system of two jump variables and one predetermined variable and then applying conditions derived by Woodford (2003) for such systems. In contrast, with skill decay the model has three forward-looking variables and three state variables. As far as we are aware, there is no straightforward way to determine the eigenvalues of a 6×6 system analytically.

15. In particular, as shown in Blanchard and Gali (2010), assuming a hiring cost such as (3) is simply a shortcut equivalent to assuming a constant-return-to-scale matching function and a flow cost of posting a vacancy, the route followed by Kurozumi and Van Zandweghe (2010).

16. We lower the steady-state match value and thus the implied value of B' in the model of Kurozumi and Van Zandweghe (2010) to its value in Blanchard and Gali (2010) by increasing the steady-state mark-up to its value in Blanchard and Gali (2010), and by increasing the flow value of unemployment. Detailed results are available upon request.

17. It is understood that increasing x also implies increasing the separation rate δ.

18. For three combinations of γ and x for which a critical value exists, the difference exceeds 0.005, and only in one case does it exceed 0.009. There is only one combination of x andγ for which the value of δ_s turning κ negative is not followed by a critical value, namely $x = \gamma = 0.2$.

19. Using the monetary policy rule, the Phillips curve, and (19), the long-run deviation of the real interest rate from its steady state can be written as $\widehat{i - \pi} = (\phi_\pi - 1)\pi - \phi_u\widehat{u} = -[(\phi_\pi - 1)\frac{\kappa}{1-\beta} + \phi_u]\widehat{u}$. Hence $\partial(\widehat{i - \pi})/\partial\widehat{u} < 0$ implies that $\phi_u > -\frac{(\phi_\pi - 1)\kappa}{1-\beta}$.

20. With $u = 0.05$, only in 0.6% of the cases does the difference between the minimum value of ϕ_u sufficient for determinacy and $-\frac{(\phi_\pi - 1)\kappa}{1-\beta}$ exceed 0.05. If we consider only those combinations of x, γ and δ_s that imply $\kappa < 0$, this fraction is still only 1.5%. For $u = 0.1$, the corresponding fractions are even lower.

REFERENCES

Agell, J. and P. Lundborg (2003) Survey evidence on wage rigidity and unemployment: Sweden in the 1990s. *Scandinavian Journal of Economics* 97, 295–307.

Babecky, J., P. Du Caju, T. Kosma, M. Lawless, J. Messina, and T. Room (2009) Downward Nominal and Real Wage Rigidity. Survey Evidence from European Firms. ECB Working Paper 1105.

Bewley, T.F. (1998) Why not cut pay? *European Economic Review* 42, 459–490.

Bewley, T.F. (1999) *Why Wages Don't Fall During a Recession.* Cambridge, MA: Harvard University Press.

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

Bilbiie, F.O. (2008) Limited asset markets participation, monetary policy and (inverted) aggregate demand logic. *Journal of Economic Theory* 140, 162–196.

Blanchard, O. and J. Gali (2010) Labor markets and monetary policy: A New Keynesian model with unemployment. *American Economic Journal: Macroeconomics* 2, 1–30.

Calvo, G.A. (1983) Staggered prices in a utility-maximising framework. *Journal of Monetary Economics* 12(3), 983–998.

Carlstrom, C.T. and T.S. Fuerst (2005) Investment and interest rate policy: A discrete time analysis. *Journal of Economic Theory* 123, 4–20.

Carrol, N. (2006) Explaining unemployment duration in Australia. *Economic Record* 82(258), 298–314.

Christiano, L., M. Trabandt, and K. Walentin (2010) DSGE Models for Monetary Policy Analysis. NBER Working Paper 16074.

Duffy, J. and W. Xiao (2008) Investment and Monetary Policy: Learning and Determinacy of Equilibrium. Department of Economics working paper 324, University of Pittsburgh,

Esteban-Pretel, J. and E. Faraglia (2010) Monetary shocks in a model with skill loss. *Journal of Money, Credit and Banking* 42(7), 1235–1265.

Fabiani, S., K. Galuscak, C. Kwapil, A. Lamo, T. Rõõm, and E. Pank (2010) Wage rigidities and labor market adjustment in Europe. *Journal of the European Economic Association* 8(2–3), 497–505.

Falk, A. and E. Fehr (1999) Wage rigidity in a competitive incomplete contract market. *Journal of Political Economy* 107(1), 106–134.

Galuščák, K., M. Keeney, D. Nicolitsas, F. Smets, P. Strzelecki, and M. Vodopivec (2010) The Determination of Wages of Newly Hired Employees. Survey Evidence on Iinternal versus External Factors. ECB working paper 1153.

Gangji, A. and R. Plasman (2007), The Matthew Effect of Unemployment: How Does It Affect Wages in Belgium? DULBA working paper 07-19.RS.

Gertler, M. and A. Trigari (2009) Unemployment fluctuations with staggered Nash wage bargaining. *Journal of Political Economy* 117(1), 38–86.

Gregg, P. and E. Tominey (2005) The wage scar from male youth unemployment. *Labour Economics* 12, 487–509.

Gregory, M. and R. Jukes (2001), Unemployment and subsequent earnings: Estimating scarring among British men 1984–94. *Economic Journal* 111(475), F607–F625.

Hall, R.E. (2005) Employment fluctuations with equilibrium wage stickiness. *American Economic Review* 95(1), 50–65.

Jackman, R., R. Layard, and S. Nickell (1991) *Unemployment. Macroeconomic Performance and the Labour Market*. Oxford, UK: Oxford University Press.

Kienzler, D. and K.D. Schmid (2013) Monetary Policy and Hysteresis in Potential Output. Working papers in economics and finance 55, University of Tübingen.

Kurozumi, T. and W. Van Zandweghe (2008) Investment, interest rate policy, and equilibrium stability. *Journal of Economic Dynamics and Control* 32, 1489–1516.

Kurozumi, T. and W. Van Zandweghe (2010) Labor market search, the Taylor principle, and indeterminacy. *Journal of Monetary Economics* 57, 851–858.

Llosa, L.-G. and V. Tuesta (2009) Learning about monetary policy rules when the cost channel matters. *Journal of Economic Dynamics and Control* 33(11), 1880–1896.

Mortensen, D.T. and C.A. Pissarides (1994) Job creation and job destruction in the theory of unemployment. *Review of Economic Studies* 61(3), 397–415.

Nickell, S., P. Jones, and G. Quintini (2002) A picture of job insecurity facing British men. *Economic Journal* 112(476), 1–27.

Phelps, E.S. (1972) *Inflation Policy and Unemployment Theory. The Cost–Benefit Approach to Monetary Planning*. New York: Norton and Company.

Pichelmann, K. and M. Riedel (1993) Unemployment Duration and the Relative Change in Individual Earnings: Evidence from Austrian Panel Data. Institute for Advanced Studies research memorandum 317.

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

Pissarides, C.A. (1992) Loss of skill during unemployment and the persistence of employment shocks. *Quarterly Journal of Economics* 107(4), 1371–1391.
Pissarides, C.A. (2009) The unemployment volatility puzzle: Is wage stickiness the answer? *Econometrica* 77, 1339–1369.
Rannenberg, A. (2009) The Taylor Principle and (In-) Determinacy in a New Keynesian Model with Hiring Frictions and Skill Loss. CDMA working paper 0909.
Ravenna, F. and C.E. Walsh (2006), Optimal monetary policy with the cost channel. *Journal of Monetary Economics* 53, 199–216.
Roed, K. and T. Zhang (2005) Unemployment duration and economic incentives—A quasi random-assignment approach. *European Economic Review* 49, 1799–1825.
Surico, P. (2008) The cost channel of monetary policy and indeterminacy. *Macroeconomic Dynamics* 12, 724–735.
Sveen, T. and L. Weinke (2005) New perspectives on capital, sticky prices, and the Taylor principle. *Journal of Economic Theory* 123(2), 21–39.
Sveen, T. and L. Weinke (2007) Firm-specific capital, nominal rigidities, and the Taylor principle. *Journal of Economic Theory* 136(1), 729–737.
Tatsiramos, K. (2009) Unemployment insurance in Europe: Unemployment duration and subsequent employment stability. *Journal of the European Economic Association* 7(6), 1225–1260.
Taylor, J.B. (1993) Discretion versus policy rules in practice. *Carnegie–Rochester Series on Public Policy* 39, 195–214.
Van den Berg, G.J. and J.C. Ours (1999) Duration dependence and heterogeneity in French youth unemployment. *Journal of Population Economics* 12(2), 273–285.
Van den Berg, G.J. and B. van der Klaauw (2001) Combining micro and macro unemployment duration data. *Journal of Econometrics* 102, 271–309.
Woodford, M. (2003) *Interest and Prices. Foundations of a Theory of Monetary Policy.* Princeton, NJ: Princeton University Press.
Zubairy, S. (in press) Interest rate rules and equilibrium stability under deep habits. *Macroeconomic Dynamics.*

APPENDIX A: STEADY-STATE VALUES

As was mentioned in the text, we start by assuming values for u and x. This allows us to write the steady state values of δ, s^i, A^L, A^A, W, and W^L:

$$\delta = \frac{ux}{(1-u)(1-x)}, s^i = x(1-x)^i, \quad A^L = \sum_{i=0}^{\infty} s^i \beta_s^i = \frac{x}{1-(1-x)\beta_s}, \quad W = \Theta' W^L,$$

$$W^L = \sum_{i=0}^{\infty} s^i \beta_s^{i(1-\gamma)} = \frac{x}{1-(1-x)\beta_s^{1-\gamma}}, \quad A^A = s^N A^L + \left(1 - s_t^N\right) = \delta A^L + 1 - \delta.$$

We can now back out Θ by first noting that in the steady state, we can write (11) as

$$A^L \left[\frac{1}{M} - g\left[1-\beta(1-\delta)\right]\right] + \beta(1-\delta)\left[\frac{1-A^L}{M}\right]$$
$$= \Theta'\left[\beta(1-\delta) + \frac{W}{\Theta'}\left[1-\beta(1-\delta)\right]\right].$$

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

Using $W^L = W/\Theta'$, we have

$$\Theta' = \frac{A^L 1\left[1/M - g\left(1 - (1-\delta)\beta\right)\right] + \frac{(1-\delta)\beta}{M}\left(1 - A^L\right)}{(1-\delta)\beta + \frac{x}{1-(1-x)\beta_s^{1-\gamma}}\left(1 - (1-\delta)\beta\right)}.$$

APPENDIX B: PROOF OF PROPOSITION 1

If $a^u = \sum_{i=1}^{\infty} a_i^u$ and $w^u = \sum_{i=1}^{\infty} w_i^u$, we have$a^u = \frac{1-x}{u(1-u)}\frac{1-\beta_s}{1-(1-x)\beta_s}$, $w^u = \frac{1-x}{u(1-u)}\frac{1-\beta_s^{1-\gamma}}{1-(1-x)\beta_s^{1-\gamma}}$. Thus $a^u > w^u$ if and only if $\frac{1}{\beta_s} > 1$, which will be true only if$\gamma > 0$ and $\beta_s < 1$. Furthermore, $\partial a^u/\partial\delta_s = \frac{1-x}{u(1-u)}\frac{x}{[1-(1-x)\beta_s]^2} > 0$ and $\partial w^u/\partial\delta_s = \frac{1-x}{u(1-u)}(1-\gamma)\frac{x\beta_s^{-\gamma}}{\left[1-(1-x)\beta_s^{1-\gamma}\right]^2} > 0$. $\partial a^u/\partial\delta_s > \frac{\partial w^u}{\partial\delta_s}$ if $\frac{1}{[1-(1-x)\beta_s]^2} > (1-\gamma)\frac{\beta_s^{-\gamma}}{\left[1-(1-x)\beta_s^{1-\gamma}\right]^2}$. This will be true if β_s is close to 1 and $\gamma > 0$. Finally,$\partial w^u/\partial\gamma = \frac{1-x}{u(1-u)}\ln\left(\beta_s^{1-\gamma}\right)\beta_s^{1-\gamma}\frac{x}{[1-(1-x)\beta_s]^2}$. $\ln\left(\beta_s^{1-\gamma}\right) < 0$ if and only if $\beta_s^{1-\gamma} < 0$. Hence $\partial w^u/\partial\gamma < 0$ if and only if $\beta_s < 1$ and $\gamma < 1$.

APPENDIX C: PROOF OF PROPOSITION 2

First, in (15), (16), and (17) we set $\widehat{\mathrm{mc}}_{t+1} = \widehat{\mathrm{mc}}_t = \widehat{\mathrm{mc}}$, $\widehat{u}_{t+1} = \widehat{u}_t = \widehat{u}_{t-1} = \widehat{u}$, $\widehat{a}_t^L = \widehat{a}_{t-1}^L = \widehat{a}^L$, $\widehat{w}_t^L = \widehat{w}_{t-1}^L = \widehat{w}^L$, and $\widehat{U}_{c,t+1}^h = \widehat{U}_{c,t}^h = \widehat{U}_c^h$ and combine the resulting expressions, which yields

$$\begin{aligned}
\lambda\widehat{\mathrm{mc}} &= -\frac{\begin{array}{c}\frac{\alpha M B' x^\alpha}{(1-u)}\left[(1-u-x)(1-\beta) + ux\right] \\ +(1-x)\left[1-\beta(1-\delta)\right]\left[\frac{-(1-\beta_s)\left(1-Bx^\alpha M\right)}{(1-(1-x)\beta_s)} + \frac{\left(1-\beta_s^{1-\gamma}\right)\frac{WM}{A^L}}{\left[1-(1-x)\beta_s^{1-\gamma}\right]}\right]\end{array}}{u(1+h_c)(1-u)}\lambda\widehat{u} \\
&= -\kappa\widehat{u}; \\
\kappa &= \frac{\left[\begin{array}{c}\frac{\alpha M B' x^\alpha}{(1-u)}[(1-u-x)(1-\beta) + ux] \\ +(1-x)[1-\beta(1-\delta)]\left[\frac{-(1-\beta_s)(1-B'x^\alpha M)}{(1-(1-x)\beta_s)} + \frac{(1-\beta_s^{1-\gamma})WM}{A^L[1-(1-x)\beta_s^{1-\gamma}]}\right]\end{array}\right]}{u(1+h_c)(1-u)}\lambda.
\end{aligned}$$

Note that for $\beta_s = 1$, we have $\kappa > 0$, because $\delta \leq 1$ implies $1 \geq u + x$.

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

We will now show that $\partial\kappa/\partial\delta_s < 0$ if β_s is close to 1 (or δ_s close to zero). A more general proof seems impossible. We have

$$
\frac{\partial \kappa}{\partial \delta_s} = \frac{\dfrac{\partial h_c}{\partial \beta_s}\kappa}{1+h_c}
$$

$$
-\frac{\lambda(1-x)}{u(1-u)}\left[\frac{[1-\beta(1-\delta)]}{(1+h_c)}\left[\begin{array}{c}
\frac{\left(1-B'x^{\alpha}M\right)[[1-(1-x)\beta_s]-(1-\beta_s)(1-x)]}{(1-(1-x)\beta_s)^2}+M \\
\left[-\beta_s^{-\gamma}\left(1-\gamma\right)W+\left(1-\beta_s^{1-\gamma}\right)\frac{\partial W}{\partial \beta_s}\right] \\
A^L\left[1-(1-x)\beta_s^{1-\gamma}\right] \\
\dfrac{-\left(1-\beta_s^{1-\gamma}\right)W\left[\begin{array}{c}\frac{\partial A^L}{\partial \beta_s}\left[1-(1-x)\beta_s^{1-\gamma}\right] \\ -A^L(1-x)\left(1-\gamma\right)\beta_s^{-\gamma}\end{array}\right]}{\left[A^L\left(1-(1-x)\beta_s^{1-\gamma}\right)\right]^2}
\end{array}\right]\right].
$$

It is easily shown that $\partial h_c/\partial\beta_s = -\beta(1-\delta)\frac{\partial A^L}{\partial \beta_s}\frac{1}{\left(A^L\right)^2} < 0$. For $\kappa > 0$, this implies that $\frac{\partial h_c}{\partial \beta_s}\kappa/(1+h_c) < 0$. Furthermore, the range of values of β_s we are interested in is the one for which κ is positive, or "just" negative. Hence $\frac{\partial h_c}{\partial \beta_s}\kappa\ (1+h_c)/(1+h_c)^2 < 0$. Setting $\beta_s = 1$ yields $W = \Theta' = \frac{1}{M} - g\,[1-\beta(1-\delta)]$, $\left(1-\beta_s^{1-\gamma}\right) = 0$, and $\left[1-(1-x)\beta_s^{1-\gamma}\right] = x$, implying that for $\partial\kappa/\partial\delta_s < 0$, we must have

$$
\gamma > \frac{B'x^{\alpha}M\beta(1-\delta)}{1-B'x^{\alpha}M\,[1-\beta(1-\delta)]}.
$$

This is easily fulfilled under the calibrations considered in this paper.One might wonder why the condition in the proposition does not simply say $\gamma > 0$. Note first that this is merely a sufficient, not a necessary and sufficient condition. The necessary and sufficient value of γ would be lower. Furthermore, it can obtained from (11) that even if there is no real-wage rigidity and thus W_t would move by the same percentage as A_t^L, the effects of a decline or increase in the average skill level would not be neutral. This is because the $t+1$ flow profit associated with hiring in t $\mathrm{mc}_{t+1}A_{t+1}^P - W_{t+1}^0$ does not depend on the skill level of the average applicant. Thus a permanent decline in A_t^L affects mc_t in some way even if there is no real-wage rigidity. The resulting effect can be obtained from (19) by setting $\gamma = 0$ in the square brackets: $\left[\left(a_1^L - a_2^L\right) - \left(w_1^L - w_2^L\right)\right]\frac{(1-x)}{u}\frac{(1-\beta_s)}{[1-(1-x)\beta_s]}$.

To derive the effect of γ on κ, let us rewrite κ as

$$
\kappa = \frac{\left[\begin{array}{c}\dfrac{\dfrac{\alpha M B'x^{\alpha}}{(1-u)}[(1-u-x)(1-\beta)+ux]}{(1-u)} \\ +[1-\beta(1-\delta)]\left[-a^u(1-B'x^{\alpha}M)+w^u\frac{WM}{A^L}\right]\end{array}\right]}{(1+h_c)}\lambda.
$$

Note that, in this expression, only the $w^n W$ term depends on γ. Let $f(\gamma) = w^n W$. Then $\partial\kappa/\partial\gamma < 0$ if and only if $f'(\gamma) < 0$. It is convenient to take the log of $f(\gamma)$ before taking

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press

the derivative with respect to γ, which yields

$$\frac{f'(\gamma)}{f(\gamma)} = \frac{\frac{\partial w^n}{\partial \gamma}}{w^n} + \frac{\frac{\partial W}{\partial \gamma}}{W}.$$

Using Proposition 1 yields $\frac{\partial w^u}{\partial \gamma}/w^u = \frac{\ln\left(\beta_s^{1-\gamma}\right)\beta_s^{1-\gamma}x}{\left(1-\beta_s^{1-\gamma}\right)(1-(1-x)\beta_s)}$. This is always negative if $\gamma < 1$ and $\beta_s < 1$, but would be zero for $\gamma = 1$ or $\beta_s = 1$. Because $W = \Theta' W^L = \frac{xA^L 1[1/M - g[1-(1-\delta)\beta]] + x\frac{(1-\delta)\beta}{M}\left(1-A^L\right)}{\left[1-(1-x)\beta_s^{1-\gamma}\right](1-\delta)\beta + x[1-(1-\delta)\beta]}$, we have $\frac{\partial W}{\partial \gamma}/W = -\frac{(1-x)\ln\left(\beta_s^{1-\gamma}\right)\beta_s^{1-\gamma}}{\left[1-(1-x)\beta_s^{1-\gamma}\right](1-\delta)\beta + x[1-(1-\delta)\beta]}$. This is always positive if $\gamma < 1$ and $\beta_s < 1$. Hence we note that $\gamma < 1$ and $\beta_s < 1$ is a necessary condition for $f'(\gamma) < 0$ and thus $\partial\kappa/\partial\gamma < 0$, though not sufficient. Plugging $\frac{\frac{\partial w^n}{\partial \gamma}}{w^n}$ and $\frac{\frac{\partial W}{\partial \gamma}}{W}$ into $f'(\gamma)/f(\gamma) < 0$ yields

$$\ln\left(\beta_s^{1-\gamma}\right)\beta_s^{1-\gamma}\left[\frac{x}{\left(1-\beta_s^{1-\gamma}\right)\left[1-(1-x)\beta_s^{1-\gamma}\right]} - \frac{1-x}{\left[1-(1-x)\beta_s^{1-\gamma}\right](1-\delta)\beta + x\left[1-(1-\delta)\beta\right]}\right] < 0.$$

For $\gamma < 1$ and $\beta_s < 1$, this implies that $f'(\gamma) < 0$ if and only if

$$x(1-\delta)\beta + \frac{x\left[1-(1-\delta)\beta\right]}{\left[1-(1-x)\beta_s^{1-\gamma}\right]} > (1-x)\left(1-\beta_s^{1-\gamma}\right).$$

This condition is easily met for the calibrations used in this paper.

https://doi.org/10.1017/S1365100513000692 Published online by Cambridge University Press