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Kant's A Priori Intuition of Space Independent of Postulates

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Abstract
Defences of Kant's foundations of geometry fall short if they are unable to equally ground Euclidean and non-Euclidean geometries. Thus, Kant's account must be separated from geometrical postulates. I argue that characterizing space as the form of outer intuition must be independent of postulates. Geometrical postulates are then expressions of particular spatializing activities made possible by the *a priori* intuition of space. While Amit Hagar contends that this is to speak of noumena, I argue that a Kantian account of space as the form of outer attention-directing remains seated in the subject.

While there have been many attempts to reinterpret Kant's *a priori* intuition of space and defend it against the criticisms that turn to advances in physics and non-Euclidean geometry,[1] on my view such defences have thus far failed to equally account for Euclidean and non-Euclidean geometry. In order for Kant's account of the relationship between space and geometry to remain viable, Kant's claim that the *a priori* intuition of space makes possible geometry as a synthetic *a priori* science must apply equally to Euclidean and non-Euclidean geometries. To do this, what is required is an account of the *a priori* intuition of space that is independent of geometrical postulates so that the Kantian account of space is not committed to any particular geometry.[2] But the question arises as to whether or not holding certain central Kantian theses about space requires affirming a Euclidean fifth postulate or any other strictly Euclidean principle. Kant considered the Euclidean fifth postulate to inhere in the *a priori* intuition of space but can a modern position that is informed of advances in non-Euclidean geometry still hold to central Kantian theses on space? In this article, I argue that upon further examination of Kant's account, one can affirm a position that is informed of modern mathematics and thus rejects the priority of Euclidean geometry while still holding, as Kant did, that space is an a priori intuition that forms all outer intuition and in turn grounds the possibility of geometry as a synthetic *a priori* science.

There are several reasons for embracing an interpretation of the Kantian account of space that does not include Euclidean postulates as a restriction on the intuition of space.[3] We can isolate the central theses of Kant's *a priori* intuitive account of space in the Metaphysical and Transcendental Expositions, where there is no mention of the Euclidean character of space. We can adhere to these central Kantian theses while still being informed of non-Euclidean geometry, as Arthur Melnick's interpretation provides for the separability of Kant's account of space from any verificational conclusions such as those in a parallel postulate. We can, however, find consequences and implementations of the central theses throughout the rest of Kant's opus. I argue that Bernard Lonergan's interpretation of the role of postulates requires us to preclude certain postulates as necessary characterizations of the *a priori* intuition of space. Further, I argue that partnered with Lonergan's interpretation Kant's own distinction between real and nominal definition requires that a fifth postulate must be considered to be independent of the central theses of Kant's expositions. Finally, I consider Amit Hagar's contention that an interpretation like mine dissolves into incoherence. Hagar holds that a single ground for Euclidean and non-Euclidean geometries needs an underlying structure that can only be found in the noumenal world. I argue that while Hagar's contention does challenge a position like Friedman's, the *a priori* intuition of space as the underlying structure that gives rise to geometry avoids the incoherence of noumenal claims and still has its seat in the subject.

I am not here arguing that Kant anticipated non-Euclidean geometry, rather that the priority of strictly Euclidean principles is not a conclusion that follows from Kant's other central theses on space. Space as singular, immediate, and *a priori* is independent of space being Euclidean. At first this conclusion might seem inconsistent in light of a multiplicity of consistent geometries. However, when this inconsistency is precluded by eliminating the fifth postulate as a necessary condition of space, the central theses about space can be affirmed and still be informed of science. That space is an *a priori* intuition is a defensible, mathematically informed claim. The claim that the *a priori* intuitive character of space necessitates strictly Euclidean principles is not.

The Euclidean postulates are intuitively plausible. That intuitive plausibility, however, concerns whether or not they admit of Kant's central theses about space as an *a priori* intuition of space and not whether we can picture them.[4] An intuition is a kind of representation. While a conceptual representation relates mediately to its object, an intuition is a representation that is singular and relates immediately to its object. The singularity and immediacy conditions require that the representation of an object not come through a concept or reflection but rather be given directly. Although such a direct representation can be sensed, it need not be. Most importantly, when it comes to the *a priori* intuition of space there is not yet any seeing or imagining, only the form that yields the possibility of sensing. Consequently, claims about the *a priori* intuition of space are not equivalent to claims about the void in which we see and imagine. To conclude that we cannot see or imagine something is not to conclude that such a thing cannot be an object of intuition. To conclude the latter, we must conclude that an object cannot be singularly and immediately represented to us. In this sense, non-Euclidean fifth postulates are just as intuitively plausible as the Euclidean one. Hypothesized objects with drastically different alternate features from Euclidean objects might not be things we can picture[5] but such a conclusion says nothing of the intuitive plausibility of such objects. Kant accepted the intuitive plausibility of a Euclidean fifth postulate and denied the intuitive plausibility of certain objects that we would today call non-Euclidean by asserting that such non-Euclidean objects were without objective validity since they could not be objects of intuition. The question is whether holding certain central Kantian theses about space requires someone informed of modern geometry to deny the intuitive plausibility of non-Euclidean geometry. Or, put another way, we must investigate whether Kant's account equally provides the tools for the foundations of Euclidean and non-Euclidean geometry. I argue that in order to defend Kant's account of the synthetic *a priori* nature of mathematics, while still being informed of modern science, one must admit the intuitive plausibility of non-Euclidean geometries.

The Infinity of A Priori Intuition

In the Metaphysical Exposition Kant argues for four points on his way to concluding that space is an *a priori* intuition: space is not empirical, space underlies all outer representation *a priori*, space is not a discursive concept, and any particular space is given as a part of the singular infinitely given space. Based solely on the Metaphysical Exposition, there is nothing that requires a Euclidean characterization of space. An interpretation of Kant's account of space without Euclidean confinement is still singular, immediate, *a priori*, and given as infinite. Kant defines the transcendental as that which is essentially involved in the coming to be of synthetic *a priori* knowledge. Thus, in the Transcendental Exposition Kant is only showing that space yields synthetic *a priori* knowledge and pointing to that knowledge. Here Kant does assert that his *a priori* intuitive account of space 'is thus the only explanation that makes intelligible the possibility of geometry, as a body of *a priori* synthetic knowledge' (B41). Kant is, as many have argued,[6] just concluding that an intuitive account of space is the only one that can account for geometry, not that the structure of geometry requires such an account. Kant here goes on to argue for the transcendental ideality of space but puts aside any further characterization of geometry.

Immediately there is the question of how an infinitely given intuition of space can equally ground various geometries, some of which are finite. This question is separate from Friedman's argument that Kant turns to intuition solely to be able to generate infinity within mathematics. Rather this question concerns how we can equally ground geometries that can be of any size but are always finite in magnitude (e.g. elliptic geometry) and geometries of infinite magnitude (e.g. Euclidean geometry) in the same infinitely given intuition of space. An aid in considering more precisely the infinity of the *a priori* intuition of space and its role in both finite and infinite geometry is Carl Posy's article 'Intuition and Infinity: A Kantian Theme with Echoes in the Foundations of Mathematics'.

Posy explains the difficulties that arise in Kant's discussion of infinity as it pertains to the *a priori* intuition of space. In the solution to the first Antinomy, Kant denies the actual infinity of any experiential intuition and consequently our ability to comprehend the universe as infinite. In the Aesthetic, however, Kant argues that space is represented as a given infinite magnitude. In considering the Antinomy, we can have no intuition of the infinite and in the Aesthetic the infinity of space is in fact intuitive. This seems to be an apparent contradiction in Kant's work and so the intuitive character of the infinite as well as precisely what is meant by infinity must be investigated. Posy argues that Kant distinguishes between the space of the antinomy and the space of the aesthetic.

Kant denies the actual infinity of empirical objects. We cannot experience the actual infinity of any objects. More precisely, Kant denies the possibility of asserting the actual infinity of the universe:

> We cannot therefore say anything at all in regard to the magnitude of the world, not even that there is in it a regressus *in infinitum*.

> All that we can do is to seek for the concept of its magnitude according to the rule which determines the empirical regress in it. This rule says no more than that, however far we may have attained in the series of empirical conditions, we should never assume an absolute limit, but should subordinate every appearance, as conditioned, to another as its condition, and that we must advance to this condition. This is the *regressus in indefinitum*, which, as it determines no magnitude in the object, is clearly enough distinguishable from the regressus in infinitum. (B548)

Posy argues that there are several reasons to believe that Kant will allow for an intuitive grasp of an actually infinite universe.[7] What prevents Kant, however, from allowing it is the singular nature of an intuition. We can think an actually infinite universe but such a contemplated universe can never be an intuitive unity with our actual experienced universe.[8] In the case of the infinitesimal, Kant is sure of its existence, we simply cannot intuit it. The potential infinity of our experiences and of particular intuitions (the kind involved in the intuition of a particular geometry) is grounded in an actual infinity, a mathematical one with objective validity. This infinity is regulative but not constitutive. 'We are speaking of pure mathematical space. We can imaginatively expand it, contract it and translate it at will' (Posy 2008: 182). The content of an intuition can never be actually infinite but the formal character must be. Here the regressus in indefinitum is contained within the regressus in infinitum. In order to do finite geometries we need a regressus in indefinitum and so a regressus in infinitum can ground finite geometries as well. The infinitesimal is also required to perform certain geometrical processes and processes of the calculus, which further emphasizes the need for a regressus in infinitum.

The Subject Matter of Geometry

For Kant, space is a pure *a priori* intuition and it is at the same time the *a priori* form of empirical intuition. The intuition of space forms the manifold of every outer intuition. Geometrical processes require construction, a productive activity. This activity is formed or guided by the *a priori* form of intuition. This productive activity serves as the subject matter of geometry for Kant. In his paper, 'The Geometry of a Form of Intuition', Arthur Melnick argues that Kant's account of the subject matter of geometry is separable from his account of geometrical verification. Melnick notes that developments in mathematics have placed 'further constraints on what a justifiable philosophical account of

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geometry would have to be' (Melnick 1992: 245).[9] Kant also holds that the verification that geometry is Euclidean is possible *a priori*. It is this position that modern mathematics renders untenable and not the position that space as the form of outer intuition is the subject matter of geometry.

Melnick argues that sensation makes up only the matter of empirical intuition. All empirical intuition, however, is more than sensation. There is also always the form of empirical intuition, namely space and time. As the formal character of empirical intuition, space guides and limits spatial and spatializing activity. Outer intuitions always involve the positing or setting of objects outside oneself (Melnick 1992: 245). This spatial activity, he argues, can be guided or prescribed without conclusions about the resulting matter of empirical intuition, that is, without conclusions about the content of how we are to be affected. The positing of objects as outside of me in space is necessary for spatial activity without the necessity for a conclusion regarding the nature of the relationship between various objects beyond that of saying that at least one such relationship exists given that they must all be in space.

Melnick distinguishes between the activity and receptivity of a spatial empirical intuition. The activity of setting objects as outside ourselves involves pointing, circumscribing, delineating, tracing out, or gesturing. It is activity that directs attention outward. Even if this activity intends or seeks a particular way of being affected it is different from that affection.

> Instead of talking of having a sensation of red, let us talk of using the sound 'R-E-D' as a reactive name; that is, as a reaction to be made only when one is red-wise affected. In this sense, for example if a dog has been trained to wag its tail only when presented with what is red, it would have the use of the reactive name 'Red'. To react red, then, is certainly different than first ostending or circumscribing properly before reacting Red. In the latter case one performs spatial-behavior and thereby ostends or indicates or directs attention, thereby producing a singular outer representation. It is spatial behavior or activity, I claim, which is the form of an empirical intuition. Ostending or delineating is productive or a matter of performing, rather than responsive or a matter of reacting. (Melnick 1992: 245–6)

In every empirical intuition there is that which is received and that which is performed. The content of what is sensed is received. That act of drawing one's attention outward is an active performance guided by the form of intuition. This interpretation is supported by Kant's account of the productive imagination in the Transcendental Deduction.

> Imagination is the faculty of representing in intuition an object that is not itself present. Now since all our intuition is sensible, the imagination, owing to the subjective condition under which alone it can give to the concepts of understanding a corresponding intuition, belongs to sensibility. But inasmuch as its synthesis is an expression of spontaneity, which is determinative and not, like sense, determinable merely, and which is therefore able to determine sense a priori in respect of its form in accordance with the unity of apperception, imagination is to that extent a faculty which determines the sensibility a priori; and its synthesis of intuitions, conforming as it does to the categories, must be the transcendental synthesis of imagination. This synthesis is an action of the understanding on the sensibility; and is its first application—and thereby the ground of all its other applications—to the objects of our possible intuition. (B151–2)

For Kant, we can represent objects in intuition that are not present. When we do so spontaneously or creatively we produce the object to be intuited and can consider only the properties we productively imagine the created object to have. Further, Melnick argues that the activities of delineating, circumscribing, pointing, tracing, or gesturing are local or small components of a larger, global spatializing activity. These local spatial activities are in fact 'limitations of an ongoing global spatial activity' (Melnick 1992: 246). These attention-directing performances must be prior to any passive reaction or receptivity.

This account of space as the form of intuition, Melnick argues, is separable from any verification that occurs within space. Spatializing behaviour can proceed without sensory reception and without prescribed reactions or names to certain affections. Further, this behaviour can be directed without reference to anticipated sensory affection. Following Kant, Melnick argues that thought regulates spatializing behaviour. The most direct way that thought regulates spatializing behaviour is by commanding it. Thought can direct or command spatializing behaviour without a prediction or conclusion about empirical sensory reaction, '[t]o say that the form of empirical intuition is itself given in a pure intuition, I claim, is to say that the very behaviour which underlies and directs our capacity to be affected, can also be carried out and ordered independent of how one may thereby be affected along the way' (Melnick 1992: 247–8). When space forms our outer intuition it can do so without offering anything in terms of content.[10]

For Melnick, this distinction of form and matter similarly applies to geometrical construction. A geometrical construction is guided by a pure rule and it is these operations that concern the geometer not the sensory result; '[i]t is, we shall argue, operations like pointing, cutting, rotating, sweeping out or flowing, that are in a sense, the subject matter of geometry. It is not the pictures left as a record of such operations that he studies, but the operations themselves' (Melnick 1992: 248). The criterion for whether or not an object possesses objective validity now becomes if we can construct it by what is made possible through spatializing behaviour. In this sense, objective validity is no less grounded in construction but is less grounded in seeing what has been constructed. For Kant, knowing is both active and receptive. Our active thought, however, is not immediately related to objects. Rather, thought produces the form through which we relate to objects. Or, to put it in Melnick's terms, thought produces the behaviour through which we relate to objects. Properly, thinking is directive or prescriptive and not descriptive (Melnick 1992: 248). Thus, spatial behaviour must be directed by thought not described by it. A proper expression of geometry then, concerns rules for spatializing and not descriptions of results. No matter how detailed, a descriptive account of an experiential reality will always be empty for Kant. This was Kant's error as he failed to see the distinction between the necessary consequences of stipulating a Euclidean fifth postulate and the descriptive nature of considering the fifth postulate to inhere in all spatializing behaviour.

For Melnick, we must now consider the extent to which these pure constructions are geometrical. To do this, Melnick thinks of a geometry as being determined or denoted by the totality of triangles. In this case, a Euclidean geometry would be distinguished from a non-Euclidean geometry by the relationships of its triangles. These totalities of triangles, however, cannot be considered to be the subject matter of geometry. In that case, the varying geometries would have different subject matters and would not be properly geometrical in virtue of the same thing,

> To keep a single subject matter for different geometries, we assume that a triangle is a matter of pairs of different constructions or operations issuing in coincidence or having coincidence as an upshot. Intuitively, such a pair must consist of a straight-line

> construction, together with a broken line construction. These construction pairs will be invariant from geometry to geometry. The only difference is which pairs of operation ensue in coincidence. (Melnick 1992: 249)

Thus a unity across geometries would be the construction of triangles by means of coinciding pairs of constructions. This Melnick equates to equivalent singular representations from varying perspectives. In other words, geometry is that by which various knowers can refer to the same singular outer representation. This notion of geometry is in accord with a Kantian interpretation in that space as the form of outer intuition makes possible geometry, the equivalent reference of outer singular representations. The operations involved in the pure constructions of geometry consequently have analogues in our empirical spatial behaviour.

What results then is an operationalist theory of geometry. As Melnick points out, an operationalist theory of geometry seems the most compatible with Kant's account since 'determinate spaces are given in construction, and constructions as well as operations are matters of performance' (Melnick 1992: 251). More precisely, Kant's account correlates the operations of interpreting geometry with the operations involved in singular representation. In having meaningful correlates these operations or performances go beyond what Kant would call mere play. This makes Kant a particular kind of operationalist but also creates an ambiguity about the separability of the subject matter of geometry and the verification of geometry.

For Kant, thought thinks the object but does not produce the object thought. Thought, however, does produce the behaviour that forms the ability to be affected by the object and thus the *a priori* properties we can consider. Any account of our thinking must be active or directive. Spatial behaviour then must be directed by thought and not described by it. Operations and constructions can be guided by thought whereas relations and entities cannot be so guided. Likewise, even descriptive empirical spatial experience will be empty if not somehow connected to thought. That is to say that there must be some way of connecting experience to thought. The only account of space that can provide such a connection is an operationalist account, '[o]nly if Space is a form of our own behavior, does that have a systematic and global field of behavior to guide, and so a full scope of reality to represent as the reactive upshot of such behavior. Put simply, there is no determinate empirical representation at all, except if Space is the activity for thought to guide' (Melnick 1992: 252). Similarly, any objectivist theory of space is empty for Kant as the possibility of a connection between thought and the objects in an absolute space is precluded. Any position on space will need to consider advances in geometry to be informed but will also have to be operationalist to be considered Kantian. Only if space forms or guides our behaviour or activity is a full reality possible for thought.

So we see from Melnick's interpretation that Kant's account of space as a form of intuition that grounds the possibility of geometry is intended to show that the form of intuition guides the processes of geometry. Let us consider the five postulates of Euclidean geometry:

1. Any two points can be joined by a straight line.
2. Any straight line segment can be extended as far as required in a straight line.
3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
4. All right angles are equal.
5. Parallel postulate. If two lines intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough. (Heath 2002: 152)

The first three postulates are attempts to prescribe or guide the activity of geometry. The first postulate prescribes the spatial activity of delineating a line segment. It says nothing of what that line segment will look like or how it will relate to other line segments. Similarly, the second postulate prescribes the indefinite extension of line segments. The third postulate prescribes the construction of a circle. This is still not to say that the first three postulates are equivalent to or constitutive of the intuition of space or the full range of all things geometrical. This is only to say that the first three postulates prescribe operations in a way that is consistent with Kant's account of space and geometry. These particular prescriptions of activity, however, should not be confused with the necessary activities that make possible all outer representation. None of these particular spatial activities constitute or make possible spatializing activity in general. These activities do not condition any of the claims for singularity, immediacy, apriority, and infinity. None of these features depends on a particular spatializing activity, rather they make spatializing activity possible.

The local spatializing activities of the first three postulates are not the same as the condition for the possibility of spatializing activity. Particular local spatial and spatializing activities—like those guided by the

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first three postulates—are instantiations of activity that are guided by the form of outer attention-directing. In much the same way that no particular space can be thought to constitute space in general, no particular spatial activity can constitute spatial and spatializing activity in general. As a result we can consider any subset of possible spatial activities so long as they are guided by the *a priori* intuition of space as the form of outer attention-directing. Different geometries can be considered by considering different subsets of possible spatial activity.

The fifth postulate, however, is in this case descriptive. It does not guide activity or operation. It describes a result or condition that would occur in a given scenario. In this case, the activity of the fifth postulate is compromised when we eliminate the descriptive account. The activity is different if it is not described by the result it attains. The activity dissolves into the activity of the second postulate. Rather than prescribe thought, the fifth postulate in this case speaks of how we are to be receptively affected. It speaks of the verification of the coincidence of pairs of construction, a verification that cannot be *a priori*, 'verification is always ultimately a local matter of having all the required information at once where and when one is. Thus, one needs physical markers or physical signals to verify coincidence results, but once these are introduced one needs empirical hypotheses about how things move or how forces operate' (Melnick 1992: 255).[11] This quality already sets the fifth postulate apart from the first four. The first three postulates can be expressed as prescriptive and consistent with the *a priori* form of intuition as spatializing activity that directs attention outward. A descriptive claim made in the fifth postulate can only result from stipulation or verification. Moreover, any verification to conclude a Euclidean character of the formal condition of outer representation must be empirical. While the fifth postulate can be formalized in its expression, the claim that the Euclidean fifth postulate holds, and alternate fifth postulates cannot, would still depend on verification. The fourth postulate is also descriptive although not descriptive in the same way as the fifth postulate. It is not prescriptive of the spatializing activity and merits further examination later.

For Kant, the *a priori* intuition of space is what makes possible geometry, the 'science which determines the properties of space synthetically, and yet a priori' (B40). To say, as Kant does, that a necessary characteristic of that intuition is a particular relationship is then to say that geometry would not be possible without this particular relationship holding. We know this not to be the case. Geometry is possible without a Euclidean conclusion about parallel lines. We may say that in doing geometry at all, some relationship concerning parallelism will always exist or can always be deduced. Such a conclusion, however, is different from saying what that relationship will be. The conclusion that it is not possible to do geometry without some relationship concerning parallelism holding is different from the conclusion that geometry is not possible without this specific relationship holding. We cannot say that geometry as a science is only possible if all lines must intersect in a particular way. Using both Kant's synthetic and analytic approach we can derive or explain geometrical propositions that do not adhere to a unique kind of intersection.

It is the case that on my view Kant's *a priori* intuition of space requires no intuition about the behaviour of parallel lines. In order for the intuition of space to ground the possibility of geometry, it need only yield the activity that makes possible reasoning spatially or geometrically. For that only a singular, immediate, ordered, and infinite intuition of space is required. This intuition does not commit us to a particular geometry. Nor does this intuition commit us to the claim that its structure is the structure that inheres in a physical universe. It commits us solely to the claim that the formal structure of our outward attention-directing grounds the possibility of geometry as a science. One can stipulate—in fact one must—any of the three conclusions about parallel lines *a priori*. The claim that one of the conclusions is possible and that the other two are not can only be an *a posteriori* claim.[12] Further, the Kantian account that results from removing the Euclidean character as a necessary characteristic of the *a priori* intuition of space refutes only those claims that are either *a posteriori* or part of the stipulative aspect of geometry.

Removing the Postulates

To aid in establishing a qualitative difference of the fourth and fifth postulates I turn to Bernard Lonergan's article 'A Note on Geometrical Possibility'. Lonergan asserts that science is most fundamentally an act of understanding and only secondarily is it expressed in definitions, postulates, and deductions. These principles are an expression of what is understood and can be divided in terms of what is understood by them. Nominal definitions are merely the understanding of a linguistic system, of the use of terms and their relationship to other terms. Essential definitions, on the other hand, are of a real system, something necessary, possible, or impossible in terms of the way things are; '[i]n both cases the understanding itself is real; but in nominal definitions the

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understood has only the reality of names; while in essential definition the understood has the reality of what names name' (Lonergan 1986: 94). Both are necessary kinds of definitions for doing science. The distinction between essential and nominal definitions resembles the distinction between form and matter and in the aggregate of our knowledge there will always be form and matter.

There is another way to consider the inevitability of nominal definitions. The *a priori* knowledge that results from experience is that of an *a priori* intelligible unity of sensed data. Lonergan argues that there will always remain a residue of sensible elements that can be generalized by nominal definitions but not universalized. Put another way, we can group or conceptualize certain sensed elements in a way that generalizes them—nominal definitions—but we can never consider that grouping of elements to be universal *a priori* knowledge. This understanding of nominal definitions mirrors Kant's account of a discursive concept. We can abstract a unity from a given sample of experienced data. That abstraction, however, will not result in a universal. It follows then that while nominal definitions suppose only the understanding of names *a priori*, they also suppose *a posteriori* knowledge that can be generalized to arrive at such names. Many of the nominal definitions in geometry suppose empirical knowledge of particular objects or conditions to be observed and generalized. These generalized conditions or objects, however, are not universal and cannot be used to determine what is possible or impossible.

Essential definitions presuppose nominal definitions and understand the formal or constitutive *a priori*. Lonergan cites the example of the formal definition of a circle,

> The formal or constitutive of a circle that grounds its circularity and other properties is the equality of radii, [i]f all radii are equal, the plane curve must be round; if any are unequal, it cannot be round and similarly for the other properties of the circle. The 'must' and the 'cannot' reveal the activity of understanding; and what is understood is not how to use the name, circle, but circularity itself. (Lonergan 1986: 97)

We can provide several descriptive and generalized accounts of a circle and can situate it relative to other objects but that which makes a circle a circle is the equality of radii from a common point. Lonergan, along with Melnick and others, argues that the first three of Euclid's postulates are operational in that they prescribe or call for geometrical activity. The fourth and fifth postulates, however, are theoretical in that they seek truth in propositions of existence and possibility. Lonergan argues that had the circle been defined nominally in terms of uniform roundness, a theoretical postulate would be needed to ensure the equality of the radii of a circle.

Lonergan argues that Euclid turns to theoretical fourth and fifth postulates to support the nominal definitions for a line and a right angle. The definition for a line reads, 'A straight line is a line which lies evenly with the points on itself' (Heath 2002: 153). This is a nominal definition as it appeals only to abstract sense data rather than to the *a priori* constitution or prescription of a line; 'this is very much like saying that a circle is a uniformly round plane figure; it enables one to use the name, straight line, correctly; it does not tell what makes straight lines straight; and it does not provide a premise for deductions about straight lines' (Lonergan 1986: 98). Further, the nominal definition of a straight line only allows for a nominal definition of straight angle which in turn allows for only a nominal definition of right angle: 'When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is right, and the straight line standing on the other is called a perpendicular to that on which it stands' (Heath 2002: 153). Combining the nominal definitions for straight line, straight angle, and right angle with the theoretical postulate of the equality of right angles yields a prescription for Euclidean straight lines. The fifth postulate partners with the nominal definition of a plane, 'a surface which lies evenly with the straight lines on itself' (Heath 2002: 153). This definition allows for referencing the plane but not for deducing properties of it. Like the fourth postulate, when combined with certain nominal definitions it can yield a prescription, '[it] is a correlation that enables one to argue from given intersections and angles to other intersections and so, through theorems to be established, to other angles and to areas' (Lonergan 1986: 100).

What results is an independence of the fourth and fifth postulates from the necessary representation of space. When partnered with nominal definitions, the theoretical fourth and fifth postulates will result in necessary consequents. The very positing of the postulates, however, is not a necessary condition required by spatial representation but rather an abstraction of experienced spatial data,

> From nominal definitions there follow necessary, though nominal, consequents. From essential definitions there follows

> the demonstration of necessary and real properties. But the transition from the material and nominal to the formal and essential cannot be necessitated by the former, else the distinction between them would be illusory; and it cannot be necessitated by the latter, for the latter is not prior to the transition. (Lonergan 1986: 100)

The first three postulates provide a prescription for spatial activity. They neither constitute nor determine a space or geometry but account for types of activity possible within the representation of space. Euclid's fourth and fifth postulates abstract from spatial experiences to provide theoretical descriptions of the space in which spatial activity occurs. It is in this way that Lonergan claims that Euclid's fourth and fifth postulates appeal to sensibility.

Thus we see that these postulates are expressed in ways that appeal to sensibility by the use of terms like straight and equal angles. Their expression in terms that appeal to sensibility does not necessarily compromise their truth as the postulates can be formalized. The fourth and fifth postulates, however, appeal to sensibility to abstract a theoretical conclusion. The conclusions of the fourth and fifth postulates can be formalized but the necessity of these postulates is not provided for by the *a priori* representation of space. Only in an abstraction from experiential images can one claim that a Euclidean fifth postulate must hold rather than a non-Euclidean fifth postulate. Turning to images is not itself an error. Geometers often turn to imagination as a tool to aid in geometrical reasoning. This visualization, however, can only serve as an aid to consider necessary properties that result from construction and not as a way of abstracting accidental features.

Kant makes a similar distinction between what he calls nominal and real definitions. In this case, nominal definitions are arbitrary and provisional and require only logical possibility while real definitions have corresponding objects or objective validity. Nominal definitions neither require nor are capable of proof; they are merely provisional, and are only intended to be turned as quickly as possible into real definitions. For Kant, the term 'straight' is a qualitative one that must be converted into a quantitative mathematical one. The conversion of the concept of straight to that of minimum distance is not an analytic conversion for Kant. One must construct *a priori* a line with constant slope and construct in intuition that the minimum distance will be attained by this line with constant slope. The straight lines must be

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constructed and by process of productive imagination they possess *a priori* the properties we productively imagine them to have. The drawn straight line, however, is empirical and the error arises when the 'look' of straightness is considered equivalent with the productively imagined quantitative property of shortest distance. In accepting this, Kant was deviating from the characterization of a pure intuition. When formalized and considered without the fifth postulate restriction, all three possible conclusions about the relationship between parallel lines can follow. Nothing from the formal condition of spatializing behaviour precludes any possible conclusion concerning parallelism. On such an interpretation, I argue that the fifth postulate becomes a part of the stipulative step in construction. Kant argues that definitions and stipulations are required for the construction of geometrical objects. In such construction, definition and stipulation provide the characteristics productively imagined to inhere in the objects treated. Kant argues that a stipulation must be made of a triangle concerning the relationship of the magnitudes of its sides (equilateral, isosceles, scalene). I argue that without a Euclidean restriction of the *a priori* intuition of space a similar stipulation must be made of figures concerning parallelism.

An Objection

A possible objection to such an account can be found in Amit Hagar's paper, 'Kant and non-Euclidean Geometry'. In it, Hagar argues that the three central components in Kant's account of space and its relationship to geometry are: '(1) space is the a priori form of pure intuition; (2) geometrical judgements are a priori and synthetic; (3) the metric of humanly intuited space is Euclidean and the propositions of Euclidean geometry are synthetic and are known a priori' (Hagar 2008: 81). Hagar grants the apparent logical relation between the first two theses but sees a difficulty in finding a similar logical necessity or relationship with the third thesis. He notes that Kant's transcendental exposition of space assures the certainty of geometry but the third thesis requires further examination of Kant's account of geometry.

For Hagar, the debate concerning Kant's account of geometry stems both from Kant's account of the role of intuition in geometry and from Kant's argument for the role of intuition in geometry. Hagar follows Broad in claiming that prior to the development of non-Euclidean geometries, there was room to challenge Kant on the role of intuition in geometry. This interpretation reads Kant as arguing that a unique condition guarantees the certainty of our concepts and a unique condition

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guarantees its application. For Hagar, the former condition is 'when we arbitrarily make up the concept for ourselves' and the latter is 'when the concept contains an arbitrary synthesis that admits of a priori construction' (Hagar 2008: 86). With regard to both conditions, this interpretation does a disservice to Kant's account. For Kant, the condition that assures the certainty of our concepts is reason or logic. In this sense, for Kant the objects of non-Euclidean geometry are no less certain as reason does not reject the existence of a space enclosed by two lines or a triangle whose angles add up to less than two right angles. Kant does turn to exhibition in intuition as the condition for ascertaining the applicability of these concepts but in that sense it is not arbitrary. What guarantees whether or not the concept of a spatial object can be applied is whether or not that object can be exhibited in space. This is far from arbitrary as if the object is to be applied—that is, if it is to be not just a definition but a geometrical object—it is to be applied in space. Questions about whether or not such a space exists, the nature of that space and the extent to which one can measure that space are to be decided but do not preclude existence in space from being the condition that assures the applicability of the concept. Mathematicians can work with and manœuvre these concepts without appealing to space but Kant's point is that in order for such manœuvring to be geometry it must be about space and the objects within it. Ultimately, Kant answered wrongly the question of whether or not some of these objects could be applied (having answered that certain non-Euclidean objects do not have objective validity when modern mathematics has shown them to have the same objective validity as Euclidean objects Kant admitted) but his appeal to the space in which these objects would occur as the condition for their applicability is justified.

Hagar and Broad are right to point to Kant's contention that this appeal to intuition allows mathematics to be demonstrative and not solely discursive like philosophy. They are also right to point to an error in Kant's account of this appeal to intuition:

> While Broad agrees with Kant that the geometrical properties of a triangle do not follow logically from the mere definition of a triangle, he points out that the former follows logically when accompanied by the concept of the space in which the triangle is imbedded, i.e., the metric signature of that space. The fact that many of Euclid's propositions do not follow deductively from his definitions, axioms and postulates, and that intuition is indeed needed in some of his proofs, is, for Broad 'a defect' in

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> Euclid's geometry, which Kant has mistaken for an inherent property in geometry as such, a view Broad shares with Russell in his *Principles of Mathematics* (1903/1937). (Hagar 2008: 86)

This is precisely the source of Kant's error and the subsequent confusion concerning his account of space. Kant considers certain accidental features to be inherent not only in geometry but also in the space in which we geometrize. Independent of the space it is in, the definition of a triangle does leave us in the dark as to the *a priori* sum of its interior angles. Partnered with a definition or stipulation concerning parallelism we can affirm *a priori* some conclusion concerning the sum of its interior angles but we would still be left with the question of that triangle's applicability. For that we must turn to its exhibition in space. Kant answered that objects like non-Euclidean triangles would fail the test of objective validity. To make that claim an empirical intuition concerning verification is required. As such, its conclusion is one that must be rejected as a feature inherent in *a priori* geometry and in the *a priori* intuition of space.

Hagar disagrees with Broad, however, on the extent to which non-Euclidean geometries have shown Kant to be wrong with regard to the synthetic *a priori*. Broad's view falls in line with many from the early twentieth century in considering pure geometry to be *a priori* but to have nothing to do with the real world, and applied geometry to be synthetic but in that case not pure enough to be *a priori*. Hagar contends that non-Euclidean geometry might actually point to Kant being right about geometry in one respect,

> Indeed, when Riemann (1866) discusses the foundations of geometry his conclusions are equivocal: space might possess a unique structure but when we try to discern this structure by way of physical measurements, we already presuppose certain hypotheses (regarding rigid rods and light rays as gauges) and a certain metric (that establishes the basic gauges). Thus at least on the latter reading of Riemann, geometry is still synthetic a priori: when applied to the world it serves as precondition for any physical experience of the world. (Hagar 2008: 87–8)

Riemann's account supports the notion of a synthetic *a priori* because certain geometrical principles guide all geometrical investigation and interpretation. Further, Hagar argues that Riemann's account precludes the need for intuition for geometrical reasoning but assures its need to establish the consistency of certain geometries. The consistencies of non-Euclidean geometries are only assured if the consistency of Euclidean geometry is assured and the only remaining appeal is to intuition. This seems to strengthen Kant's account,

> It seems that the main theme of the discussion so far is that Kant might have lost a battle, betting on Aristotelian logic, Newtonian physics and Euclidean geometry, but nevertheless won the war: not only did his legacy of transcendental philosophy come out of the nineteenth and twentieth centuries intact but it was also reinforced. (Hagar 2008: 89)

Hagar then works to reject Friedman's defence of Kant by setting what he considers to be a trap. He sets this trap by arguing that tangible experiences are Euclidean while not all visual appearances are Euclidean, offering certain examples from two-dimensional space such as looking at the angles of a ceiling. This is intended to reject the Kantian claim that there is only one geometry or metric that applies to the phenomenal world. Hagar recognizes that many modern Kant scholars might not embrace this positive thesis but rather a negative one that denies not the construction of non-Euclidean objects but rather the possibility of non-Euclidean metrics in the phenomenal world. He contends that the Kantian requires this uniqueness to provide the objective validity of the propositions of Euclidean geometry.

> This uniqueness ensures the correspondence between our knowledge and its objects in experience: the truth of the Euclidean propositions, under this account, is secured by the coherence of Euclidean geometry as a complete system of knowledge according to which we organize our experience. For this reason, as Brittan argues, the uniqueness of the Euclidean metric is crucial for establishing objective knowledge of appearances. (Hagar 2008: 91)

Hagar argues that both the positive and negative theses are wrong. There are non-Euclidean phenomenal experiences and the Euclidean metric is not unique. If the uniqueness of the Euclidean metric is required to ensure the categorical application to experience then its non-uniqueness precludes a categorical application.

Such a conclusion is intended to draw the Kantian into admitting a non-arbitrary structure for noumena that yields tangible and visual appearances. Given the discord between certain non-Euclidean visual appearances and Euclidean tangible appearances, Hagar argues that the only way to still hold for a categorical application of a metric is to argue that there is an underlying structure or metric that gives rise to both. Under this interpretation, tangible space would remain invariant and consistent with the underlying structure while visual space would vary with perspective. Such a claim, however, aims to speak intelligently about the noumenal world by assigning properties to it, something Kant argues we cannot do. The move to admit of an underlying structure or metric is one that changes the account of truth from coherence to correspondence, a correspondence that cannot be attained given Kant's separation of phenomena and noumena. Here Hagar argues that the only move left for the Kantian is to grant that an underlying structure of noumena exists but that we cannot ascribe any properties to it. But this separation of phenomena and noumena is an essential component of Kant's metaphysical position of transcendental idealism. Any argument that turns to non-trivial information or correspondence concerning noumena is already a departure from transcendental idealism.

Hagar is right to contend that the Euclidean metric is not unique. Any account that is informed of mathematics must reject such a claim. It is clear that an attempt to defend a Kantian account of space and its relation to geometry will fall apart if it remains wedded to Euclidean geometry in the way Friedman suggests. This only makes clearer the need to interpret Kant in a way that dissociates space from a commitment to a particular geometry or metric. Note that Hagar's trap only works if we pursue Friedman's interpretation.[13] If, however, we reject the need to commit to a particular geometry or metric, there is no trap. The only work that remains is that of explaining how Kant's theses concerning space and its relationship to geometry still apply to all geometries or geometry more generally. If we can account for these theses then a particular geometry or metric being relevant in visual space and another in tangible space does not compromise the categorical applicability of geometry to all experiences. Spatializing activity grounds the possibility of geometry as the processes of outward attention-directing that remain constant across perspectives. In particular cases, such processes may require stipulations concerning parallelism and curvature but these stipulations do not compromise *apriority*. Euclidean and non-Euclidean geometries are geometries in virtue of the same thing, namely, they make possible the referencing of outer objects across perspectives. For this only the formal condition of spatializing behaviour is required. The threefold thesis then becomes (1) space is an *a priori* intuition and the *a priori* form of all outer intuition, (2) geometrical propositions are synthetic *a priori*, and (3) the intuition of space is not a particular metric of space but rather the form of spatializing behaviour that makes possible any and all spaces and any and all geometries by virtue of their being about space.

My thesis does not require the claim that all appearances are Euclidean in the sense of an intuition of tangible space. Hagar and Hopkins are right in their assertion that we have experiences and images of things that look non-Euclidean. It is in this way that I reject Kant's conclusions. We can determine *a priori* what distinguishes hyperbolic geometry from Euclidean geometry and we can derive *a priori* the formula for calculating a curvature tensor but we cannot say that this experience or that tangible world is Euclidean or non-Euclidean *a priori*. It requires measurement and application.

Hagar is right to recognize the difference between the phenomena of visible or visualized space and tangible space.[14] The distinction, however, between a Euclidean tangible space and a non-Euclidean visual space is one that relies on measurement and verification which must be empirical. Many read Kant as starting from a synthetic *a priori* account of geometry and moving to a necessary intuitive account of space. Unilaterally, those that read Kant in this way deny the validity of such a move. Likewise, any *a posteriori* measurements of Euclidean or non-Euclidean characteristics cannot be the basis of an *a priori* account of a formal condition. Such moves will only reject a Kantian position that makes the move from a synthetic geometry to an intuitive account of space.

If we adhere to Hagar's terminology then in fact there may be varying metrics for visible and tangible space. This is only a problem, however, if we believe that the way in which an *a priori* intuitive account of space is supposed to make possible a synthetic *a priori* account of geometry or the way in which the formal condition of space forms our experience is by providing a metric. If by metric we mean solely a way or type of measuring, then this is not what the *a priori* intuition is intended to provide. The geometer can construct a triangle with sides of equal length even if she cannot measure because the equality of the lengths of the sides of the triangles is assured by *a priori* stipulation. Even the metric within which she is working is likewise assured. A stipulation about parallelism is required in the construction of a triangle. Geometrical novices are only introduced to Euclidean geometry and so when we say 'triangle' the Euclidean stipulation concerning parallelism is assumed but still required. Also, granting different metrics for visible and tangible space is not itself a problem as it does not ensure non-isomorphic behaviour.

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The objects of our experiences in the phenomenal world may still be translatable between visible and tangible. If by metric we mean something more general like say 'paradigm' or 'framework' then the common metric is precisely geometric, that activity by which we refer uniformly to outer objects.

Suppose, however, that we grant Hagar's contention that an insurmountable hurdle exists between a non-Euclidean visual space and a Euclidean tangible space, what then would be the fundamental problem? Hagar seems to be concerned with a fundamental structure, 'if tangible space and visual space have different characteristics, and these characteristics are a product of an existent non-arbitrary underlying structure, then the Kantian silence with respect to "things-in-themselves" becomes even more puzzling. Of course the Kantian can always claim that if such structure does exist it is still inaccessible to us' (Hagar 2008: 93). Here Hagar is arguing that the only way a Kantian can unite the apparently discordant spaces is to find a fundamental structure which gives rise to both. Such a move, Hagar argues, requires an appeal to noumena that forces the Kantian to compromise Kant's account of truth. But an *a priori* intuitive account of space dissociated from a Euclidean fifth postulate is precisely that underlying structure. This structure still has its seat in the subject and does not require any appeal to noumena. Even if we grant that visual space and tangible space cannot be experientially reconciled, an *a priori* intuition of space still conditions both forms of spatializing activity. The *a priori* intuition of space makes possible Euclidean and non-Euclidean geometry.

Our geometrical knowledge of the tangible phenomenal world and our geometrical knowledge of the visual phenomenal world equally rely on the *a priori* intuition of space as the condition for outward attention-directing. Thus, speaking of an underlying structure that gives rise to both is not a venture into transcendental realism where we ascribe properties to things in themselves. It is not a claim that things in themselves admit of a certain geometrical relationship which gives rise to both the geometries of the visual and tangible world. Rather, it is a claim about the reality of the structure of knowing and particularly the science of geometry. All geometrical knowing is knowing that relies on the *a priori* intuition of space. This intuitive ground is the source of the syntheticity of geometry and not some correspondence to the noumena of a physical world. In referencing visual phenomena and tangible phenomena there are various stipulations employed but the geometries are equally grounded in the *a priori* condition of outward attention-directing. If the claim about an underlying structure were in fact a transcendentally real claim not only would it would depart from a Kantian position but it would make the rather odd claim that the structure of things in themselves had its seat in the subject. Instead, the science of geometry whether it concerns visual or tangible objects has its seat in the outward attention-directing of the subject. While noumenal relationships may have little or nothing to do with the *a priori* intuition of space, all our geometrical knowledge has its ground in the *a priori* intuition of space as the condition for outward attention-directing.

One can make appeals to a meta-mathematics or meta-geometry as a way of talking about that in virtue of which Euclidean and non-Euclidean geometries are still geometry and such projects have proved to be, and will continue to be, fruitful for the philosophy of mathematics. For the purposes of my project, something far less comprehensive, I think, will suffice. The *a priori* intuition of space grounds the possibility of geometry. Separated from a Euclidean fifth postulate, this intuition grounds the possibility of non-Euclidean geometry in the same way that it grounds the possibility of Euclidean geometry. This intuition provides the formal character of all outer experiences. Similar to the case of geometry, this formal condition characterizes our non-Euclidean experiences in the same way that it characterizes our Euclidean experiences; it makes possible spatial and spatializing activity. As a result any conclusion about the parallelism of our experiences or about the curvature of the universe has no influence on how we consider the *a priori* intuition of space to form or condition our experiences. Such a project is mine and is developed in light of advances in mathematics and physics but is one I consider to be truly Kantian in its motivation.

Defences, like Barker's, that argue for an epistemological or phenomenological priority of Euclidean geometry compromise Kant's account in that he can only account for a (small) slice of geometry and must consider differing geometries to have different sources, processes, and value. On my interpretation, Kant can account for the derivation of all geometries. Kant can now be more in tune with the geometer that considers the Euclidean and non-Euclidean geometries to be of the same value and objective validity. In this way, any subsequent arguments for construction, syntheticity, or any other trait of mathematics can be made of geometry more generally.

Defences that argue for the constructive nature of Euclidean geometry are susceptible to the objection that non-Euclidean geometries can accomplish the same without construction. On my account, however, such an objection cannot be made. First, the same account of construction is ascribed to all geometries, thus eliminating the possibility of a geometry arriving at the same set of theorems and propositions by a process other than construction. It further clarifies the role of construction by making a distinction between geometrical construction and the images of the objects constructed. Moreover, the stipulative aspect of construction is made explicit and necessary. Where the geometry within which we are working is often taken for granted, it must now be stipulated. My interpretation now makes possible and necessary the construction of non-Euclidean figures while granting them both objective validity.

Defences of Kant that argue for the syntheticity of *a priori* Euclidean geometry are unable to account for the validity and objects of non-Euclidean geometry. On my interpretation, Kant's account of space can now account for the possibility and validity of all geometry. Further, by separating it from any particular geometry or set of geometries, it need not worry about the next geometry. So long as the next geometry is still about space—as it must be in order to be properly deemed a geometry—it will depend upon the *a priori* intuition of space for syntheticity and objective validity. My interpretation allows pure space to yield Euclidean and non-Euclidean axioms. Since there is no longer a distinction between the source of the objects and validity of Euclidean geometry as opposed to non-Euclidean geometry, the epistemological role of the intuition of space need not yield a restriction to Euclidean space that it otherwise would.

Such an interpretation has been shown to be consistent with Kant's account and to help defences for the constructivity and syntheticity of geometry to avoid the inconsistencies that result from treating Euclidean and non-Euclidean geometries differently. Given the role mathematics plays in Kant's philosophy, avoiding these difficulties makes Kant's *a priori* intuitive account of space a worthwhile model for game theory semantics and a fruitful ground for structuralism in mathematics.[15]

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Notes

1 Michael Friedman provides a first rate account of the incompatibilities with Kant's account of the constructive and synthetic aspects of geometry in *Kant and the Exact Sciences*. He also provides a compelling account of why Kant must be taken at his word and not interpreted anachronistically using modern accounts of mathematics. The defences of various aspects of Kant's account are many but in my estimation Lisa Shabel, David Sherry, Joongol Kim, and Emily Carson provide the most comprehensive. Shabel accounts for the constructive nature of algebra as well as geometry. Sherry argues for reductio ad absurdum proofs in geometry. Kim provides for the role of the productive imagination in

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the construction of a priori geometrical objects. Carson argues for the epistemological necessity of the intuitive account of space.

2 This does not, however, obviate the defences for the syntheticity and construction in geometry but rather grounds them.

3 Properly speaking an *a priori* intuition, as Kant discusses it, is not restricted and I use this terminology here to indicate that one restricts the intuition of space if it is taken to be solely Euclidean. I do this to avoid using the terminology of 'necessity' because if any particular fifth postulates are stipulated then these will be necessary propositions.

4 We must be careful when we speak of the intuitive plausibility of an object or of a geometry. We cannot equate intuitive plausibility with picture thinking or imagining. Although such an account may fit in with our common-sense accounts of intuitive plausibility and with certain other philosophical accounts, I hope my work has shown—as has the work of many others like Hintikka—that the relationship between picture thinking and intuitive plausibility, intricate as it may be, is not one of equality.

5 Many such objects we can imagine and picture. Others we can imagine but only within a Euclidean space or graph. Others still we cannot picture or imagine at all.

6 See Allison (2004) and Shabel (1998).

7 Among them, Kant's assertibilism and internal realism.

8 By means of intuition we would be unable to distinguish between a universe that was only slightly larger than the one we can intuit—and thus not infinite—and a universe that was actually infinite.

9 In Kant's day these constraints are merely that space was described or regulated by the laws of geometry. Since Kant's day, the geometry we use to describe space has developed and space has shown itself to be more intricately connected to time and to matter.

10 This is in fact precisely why Kant argues that space cannot be transcendentally real.

11 In fact, as we will later see, any account that turns to coincidence or correspondence for *a priori* truth will be in conflict with the Kantian position as it will stray from the position of transcendental idealism.

12 It also must be only a local claim about a particular space in which such possibility or impossibility exists since no such global conclusion can be made.

13 Or any other interpretation that aims to commit Kant's *a priori* intuitive account of space to any particular geometry or set of geometries.

14 I disagree with the assertion that tangible space is always three-dimensional Euclidean. Microwave anisotropy measurements (considered the most advanced measurements we have thus far) point to the global shape of our universe being flat. The precision of these measurements cannot yet rule out a hyperbolic curvature. They also can rule out neither local non-Euclidean behaviour (black holes, etc.) nor the possibility that our measurable universe is merely a local topology in a larger space-time continuum. There is also no reason for excluding larger topologies or black holes as part of tangible space. But such considerations are beside the point; any such measurement must be an *a posteriori* endeavour and thus not qualified to be the basis of the rejection of an *a priori* condition.

15 I elsewhere discuss why removing the postulates is a necessary step in such applications but Jaako Hintikka discusses the fruitfulness of Kant's work for game theory semantics and Graham Bird discusses its usefulness for structuralism in mathematics.

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