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AN ASSESSMENT OF THE EFFECT OF COVARIANCES OF PLOT ERRORS OVER TIME ON THE PRECISION OF MEANS OF ROTATIONS WITH WHEAT

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SUMMARY

The introduction of appropriate crop rotations is known to be beneficial in many farming systems. One feature of rotations is that it takes a valuable length of time for the advantage of the rotation to take effect. In long-term rotation trials, the observations from the same plot over years are correlated; ignoring such correlations may affect the precision of the estimates of rotation effects. We examined five covariance structures between the plot errors over time to assess the effect of correlations on the standard errors of rotation means and rotation x cycle combination (interaction) means on wheat yields using eight years of data from six two-phase rotations with wheat. Based on wheat yield data from the four cycles of the rotations considered, the compound symmetry covariance structure (constant correlation) between plot errors arising over alternate years gave more efficient estimates of rotation means compared with the other four covariance structures.

INTRODUCTION

In a crop rotation, different crops are grown in sequence over time on the same piece of land. An appropriately selected rotation provides a degree of natural control over crop yield-limiting factors such as insects, pests and weeds and a general deterioration of soil conditions, while continuous cropping with the same crop may encourage the build up of such biotic factors which may lead to a decline in yield. The aspects of design and analysis of long-term rotational trials (LTRT) have been developed and discussed at length by Cochran (1939), Yates (1949; 1954), Patterson (1964) and others. Patterson (1964) presents a review of statistical problems arising in the design and analysis of long-term cyclic experiments for comparing different rotations; Cady and Mason (1964) suggest that the year × treatment interaction indicates the cumulative effects. In fixed-rotation experiments the interest lies in cumulative effects not over years *per se* but over entire cycles of rotation. Cady and Mason (1964) suggest partitioning the year into cycle, series (that is, different crop phases) and cycle × series interaction. Thus an assessment of rotation means and rotation × cycle means is of considerable interest. An important feature of a good design for LTRT is that each phase in a rotation must appear each year. A problem which does not appear in the analysis of seasonal trials but occurs in the analysis of LTRT data is that the observations from the same plot over years are correlated (Cochran, 1939). Ignoring such correlations may affect the precision of the estimates of rotation effects.

Patterson (1964) approximates the correlation between plot errors over years using mean correlation, although the actual correlation between two years may decrease with interval between them. By considering the experimental errors to consist of two parts – a plot error sum of squares derived from plot totals and a plot × year error sum of squares – Patterson (1964) derived the correlation from the two error mean squares.

In this work five covariance structures between the plot errors were examined over time to assess the effects of correlations on the standard errors of rotation means and rotation × cycle combination (interaction) means on wheat yields, using eight years of data from six two-course rotations with wheat at the International Center for Agricultural Research in the Dry Areas (ICARDA), Syria.

COVARIANCE STRUCTURES AND STANDARD ERRORS OF MEANS

Considering only one cropping season per year, let there be R two-phase rotations (of wheat) evaluated in B complete blocks and let both phases of the rotations be present in each of the T years. The T years would give rise to c cycles of the two-phase rotations. We assume an even T, thus $T = 2c$. Each of the two phases of rotations represents a series. An example would be to consider a rotation of wheat (W) with medic, an annual pasture legume, grazed at a low stocking rate (M).

The two plots with wheat yield under wheat/medic rotation (W/M) in one of the replications can be exhibited as in the following:

Year	Phases		Series	Cycle	Yield
	Plot 1	Plot 2			
1	W	M	1	1	Y_{111}
2	M	W	2	1	Y_{112}
3	W	M	1	2	Y_{113}
4	M	W	2	2	Y_{114}
5	W	M	1	3	.
6	M	W	2	3	.
7	W	M	1	4	.
8	M	W	2	4	Y_{118}
⋮	⋮	⋮	⋮	⋮	⋮
T	M	W	2	c	Y_{11T}

Suppose we have data for c cycles of the rotations. Further, let Y_{jlt} be the yield of wheat in the t-th year in the plot under the j-th rotation and the l-th replication ($t = 1 \ldots T$; $j = 1 \ldots R$; $l = 1 \ldots B$). Following Patterson (1964) and using the partitioning of year in terms of series and cycle effects and their interaction (Cady and Mason, 1964), we can write the model for Y_{jlt} as:

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Υ_{jlt} = *Constant* + *Series effect* + *Replication within series effect* + *Rotation effect* + *Series* × *Rotation* + *Rotation* × *Replication interaction within series (= plot error)* + *Cycle effect* + *Series* × *Cycle interaction* + *Rotation* × *Cycle interaction* + *Series* × *Cycle* × *Rotation interaction* + *Replication* × *Rotation* × *Cycle within series (= plot* × *year error)*

Let the *plot* × *year* error for Υ_{jlt} be denoted by $\epsilon_{[P]t}$, where P is the plot number corresponding to the j-th rotation and the l-replicate in the wheat phase in the t-th year. For a given plot P, the c errors $\epsilon_{[P]t}$, where $t = 1, 3, \ldots T-1$ (if the first year is in the wheat phase) or $t = 2, 4, \ldots T$ (otherwise), would be correlated. Further arrange the errors $\epsilon_{[P]t}$ according to series and time within series, that is, for a given plot P, all errors $\epsilon_{[P]t}$ are written for odd years $t = 1, 3, \ldots$ in sequence and then for even years. The variance–covariance matrix of vector of errors $\underline{\epsilon}_P = (\epsilon_{[P]1}, \epsilon_{[Pj]3}, \ldots \epsilon_{[P](T-1)}, \epsilon_{[P]2}, \epsilon_{[P]4}, \ldots \epsilon_{[P]T})'$ can be written as

$$\mathrm{Cov}(\underline{\epsilon}_P) = \begin{pmatrix} \boldsymbol{\Sigma} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma} \end{pmatrix}$$

where $\boldsymbol{\Sigma} = c \times c$ variance–covariance matrix of vector $(\epsilon_{[P]1}, \epsilon_{[P]3}, \ldots \epsilon_{[P](T-1)})'$ assumed to be the same as that of vector $(\epsilon_{[P]2}, \epsilon_{[P]4}, \ldots \epsilon_{[P]T})'$ on any given plot P. We have considered the following five structures of $\boldsymbol{\Sigma}$.

Structure	*Form of* $\boldsymbol{\Sigma}$
S_1: uncorrelated errors	$\sigma^2 \mathbf{I}_c$
S_2: compound symmetry	$\begin{bmatrix} \sigma^2+\sigma_1 & \sigma_1 & \ldots & \sigma_1 \\ & \sigma^2+\sigma_1 & \ldots & \sigma_1 \\ & & & \\ & & & \sigma^2+\sigma_1 \end{bmatrix}_{c \times c}$
S_3: first order auto-correlation	$\sigma^2 \begin{bmatrix} 1 & \rho & \rho^2 & \ldots & \rho^{c-1} \\ & 1 & \rho & \ldots. & \rho^{c-2} \\ & & & & \\ & & & & 1 \end{bmatrix}_{c \times c}$
S_4: Toeplitz with two bands	$\begin{bmatrix} \sigma^2 & \sigma_1 & 0 & .. & 0 \\ \sigma_1 & \sigma^2 & \sigma_1 & .. & \\ 0 & \sigma_1 & \sigma^2 & & \sigma_1 \\ . & & & & \\ . & & & \sigma_1 & \sigma^2 \end{bmatrix}_{c \times c}$

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S_5: unstructured

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \ldots & \sigma_{1c} \\ \cdot & \cdot & & \cdot \\ & \cdot & & \cdot \\ & & & \sigma_{cc} \end{bmatrix}_{c \times c}$$

In all the structures except S_5 the error variances are considered homogeneous, while in S_5 the error variances $\sigma_{ii}(i = 1 \ldots c)$ are allowed to vary with cycle number i and the error covariances $\sigma_{ii'}$ with the pair of cycles $i \neq i' = 1 \ldots c$. Under structure S_1, the plot-errors are uncorrelated. Since the same plot is measured over cycles, S_1 may not be a realistic structure in the context of long-term rotational trials but can be examined as a control structure for comparing with the other structures. Structure S_2 comprises a constant covariance (σ_1) and hence a constant correlation (given as $\rho = \sigma_1/(\sigma^2 + \sigma_1)$ in Table 1) between the errors over cycle pairs and may be suitable for a moderate number of cycles. This structure has been considered by Patterson (1964). In structure S_3, correlation between the errors from two consecutive cycles is ρ (also in Table 1). This structure generates a power series $\rho^{|i-i'|}$ in ρ for correlations between errors arising under cycles i and i' on the same plot and may be suitable for modelling data based on a large number

Table 1. *Estimates of parameters of covariance structures in long-term rotational trials for wheat yields (kg ha^{-1})*

Structures†		Grain yield			Straw yield		
		Cycles			Cycles		
		1–4	1–3	2–4	1–4	1–3	2–4
(multiply the estimates of σ^2 and σ_1, and their standard errors by 10^4 for cycles under grain yield and by 10^5 for cycles under straw yield)							
S_1	σ^2	8.75	7.52	9.29	3.35	2.94	2.60
	s.e.	1.38	1.37	1.70	0.530	0.657	0.474
S_2	σ^2	9.31	7.78	9.97	3.42	2.94	2.86
	s.e.	1.70	1.74	2.23	0.624	0.657	0.640
	σ_1	−0.557	−0.297	−0.869	−0.0655	0.000	−0.0262
	s.e	0.102	0.664	0.194	0.0120	0.000	0.0587
	ρ	−0.0636	−0.0397	−0.0955	−0.0196	0.000	−0.1009
S_3	σ^2	8.75	7.52	9.29	3.35	2.81	2.60
	s.e.	1.39	1.38	1.70	0.530	0.699	0.474
	ρ	−0.0406	−0.0639	−0.0382	0.0028	−0.0842	−0.0150
	s.e.	0.1345	0.1826	0.1485	0.1508	0.2555	0.1681
S_4	σ^2	8.76	7.52	9.29	3.35	2.79	2.60
	s.e.	1.39	1.38	1.70	0.530	0.755	0.475
	σ_1	−0.40	−0.49	−0.40	−0.0098	−0.256	−0.0565
	s.e.	1.26	1.38	1.47	0.5166	0.8138	0.5266
	ρ	−0.0461	−0.0651	−0.0430	0.0029	−0.0916	−0.0217

†Structures: S_1 = uncorrelated; S_2 = compound symmetry; S_3 = first order auto-correlation; S_4 = Toeplitz with two bands; S_5 = unstructured; s.e. = standard error of the associated estimate.

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of cycles. Structure S_4 is based on the assumption that the errors between only consecutive cycles are correlated (correlation given as $\rho = \sigma_1/\sigma^2$ in Table 1) and are independent of the errors from the other cycles. This may be suitable for situations with relatively low correlation values. When the trial has undergone only two cycles (that is, $c = 2$), the three correlation structures S_2, S_3 and S_4 would be equivalent. However, it is less likely that any reasonable build-up of the cumulative effects may be reflected in such a short rotation. Therefore, we kept our comparisons based on data sets from a minimum of three cycles.

Estimation of parameters was done using SAS software. The variance–covariance parameters were estimated using the restricted maximum likelihood method (REML), a default option in the PROC MIXED procedure of SAS. In the experiment described in the next section, $T = 8$, $c = 4$, $R = 6$, $B = 3$.

AN EXPERIMENT

A long-term trial on rotations with wheat

An experiment was started in the 1986/87 season at Hemo Experimental Station near Kamishly in north-east Syria. This work, undertaken by the Pasture, Forage and Livestock Programme of ICARDA, aimed to determine the productivity of wheat in rotation with six cropping treatments (three rates of grazing pressure by sheep on a mixture of medics, a common vetch, fallow and wheat). There were three replicates, with treatments being allotted randomly within each complete block. Both phases of the rotations were present each year. Seed and straw yields of wheat per plot were measured, together with other variables. For the purpose of this paper, we considered wheat yield from six rotations over the eight years (1986/87 to 1993/94) under a single fertility level.

Estimation of covariance structures parameters was done for each of the three sets consisting of data from (i) all eight years (4 cycles), (ii) the first three cycles (1986/87 to 1991/92) and (iii) the last three cycles (1988/89 to 1993/94). In the first two data sets (i, ii) we ignored the role of preliminary years while in (iii) we allowed for an appropriate plot history of rotations.

RESULTS AND DISCUSSION

Long-term trials are very expensive and three to four cycles of a rotation trial represent a sizeable commitment of resources. We present estimates of parameters of the first four covariance structures for each data set in Table 1 and the precision of rotation means and rotation × cycle means in terms of their standard errors in Table 2. The means of rotations over all cycles on wheat grain yield varied from 1030 kg ha^{-1} for the wheat–wheat to 2352 kg ha^{-1} for the wheat–fallow rotation. The mean over all rotations was 1681 kg ha^{-1} for the wheat grain and 3545 kg ha^{-1} for straw.

Except for the structure S_1, where errors are assumed uncorrelated, and S_5, where they are allowed to vary over all pairs of cycles, the other three structures

Table 2. *Estimates of standard errors of rotation and rotation × cycle means under the five covariance structures (S_1 = uncorrelated; S_2 = compound symmetry; S_3 = first order auto-correlation; S_4 = Toeplitz with two bands; S_5 = unstructured) on plot errors over times*

Yield	Cycles	Mean	S_1	S_2	S_3	S_4	S_5
Grain	1–4	rotation	60.4	54.4	58.6	58.3	65.1
		rotation × cycle	120.8	120.8	120.8	120.8	(106.1–130.6)
	1–3	rotation	64.6	62.3	61.9	61.8	74.9
		rotation × cycle	111.9	111.9	111.9	112.0	(88.6–145.9)
	2–4	rotation	71.8	66.3	70.0	69.7	78.2
		rotation × cycle	124.4	124.4	124.4	124.4	(106.1–161.0)
			(Mean of all rotations = 1681 kg ha^{-1})				
Straw	1–4	rotation	118.2	115.0	118.5	118.5	111.1
		rotation × cycle	236.5	236.5	236.5	236.5	180.0–304.7
	1–3	rotation	144.7	144.7	143.3	143.3	166.7
		rotation × cycle	231.7	231.5	231.5	231.2	232.0–338.0
	2–4	rotation	120.2	107.4	119.0	118.4	119.4
		rotation × cycle	208.1	208.1	208.1	208.1	190.0–256.0
			(Mean of all rotations = 3545 kg ha^{-1})				

Degrees of freedom for s.e. (rotation) = 20; degrees of freedom for s.e. (rotation × cycles) = 40 for 1–3 or 2–4 cycles and 60 for 1–4 cycles.

S_2, S_3 and S_4 include one correlation parameter, which measures the correlation between plot errors from two consecutive cycles (for example, between year 1 and year 3, or between year 2 and year 4, for any given plot). The estimates of correlation were generally negative (Table 1) and rather small in magnitude: from −0.04 to −0.10 for grain and from 0 to −0.10 for straw. For correlations, there was no pattern across the three data sets selected. For example, the correlation varied from −0.064 to −0.095 over the data sets for grain yield under the compound symmetry model (S_2), from −0.038 to −0.064 under the first order auto-correlation model (S_3) and from −0.043 to −0.065 for the Toeplitz with two bands model (S_4). The magnitude of the correlation under compound symmetry was generally higher than the correlations under autoregressive and Toeplitz models for both grain and straw yields and also for the data sets based on the four cycles and the last three cycles.

Table 2 indicates that the standard errors of rotation means under uncorrelated errors (structure S_1) were higher than those under other structures except S_5. Thus there was an increase in precision of rotation means by considering any of the correlation structures (S_2, S_3 or S_4). Furthermore, under the compound symmetry model (S_2), estimates of standard errors of rotation means were smaller than with any other correlation structure, when data from either the last three cycles or all four cycles were used. However, when data from the first three cycles were analysed we found first order autocorrelation or Toeplitz models gave a slightly lower standard error of rotation mean than that under compound symmetry. Standard errors for rotation × cycle means were the same under the first four models for any of the subsets of data. We also noticed (Tables 1 and 2) that the cases where S_2 did not perform as well as S_3 and/or S_4, the magnitude of the correlation (ρ) was lower than in S_3 and S_4. Under the unstructured covariance model (S_5), standard errors varied over the combinations of rotation and cycle, as can be seen by the range of values shown in Table 2.

The data set examined here was rather small. However, based on these data, we found that compound symmetry gave more precise estimates than other covariance structures. Thus, further evaluation of crop rotation systems in terms of rainfall and number of cycles of rotations may be carried out under the above correlation structure.

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