RUN OF THE SCALE FACTOR R(t) IN SOME UNIVERSES WITH ZERO AND NONZERO PRESSURE

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We have a program to integrate and show graphically the run of the scale factor R(t) in generalized Friedmann-Lemaître universes containing two non-interacting fluids, with densities ρ_{nr} and ρ_r, plus a cosmological constant Λ. One fluid exerts no pressure ("ordinary matter"); the second exerts a pressure $p = (\nu - 1)\rho_r c^2$, where ν is an arbitrary constant (the Zeldovich parameter). Then ρ_r varies as $R^{-3\nu}$. If $\nu = 4/3$, the second fluid is radiation or relativistic particles; if $\nu = 1$ it is pressureless like the first. For other values of ν, the second fluid is some kind of exotic matter. The cosmological constant, which we write in the "reduced form" $\lambda_0 \equiv \Lambda/3H_0^2$, may be regarded as a "third fluid" having $\nu = 0$ ($p = -\rho c^2$) and density $\Lambda/8\pi G$ (positive or negative).

We normalize R(t) to unity at present time $t_0 = 0$: $R_0 = 1$. Then the run of R(t) vs. time in units H_0^{-1} is completely determined by four disposable constants, which we can take to be Ω_{nr0}, Ω_{r0}, ν, and λ_0, where $\Omega_{nr0} \equiv 8\pi G\rho_{nr0}/3H_0^2$ and $\Omega_{r0} \equiv 8\pi G\rho_{r0}/3H_0^2$. Examples of graphs of this kind are useful in visualizing the wide range of behavior of R(t) permitted by the generalized Lemaître equation, particularly if $\lambda_0 \neq 0$, Ω_0 need not be unity, and/or space need not be flat. One can always calculate a critical value of λ_0, $\lambda_s(\Omega_{nr0}, \Omega_{r0}, \nu)$, such that $\lambda_0 > \lambda_s$ produces a "bounce" (no big bang) in the past. If $\lambda_0 = \lambda_s + \varepsilon$, where ε is small, the bounce has a coasting period. $\lambda_0 < \lambda_s$ gives a big bang, and if $\lambda_0 = \lambda_s - \varepsilon$, one has a big bang followed by a coasting period. A bounce at large z usually implies a very low density at present, but this is not true for all pressure laws.

QSO observers can read from these graphs the approximate age of the universe when it contained a QSO observed at red shift z, for any given model. We can supply graphs, accurate model ages, and other particulars for any desired model. The following graphs show some interesting examples of R(t). We wish to thank Dave Bazell for help with these figures.

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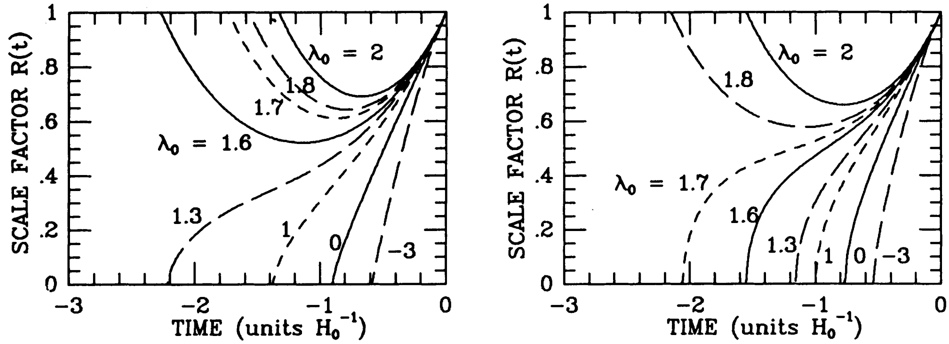
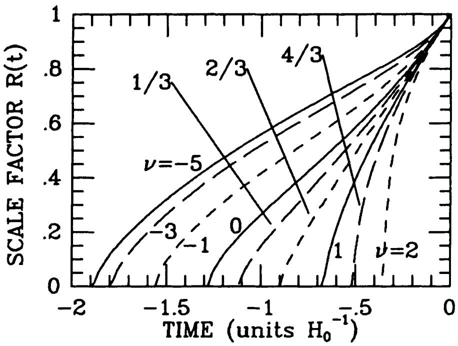


Figures 1a-1b. The left-hand models (Fig. 1a) are Friedmann models ($p = 0$) having $\Omega_0 = 0.1$ and various values of λ_0 (shown on the curves). Figure 1b (right) shows how these same models change if the pressure is relativistic ($p = (1/3)\rho c^2$ instead of $p = 0$). The pressure shortens the age of each model, and two ages which were infinite become finite.

Figure 2(below, left). These are "standard inflationary" models with $\Omega_{nr0} = 0.1$, $\Omega_{r0} = 0.9$, $\lambda_0 = 0$, and various values of ν (shown on the curves); i. e., the universe contains some ordinary matter and is "closed" by exotic matter having various equations of state. The model age is seen to be a monotonic function of ν.



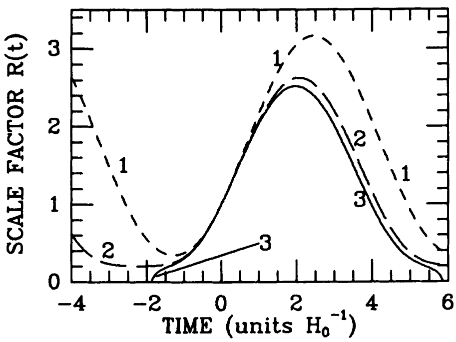


Figure 3(above, right). These three are among the curious models which exist in the realm of negative pressure. All three have $\Omega_{nr0} = 0.1$, $\Omega_{r0} = 2.9$, and $\nu = 1/3$; i. e., the universe contains some ordinary matter plus a lot of exotic matter with $p = -(2/3)\rho_r c^2$. The top model (curve 1) has $\lambda_0 = -0.8$. It oscillates between finite maximum and minimum values of R(t). It turns around at the bottom because the repulsive (negative) pressure dominates, and at the top because the attractive (negative) λ_0 dominates. By "fine-tuning" λ_0 to be near a critical value (curve 2, $\lambda_0 = -0.958$), we can make the model coast at minimum. If λ_0 is more negative (curve 3, $\lambda_0 = -1$), the model has a big bang and a big crunch.

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