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WILLIAM ALONSO, RICHARD MUTH, RESOURCES FOR THE FUTURE, AND THE FOUNDING OF URBAN ECONOMICS

BY
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I. INTRODUCTION

Frequent reference is made to the Alonso-Muth model of the urban land and housing markets, and the creation of this model often is considered to be the founding act in the field of urban economics. One purpose of this paper is to examine closely the original writings of these two distinguished authors, William Alonso and Richard Muth, and to show that their contributions to the development of the model were not identical. Both authors focused on the idea that there is a trade-off between access to a central point (for employment) and the price of residential land or housing, but their models were designed to serve somewhat different purposes. A second objective of this paper is to evaluate the proposition that theirs was indeed the founding act in the field by concentrating on the earliest versions of their work on the model, rather than on the enormous literature that followed. See Masahisa Fujita (1989) for a comprehensive treatment of the subsequent theoretical literature that was stimulated by Alonso and Muth.

At the outset, the critical roles of Resources for the Future and the Ford Foundation in the founding of the field of urban economics must be acknowledged. Resources for the Future was founded in 1952 as a non-profit corporation for research and education in the development, conservation, and use of natural resources. Its work during the early years was financed by grants from the Ford Foundation. The program in regional studies (later renamed regional and urban studies) was directed by Harvey S. Perloff, and Lowdon Wingo, Jr. worked as a research associate in the field of regional economics and later succeeded Perloff as director of the program. The Committee on Urban Economics (CUE) of Resources for the Future was active during the 1960s in providing doctoral dissertation fellowships in urban economics and sponsoring research and research conferences. Harvey Perloff served as chairman of CUE. The basic research of both Alonso and Muth was supported by Resources for the Future in the late 1950s. Resources for the Future published a series of volumes in the field, including Wingo (1961), Wilbur Thompson (1965), Perloff and Wingo (1968), Edwin Mills (1972), and others.

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The paper begins with a brief discussion of the antecedents to the Alonso-Muth model. The doctoral dissertation by Alonso (1960a) is the first extant version of the model, so the antecedents refer to work before 1960. The next section is a detailed examination of Alonso's work (1960a, 1960b, 1964), the original dissertation and the article and book based on the dissertation. This is followed by a close examination of the original contributions by Muth (1961a, 1961b) that were written roughly simultaneously with and independently of Alonso. Muth's work on the model culminated in his classic book, *Cities and Housing* (1969). The paper concludes by placing the Alonso-Muth model in the broader context of applied microeconomic theory. It is suggested that they, along with several others, developed expanded versions of the standard model of consumer choice that has led to a wide variety of applications.

II. ANTECEDENTS

Both the dissertation and the book by Alonso (1960a, 1964) provide extensive discussions of the early writings on the economics of urban land. The most influential early writings on the economics of land use and land rent are, of course, David Ricardo (1817) and Johann von Thünen (1826). Ricardo's theory of agricultural land rent is based on differential fertility and fixed supply of land at each level of fertility. The land actually in use with the lowest level of fertility earns no rent. Land with greater fertility earns rent per acre equal to the value of its output minus the value of the output of the least fertile land actually in use. von Thünen's theory bases agricultural land rent on distance from the market. The land in use at the greatest distance from the market earns no rent, and other land in use earns rent equal to the cost savings in transporting the output to the market. Both of these theories were common knowledge, and the von Thünen model had been used extensively in the related field of land economics. Indeed, in his first contribution, Muth (1961a) simply asserted a von Thünen plain without further attribution.

According to Alonso (1960a, 1964), little progress had been made in the analysis of urban land markets, especially the market for residential land—by far the largest private user of urban land. However, Alonso (1960a, p. 22) gave credit to an unpublished paper by Martin Beckmann (1957a) as "in many respects the most similar to that which will be presented in this dissertation." Alonso (1960a) discussed Beckmann's model in detail and concluded that it is a special case of his more general model. Beckmann's model is discussed below after Alonso's model is examined. On the other hand, Muth never cited the unpublished Beckmann (1957a) paper. Beckmann (1957b) had also published a paper on the equilibrium distribution of population in space, but this paper is not a model of land markets and is not cited by either Alonso or Muth.

In a 1979 interview, Muth (1979) gave credit to the book by Wingo (1961) that he read in manuscript form. Referring to his classic book (1969), Muth states that, "The one feature of the book that has seemed to me to inspire more professional attention than any other is Chapter 2 on the spatial equilibrium of the household. This analysis resulted largely from earlier conversations with Lowdon Wingo in Washington and reviewing his manuscript for publication" (1979).

Alonso (1960a) did not cite Wingo's manuscript in his dissertation, but he did cite Wingo's book (1961) in his own (1964, p. 15) as follows: "Mr. Wingo's and my own work were carried out simultaneously and independently, neither of us aware of the other's activity." The book by Wingo existed in draft form prior to its publication in 1961. The published version cites both the Alonso dissertation (1960a) and Muth's first article (1961a) in footnotes. One of Wingo's (1961, p. 82) references to Alonso is a basic description of Alonso's model of residential bid prices, but the reference to Muth (1961a) is simply an acknowledgement.

Wingo's (1961) model of the household in urban space specifies that the total amount spent by a household on land and transportation is equal to a constant. Transportation cost is the cost of traveling to work and is a function of distance to the city center. The price of land at the most distant occupied location is zero, so

$$PQ + k(t) = k(t_0),$$

where P is the rental price of a unit of land, Q is the quantity of land, $k(t)$ is the transportation cost at distance t, and t_0 is the transportation cost at the edge of the urban area. A simple demand function for land is assumed:

$$Q = (a/P)^b,$$

where $b>0$ is the price elasticity of demand for land. At any given location the first equation fixes the amount of money spent on land, and the second equation determines quantity. However, there is a basic difficulty with this model. Solving for P yields

$$P^{1-b} = [k(t_0) - k(t)]/a^b.$$

If the demand for land is of unitary price elasticity ($b = 1$), then this equation yields the implausible result that

$$k(t_0) - k(t) = a^b \quad (= \text{a constant}).$$

In other words, transportation cost at any distance $t < t_0$ inside the urban area must be the same as at any other distance inside the urban area. Further, if demand is inelastic the rental price of land declines with distance to the city center, but if demand is elastic the P increases with distance.

III. WILLIAM ALONSO'S MODEL OF THE URBAN RESIDENTIAL LAND MARKET

William Alonso's work on the model is contained in his dissertation (1960a), completed in 1959, one short article (1960b) that was presented at the 1960 meetings of the Regional Science Association, and in the classic book *Location and Land Use* (1964). The book is a very minor revision of the dissertation, so the publication lag

was quite long. Alonso's dissertation was the first one completed in Regional Science in the newly formed Department of Regional Science at the University of Pennsylvania. Walter Isard and Benjamin H. Stevens both signed as supervisors of the dissertation. Alonso (1960, p. xvii) acknowledged the fellowship support of a grant from Resources for the Future.

Alonso stated his purpose as the development of "self-consistent explanatory theory ... which will shed light on some aspects of the internal structure of cities" (1960, p. 2). In all three versions of his work he made no attempt to draw policy conclusions from the model. The last chapter in both the dissertation and the book is titled "Some applications of the model and an outline for empirical research." As we shall see, Muth's (1961a, 1961b) early objective also was the development of explanatory theory, although he did draw some policy conclusions in the later book (1969). The focus on the development of explanatory theory is not surprising, of course, but it is worth recalling that Alonso and Muth came from two different traditions or schools of thought in economics—Alonso from the mainstream (liberal) school associated with Harvard and other Ivy League universities, and Muth from the conservative Chicago school. Alonso was supervised by Walter Isard, who had just arrived at Pennsylvania from Harvard. Alonso joined the faculty at Harvard and served there and at Berkeley for his entire career until his death in 1999. Muth earned his Ph. D. from the University of Chicago in 1958, and returned as an Assistant Professor in 1959. It is clear that both were influenced by the research agenda that was being promoted by Harvey Perloff and Lowdon Wingo at Resources for the Future, but it is interesting that two researchers from different schools of thought took similar approaches to problem definition and solution.

Alonso (1960, p. 2) stated that the focus of his attention is on the market for residential land within the city because four-fifths of privately developed land in cities is devoted to housing, and because the economic theory of this market has been neglected. He examined urban firms more briefly, and this aspect of his work will not be discussed in detail here. The basic assumptions of his theory were stated clearly:

> The city in which the individual arrives is a simplified city. It lies on a featureless plain and transportation is possible in all directions. All employment and all goods and services are available only at the center of the city. Land is bought and sold by free contract, without any institutional restraints and without having its character fixed by any structures on the ground. Municipal services and tax rates are uniform throughout the city. The individual knows the price of land at every location, and from his point of view it is a given fact not affected by his actions (1960a, p. 28).

A monocentric city is assumed (all employment located at the center), and the model ignores structures. The first modeling task is to examine the individual choice problem.

The budget constraint faced by the individual is (using Alonso's notation)

$$y = p_z z + P(t)q + k(t),$$

where y is income, z is the composite good with price p_z, q is the quantity of land with price $P(t)$ at distance t from the center of the city, and $k(t)$ is the monetary cost of

commuting from distance t to the center. The utility function that is maximized subject to the budget constraint is

$$u = u(z, q, t).$$

Both z and q are goods of the usual sort, but good t is unusual. Alonso states that:

> We assume that, all other things being equal, a rational individual will prefer a more accessible location to a less accessible one. Since t represents the distance from the center of the city, and thus the distance he must commute to the principal place of shopping, amusement, and employment, we may say that accessibility decreases as t increases. In other words, the individual would prefer t to be smaller rather than larger, so that t may be thought of as a good with negative utility. Increases in t produce disutility (1960a, p. 44).

The specification of the consumer choice problem in this way is at the heart of Alonso's enduring contribution to the field because the standard model is expanded to include the choice of location. Alonso was one of the first to adapt the basic model to the study a topic that was not a traditional part of consumer choice theory. The traditional theory of choice, as outlined and criticized by Gary Becker and Robert Michael (1973), specifies that the consumer maximizes utility, which is obtained directly from the services of goods purchased in the marketplace, subject to a money budget constraint. The single important behavioral "law" that is derived is that an income-compensated change in a relative price of any good changes the quantity demanded of that good in the opposite direction.

Alonso (1960a, Chapter II) used some rather cumbersome graphical and mathematical methods to solve the consumer choice problem, but the solution is actually quite straightforward. Maximization of utility subject to the budget constraint with respect to z, q, and t produces the following first-order conditions:

$$\begin{aligned} u_z - \lambda p_z &= 0, \\ u_q - \lambda P(t) &= 0 \\ u_t - \lambda[q(dP/dt) + dk/dt] &= 0, \end{aligned}$$

where u_z, u_q, and u_t are the partial derivatives of the utility function with respect to z, q, and t; and λ is the Lagrange multiplier associated with the constrained maximization problem (i.e., the marginal utility of another dollar of income y). Manipulation of these conditions produces two consumer equilibrium conditions:

$$\begin{aligned} u_q/u_z &= P(t)/p_z \text{ and} \\ u_t/u_z &= [qdP/dt + dk/dt]/p_z. \end{aligned}$$

The first condition simply states the usual condition that the marginal rate of substitution of the two standard goods equals their price ratio. The second condition is the equilibrium condition for individual's choice of location, and states that the marginal rate of substitution of distance t for composite good z equals what Alonso

(1960a, p. 73) calls the marginal cost of spatial movement divided by the price of z. Without loss of generality, we can assume that the good z is expenditures on goods other than land and commuting (that is, the price of z is unity). Recall that u_t is assumed to be less than zero (and u_z is positive, of course). The second condition can be rewritten as:

$$q(dP/dt) = [(u_t/u_z) - dk/dt].$$

This condition states that when the individual is in equilibrium, the change in the amount paid for land as distance increases equals the additional cost of distance, which is broken down into two components: the dollar value of its marginal disutility and the added monetary cost of distance. In short, the individual is in equilibrium when the marginal benefit of an increase in distance equals its marginal cost. The right-hand side of this equation is negative (and q is positive), so dP/dt must be negative as well. The price of land must decline with distance to the center of the city for individuals to be in equilibrium. What an elegant piece of deductive reasoning this is.[1]

Alonso (1960a, Chapter IV) next turned the consumer choice problem around to derive what he called residential bid price curves. A bid price curve shows the price the individual is willing to pay per unit of land as a function of distance to the center of the city and remain at the same level of satisfaction. To solve this problem one minimizes the expenditures on z, q, and t subject to the constraint that utility is held constant at some arbitrary level. In this case, the bid price is one of the unknowns along with z, q, and t, and will be denoted $p_i(t)$ for individual i. The slope of a bid price curve for individual i at some level of utility is simply:

$$dp_i/dt = [(u_t/u_z)_i - dk/dt]/q,$$

where the subscript refers to individual i. The bid price curve has a slope that leaves the individual indifferent to alternative locations because the reduction in expenditure on land equals the increase in the cost of distance. As Alonso (1960a, p. 127) put it: "Bid price, then, has been defined so that the income effect of cheaper land will counteract the depressing effect of commuting costs on income, and will permit the consumer to maintain a given level of satisfaction by manipulating the substitution of land and the composite good at any location."

[1]Alonso (1960a, p. 64) put Beckmann's (1957a) model into his notation. Beckmann's individual with a given income level spends a constant amount on land and commuting; $pq + k(t) = K$, but this amount will increase with increases in income. Utility is a function only of land, so $u = u(q)$. Maximization of utility subject to this budget constraint produces the equilibrium condition that $q(dp/dt) = -dk/dt$. Writing this as $dp/dt = -(dk/dt)/q$, Alonso noted that an increase in income, which will increase q, requires that the slope of the land price function become unambiguously "flatter." As is shown below, this result means that higher income people will locate farther from the center of the city than lower income people (so also spend more on commuting). However, Alonso (1960a, p. 23) believed that there is a basic flaw in the mathematics of Beckmann's model as originally formulated, but he was unable to pinpoint the source of the problem.

Market equilibrium at any distance t requires that

$$dP/dt = dp_i/dt$$

for those individuals who occupy land at that distance.

Alonso's treatment of the firm is fairly brief but generalizes the existing literature in the von Thünen tradition. Profits for a firm located at some distance t from the center of the city are

$$G = V(t, q) - C(V, t, q) - P(t)q,$$

where V is total revenue (a negative function of t and a positive function of q), C is operating costs (a positive function of V and t and effect of q of indeterminate sign). At a given level of profits (for example, $G = 0$), the location equation for firm i is

$$V_t - C_V V_t - C_t = q(dp_i/dt).$$

The first term is the loss in total revenue that results from moving away from the center of the city, the next two terms are the change in operating costs, and the last term is the familiar change in the amount spent on land. Therefore, the slope of the bid price for the firm is:

$$dp_i/dt = (V_t - C_V V_t - C_t)/q,$$

the change in the volume of sales minus the change in operating costs, divided by the quantity of land. If the output per unit of land is a constant (as in some versions of the model) and operating costs do not vary with location, then the slope of the bid price curve is reduced to $dp_i/dt = (Q_i dP_i/dt)/q$, where Q_i and P_i are output quantity and price for the firm.

Alonso's chapter V in both the dissertation and the book (1960a, 1964) contain his treatment of market equilibrium. He emphasized the importance of the relative steepness of the bid price curves for the various sectors in the determination of the pattern of land use. A brief statement is in the article (Alonso 1960b). It is recognized that his treatment of market equilibrium is incomplete, but subsequent work by others has produced a complete general equilibrium system based on Alonso's principles. As Mills (1972) and others have pointed out, Alonso's system is incomplete because it does not attempt to make the bid-price curves of the various sectors consistent with each other. Firms and households locate in the city, but there is no equating of demand for and supply of labor. Furthermore, employment is no longer all located at the center, so travel to work presumably does not only involve trips to the center. Mills (1972) also offers a more technical critique of chapter V that essentially amounts to saying that Alonso neglected to assume perfect competition in the land market, and that this assumption is needed to obtain Alonso's results as presented.

Alonso (1960a, 1964; chapter VI) concluded the body of his treatise with applications and suggestions for empirical research. He was particularly interested in

tracing out the effects of income, improvements in urban transportation, and population growth on the residential land market, as well as considering causes of city shapes that depart from the circular city implied by his basic model. He gave detailed attention to the effect of income on the slope of the residential bid price function, and he concluded that individuals with higher incomes do not necessarily have flatter bid price functions (and therefore do not necessarily reside at greater distances from the center than individuals with lower incomes). Rewriting the equation for the slope of the residential bid price function from above,

$$dp_i/dt = (u_t/u_z)_i/q - (dk/dt)/q.$$

Since land is a normal good, an increase in income will increase q and therefore tend to make the bid price function "flatter." However, an increase in income likely will increase the monetary value associated with the disutility of an increase in distance; that is, $(u_t/u_z)_i$ becomes a larger negative number and tends to make the bid price function steeper. He concluded with a limited empirical test of the residential model in which the amount spent on land *(pq)* was hypothesized to be a function of income and distance to the center of Philadelphia. The test showed that income has a positive effect, and distance a negative effect on the amount spent on land. Alonso (1960a, p. 208) stated that both of these results are as expected by the model. However, at a given income level, an increase in distance reduces price and increases quantity of land, leaving the sign of the effect indeterminate. Essentially Alonso left empirical testing of the model to others, including Richard Muth.

IV. RICHARD MUTH AND THE URBAN HOUSING MARKET

Richard Muth began his career as a graduate student in economics at the University of Chicago. There he developed an affiliation with the Cowles Foundation for Research in Economics, and he is the author of Cowles Foundation Discussion Paper No. 2, dated 1955 and titled "Comments on organizational aspects of the inventory control problem." Cowles Foundation Discussion Paper No. 1 is "The application of multivariate probit analysis to economic survey data" by James Tobin, who served as Director of the Cowles Foundation upon its move from Chicago to Yale. Muth's work on inventory policy included two papers with Martin Beckmann (Beckmann and Muth 1956, 1957). However, Muth's (1958, 1960) doctoral dissertation is not on inventory theory or spatial patterns of urban housing markets, but rather is "The demand for non-farm housing." In this work he devised his concept of housing, which is "the quantity of service yielded by one unit of housing stock per unit of time" (1960, p. 32). Housing is a bundle of services that is produced by stocks of housing capital and land, and the price of housing is the expenditure needed to purchase a standardized basket of those services. As we shall see, in his work on the spatial patterns of urban housing, Muth focused on two attributes: capital embodied in the house and land (lot size). We pick up the story in an interview with Professor Muth (1979). He stated that:

> My work in this field began over 20 years ago while at Resources for the Future in Washington, D.C. One day while reading a book by a good friend and colleague,

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> I came across the claim that the rental value of farmland is determined without reference to external markets. Thinking this to be surely incorrect, I sat down to work out what I thought to be a correct analysis of how markets determine land values, especially in a spatial context. Much of this was worked out over a two-day period while more-or-less confined to my apartment by a severe snowstorm. It resulted in my first paper on the subject (1979).

That first paper appeared in *Econometrica* in 1961 (Muth 1961a)—another lengthy publication lag. Muth (1979) goes on to say that he was visiting at Vanderbilt about six months later when Milton Friedman, his former teacher, came to give a lecture. Muth and Friedman had a chat:

> After describing my recent work, I mentioned that under certain circumstances, it implied that urban population densities would decline negative exponentially with distance from the city center. Milton remarked that Colin Clark had found this to be the case empirically in a paper published eight years earlier [Clark (1951)], of which I had been previously unaware. Finding true confirmation of the analysis, I determined to pursue it further.

Shortly after this encounter, Muth accepted a position at the University of Chicago Graduate School of Business and moved to Chicago in the summer of 1959. Working backward from the timing of the move to Chicago, Muth's conversation with Milton Friedman in Nashville took place sometime early in the 1958-59 academic year. Six months prior to that conversation would place the two-day snowstorm in the late winter of 1958. Recall that Alonso's dissertation was completed in 1959, after what he described (1960a, p. xvii) as "a long and arduous process." So it is possible that Alonso was at work on the topic in early 1958, but if I had to pick a founding moment for the field of urban economics, I would nominate that snowy two-day period in which Richard Muth worked out what he thought to be "a correct analysis of how markets determine land values, especially in a spatial context."

Muth (1961a) constructed a model of two industries located on a featureless urban plain. The price of output received declines exponentially with distance from the center of the city. Firms produce output with two inputs, land and another input he called labor, but Muth (1961a, p. 5) stated that, "Now here, of course, labor stands for all inputs other than land: by assuming that the relative factor prices of all factors except land are fixed I may treat them as a single input."

Muth (1961a, p. 1) identified one of the industries as housing services and the other as an agricultural commodity, and he focused on the distance from the center of the city at which the land use changes from urban to rural. The second input is called "non-land" in the later article (Muth 1961b) that concentrated on urban housing, and in fact Muth did not refer to the second input into the production of housing as "built structures" until a decade later (1971).[2] In this article Muth called housing the capacity to produce housing services. The notion that housing, which represents the capacity to produce housing services, is produced by stocks of land and built structures is a particularly apt invention because such a bundle of land and capital is bought and sold in a very active market—the market for residential real estate.

[2]Mills (1967), in his first article in the field, assumed that housing is produced using land, labor, and capital.

Muth (1961a) assumed that urban housing services are produced according to a Cobb-Douglas production function:

$$Q = aL^{\alpha}N^{\beta} \quad (\alpha + \beta = 1),$$

with Q = output, L = land, and N = non-land. The price of output is p, the price of land is r, and the price of non-land is w. On the grounds that transportation costs generally increase at a decreasing rate with distance, the price of output is assumed to decline exponentially from the center of the city according to

$$p(t) = p_0 e^{-ct}.$$

The assumption of competitive input and output markets leads to the input demand functions for the firm

$$L = \alpha pQ/r \quad \text{and} \quad N = \beta pQ/w$$

Substitution of these two input demand functions into the production function and rearranging terms produces

$$r = [(a\alpha^{\alpha}\beta^{\beta})^{1/\alpha}]p^{1/\alpha}w^{-\beta/\alpha},$$

i.e.,

$$lnr = \text{constant} + 1/\alpha \ \ lnp - \beta/\alpha \, lnw.$$

Given that the price of output declines exponentially with distance,

$$dlnr/dt = -c/\alpha.$$

The ratio of non-land to land (a measure of the intensity of land use) is

$$N/L = (\beta/\alpha)(r/w),$$

so

$$ln(N/L) = \text{constant} + ln\ r - ln\, w.$$

Given that w is constant by assumption,

$$dln(N/L)/dt = -c/\alpha.$$

Muth (1961a) also showed that the ratio of output to land varies according to

$$dln(Q/L)/dt = -\beta c/\alpha.$$

In short, if the price of output declines exponentially with distance to the center of the city, then (given Cobb-Douglas technology) so do the price of land, the ratio of non-land to land inputs, and the ratio of output to land.

Muth's purpose in the first article (1961a) was to show how the boundary between urban and rural land use changes with changes in underlying variables in the model. He assumed that the two industries faced demand functions at the city center, and that these demand functions have finite elasticities with zero cross elasticities. He examined the effects of several variables, and his results include the following:

(1) An increase in the demand for housing moves the "city limits" outward, thereby reducing the supply of the agricultural product and increasing its price. The market prices of both commodities increase, so in that sense the markets for the two commodities are related.
(2) An increase in the demand for both goods has an ambiguous effect on the city limits, with the net effect depending upon the relative demand elasticities of the two goods. For example, if the demand for the agricultural product is highly elastic and the demand for housing is not highly elastic, then land will be converted from rural to urban use.
(3) The net effect of an increase in the price of non-land inputs depends upon the extent to which the price of each product depends upon this input price and upon the relative demand elasticities.
(4) If the housing price gradient becomes steeper (flatter), the city limits move inward (outward).

Muth did not pursue the empirical work on urban-rural boundaries suggested by the first article (1961a). Instead, he adapted the model to the study of urban population densities in the second article (1961b) and provided the first formal economic model in which population density declined exponentially with distance to the center of the city. He added the assumption to the model that the per-capita income-constant (that is, compensated) elasticity of demand for housing (denoted $e < 0$) is a constant. Population density can be written $D = Pop/L = (Pop/Q)(Q/L)$, so:

$$dlnD/dt = dln(Q/L)/dt - [(dln(Q/Pop)/dlnp)(dlnp/dt)].$$

The population density gradient is therefore

$$dlnD/dt = -\beta c/\alpha + ec = -g < 0.$$

The central density in the function $D = D_0 e^{-gt}$ is found by integrating, so

$$D_0 = \boldsymbol{P} g^2/2\pi,$$

Where $\boldsymbol{P}$ is the population of the urban area and 2π appears if one assumes a circular city with all land used for housing.

A critical part of the analysis is the statement of equilibrium for the individual. Muth (1961b, p. 208) stated: "For any pattern of residential location to be an equilibrium one, for each consumer at his optimal location the saving on housing costs from a small change in distance must exactly equal the change in transport costs."

In the notation used here:

$$-Q(dp/dt) = dk/dt.$$

This condition, which is often called Muth's condition, obviously is very similar to Alonso's statement for the same equilibrium condition; the marginal benefit of an increase in distance equals its marginal cost. In other words, Alonso and Muth arrived at the same consumer equilibrium condition independently. Furthermore, Muth went on to say that:

> If one assumes, as I shall, that all households are identical—the same size and with the same tastes and incomes—then all households must be on the same indifference curve in equilibrium, regardless of their location. It follows that the per capita consumption of housing increases with distance, and its change per unit distance depends upon the real-income-constant price elasticity of demand for housing and the change in the price per unit of distance (1961b, pp. 208–209).

Rewriting Muth's equilibrium as

$$dp/dt = -(dk/dt)/Q,$$

we see that Muth based the assumption of a constant exponential decline in the price of housing with distance on a particular form for the transportation cost function. If p declines exponentially with t, then $dlnp/dt = -c$ and $dp/dt = -cp$. From Muth's condition $dk/dt = cpQ$. For dk/dt to decline as t increases ($d^2k/dt^2 < 0$), pQ must decline as t increases. For pQ to decline as t increases (p declines), the income-constant demand must be inelastic; that is, e falls in between -1 and 0. It is not clear that Muth was aware of the particular feature of the model, but the assumption of inelastic demand for housing has been confirmed by numerous subsequent empirical studies.[3]

Muth's (1961b) second article is an empirical study of population density functions for forty-six cities in the U.S. in 1950, and it is in fact the first empirical study in urban economics based on a precise economic model. The estimated equation is

$$lnD(t) = lnD_0 - gt + u,$$

where u is a random error term. The gradients varied from a low of -0.07 for Nashville and -0.08 for Los Angeles to a high of -1.2 for Utica, New York. These estimated gradients varied with car registrations (flatter gradient), proportion of manufacturing employment in the urban area located in the central city (steeper gradient), and proportion of central city dwellings in substandard condition (flatter gradient). These results are as expected.

[3]Note that unitary real-income-constant price elasticity of demand is consistent with constant marginal transportation cost, and elastic demand matches with increasing marginal transportation cost. In later years the model was often specified with constant marginal transportation cost and unitary real-income-constant price elasticity. See Jan Brueckner (1982) and Kyung-Hwan Kim and John McDonald (1987).

Muth's two articles (1961a, 1961b) initiated both theoretical and empirical work in urban economics. Indeed, the first article provided (among several new results) the first proofs of negative exponential housing price, input/land and output/land functions. The second article included the first proof of the negative exponential urban population density function and was the first empirical study in the field based on an explicit economic model. Muth's condition for individual equilibrium distance was stated in mathematical terms, but Alonso stated the same condition one year earlier in non-mathematical form as:

> Along any bid rent curve, the price the individual will bid for land will decrease with distance from the center at a rate just sufficient to produce an income effect which will balance to this satisfaction the increased costs of commuting and the bother of a long trip. This slope may be expressed quite precisely in mathematical terms, but it is a complex expression, the exact interpretation of which is beyond the scope of this paper (1960b, p. 154).

V. THE PLACE OF THE ALONSO-MUTH MODEL IN APPLIED MICROECONOMIC THEORY

Alonso and Muth expanded the conventional economic model of the consumer to include the choice of location within an urban area. Their model of the urban land market, with the tradeoff between "place and space" is at the core of urban economics, a field that has become a branch of applied microeconomics. Before Alonso and Muth, major comprehensive studies of urban economies were done without discussion of how the urban land market operates to assign activities to sites and how the intensity of land use is determined by market forces. For example, the New York Metropolitan Region Study took place in the 1950s, and the summary volume was published in 1959 (Edgar Hoover and Raymond Vernon). While this volume does include discussion of land-use surveys conducted by urban planners, there is no analysis of the market for urban land. The book includes data on population density patterns, but no economic model explaining those patterns is used. Some discussion of zoning is included, but there is no examination of the interaction of market forces and zoning constraints. It is inconceivable that economists would approach the study of a metropolitan area in a like manner today.

Alonso and Muth based their models on simplified versions of the consumer choice problem. Alonso divided consumer choice into land, location, and a composite good, while Muth used his concept of housing services, location, and a composite good. At virtually the same time other researchers were expanding the standard consumer choice model in other directions. Gary Becker and others (including Jacob Mincer and T. W. Schultz) had already begun work on the theory of human capital and the economics of fertility that pushed the boundaries of applied microeconomic theory. Zvi Griliches (1961) and Kelvin Lancaster (1966) were developing the hedonic approach to the study of complex consumer goods. The hedonic approach had been used much earlier by F. V. Waugh (1929) to study vegetable prices and by Andrew Court (1939) to explain automobile prices, but Griliches (1961) gets credit for bringing the concept of hedonic prices into general use by economists, most prominently by

housing and urban economists. In fact, Muth (1966) made an important early theoretical contribution to this literature. The hedonic approach emphasizes the idea that housing, or automobiles, or many other consumer goods are bundles of numerous characteristics that have value to households. The market supplies these complex bundles, and the market price is the price of the entire bundle. Demand, supply, and competitive equilibrium for the hedonic model were presented formally by Sherwin Rosen (1974). Stephen Shephard (1999) provides an excellent survey of the hedonic approach to the study of housing markets. Becker (1965) developed a general model of consumer choice that includes all uses of the household's time, including production of consumer goods in the home. Indeed, Becker (1965) included a discussion of the use of time for commuting to work and derived a version of Muth's condition.

The hedonic approach to the study of housing has purposes that differ from those of Alonso and Muth. Shephard described those purposes as follows:

> Imagine, for a moment, that you are a private investigator or market researcher studying the demand for food. You have a particular disadvantage, however, in that you have been banned from entering the local grocer. You have found a place outside where you can sit and photograph shoppers as they approach the checkout counter, and from these photographs you can pretty much tell what foods each customer has purchased (although some of the items may be obscured in the shopping basket) and the total cost of all items combined. By bribing a contact in the local bank, you are able to find out each shopper's income. That is all the information you have. From this, can you infer the demand for eggs? Can you determine how much households would be willing to pay to remove sugar import quotas? (1999, p. 1596).

As Shephard (1999) explained, the hedonic approach to housing market analysis focuses attention on the implicit prices of housing and environmental characteristics and the use of these implicit prices to infer household demand for housing attributes and environmental characteristics. The emphasis on environmental characteristics is important in the housing market because the decision to consume housing is also the decision to consume all of the features of that location, including public goods and services (and the taxes needed to pay for them), environmental quality, and the neighbors, to name a few. In empirical work the hedonic approach often involves running a multiple regression with the housing price as the dependent variable and numerous attributes of the residential property and its location as the independent variables. This will produce an estimate of the hedonic price function, but as Shephard (1999) explained in depth, the consistent estimation of demand functions for housing and environmental attributes can pose very difficult econometric problems that are beyond the scope of this essay.

It can be shown that the Alonso-Muth and the hedonic approaches are special cases of an expanded theory of consumer choice. John McDonald (1979, pp. 7–14) provided one synthesis in which the household, which might include more than one worker, maximizes utility subject to both budget and time constraints. Following Lancaster (1966) and Muth (1966), utility is a function of a composite good, a vector of housing and locational characteristics, and a vector of time allocations that includes leisure, work time, and commuting time of the household members. Housing characteristics are produced by a vector of purchased goods and locational attributes as well as time inputs of the household. The first-order conditions from this model include a

set of equations for the marginal value of each purchased good that enter into the production of housing characteristics. The model also produces equations for the marginal value of each use of time and for locational equilibrium of the household. The condition for locational equilibrium is an expanded version of the Alonso-Muth condition that the marginal benefit of moving a unit of distance equals its marginal cost. The marginal benefit of increasing distance includes the quantity of each purchased good times the change in its price as distance increases. Alonso and Muth argued that land is the only purchased good with a price that is a function of distance, but the more general analysis includes the possibility that other purchased goods vary in price over space.

The Alonso-Muth model can be presented as a simple example of this more general model. Assume that the household contains one individual who commutes to a job in the central business district. Utility is a function of the composite commodity z and housing services Q that are produced by the services of land L and capital K with constant returns to scale. Housing is a complex good that is a bundle of land and capital valued according to a hedonic function. The bundle of land and capital called housing is purchased in a very active market called the residential real estate market. In this model the individual's own time does not enter into the production of Q, and utility is not a function of commuting time. The composite commodity includes expenditures (wage income foregone) on leisure. Using Alonso's notation, the household maximizes the Lagrangian expression

$$U^* = u[z, Q(L,K)] + \lambda[y - z - p(t)Q(L, K) - k(t)].$$

Recall that $k(t)$ is transportation cost as a function of distance t, but now this cost includes the opportunity cost of commuting time, expressed in money terms. The first-order conditions include

$$u_z - \lambda = 0,$$
$$u_Q - \lambda p(t) = 0,$$

and

$$-\lambda[p'(t)Q + k'(t)] = 0 \text{ [Muth's condition] } .$$

In this model the marginal value of land (capital) equals the marginal value of housing services times the marginal product of land (capital) in the production of housing services. If the price of land P varies with distance, but the price of capital does not, then

$$p'(t) = P'(t)S_L,$$

where S_L is the share of land as a proportion of the value of the output of housing services (assuming $S_L + S_K = 1$). The price V of a housing unit is the hedonic function

$$V(t,Q) = p(t)Q = p(t)Q(L, K),$$

which can be estimated as

$$lnV(t,Q) = lnp(t) + lnQ(L, K) = lnp_0 - ct + lnQ(L, K).$$

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The best empirical evidence [Thorsnes (1997)] indicates that *Q(L, K)* can be assumed to be a Cobb-Douglas function, in which case the hedonic function becomes

$$lnV(t,Q) = lnp_0 - ct + \alpha lnL + (1 - \alpha)lnK.$$

Since the price of housing must fall with distance, according to Muth's condition, for household equilibrium the price of land must fall according to

$$P'(t) = -k'(t)/QS_L.$$

This basic example shows that the Alonso-Muth model can be considered a special case, albeit an important special case, of a more general model of consumer choice.

VI. CONCLUSION

This essay has shown that by 1958, William Alonso and Richard Muth were at work creating the basic theoretical approach that urban economists have used ever since. In my view they can be called the cofounders of the field as it is practiced today. Richard Muth followed up on the basic theoretical insights with extensive empirical studies and further theoretical developments. He is a founder who also showed the rest of the profession how to do urban economics, both formal microeconomic theory and applied econometric work.

At roughly the same time Griliches (1961), Becker (1965), Lancaster (1966), Muth (1966) himself, and others, were recasting consumer theory in the more general terms that included household time allocation and household production as well as the hedonic approach to complex consumer goods. An expanded theory of consumer choice has emerged that includes household production, time allocation (including time allocated to work, home production, leisure, and commuting to work), location choice, complex goods, investment in human capital, fertility, and many other decisions. See Becker and Michael (1973) for an early overview of the development of this theory. The task of the researcher now is to choose the special case of this expanded theory to suit the question at hand. What aspects of the expanded theory are necessary for an understanding of a particular phenomenon, and what can be assumed safely to be unimportant? And then, can the researcher devise tests of the validity of the simplifying assumptions? These can be difficult questions to answer. But it is very clear that Alonso and Muth extended the standard model of consumer choice of the 1950s in a manner that was needed to explain the behavior of households in urban space, and in so doing, founded a new field.

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