ON THE DISTRIBUTION OF THE EXCEDENTS OF FUNDS WITH ASSETS AND LIABILITIES IN PRESENCE OF SOLVENCY AND RECOVERY REQUIREMENTS

BY

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ABSTRACT

We consider a profitable, risky setting with two separate, correlated asset and liability processes (first introduced by Gerber and Shiu, 2003). The company that is considered is allowed to distribute excess profits (traditionally referred to as dividends in the literature), but is regulated and is subject to particular regulatory (solvency) constraints. Because of the bivariate nature of the surplus formulation, such distributions of excess profits can take two alternative forms. These can originate from a reduction of assets (and hence a payment to owners), but also from an increase of liabilities (when these represent the wealth of owners, such as in pension funds). The latter is particularly relevant if distributions of assets do not make sense because of the context, such as in regulated pension funds where assets are locked until retirement. In this paper, we extend the model of Gerber and Shiu (2003) and consider recovery requirements for the distribution of excess funds. Such recovery requirements are an extension of the plain vanilla solvency constraints considered in Paulsen (2003), and require funds to reach a higher level of funding than the solvency level (if and after it is triggered) before excess funds can be distributed again. We obtain closed-form expressions for the expected present value of distributions (asset decrements or liability increments) when a distribution barrier is used.

KEYWORDS

Dividends, funding ratio, regulation, solvency, recovery requirements.
JEL codes: C44, G24, G32, G35.

1. INTRODUCTION

1.1. Motivation and main contributions

In actuarial risk theory, the *stability problem* is about modelling the dynamics of risky businesses in a stylised fashion, in order to help them make decisions

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about how to manage their risks; see Bühlmann (1970) for a classical reference. Over the past century, a variety of (decision) criteria were considered, including the probability of ruin (see Asmussen and Albrecher, 2010, for a recent comprehensive review) or the expected present value of dividends (see Albrecher and Thonhauser, 2009; Avanzi, 2009, for recent reviews). In their purest form, these criteria have various shortcomings that researchers have tried to address over time. The criteria are also sometimes modified or augmented to better fit some specific contexts.

In this paper, we consider a profitable, risky setting with two separate, correlated asset and liability processes; see Section 1.2. One of the criticisms formulated against the expected present value of dividends criterion (see Gerber, 1974, for instance) as introduced in de Finetti (1957) is the lack of explicit focus on (or consideration of) solvency in the criterion, and in its optimisation. The company considered by our model is allowed to distribute excess profits (traditionally referred to as *dividends* in the literature), but is regulated and is subject to particular regulatory (solvency) constraints. Importantly, because of its bivariate nature, such *distributions* of excess profits can take two alternative forms. These can originate from a reduction of assets (and hence a payment to owners), but also from an increase of liabilities (when these represent the wealth of owners, such as in pension funds), see also Section 1.3. The latter is particularly relevant if leakages do not make sense because of the context, such as in regulated pension funds where assets are locked until retirement (see, for instance, Müller and Wagner, 2017, who consider the case of Swiss pension funds).

Mathematically, both distribution avenues are treated in a very similar way (although there are material differences in some cases). For sake of brevity, we will provide full details only for one case, and only results for the other. We elected to focus primarily on the 'increase of liabilities' case, as we believe this is the most innovative in this context.

In this paper, we extend the model of Gerber and Shiu (2003) and consider recovery requirements (see, for instance, Avanzi and Wong, 2012) for the distribution of excess funds. The recovery requirements are an extension of the plain vanilla solvency constraints considered in Paulsen (2003) and Avanzi *et al.* (2017), and require funds to reach a higher level of funding than the solvency level (if and after it is triggered) before excess funds can be distributed again. This is further developed and motivated in Section 1.4. In Section 2, we obtain closed-form expressions for the expected present value of distributions (asset decrements or liability increments) when a distribution barrier is used. Section 3 illustrates our results. The solvency requirements considered in this paper improves the stability of the company substantially, for only minimal reductions in expected present value of distributions. We also illustrate how different the optimal barrier of our model is, when compared to that in the existing literature (without or with simple solvency constraints), as this relationship is non-trivial (see Section 2.4).

1.2. A bivariate asset and liability process

As mentioned earlier, we consider a bivariate surplus process, where assets and liabilities are modelled as correlated geometric Brownian motions. Such a bivariate geometric Brownian motion was introduced by Gerber and Shiu (2003). They considered two problems: (a) to keep the funding ratio (ratio of assets to liabilities) within a band, by equalising inflows and outflows at the boundaries of the band — they conjectured a fund "should" do so; and (b) to maximise (in absence of inflows) the expected present value of outflows (dividends). They conjectured that a barrier dividend strategy should be optimal. Decamps *et al.* (2006) extended (a) to finite time horizon, while Decamps *et al.* (2009) proved that the conjecture in (b) is correct. Also, Chen and Yang (2010) extended the results of Gerber and Shiu (2003) to a regime-switching environment. Avanzi *et al.* (2017) determined that barrier-type distributions are optimal in presence of a solvency constraint (such as in Paulsen, 2003) or in presence of forced rescue measures below a pre-specified level.

The dynamics of the assets $\{A(t)\}$ and liabilities $\{L(t)\}$, which we also denote by $X_1 := A$ and $X_2 := L$, respectively, are given by

$$\begin{aligned} d\vec{X}(t) &= d\begin{pmatrix} A(t) \\ L(t) \end{pmatrix} = \begin{pmatrix} \mu_A & 0 \\ 0 & \mu_L \end{pmatrix} \begin{pmatrix} A(t) \\ L(t) \end{pmatrix} dt + \begin{pmatrix} \sigma_A A(t) & 0 \\ \rho\sigma_L L(t) & \sqrt{1-\rho^2}\sigma_L L(t) \end{pmatrix} dW(t) \\ &= \mu(\vec{X}(t))dt + \sigma(\vec{X}(t))dW(t), \end{aligned} \tag{1.1}$$

where $A(0) = A_0$, $L(0) = L_0$, and where W is a standard two-dimensional Brownian motion and $\rho \in]-1, 1[$. Following the lines of Gerber and Shiu (2003), we assume that the time value of money δ (for the decision maker) is greater than the drift of the assets, which itself is greater than the drift of the liabilities. That is,

$$\delta > \mu_A \tag{1.2}$$

and

$$\mu_A > \mu_L. \tag{1.3}$$

Equation (1.2) is required for the expected present value of dividends to be finite, and (1.3) is required for the problem to be non-trivial (otherwise immediate liquidation and ruin are optimal), see Gerber and Shiu (2003, Section 9). The funding ratio $\{Y(t)\}$ is defined as the ratio of assets to liabilities, that is,

$$Y(t) = \frac{A(t)}{L(t)}, \quad t \geq 0.$$

This model setting is identical to that of Gerber and Shiu (2003). Note that Sethi and Taksar (2002) considered dividends and capital injections for a company whose surplus is modelled by a (univariate) geometric Brownian motion, which is a more traditional, unidimensional formulation.

1.3. Distribution of excess profits

In what follows, we will consider the following two ways of distributing excess profits:

A. Increase liabilities.
B. Decrease assets.

For the rest of the paper, we will refer to both cases as to "Case A" and "Case B", respectively. The result of either will be referred to as a "distribution".

The increase of liabilities (Case A) could be used in the modelling of non-for-profit mutual funds or pension funds. On the other hand, the decrease of assets (Case B) could be used in the modelling of for-profit companies. The latter case is equivalent to paying out dividends, which is the standard assumption in the actuarial dividend literature.

In this paper, we consider barrier-type distributions (defined in Section 1.4 below). Note that we do not show that such a strategy is optimal in this paper (although we conjecture it is), but we do determine the optimal level of the barrier, which turns out to be surprisingly non-trivial. Importantly, the barrier is defined here on the funding *ratio* (distributions are made if the funding ratio is beyond a certain barrier level, so that the funding ratio is brought back to that particular level). Such a policy is observed in practice. While some insurance companies define their dividend payout strategy based on a target capital in dollars, others do define their dividend policy as a target ratio of the minimum capital requirements as defined by the regulator (see, e.g. Australian Actuaries Institute, 2016).

The model is illustrated in Figure 1, where assets, liabilities, and funding ratio are displayed (for a given sample path of the two-dimensional Brownian motion), both for the ratio without distributions (black lines) and the ratio with distributions (grey lines). Moreover, the dotted line in the figures show the positive distribution processes $\{D^{\pi}_{\mathrm{A}}(t)\}$ and $\{D^{\pi}_{\mathrm{B}}(t)\}$ defined below when a barrier β^* is applied. Note that while the funding ratio after distributions is the same for both cases, the respected expected present values of distributions will be different. This is because the two types of distributions have a different impact on the scale of the processes.

Distributions will either translate into increasing liabilities (Case A) or decreasing assets (Case B). The asset and liability processes after distributions, which we denote $\{\vec{X}^{\pi}_{A}(t)\}$ and $\{\vec{X}^{\pi}_{B}(t)\}$, have dynamics

$$d\vec{X}^{\pi}_{\mathrm{A}}(t) = \mu\left(\vec{X}^{\pi}_{\mathrm{A}}(t)\right)dt + \sigma\left(\vec{X}^{\pi}_{\mathrm{A}}(t)\right)dW(t) + \begin{pmatrix} 0 \\ dD^{\pi}_{\mathrm{A}}(t) \end{pmatrix} \tag{1.4}$$

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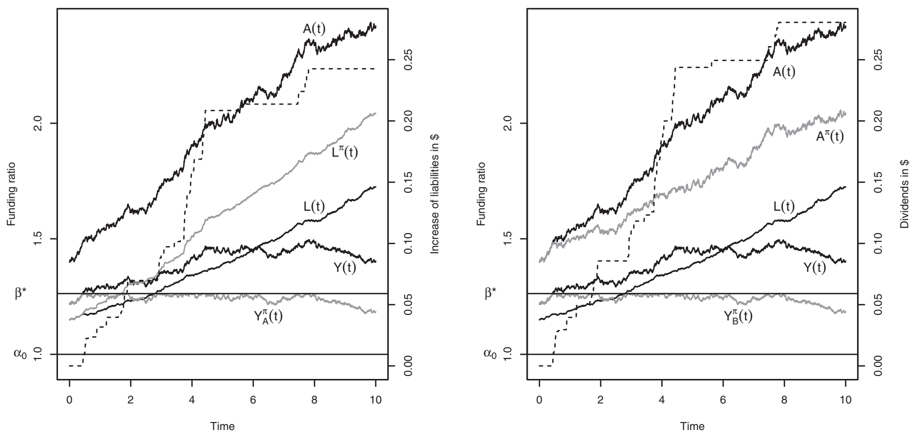

FIGURE 1: Figure illustrating the model. Left: Case A. Right: Case B. The asset and liability processes before distributions are in black, and after distributions in grey. The dotted lines depict the undiscounted, aggregated payment processes.

and

$$d\vec{X}_{\mathrm{B}}^{\pi}(t) = \mu\left(\vec{X}_{\mathrm{B}}^{\pi}(t)\right)dt + \sigma\left(\vec{X}_{\mathrm{B}}^{\pi}(t)\right)\mathrm{dW}(t) - \begin{pmatrix} dD_{\mathrm{B}}^{\pi}(t) \\ 0 \end{pmatrix}, \tag{1.5}$$

where D_{A}^{π} represents the process of aggregate increases of the liabilities (Case A) and D_{B}^{π} is the aggregate dividends process. We denote by $\{L^{\pi}(t)\}$ liabilities after addition of D_{A}^{π} and by $\{A^{\pi}(t)\}$ assets after subtraction of D_{B}^{π}. The funding ratios of the asset and liability processes after distributions are then given by

$$Y_{\mathrm{A}}^{\pi}(t) = \frac{A(t)}{L^{\pi}(t)} \text{ and } Y_{\mathrm{B}}^{\pi}(t) = \frac{A^{\pi}(t)}{L(t)}, \quad t \geq 0, \tag{1.6}$$

respectively.

Remark 1.1. *The model (before dividends) allows for deterministic (multiplicative) increases of the assets and liabilities, plus (correlated) random variations. Because of the nature of the processes, these variations are continuous, and one might argue that abrupt changes in assets and liabilities (jumps) should be allowed in order to reflect the random nature of the businesses, and/or expected changes in scale. Beyond the fact that these would require developments beyond the scope of a single paper, we believe our model is still reasonable for the following reasons:*

1. *Case A: the formulation of our model means that we consider an accumulation scheme in equilibrium, that is, where contributions are continuously offset by payouts. This is an approximation, but we believe it is good enough for our analysis. If significant assets and liabilities were to enter or leave the fund, this typically would lead to a specific procedure and distribution rule (e.g. partial liquidation).*

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2. *Case B: additional contributions to the company can be made from time to time without affecting the conclusions as long as these are made so as not to make existing shareholders richer or poorer. In terms of our model, this means that they would be made at the existing funding ratio. We will see later that a change of scale that does not impact the funding ratio has no impact on how to control the process.*

1.4. Bankruptcy and recovery requirements

Of course, the fund may become bankrupt. This will occur as soon as the funding ratio reaches a given level α_0. For either of the cases A and B, we denote by τ_{α_0} the time of ruin, which is the stopping time defined as the first time the funding ratio of the processes after distributions equals α_0. For the rest of the paper, we will use the notation $\varpi \in \{A, B\}$ to simplify notation where possible. Using this notation, the bankruptcy time for the two cases is given by

$$\tau_{\alpha_0} = \inf \left\{ t \geq 0 | Y_{\varpi}^{\pi}(t) = \alpha_0 \right\}.$$

For Case A, α_0 could be below or above 1, depending on the nature of the fund (partially funded public or fully funded private, for instance). For Case B, the level α_0 would typically be at least 1 (higher for financial institutions).

A solvency constraint of the type introduced by Paulsen (2003) is considered in Avanzi *et al.* (2017). In this framework, distributions cannot bring the funding ratio below a level $\alpha_1 \geq \alpha_0$ (a target buffer above a given minimum capital requirement). With this, we have two areas: between α_0 and α_1, where no distribution is allowed, and beyond α_1, where distributions are allowed.

In this paper, we consider an extension to the solvency constraint described above. In Australia, insurers know that the regulator will take action if their level of capitalisation downcrosses a certain trigger ratio (α_1), which is company specific (e.g., companies do not have the same level of risk appetite when investing their assets, and this is recognised). We do not expect that companies would want to pay dividends down to that level to avoid the trigger. Perhaps, more importantly, the regulator would not let them pay dividends if that alarm was raised. Instead, the company should recover and recapitalise up to a certain level α_2 before being able to freely pay dividends again. This is what we call the "recovery requirement". Note that this mechanism further incentivises the company to not pay to levels that are too close to α_1, which makes the determination of the optimal barrier both interesting and difficult.

The solvency and recovery requirements $\{\alpha_0, \alpha_1, \alpha_2\}$ are illustrated on Figure 2. When the process is in the region between α_1 and α_2 and coming from α_2 — that is, when the last visit at either α_1 or α_2 was α_2 — distributions are allowed. When the last visit was α_1, we consider that the process is in recovery and no distribution is allowed.

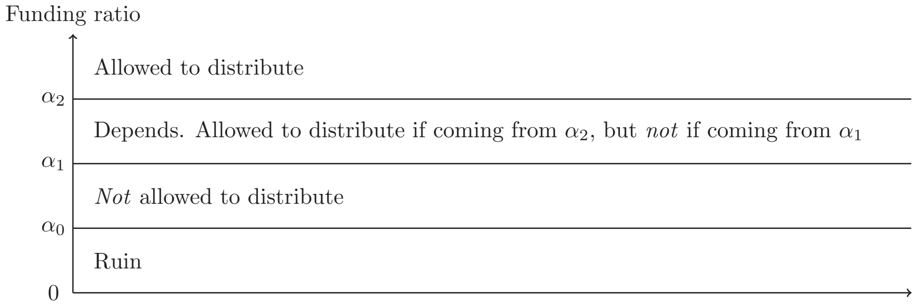

FIGURE 2: Graphical illustration of a model with recovery requirements $\{\alpha_0, \alpha_1, \alpha_2\}$.

Note that this recovery requirement has another advantage, which is one reason which had motivated Avanzi and Wong (2012) to introduce it at first. In a diffusion environment (such as in this paper), absence of recovery requirement would lead to erratic periods of dividend payments if the barrier is equal to α_1 (this will sometimes be the case, see Avanzi *et al.*, 2017), which is unrealistic (see also Avanzi *et al.*, 2016, for a discussion of this). Such erratic dividend payments with optimal barrier strategies was another criticism formulated by Gerber (1974).

2. BARRIER STRATEGIES UNDER RECOVERY REQUIREMENTS

In this section, we consider recovery requirements with the boundaries $\{\alpha_0, \alpha_1, \alpha_2\}$ as introduced and described in Section 1.4, see in particular Figure 2. Here, we are allowed to pay dividends when the funding ratio is in the interval (α_1, α_2) if and only if the funding ratio last crossed α_2, rather than α_1 (as in Avanzi and Wong, 2012). In this paper, we consider barrier strategies only. Main challenges are to formulate the constraint mathematically (Section 2.1), to obtain a value function explicitly (Sections 2.2 and 2.3 for cases A and B, respectively), and to determine the optimal barrier level (Section 2.4). Note that we initially consider $\alpha_1 < \beta < \alpha_2$, where β is the barrier level for the funding ratio at which the company either pays out dividends or increases liabilities, respectively; discussion of other cases is deferred to Section 2.4.

2.1. Mathematical formulation of the recovery requirements

We start by defining a set of stopping times $\tau^{\pi}_{\varpi,n}$, $n \in \mathbb{N}_0$ given by

$$\tau^{\pi}_{\varpi,0} = \inf\left\{t \geq 0 : (Y^{\pi}_{\varpi}(t) \leq \alpha_1) \vee (Y^{\pi}_{\varpi}(t) \geq \alpha_2)\right\}, \tag{2.1}$$

$$\begin{aligned}\tau^{\pi}_{\varpi,i} = \inf\Big\{t > \tau^{\pi}_{\varpi,i-1} : &\big((Y^{\pi}_{\varpi}(t) \leq \alpha_1) \wedge \big(Y^{\pi}_{\varpi}(\tau^{\pi}_{\varpi,i-1}) \geq \alpha_2\big)\big) \\ &\vee \big((Y^{\pi}_{\varpi}(t) \geq \alpha_2) \wedge \big(Y^{\pi}_{\varpi}(\tau^{\pi}_{\varpi,i-1}) \leq \alpha_1\big)\big)\Big\},\end{aligned} \tag{2.2}$$

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for $i = 1, 2, \ldots$. Here, $\vee$ is the logical "or" operator and $\wedge$ is the logical "and" operator. That is, the first stopping time is the first time the funding ratio is either below α_1 or above α_2. The next stopping times are given by the time points where the funding ratio either becomes greater than or equal to α_2 after the value in the previous stopping having been less than or equal to α_1, or at the time points where the funding ratio become less than or equal to α_1 after the value in the previous stopping having been greater than or equal to α_2. Now that we are equipped with those stopping times, we define a 0–1 process ϕ_ϖ such that

$$\phi_\varpi(t) = 1_{\{t<\tau^\pi_{\varpi,0} \cup Y^\pi_\varpi(t) \geq \alpha_1\}} + \sum_{i=1}^{\infty} \left(1_{\{t \in [\tau^\pi_{\varpi,i-1}, \tau^\pi_{\varpi,i}[\}} 1_{\{Y^\pi_\varpi(\tau^\pi_{\varpi,i-1}) \geq \alpha_2\}} \right). \tag{2.3}$$

The process ϕ_ϖ is 1 when we are allowed to control the funding ratio (distribute) and 0, when we are not. With this, we can formulate the recovery requirement as

$$\int_0^{\tau_{\alpha_0}} (1 - \phi_\varpi(s)) dD^\pi_\varpi(s) = 0. \tag{2.4}$$

Due to the form of the function ϕ_ϖ given by (2.3), we assume that distributions are initially allowed if $\alpha_1 < Y^\pi_\varpi(0) < \alpha_2$ (until the first stopping time $\tau^\pi_{\varpi,0}$).

An example of a sample path for $\alpha_1 < \beta < \alpha_2$ is found in Figure 3 where β is the optimal barrier β^*_2 (which will be determined later). The dashed and dotted lines show the distributions in Case A and Case B, respectively. The grey parts of the funding ratio process illustrates time spans where it is *not* allowed to pay out dividends ($\phi = 0$) and the black parts of the line illustrates time spans where it is allowed to pay out dividends ($\phi = 1$). We observe that distributions consist of infinitesimal payments at the barrier β^*_2 (when $\phi = 1$) and lump sum payments of size $(1/\beta^*_2 - 1/\alpha_2)A$ (Case A) and $(\alpha_2 - \beta^*_2)L$ (Case B), when the funding ratio hits α_2.

2.2. Value of distributions when liabilities are increased (Case A)

First, we consider the value function of a barrier strategy with an arbitrary barrier level β. We assume that $\alpha_1 \leq \beta$, since a control strategy within the present solvency regime clearly does not allow $\beta < \alpha_1$. We denote by $V^\beta_{\mathrm{A}}(\vec{x})$ the value function for a barrier strategy with barrier β on the funding ratio, that is,

$$V^\beta_{\mathrm{A}}(\vec{x}) = \mathbb{E}^{\vec{x}} \left[\int_{0-}^{\tau_{\alpha_0}} e^{-\delta s} dC_\beta(s) \right],$$

where C_β is notation for a strategy, where we increase liabilities in order to keep the funding ratio below β. Using the martingale approach of Gerber and Shiu

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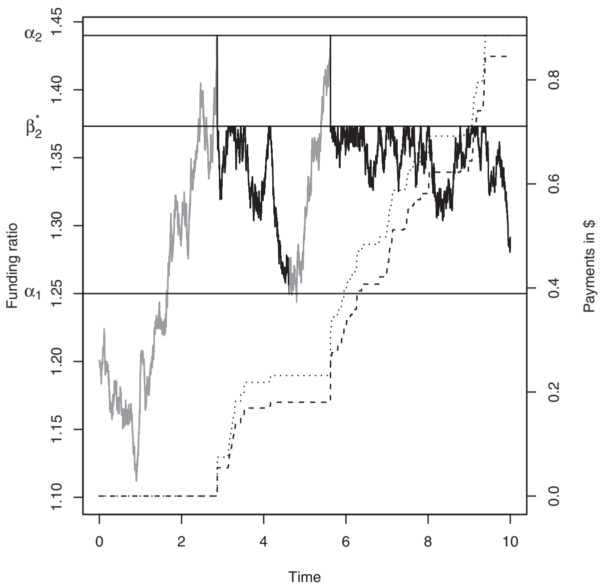

FIGURE 3: Illustration of the funding ratio and distributions in case of recovery requirements.

(2003) and results from Decamps *et al.* (2009), the value of V_A^β is given by

$$V_A^\beta(\vec{x}) = \begin{cases} V_A^0(\vec{x};\beta), & x_1 \in [\alpha_0 x_2, \alpha_2 x_2[\wedge \phi = 0, \\ V_A^1(\vec{x};\beta), & x_1 \in [\alpha_1 x_2, \beta x_2] \wedge \phi = 1, \\ \frac{x_1}{\beta} - x_2 + V_A^1\left(x_1, \frac{x_1}{\beta}; \beta\right), & x_1 > \beta x_2 \wedge \phi = 1, \end{cases} \tag{2.5}$$

where $V_A^0(\cdot;\beta)$ and $V_A^1(\cdot;\beta)$ fulfil the following systems of PDEs:

$$\begin{aligned} (\mathcal{A}-\delta)V_A^0(\vec{x};\beta) = 0 \text{ for } \alpha_0 \le \frac{x_1}{x_2} \le \alpha_2, \quad & V_A^0(\alpha_0 x_2, x_2;\beta) = 0, \\ & V_A^0\left(x_1, \frac{x_1}{\alpha_2};\beta\right) = V_A^1\left(x_1, \frac{x_1}{\alpha_2};\beta\right), \\ (\mathcal{A}-\delta)V_A^1(\vec{x};\beta) = 0 \text{ for } \alpha_1 \le \frac{x_1}{x_2} \le \beta, \quad & V_A^1\left(x_1, \frac{x_1}{\alpha_1};\beta\right) = V_A^0\left(x_1, \frac{x_1}{\alpha_1};\beta\right), \\ & \frac{\partial}{\partial x_2} V_A^1(x_1, x_2;\beta)|_{x_2 = \frac{x_1}{\beta}} = -1. \end{aligned} \tag{2.6}$$

The boundary conditions are due to ruin, continuity (twice) and the oscillation of the process. For the latter (which is the last line in (2.6)), a proof is included in Gerber (1972, Section 8), see also Gerber and Shiu (2003, heuristic justification

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of (6.1)) or Decamps, Schepper, and Goovaerts (2009, Proposition 1). Now,

$$\begin{aligned}\mathcal{A}f(\vec{x}) =&\mu_A x_1 \frac{\partial}{\partial x_1} f(\vec{x}) + \mu_L x_2 \frac{\partial}{\partial x_2} f(\vec{x}) + \frac{1}{2}\sigma_A^2 x_1^2 \frac{\partial^2}{\partial x_1^2} f(\vec{x}) \\ &+ \frac{1}{2}\sigma_L^2 x_2^2 \frac{\partial^2}{\partial x_2^2} f(\vec{x}) + \rho\sigma_A\sigma_L x_1 x_2 \frac{\partial^2}{\partial x_1 \partial x_2} f(\vec{x}).\end{aligned} \tag{2.7}$$

Note that for $x_2 \leq x_1/\alpha_2$ we automatically have that $\phi = 1$. The notation V_{A}^0 highlights that this is the value function for $\phi = 0$ and the notation V_{A}^1 highlights that this is the value function for $\phi = 1$. Moreover, for the optimal level of the barrier, β_2^*, we have the smooth fit condition that

$$\frac{\partial^2}{\partial x_2^2} V_{\mathrm{A}}^1 (x_1, x_2; \beta_2^*)\, |_{x_2 = \frac{x_1}{\beta_2^*}} = 0. \tag{2.8}$$

The boundary conditions $V_{\mathrm{A}}^1\left(x_1, \frac{x_1}{\alpha_1}; \beta\right) = V_{\mathrm{A}}^0\left(x_1, \frac{x_1}{\alpha_1}; \beta\right)$ and $V_{\mathrm{A}}^1\left(x_1, \frac{x_1}{\alpha_2}; \beta\right) = V_{\mathrm{A}}^0\left(x_1, \frac{x_1}{\alpha_2}; \beta\right)$ are due to the continuity of the diffusion term. Since $V_{\mathrm{A}}^1\big(x_1, \frac{x_1}{\alpha_1}; \beta\big) = V_{\mathrm{A}}^0\big(x_1, \frac{x_1}{\alpha_1}; \beta\big)$ and $V_{\mathrm{A}}^1\big(x_1, \frac{x_1}{\alpha_2}; \beta\big) = V_{\mathrm{A}}^0\big(x_1, \frac{x_1}{\alpha_2}; \beta\big)$ depend on each other, the system of PDEs needs to be solved simultaneously. A graphical representation of the value function for a given barrier β is illustrated in Figure 4. The left rectangle illustrates the domain of V_{A}^0 and the right "open" rectangle illustrates the domain of V_{A}^1.

Before calculating the value functions, we state the following lemma:

Lemma 2.1. *The solution to*

$$(\mathcal{A} - \delta) G_{\mathrm{A}}(\vec{x}; \beta) = 0 \quad \textit{for} \quad \alpha_0 \leq \frac{x_1}{x_2} \leq \beta, \tag{2.9}$$

(without any boundary conditions) is given by

$$G_{\mathrm{A}}(x_1, x_2; \beta) = C_1 x_1^{\zeta_1} x_2^{1-\zeta_1} + C_2 x_1^{\zeta_2} x_2^{1-\zeta_2}, \tag{2.10}$$

where

$$\begin{aligned}\tilde{\sigma}^2 &= \sigma_A^2 + \sigma_L^2 - 2\rho\sigma_A\sigma_L, \\ \zeta_1 &= \frac{\frac{1}{2}\tilde{\sigma}^2 - (\mu_A - \mu_L) - \sqrt{\frac{1}{4}\tilde{\sigma}^4 + (\mu_A - \mu_L)^2 - \tilde{\sigma}^2 (\mu_A + \mu_L - 2\delta)}}{\tilde{\sigma}^2}, \\ \zeta_2 &= \frac{\frac{1}{2}\tilde{\sigma}^2 - (\mu_A - \mu_L) + \sqrt{\frac{1}{4}\tilde{\sigma}^4 + (\mu_A - \mu_L)^2 - \tilde{\sigma}^2 (\mu_A + \mu_L - 2\delta)}}{\tilde{\sigma}^2},\end{aligned} \tag{2.11}$$

and C_1 and C_2 are some constants.

Proof. See Appendix A.1. ■

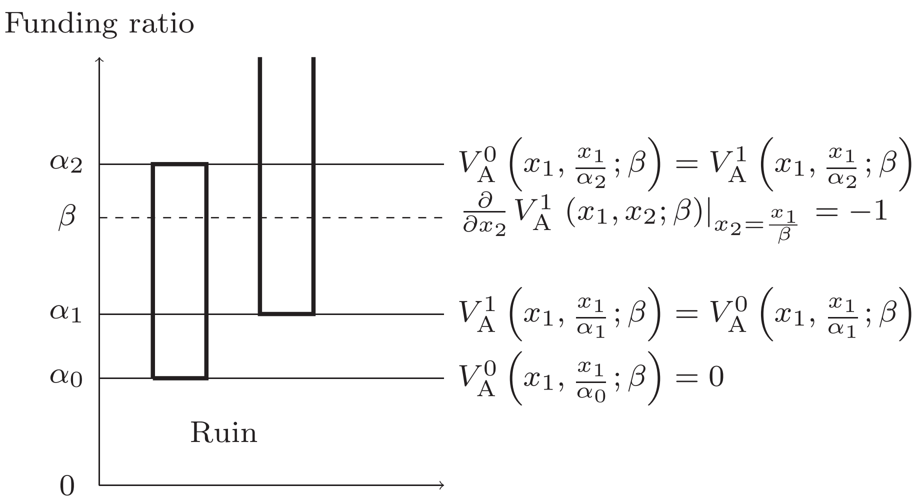

FIGURE 4: Illustration of the domains and boundary conditions in the case of recovery requirements.

We know from Lemma 2.1 that V_A^i, $i = 0, 1$, are given by

$$V_A^i(x_1, x_2; \beta) = K_{1,i}^A x_1^{\zeta_1} x_2^{1-\zeta_1} + K_{2,i}^A x_1^{\zeta_2} x_2^{1-\zeta_2}, \tag{2.12}$$

for some constants $K_{1,i}^A$ and $K_{2,i}^A$ that fulfil the equation $(\mathcal{A} - \delta)V_A^i = 0$. That is, including the boundary conditions, we get that the value function is specified by

$$V_A^0(x_1, x_2; \beta) = K_{1,0}^A x_1^{\zeta_1} x_2^{1-\zeta_1} + K_{2,0}^A x_1^{\zeta_2} x_2^{1-\zeta_2}, (x_1 \in [\alpha_0 x_2, \alpha_2 x_2]), \tag{2.13}$$

$$V_A^1(x_1, x_2; \beta) = K_{1,1}^A x_1^{\zeta_1} x_2^{1-\zeta_1} + K_{2,1}^A x_1^{\zeta_2} x_2^{1-\zeta_2}, (x_1 \in [\alpha_1 x_2, \beta x_2]), \tag{2.14}$$

$$V_A^0(\alpha_0 x_2, x_2; \beta) = 0, \tag{2.15}$$

$$V_A^0\left(x_1, \frac{x_1}{\alpha_2}; \beta\right) = V_A^1\left(x_1, \frac{x_1}{\alpha_2}; \beta\right), \tag{2.16}$$

$$V_A^1\left(x_1, \frac{x_1}{\alpha_1}; \beta\right) = V_A^0\left(x_1, \frac{x_1}{\alpha_1}; \beta\right), \tag{2.17}$$

$$\frac{\partial}{\partial x_2} V_A^1(x_1, x_2; \beta)|_{x_2=\frac{z x_1}{\beta}} = -1, \quad z \le 1. \tag{2.18}$$

The solution to this system is provided in Theorem 2.2.

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Theorem 2.2. *The solutions V_{A}^0 and V_{A}^1 to the system of equations given by (2.13)–(2.18) are given by*

$$V_{\mathrm{A}}^0(x_1, x_2; \beta) = K_{1,0}^{\mathrm{A}} \left(x_1^{\zeta_1} x_2^{1-\zeta_1} - {\alpha_0}^{\zeta_1-\zeta_2} x_1^{\zeta_2} x_2^{1-\zeta_2} \right) \text{ and}$$

$$V_{\mathrm{A}}^1(x_1, x_2; \beta) = \left(K_{1,0}^{\mathrm{A}} \left(1 - {\alpha_0}^{\zeta_1-\zeta_2} {\alpha_1}^{\zeta_2-\zeta_1}\right) - K_{2,1}^{\mathrm{A}} {\alpha_1}^{\zeta_2-\zeta_1} \right) x_1^{\zeta_1} x_2^{1-\zeta_1} + K_{2,1}^{\mathrm{A}} x_1^{\zeta_2} x_2^{1-\zeta_2},$$

where

$$\xi_A = \frac{\min(\beta, \alpha_2)^{\zeta_2-1} - {\alpha_1}^{\zeta_2-\zeta_1} \min(\beta, \alpha_2)^{\zeta_1-1}}{(1-\zeta_2)\,\beta^{\zeta_2} - (1-\zeta_1)\,\alpha_1^{\zeta_2-\zeta_1}\beta^{\zeta_1}},$$

$$K_{1,0}^{\mathrm{A}} = \frac{\left(\frac{1}{\beta} - \frac{1}{\alpha_2}\right)^+ - \xi_{\mathrm{A}}}{{\alpha_2}^{\zeta_1-1} - {\alpha_0}^{\zeta_1-\zeta_2}{\alpha_2}^{\zeta_2-1} + \left({\alpha_0}^{\zeta_1-\zeta_2}{\alpha_1}^{\zeta_2-\zeta_1} - 1\right)\min(\beta, \alpha_2)^{\zeta_1-1} + \xi_{\mathrm{A}}(1-\zeta_1)\left(1 - {\alpha_0}^{\zeta_1-\zeta_2}{\alpha_1}^{\zeta_2-\zeta_1}\right)\beta^{\zeta_1}},$$

$$K_{2,1}^{\mathrm{A}} = \frac{-1 - (1-\zeta_1)\left(K_{1,0}^{\mathrm{A}} - K_{1,0}^{\mathrm{A}}{\alpha_0}^{\zeta_1-\zeta_2}{\alpha_1}^{\zeta_2-\zeta_1}\right)\beta^{\zeta_1}}{(1-\zeta_2)\,\beta^{\zeta_2} - (1-\zeta_1)\,\alpha_1^{\zeta_2-\zeta_1}\beta^{\zeta_1}}.$$

Proof. See Appendix A.2. ■

2.3. Value of distributions when assets are decreased (Case B)

For Case B, the value function is given by

$$V_{\mathrm{B}}^{\beta}(\vec{x}) = \begin{cases} V_{\mathrm{B}}^0(\vec{x}; \beta), & x_1 \in [\alpha_0 x_2, \alpha_2 x_2[\wedge \phi = 0, \\ V_{\mathrm{B}}^1(\vec{x}; \beta), & x_1 \in [\alpha_1 x_2, \beta x_2] \wedge \phi = 1, \\ x_1 - \beta x_2 + V_{\mathrm{B}}^1(\beta x_2, x_2; \beta), & x_1 > \beta x_2 \wedge \phi = 1, \end{cases} \tag{2.19}$$

where V_{B}^0 and V_{B}^1 are given by the following specification:

$$V_{\mathrm{B}}^0(x_1, x_2; \beta) = K_{1,0}^{\mathrm{B}} x_1^{\zeta_1} x_2^{1-\zeta_1} + K_{2,0}^{\mathrm{B}} x_1^{\zeta_2} x_2^{1-\zeta_2}, (x_1 \in [\alpha_0 x_2, \alpha_2 x_2]), \tag{2.20}$$

$$V_{\mathrm{B}}^1(x_1, x_2; \beta) = K_{1,1}^{\mathrm{B}} x_1^{\zeta_1} x_2^{1-\zeta_1} + K_{2,1}^{\mathrm{B}} x_1^{\zeta_2} x_2^{1-\zeta_2}, (x_1 \in [\alpha_1 x_2, \beta x_2]), \tag{2.21}$$

$$V_{\mathrm{B}}^0(\alpha_0 x_2, x_2; \beta) = 0, \tag{2.22}$$

$$V_{\mathrm{B}}^0(\alpha_2 x_2, x_2; \beta) = V_{\mathrm{B}}^1(\alpha_2 x_2, x_2; \beta), \tag{2.23}$$

$$V_{\mathrm{B}}^1(\alpha_1 x_2, x_2; \beta) = V_{\mathrm{B}}^0(\alpha_1 x_2, x_2; \beta), \tag{2.24}$$

$$\frac{\partial}{\partial x_1} V_{\mathrm{B}}^1(x_1, x_2; \beta)|_{x_1 = z x_2} = 1, z \geq \beta. \tag{2.25}$$

Theorem 2.3. *The solutions V_{B}^0 and V_{B}^1 to the system of equations given by (2.20)–(2.25) are given by*

$$V_{\mathrm{B}}^0(x_1, x_2; \beta) = K_{1,0}^{\mathrm{B}} \left(x_1^{\zeta_1} x_2^{1-\zeta_1} - {\alpha_0}^{\zeta_1-\zeta_2} x_1^{\zeta_2} x_2^{1-\zeta_2} \right) \text{ and}$$

$$V_{\mathrm{B}}^1(x_1, x_2; \beta) = \left(K_{1,0}^{\mathrm{B}} \left(1 - {\alpha_0}^{\zeta_1-\zeta_2} {\alpha_1}^{\zeta_2-\zeta_1}\right) - K_{2,1}^{\mathrm{B}} {\alpha_1}^{\zeta_2-\zeta_1} \right) x_1^{\zeta_1} x_2^{1-\zeta_1} + K_{2,1}^{\mathrm{B}} x_1^{\zeta_2} x_2^{1-\zeta_2},$$

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where

$$\xi_{\mathrm{B}} = \frac{\min(\beta, \alpha_2)^{\zeta_2} - \alpha_1{}^{\zeta_2-\zeta_1} \min(\beta, \alpha_2)^{\zeta_1}}{\zeta_2 \beta^{\zeta_2-1} - \zeta_1 \alpha_1^{\zeta_2-\zeta_1} \beta^{\zeta_1-1}},$$

$$K_{1,0}^{\mathrm{B}} = \frac{(\alpha_2 - \beta)^+ + \xi_{\mathrm{B}}}{\alpha_2{}^{\zeta_1} - \alpha_0{}^{\zeta_1-\zeta_2}\alpha_2{}^{\zeta_2} + (\alpha_0{}^{\zeta_1-\zeta_2}\alpha_1{}^{\zeta_2-\zeta_1} - 1)\min(\beta, \alpha_2)^{\zeta_1} + \xi_{\mathrm{B}}\,(1 - \alpha_0{}^{\zeta_1-\zeta_2}\alpha_1{}^{\zeta_2-\zeta_1})\,\zeta_1 \beta^{\zeta_1-1}},$$

$$K_{2,1}^{\mathrm{B}} = \frac{1 - \zeta_1\left(K_{1,0}^{\mathrm{B}} - K_{1,0}^{\mathrm{B}}\alpha_0{}^{\zeta_1-\zeta_2}\alpha_1{}^{\zeta_2-\zeta_1}\right)\beta^{\zeta_1-1}}{\zeta_2 \beta^{\zeta_2-1} - \zeta_1 \alpha_1^{\zeta_2-\zeta_1} \beta^{\zeta_1-1}}.$$

2.4. Discussion of optimal barrier levels

We denote the optimal barrier in the model without any solvency constraints by β_0^* (such as considered in Gerber and Shiu, 2003), and we denote the optimal barrier in the model with recovery requirements by β_2^*.

In order to formulate a conjecture about an optimal barrier strategy, we consider different possibilities for the relationship between β_0^* and $\{\alpha_0, \alpha_1, \alpha_2\}$. We know that $\beta_0^* > \alpha_0$, so we only need to consider the following cases:

Case 1: $\alpha_0 < \beta_0^* < \alpha_1$.
Case 2: $\alpha_1 \leq \beta_0^* < \alpha_2$.
Case 3: $\beta_0^* \geq \alpha_2$.

Let us consider each of the three cases separately:

Case 1: The conjecture is *not* that you distribute as much as you are allowed to (down to α_1) — that is, that $\beta_2^* = \alpha_1$ — as is shown to be optimal in Avanzi *et al.* (2017). Instead the conjecture is that the optimal strategy is a barrier strategy with level $\Lambda > \alpha_1$ (strictly higher), which enables you to keep paying dividends for some time. If the barrier was α_1, we would lose any opportunity to pay dividends until we reach α_2 again.

Case 2: Following the lines of the previous point, we conjecture that the optimal strategy is a barrier strategy with barrier $\Lambda \geq \beta_0^*$ because of that new danger of hitting α_1. The supplement $\Lambda - \beta_0^*$ would be larger as β_0^* is close to α_1. This leads to a mix of lump sum distributions of size $(1/\Lambda - 1/\alpha_2)A$ for Case A and $(\alpha_2 - \Lambda)L$ for Case B (when ϕ switches from 0 to 1) and payments at the barrier according to the oscillation of the Brownian motion (when the process ϕ is in state 1). This control can be seen both as alternating between a non-singular and an impulse control, or it can be seen as a singular control.

Case 3: In this case, the solvency constraint is no constraint at all in terms of optimal dividend strategies, and we get the same result as in the unconstrained case. That is, $\beta_2^* = \beta_0^*$.

Numerical studies concur with the above-mentioned conjectures. The optimal barrier β_2^* is obtained by maximising the value function V^1 with respect to β. Unfortunately, a closed-form expression for β_2^* does not seem to be available,

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but numerical studies suggest that the optimisation is not problematic. Furthermore, it turns out that maximising V^1 is equivalent to maximising V^0, or to solve (2.8). This means that the easiest way to obtain β_2^* is to minimise $K_{1,0}^{\mathrm{A}}$, which we also note does not depend on the initial surplus or ratio. As for β_0^* and β_1^*, we omit the superscript A for β_2^* to simplify notation, because the optimal barrier is the same for Case A and Case B. The fact that the funding ratio after distribution is the same in both cases supports this claim, but to prove this formally is surprisingly challenging. However, based on numerical studies, β_2^* is indeed the same in both cases. Furthermore, the optimal barrier seems to behave nicely and we did not encounter any problems with existence or uniqueness.

Remark 2.1. Let β_1^* be the optimal barrier level as established in Avanzi *et al.* (2017) for a simple solvency constraint (without α_2). As often with the introduction of a simple solvency constraint α_1, $\beta_1^* = \beta_0^*$ as long as $\beta_0^* > \alpha_1$. Otherwise, $\beta_1^* = \alpha_1$. Interestingly, it appears that $\beta_2^* > \beta_1^*$ in both Cases 1 and 2, which is not trivial. In Case 3, $\beta_2^* = \beta_1^*$.

3. Numerical studies

In this section, we illustrate numerically the impact of imposing recovery requirements. For all the numerical illustrations (where applicable), the value process of the assets starts at (1.2), whereas the value process of the liabilities starts at 1.0.

3.1. Impact of the introduction of a recovery constraint

We start by investigating the overall effects of imposing recovery requirements on aggregated distributions and ruin times. In return for accepting a small decrease in expected aggregated distributions, recovery requirements achieve much more stable outcomes and reduce the likelihood of early ruin significantly.

Figure 5 consists of comparisons of models without constraints (as in Gerber and Shiu, 2003), with simple solvency constraints (as in Avanzi *et al.*, 2017), and with solvency and recovery requirements (as in this paper). We focus on the aggregate value of dividends (first column) and time to ruin (second column), using 10,000 simulated couples (censored at 15,000). The first row compares absence of constraints with solvency and recovery requirements, whereas the second row compares solvency constraints with and without recovery requirements. The numbers in the axes' labels correspond to (mean, standard deviation) of the related outcomes.

While aggregate dividends are slightly less without constraints (in expected value), differences are strongly in favour of the recovery requirements (under the diagonal) when these are large (most likely because of a related rescue that led to more dividends). Furthermore, we observe that recovery requirements lead to significant improvements with respect to ruin times compared to the solvency constraint, of a nature that is qualitatively similar to as what was discussed in

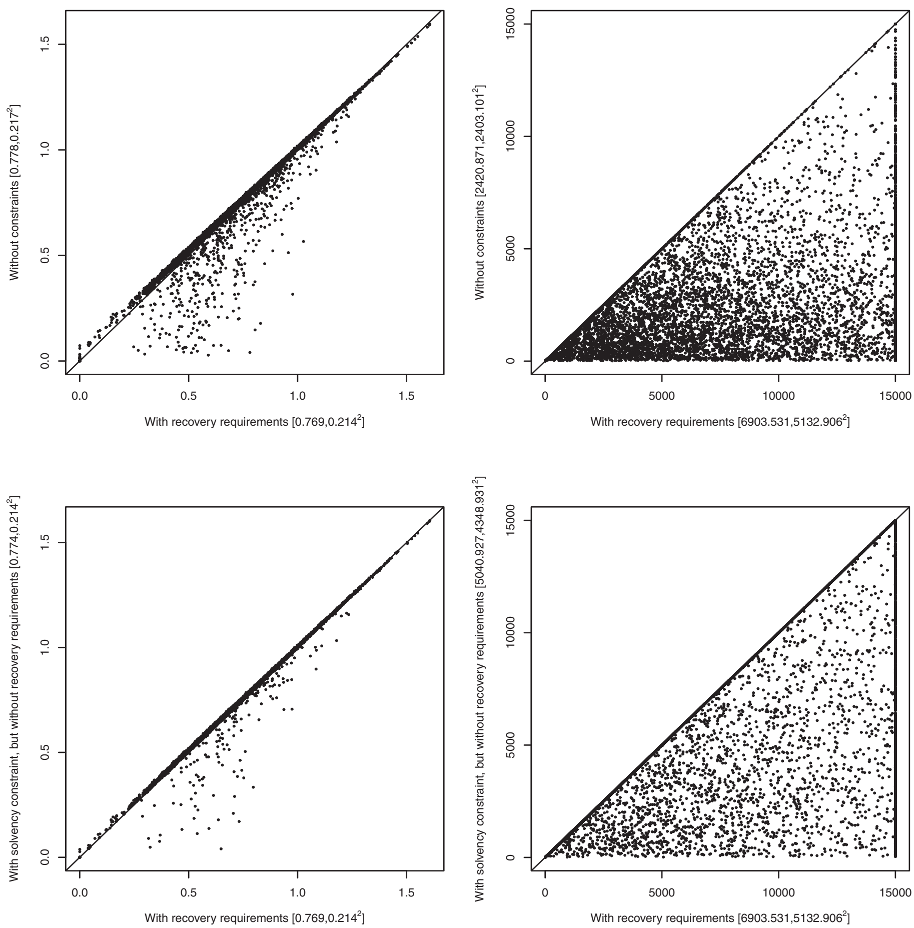

FIGURE 5: Scatterplots for unconstrained and recovery requirements (above) and for solvency constraint and recovery requirements (below), respectively. Aggregate dividends are displayed on the LHS, whereas times to ruin are displayed on the RHS. For parameters used, see Table B.1 in Appendix B, set no. 1.

Avanzi *et al.* (2017). In other words, there is an additional, substantial difference between recovery requirements and solvency constraints.

Next, we investigate whether most of the differences between solvency constraints and recovery requirements occur when α_2 is closest to α_1, or whether they occur slowly as α_2 moves away from α_1. We also investigate this as α_1 moves away from α_0 in the simple framework. This is illustrated in Figure 6 for Case B. We see that small spacing between the αs have marginal impact on the expected present value of distributions initially, even though they can have a large impact on stability as discussed above.

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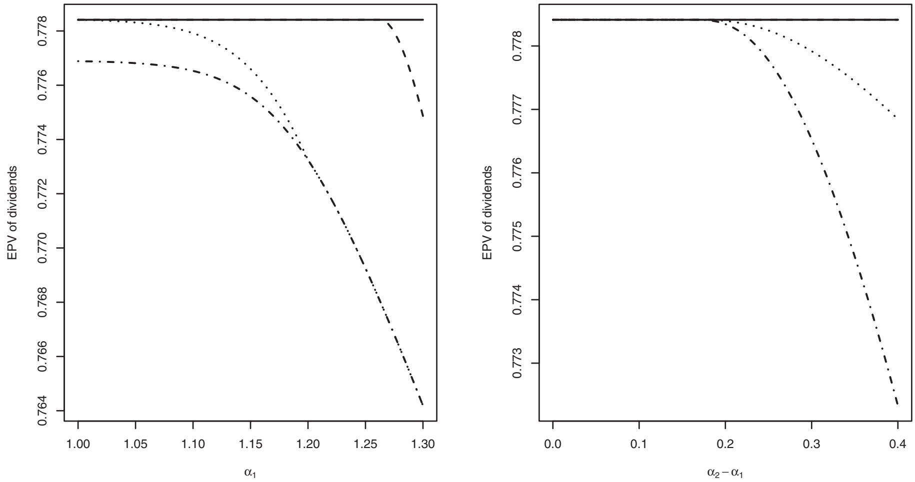

FIGURE 6: Value functions for Case B for the unconstrained case, solvency constraint and recovery requirements ($\phi \in \{0, 1\}$) as a function of α_1 and α_2, respectively. For both plots, the solid line is the value function for the unconstrained case, the dashed line is the value function under the solvency constraint, the dotted line is the value function under the recovery requirements with $\phi = 1$, and the dot-dashed line is the value function under the recovery requirements with $\phi = 0$. For parameters used, see Table B.1 in Appendix B, set no. 1 with the modification that $\alpha_2 = 1.4$ for the left plot and $\alpha_1 = 1.1$ for the right plot.

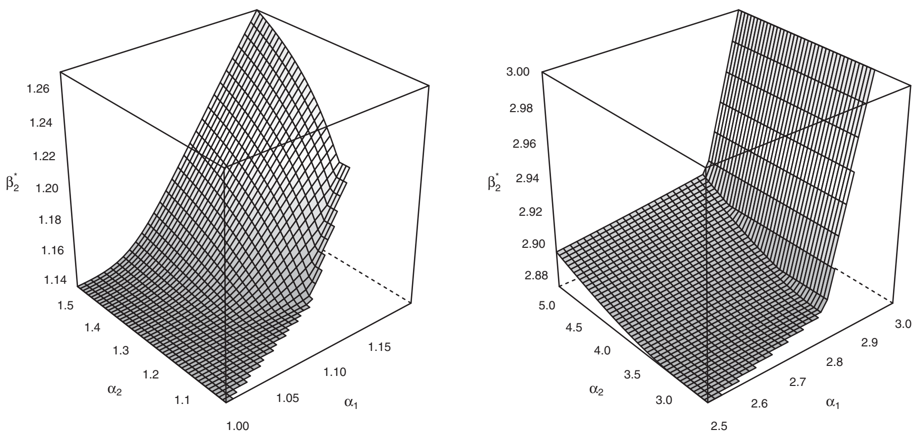

FIGURE 7: Plot of optimal barrier level for different combinations of α_1 and α_2. Left: $\sigma_A = 0.02$ and $\beta_0^* = 1.13588$. Right: $\sigma_A = 0.3$ and $\beta_0^* = 2.87326$. For parameters used, see Table B.1 in Appendix B, set no. 2 (left plot) and set no. 3 (right plot).

3.2. Moving from β_0^* to β_2^*

The relationship between β_1^* and β_0^* is trivial, but the relationship between β_2^* and β_0^* is not, as explained early in the paper; see Section 2.4. Figure 7 compares the optimal barrier level in a model without solvency constraints with the optimal barrier level in the model with recovery requirements, and shows how the

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optimal barrier without constraints, β_0^*, is no longer optimal. Instead we get the optimal barrier β_2^* (adjusted compared to β_0^*) when recovery requirements are introduced. Note that on both graphs, the floor is at β_0^*.

Note that it is reasonable to assume that $\beta_2^* \to \beta_1^*$ when $\alpha_2 \to \alpha_1$. This is observed on the right edge of the surfaces, which correspond to this limit. There we can see the trivial, linear relationship between β_1^* and β_0^*, which is flat as long as $\beta_0^* > \alpha_1$, and then increases linearly such that $\beta_1^* = \alpha_1$. On the LHS, one can see that the kink occurs when $\alpha_1 = \beta_0^*$, which is indicated with a pin.

When the volatility is rather low (as on the left plot), and as we move towards the left of the surface (α_2-wards), for given low α_1, the optimal barrier does not change and is very close to β_0^*. This is because β_0^* is far enough from α_1, and the process is very stable. When we increase volatility (moving to the right plot), β_0^* increases (the floor is higher) and even for low values of α_1, β_2^* increases with α_2.

Now, if we move towards the right of the surface for given α_2, we can observe an increase of the optimal barrier even before the kink. This is because moving α_1 towards the barrier level makes periods when no distributions are allowed more likely, which is a problem particularly for low volatility (a low volatility means that the process can get stuck in a non-distribution state for a very long period of time). This effect seems to dominate the "kink" effect especially for low volatilities.

3.3. The cost of not being able to distribute

Under the recovery requirements, we have two different value functions when the funding ratio is in the interval between α_1 and α_2. One can interpret the differences between these two value functions as the cost of being in the undesirable *no distribution* state. The difference between the value functions for $\phi = 0$ (the "undesirable" state) and $\phi = 1$ (the "good" state) in Case A is illustrated in Figure 8. The left plot is for a high value of σ_A (0.25), whereas the right figure is for a low value of σ_A (0.01). The reason that the differences are smallest for the most volatile model is that higher volatility leads to more switches between both environments, decreasing the influence of whether $\phi = 0$ or $\phi = 1$.

3.4. Sensitivity analysis for the volatility and correlation

Figure 9 shows the impact of the volatility $\tilde{\sigma}$ defined in (2.11) (first row) and the correlation ρ (second row) on the optimal barrier in absence of recovery constraints (first column), or with recovery requirements (second column). The immediate observation is that effects that were trivial before the introduction of recovery requirements are not trivial anymore; because the introduction of recovery requirements has mixed effects on the level of the optimal barrier.

In terms of volatility (first row), absence of recovery requirements means that higher volatility levels will generally lead to higher barrier levels. Note that the

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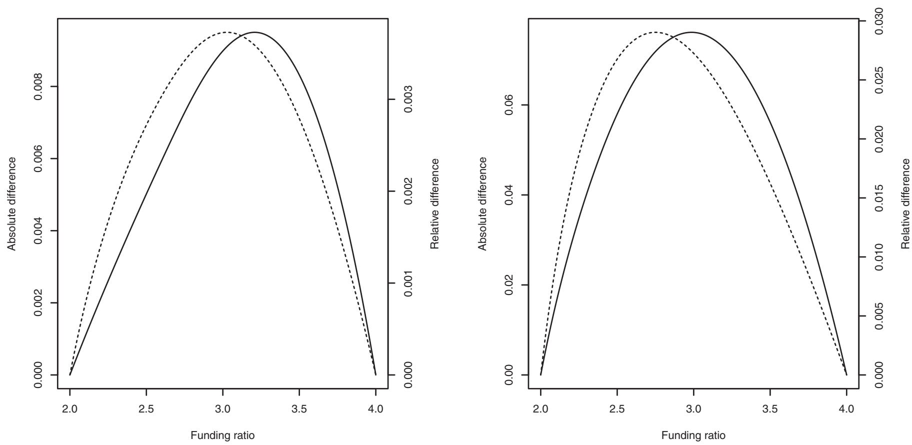

FIGURE 8: Differences between V_A^0 and V_A^1. Note that $V_A^0(\alpha_1) = V_A^1(\alpha_1)$ and $V_A^0(\alpha_2) = V_A^1(\alpha_2)$. The differences illustrated with the dotted line are relative to the values of V_A^0. Note that magnitudes of the differences are very different in the two plots. For parameters used, see Table B.1 in Appendix B, set no. 4 (left plot) and set no. 5 (right plot).

flat segments correspond to those cases where β^* has been "floored" at α_1, see Section 2.4. When we include recovery requirements, things get more interesting. First, optimal barrier levels are higher, and not "floored" at α_1 any more. Furthermore, something drags the increasing effect down for moderate levels of volatility. For low levels of $\tilde{\sigma}$, the barrier is increasing in volatility which is similar to the effect in the model without recovery requirements. However, for a $\tilde{\sigma}$ that is not too high, there is an advantage in decreasing β_2^* for increasing volatility as it will make it easier to leave state $\phi = 0$ if you fall into a no distribution period. As $\tilde{\sigma}$ tends to infinity, this effect fades and the risk of bankruptcy becomes dominant again.

In terms of correlation (second row), one can observe that higher correlation levels will lead to decreasing levels of the optimal barrier (except when it is "floored" as explained above). This is because high correlation makes the funding ratio evolve in a (relatively) stable manner, such that we can choose a barrier that is not too far away from α_1. With the introduction of recovery constraints, one should avoid down-crossing α_1 as it will switch the process into recovery mode. This leads to a global increase in the optimal barriers, especially when those were close to their respective α_1. For instance, for $\alpha_1 = 1.35$, the optimal barrier is "floored" at α_1 in absence of recovery constraints, but strictly higher than α_1 for any levels of correlation when recovery constraints are introduced.

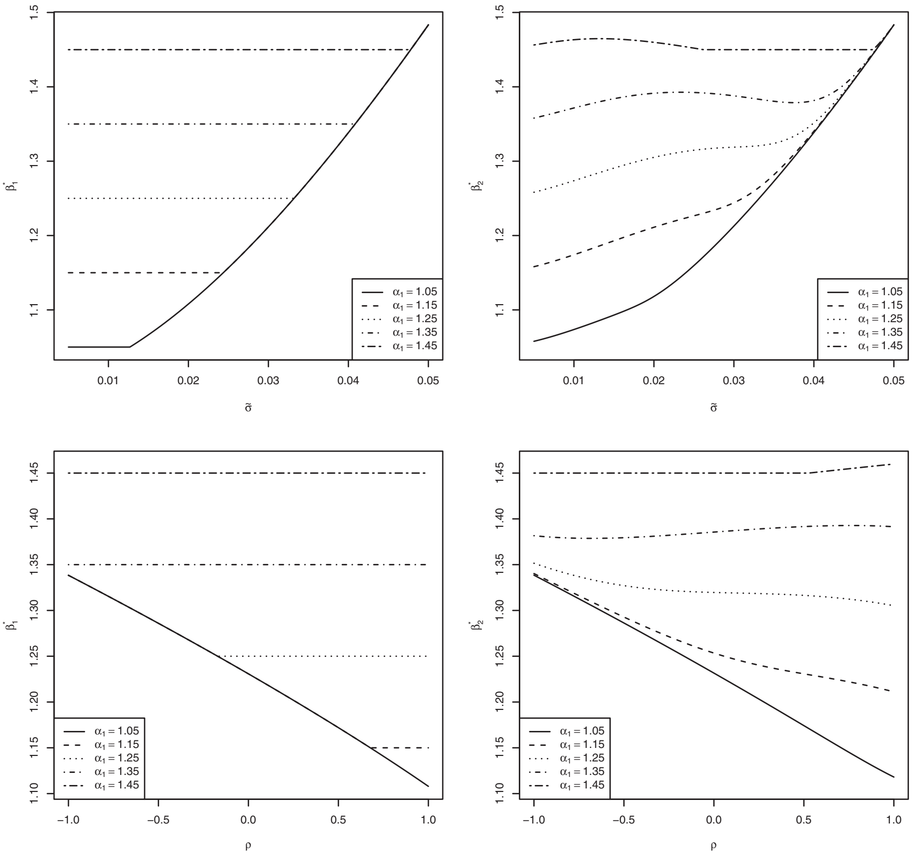

FIGURE 9: Sensitivity plots with respect to $\tilde{\sigma}$ (above) and ρ (below). For parameters used, see Table B: Parameter values, set no. 6.

4. CONCLUSION

In this paper, we considered a model for the dynamics of risky businesses, whose assets and liabilities can be approximated by two correlated geometric Brownian motions. Because we assume that those businesses are profitable, excess levels of profits are likely to be available for distribution in the future. Such distributions can materialise either as asset payouts (case B), or liability increases (case A).

Inspired by the regulatory frameworks of some jurisdictions, we assumed that fund distributions are subject to the following constraints. Because a funding ratio below α_0 will lead to bankruptcy, no surplus can be distributed if the surplus is inferior to α_1 (which is assumed to be strictly greater than α_1). Furthermore, if the funding ratio down-crosses that same trigger ratio α_1, the fund is required to recover — that is, to reach a higher level $\alpha_2 > \alpha_1$ — before it is allowed to distribute surplus again.

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We derived explicit expressions for the expected present value of distributions under that framework, and for both cases A and B. This allowed us to show how such regulation increases the stability of the fund substantially, while only minimally reducing the expected present value of distributions when compared to an optimal distribution with no regulation (as discussed in Decamps *et al.*, 2009; Avanzi *et al.*, 2017).

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APPENDIX A: PROOFS

A.1. Proof of Lemma 2.1

We introduce the notation $\tilde{G}(\cdot;\beta)$ by

$$G_{\mathrm{A}}(x_1,x_2;\beta) = (x_1+x_2)G_{\mathrm{A}}\left(\frac{x_1}{x_1+x_2},\frac{x_2}{x_1+x_2};\beta\right) = (x_1+x_2)G_{\mathrm{A}}\left(y,1-y;\beta\right)$$
$$= (x_1+x_2)\tilde{G}\left(y;\beta\right),$$

where $y=\frac{x_1}{x_1+x_2}$. For reformulating (2.9), we need the following derivatives:

$$\frac{\partial}{\partial x_i}G_{\mathrm{A}}(\vec{x};\beta) \text{ and } \frac{\partial^2}{\partial x_i\partial x_j}G_{\mathrm{A}}(\vec{x};\beta), \quad i,j=1,2.$$

We get

$$\frac{\partial}{\partial x_1}G_{\mathrm{A}}(x_1,x_2;\beta) = \tilde{G}\left(\frac{x_1}{x_1+x_2};\beta\right) + (x_1+x_2)\,\tilde{G}'\left(\frac{x_1}{x_1+x_2};\beta\right)\left(\frac{1}{x_1+x_2}-\frac{x_1}{(x_1+x_2)^2}\right)$$
$$= \tilde{G}(y;\beta)+\tilde{G}'(y;\beta)(1-y),$$
$$\frac{\partial}{\partial x_2}G_{\mathrm{A}}(x_1,x_2;\beta) = \tilde{G}\left(\frac{x_1}{x_1+x_2};\beta\right) + (x_1+x_2)\,\tilde{G}'\left(\frac{x_1}{x_1+x_2};\beta\right)\left(-\frac{x_1}{(x_1+x_2)^2}\right)$$
$$= \tilde{G}(y;\beta)+\tilde{G}'(y;\beta)(-y),$$
$$\frac{\partial^2}{\partial x_1^2}G_{\mathrm{A}}(x_1,x_2;\beta) = \tilde{G}'\left(\frac{x_1}{x_1+x_2};\beta\right)\left(\frac{1}{x_1+x_2}-\frac{x_1}{(x_1+x_2)^2}-\frac{x_2}{(x_1+x_2)^2}\right)$$
$$+\frac{x_2}{x_1+x_2}\tilde{G}''\left(\frac{x_1}{x_1+x_2};\beta\right)\frac{x_2}{(x_1+x_2)^2}$$
$$= \tilde{G}''(y;\beta)\frac{x_2^2}{(x_1+x_2)^3}.$$

Likewise,

$$\frac{\partial^2}{\partial x_2^2}G_{\mathrm{A}}(x_1,x_2;\beta) = \tilde{G}''(y;\beta)\frac{x_1^2}{(x_1+x_2)^3},$$
$$\frac{\partial^2}{\partial x_1\partial x_2}G_{\mathrm{A}}(x_1,x_2;\beta) = -\,\tilde{G}''(y;\beta)\frac{x_1x_2}{(x_1+x_2)^3}.$$

In total, we get that

$$\frac{\mathcal{A}G_{\mathrm{A}}(\vec{x};\beta)-\delta G_{\mathrm{A}}(\vec{x};\beta)}{x_1+x_2}$$
$$= \frac{1}{2}\left(\sigma_A^2+\sigma_L^2-2\rho\sigma_A\sigma_L\right)y^2(1-y)^2\tilde{G}''(y;\beta) + (\mu_A-\mu_L)\,y(1-y)\tilde{G}'(y;\beta)$$
$$+\left(\mu_A y+\mu_L(1-y)-\delta\right)\tilde{G}(y;\beta). \tag{A.1}$$

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We guess that the solution to the right-hand side of Equation (A.1) equal to 0 has a solution of the form

$$\tilde{G}(y;\beta) = y^{\vartheta}(1-y)^{\varphi}.$$

Inserting this in (A.1) gives

$$\begin{aligned}
&\frac{\mathcal{A}G_{\mathrm{A}}(\vec{x};\beta) - \delta G_{\mathrm{A}}(\vec{x};\beta)}{x_1+x_2}\\
=&\frac{1}{2}\left(\sigma_A^2+\sigma_L^2-2\rho\sigma_A\sigma_L\right)y^2(1-y)^2\\
&\times\left(y^{\vartheta-2}\left(\vartheta^2-\vartheta\right)(1-y)^{\varphi}-2\,y^{\vartheta-1}\vartheta\,(1-y)^{\varphi-1}\varphi+y^{\vartheta}\,(1-y)^{\varphi-2}\left(\varphi^2-\varphi\right)\right)\\
&+(\mu_A-\mu_L)\,y(1-y)\left(\vartheta y^{\vartheta-1}(1-y)^{\varphi}-\varphi y^{\vartheta}(1-y)^{\varphi-1}\right)\\
&+(\mu_A y+\mu_L(1-y)-\delta)\,y^{\vartheta}(1-y)^{\varphi}.
\end{aligned}$$

Dividing the above equation with $y^{\vartheta}(1-y)^{\varphi} = \tilde{G}(y)$ and setting $\tilde{\sigma}^2 = \sigma_A^2+\sigma_L^2-2\rho\sigma_A\sigma_L$ yields that the right-hand side is equal to

$$\begin{aligned}
&\underbrace{\frac{1}{2}\left(\vartheta^2-\vartheta\right)\tilde{\sigma}^2+\vartheta(\mu_A-\mu_L)+\mu_L-\delta}_{:=(*)}\\
&+\underbrace{(\vartheta+\varphi-1)}_{:=(**)}\left((-\vartheta\tilde{\sigma}^2-\mu_A+\mu_L)y+\frac{1}{2}\tilde{\sigma}^2(\vartheta+\varphi)y^2\right).
\end{aligned} \tag{A.2}$$

Setting the part of (A.2) not depending on y equal to 0 gives us a quadratic equation for ϑ:

$$\frac{1}{2}\tilde{\sigma}^2\vartheta^2+\left(\mu_A-\mu_L-\frac{1}{2}\tilde{\sigma}^2\right)\vartheta+\mu_L-\delta=0, \tag{A.3}$$

and setting the last term of (A.2) equal to 0, we get that $\varphi = 1-\vartheta$. We denote by ζ_1 and ζ_2 the two solutions to the quadratic equation. The solutions are given by

$$\begin{aligned}
&\frac{-\left(\mu_A-\mu_L-\frac{1}{2}\tilde{\sigma}^2\right)\pm\sqrt{\left(\mu_A-\mu_L-\frac{1}{2}\tilde{\sigma}^2\right)^2-4\frac{1}{2}\tilde{\sigma}^2\left(\mu_L-\delta\right)}}{\tilde{\sigma}^2}\\
&=\frac{\frac{1}{2}\tilde{\sigma}^2-(\mu_A-\mu_L)\pm\sqrt{\frac{1}{4}\tilde{\sigma}^4+(\mu_A-\mu_L)^2-(\mu_A-\mu_L)\,\tilde{\sigma}^2-2\tilde{\sigma}^2\left(\mu_L-\delta\right)}}{\tilde{\sigma}^2}\\
&=\frac{\frac{1}{2}\tilde{\sigma}^2-(\mu_A-\mu_L)\pm\sqrt{\frac{1}{4}\tilde{\sigma}^4+(\mu_A-\mu_L)^2-\tilde{\sigma}^2\left(\mu_A+\mu_L-2\delta\right)}}{\tilde{\sigma}^2}.
\end{aligned} \tag{A.4}$$

Because the coefficient of the quadratic term of (A.3), $\frac{1}{2}\tilde{\sigma}^2$, is greater than 0 and because the left-hand side of (A.3) is negative for $\vartheta = 0$ by (1.2) and (1.3) the quadratic equation (A.3) has a positive solution, which we denote ζ_2, and a negative solution, which we denote ζ_1. Because the left-hand side of (A.3) is equal to $\mu_A-\delta<0$ for $\vartheta=1$, we get that $\zeta_2>1$.

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By using that $\tilde{G}(y;\beta) = G_{\mathrm{A}}(y, 1-y;\beta)$, we get that a general solution is given by

$$G_{\mathrm{A}}(x_1, x_2;\beta) = C_1 x_1^{\zeta_1} x_2^{1-\zeta_1} + C_2 x_1^{\zeta_2} x_2^{1-\zeta_2}. \tag{A.5}$$

This ends the proof.

A.2. Proof of Theorem 2.2

Using (2.13) and (2.15), we get

$$K_{2,0}^{\mathrm{A}} = -K_{1,0}^{\mathrm{A}} {\alpha_0}^{\zeta_1-\zeta_2}, \tag{A.6}$$

such that

$$V_{\mathrm{A}}^{0}(x_1, x_2;\beta) = K_{1,0}^{\mathrm{A}} x_1^{\zeta_1} x_2^{1-\zeta_1} - K_{1,0}^{\mathrm{A}} {\alpha_0}^{\zeta_1-\zeta_2} x_1^{\zeta_2} x_2^{1-\zeta_2}.$$

Condition (2.17) states that

$$K_{1,0}^{\mathrm{A}} x_1^{\zeta_1} \left(\frac{x_1}{\alpha_1}\right)^{1-\zeta_1} + K_{2,0}^{\mathrm{A}} x_1^{\zeta_2} \left(\frac{x_1}{\alpha_1}\right)^{1-\zeta_2} = K_{1,1}^{\mathrm{A}} x_1^{\zeta_1} \left(\frac{x_1}{\alpha_1}\right)^{1-\zeta_1} + K_{2,1}^{\mathrm{A}} x_1^{\zeta_2} \left(\frac{x_1}{\alpha_1}\right)^{1-\zeta_2}.$$

This is a binding condition for either $K_{1,1}^{\mathrm{A}}$ or $K_{2,1}^{\mathrm{A}}$, whereas the other parameter can vary freely. That is, we choose to represent $K_{1,1}^{\mathrm{A}}$ as

$$\begin{aligned} K_{1,1}^{\mathrm{A}} &= \frac{K_{1,0}^{\mathrm{A}} x_1^{\zeta_1} \left(\frac{x_1}{\alpha_1}\right)^{1-\zeta_1} + K_{2,0}^{\mathrm{A}} x_1^{\zeta_2} \left(\frac{x_1}{\alpha_1}\right)^{1-\zeta_2} - K_{2,1}^{\mathrm{A}} x_1^{\zeta_2} \left(\frac{x_1}{\alpha_1}\right)^{1-\zeta_2}}{x_1^{\zeta_1} \left(\frac{x_1}{\alpha_1}\right)^{1-\zeta_1}} \\ &= K_{1,0}^{\mathrm{A}} + K_{2,0}^{\mathrm{A}} x_1^{\zeta_2-\zeta_1} \left(\frac{x_1}{\alpha_1}\right)^{-\zeta_2+\zeta_1} - K_{2,1}^{\mathrm{A}} x_1^{\zeta_2-\zeta_1} \left(\frac{x_1}{\alpha_1}\right)^{-\zeta_2+\zeta_1} \\ &= K_{1,0}^{\mathrm{A}} + K_{2,0}^{\mathrm{A}} \alpha_1^{\zeta_2-\zeta_1} - K_{2,1}^{\mathrm{A}} \alpha_1^{\zeta_2-\zeta_1}. \end{aligned} \tag{A.7}$$

Using (2.14), (A.7) and (A.6), we get that V_{A}^{1} has the representation

$$V_{\mathrm{A}}^{1}(x_1, x_2;\beta) = \left(K_{1,0}^{\mathrm{A}} - K_{1,0}^{\mathrm{A}} {\alpha_0}^{\zeta_1-\zeta_2} {\alpha_1}^{\zeta_2-\zeta_1} - K_{2,1}^{\mathrm{A}} {\alpha_1}^{\zeta_2-\zeta_1}\right) x_1^{\zeta_1} x_2^{1-\zeta_1} + K_{2,1}^{\mathrm{A}} x_1^{\zeta_2} x_2^{1-\zeta_2}. \tag{A.8}$$

That is, we have the representations for V_{A}^{0} and V_{A}^{1} given in the theorem but we still need to determine $K_{1,0}^{\mathrm{A}}$ and $K_{2,1}^{\mathrm{A}}$. The partial derivative of V_{A}^{1} with respect to x_2 in the point $\frac{x_1}{\beta}$ is given by

$$\begin{aligned} \frac{\partial}{\partial x_2} V_{\mathrm{A}}^{1}(x_1, x_2;\beta)|_{x_2=\frac{x_1}{\beta}} &= (1-\zeta_1)\left(K_{1,0}^{\mathrm{A}} - K_{1,0}^{\mathrm{A}} {\alpha_0}^{\zeta_1-\zeta_2} {\alpha_1}^{\zeta_2-\zeta_1} - K_{2,1}^{\mathrm{A}} {\alpha_1}^{\zeta_2-\zeta_1}\right) x_1^{\zeta_1} \left(\frac{x_1}{\beta}\right)^{-\zeta_1} \\ &\quad + (1-\zeta_2) K_{2,1}^{\mathrm{A}} x_1^{\zeta_2} \left(\frac{x_1}{\beta}\right)^{-\zeta_2} \\ &= (1-\zeta_1)\left(K_{1,0}^{\mathrm{A}} - K_{1,0}^{\mathrm{A}} {\alpha_0}^{\zeta_1-\zeta_2} {\alpha_1}^{\zeta_2-\zeta_1} - K_{2,1}^{\mathrm{A}} {\alpha_1}^{\zeta_2-\zeta_1}\right) \beta^{\zeta_1} \\ &\quad + (1-\zeta_2) K_{2,1}^{\mathrm{A}} \beta^{\zeta_2}. \end{aligned}$$

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This means that we get the following equation by using condition (2.18):

$$(1-\zeta_1)\left(K_{1,0}^{\mathrm{A}} - K_{1,0}^{\mathrm{A}}\alpha_0{}^{\zeta_1-\zeta_2}\alpha_1{}^{\zeta_2-\zeta_1} - K_{2,1}^{\mathrm{A}}\alpha_1{}^{\zeta_2-\zeta_1}\right)\beta^{\zeta_1} + (1-\zeta_2)\,K_{2,1}^{\mathrm{A}}\beta^{\zeta_2} = -1.$$

From this, it follows that

$$K_{2,1}^{\mathrm{A}}\left((1-\zeta_2)\,\beta^{\zeta_2} - (1-\zeta_1)\,\alpha_1^{\zeta_2-\zeta_1}\beta^{\zeta_1}\right) + (1-\zeta_1)\left(K_{1,0}^{\mathrm{A}} - K_{1,0}^{\mathrm{A}}\alpha_0{}^{\zeta_1-\zeta_2}\alpha_1{}^{\zeta_2-\zeta_1}\right)\beta^{\zeta_1} = -1,$$

such that $K_{2,1}^{\mathrm{A}}$ is given by

$$K_{2,1}^{\mathrm{A}} = \frac{-1-(1-\zeta_1)\left(K_{1,0}^{\mathrm{A}} - K_{1,0}^{\mathrm{A}}\alpha_0{}^{\zeta_1-\zeta_2}\alpha_1{}^{\zeta_2-\zeta_1}\right)\beta^{\zeta_1}}{(1-\zeta_2)\,\beta^{\zeta_2} - (1-\zeta_1)\,\alpha_1^{\zeta_2-\zeta_1}\beta^{\zeta_1}}. \tag{A.9}$$

We can represent $V_{\mathrm{A}}^0\left(x_1, \frac{x_1}{\alpha_2}; \beta\right)$ by

$$\begin{aligned} V_A^0\left(x_1, \frac{x_1}{\alpha_2}; \beta\right) &= K_{1,0}^{\mathrm{A}} x_1^{\zeta_1}\left(\frac{x_1}{\alpha_2}\right)^{1-\zeta_1} - K_{1,0}^{\mathrm{A}}\alpha_0{}^{\zeta_1-\zeta_2}x_1^{\zeta_2}\left(\frac{x_1}{\alpha_2}\right)^{1-\zeta_2} \\ &= \left(\frac{x_1}{\alpha_2}\right)K_{1,0}^{\mathrm{A}}\left(\alpha_2{}^{\zeta_1} - \alpha_0{}^{\zeta_1-\zeta_2}\alpha_2{}^{\zeta_2}\right), \end{aligned} \tag{A.10}$$

and $V_{\mathrm{A}}^1\left(x_1, \frac{x_1}{\alpha_2}; \beta\right)$ by

$$\begin{aligned} V_{\mathrm{A}}^1\left(x_1, \frac{x_1}{\alpha_2}; \beta\right) &= \left(\frac{1}{\beta} - \frac{1}{\alpha_2}\right)^+ x_1 + V_A^1\left(x_1, \left(\frac{x_1}{\min(\beta, \alpha_2)}\right); \beta\right) \\ &= x_1\left(\left(\frac{1}{\beta} - \frac{1}{\alpha_2}\right)^+ + \left(K_{1,0}^{\mathrm{A}} - K_{1,0}^{\mathrm{A}}\alpha_0{}^{\zeta_1-\zeta_2}\alpha_1{}^{\zeta_2-\zeta_1} - K_{2,1}^{\mathrm{A}}\alpha_1{}^{\zeta_2-\zeta_1}\right)\min(\beta, \alpha_2)^{\zeta_1-1}\right. \\ &\quad \left. + K_{2,1}^{\mathrm{A}}\min(\beta, \alpha_2)^{\zeta_2-1}\right), \end{aligned} \tag{A.11}$$

where we have used (A.8). By condition (2.16), we have that (A.10) equals (A.11). Using this and rearranging the terms give us that

$$\begin{aligned} &K_{1,0}^{\mathrm{A}}\left(\alpha_2{}^{\zeta_1-1} - \alpha_0{}^{\zeta_1-\zeta_2}\alpha_2{}^{\zeta_2-1} + (\alpha_0{}^{\zeta_1-\zeta_2}\alpha_1{}^{\zeta_2-\zeta_1} - 1)\min(\beta, \alpha_2)^{\zeta_1-1}\right) \\ &= \left(\frac{1}{\beta} - \frac{1}{\alpha_2}\right)^+ - K_{2,1}^{\mathrm{A}}\left(\alpha_1{}^{\zeta_2-\zeta_1}\min(\beta, \alpha_2)^{\zeta_1-1} - \min(\beta, \alpha_2)^{\zeta_2-1}\right). \end{aligned} \tag{A.12}$$

Inserting $K_{2,1}^{\mathrm{A}}$ given by (A.9) in (A.12) leads to the equation

$$\begin{aligned} &K_{1,0}^{\mathrm{A}}\left(\alpha_2{}^{\zeta_1-1} - \alpha_0{}^{\zeta_1-\zeta_2}\alpha_2{}^{\zeta_2-1} + (\alpha_0{}^{\zeta_1-\zeta_2}\alpha_1{}^{\zeta_2-\zeta_1} - 1)\min(\beta, \alpha_2)^{\zeta_1-1}\right) \\ &= \left(\frac{1}{\beta} - \frac{1}{\alpha_2}\right)^+ + \xi_A\left(-1-(1-\zeta_1)\left(K_{1,0}^{\mathrm{A}} - K_{1,0}^{\mathrm{A}}\alpha_0{}^{\zeta_1-\zeta_2}\alpha_1{}^{\zeta_2-\zeta_1}\right)\beta^{\zeta_1}\right), \end{aligned}$$

where

$$\xi_A = \frac{\min(\beta, \alpha_2)^{\zeta_2 - 1} - {\alpha_1}^{\zeta_2 - \zeta_1} \min(\beta, \alpha_2)^{\zeta_1 - 1}}{(1-\zeta_2)\, \beta^{\zeta_2} - (1-\zeta_1)\, \alpha_1^{\zeta_2-\zeta_1} \beta^{\zeta_1}}. \tag{A.13}$$

Solving with respect to $K^{\mathrm{A}}_{1,0}$ gives us that

$$K^{\mathrm{A}}_{1,0} = \frac{\left(\frac{1}{\beta} - \frac{1}{\alpha_2}\right)^+ - \xi_A}{{\alpha_2}^{\zeta_1 - 1} - {\alpha_0}^{\zeta_1-\zeta_2}{\alpha_2}^{\zeta_2-1} + ({\alpha_0}^{\zeta_1-\zeta_2}{\alpha_1}^{\zeta_2-\zeta_1} - 1)\min(\beta,\alpha_2)^{\zeta_1-1} + \xi_A\left((1-\zeta_1)\left(1 - {\alpha_0}^{\zeta_1-\zeta_2}{\alpha_1}^{\zeta_2-\zeta_1}\right)\beta^{\zeta_1}\right)}.$$

This concludes the proof.

APPENDIX B: PARAMETER VALUES

TABLE B.1

PARAMETER VALUES FOR NUMERICAL ILLUSTRATIONS.

No.	ρ	δ	μ_A	μ_L	σ_A	σ_L	α_0	α_1	α_2	κ	A_0	L_0
1	0.5	0.055	0.05	0.04	0.03	0.01	1	1.3	1.35	1.05	1.2	1
2	0.5	0.055	0.05	0.04	0.02	0.01	1	–	–	1.05	1.2	1
3	0.5	0.055	0.05	0.04	0.3	0.01	1	–	–	1.05	1.2	1
4	0.5	0.05	0.04	0.02	0.25	0.1	1	2	4	1.05	–	–
5	0.5	0.05	0.04	0.02	0.01	0.1	1	2	4	1.05	–	–
6	0.5	0.055	0.05	0.03	0.03	0.01	1	1.3	1.5	1.05	1.2	1

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