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TWO-DIMENSIONAL HELIOSEISMIC INVERSIONS

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INTRODUCTION

We present results of investigations of different inversion methods for inferring from helioseismic observations the rotation rate in the solar interior as a function of radius and latitude, using a mode set similar to that which is expected from the GONG network.

The rotation of a star perturbs the frequency ω_{nlm} of a normal mode by

$$\Delta\omega_{nlm} = \omega_{nlm} - \omega_{nl0} = \int_0^R \int_0^\pi K_{nlm}(r,\theta)\Omega(r,\theta) r dr d\theta \,, \qquad (1)$$

where n, l and m are the mode 'quantum' numbers, θ the colatitude, Ω the rotation rate and K_{nlm} a known function. Until now the observations have not been in terms of individual $\Delta\omega_{nlm}$'s; rather, coefficients $a_i(n,l)$ of a parametrization of the m-dependence of $\Delta\omega_{nlm}$ have been given. By representing the latitudinal dependence of the rotation rate in terms of an expansion in $\cos\theta$, an inversion of the a_i can be carried out as a series of 1-dimensional inversions. Such inversions have been dubbed "1.5-dimensional". Our present implementation of a 1.5D procedure follows Schou *et al.* (1992), and uses a_1, a_3 and a_5.

By contrast a full 2-dimensional inversion uses all the individual $\Delta\omega_{nlm}$. Our implementation minimizes a discretized version of

$$\sum_i \left(\frac{\int K_i \bar{\Omega} r d\theta dr - \Delta\omega_i}{\sigma_i}\right)^2 + \mu \int \left[f(r,\theta)\left(\frac{\partial^2\bar{\Omega}}{\partial r^2}\right)^2 + g(r,\theta)\left(\frac{\partial^2\bar{\Omega}}{\partial\theta^2}\right)^2\right] d\theta dr \qquad (2)$$

(Schou 1991; cf. Sekii 1990, 1991). Here $_i$ stands for the combination $_{nlm}$, μ is a trade-off parameter, σ_i are the errors on the observed splittings, f and g are functions of r and θ that can be specified (here $f = 1$ and $g = r^{-4}$), and $\bar{\Omega}$ is the rotation rate inferred from the inversion.

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RESULTS

Figure 1 shows the inferred rotation rate $\bar{\Omega}(r_0, \theta_0)$ obtained with the two methods from splittings computed for an assumed "true" rotation rate. We have used a mode set comprising all modes with $l = 0 - 200$ and $\nu = 1 - 4$mHz, which is roughly what is expected from the GONG network. The assumed errors were estimated from the mode linewidths, and should represent the errors in real data. The trade-off parameters have been adjusted to obtain the same error for both methods. Superficially both methods reproduce the original rotation rate with reasonable precision. Error estimates show that with a realistic dataset one should be able to determine the rotation rate with considerable precision: for example, the standard error in $\bar{\Omega}$ is approximately 1 nHz at $r_0 = 0.5R$ on the equator.

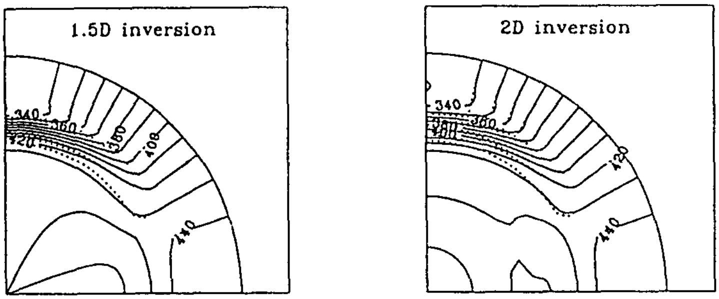

Fig. 1. The inferred rotation rate from 1.5D and 2D inversions, in a single quadrant with the North pole towards the top. Contours are labelled by the rotation rate, in nHz. Dotted contours show the "true" rotation rate.

For both methods *averaging kernels* $\mathcal{K}(r_0, \theta_0, r, \theta)$ exist which relate the inferred and true rotation rates (Schou *et al.* 1992):

$$\bar{\Omega}(r_0, \theta_0) = \int_0^R \int_0^\pi \mathcal{K}(r_0, \theta_0, r, \theta)\Omega(r, \theta) r dr d\theta \, . \quad (3)$$

As illustrated in Figures 2 and 3 these provide a more sensitive impression of the properties of the inversions. It is evident that the 2D inversion has better latitudinal resolution: the reason is the 2D method uses information that is effectively thrown away when expanding in a-coefficients. Note that the improvement in resolution is achieved with the same errors in the data, and similar errors in the inferred $\bar{\Omega}$, as for the 1.5D case.

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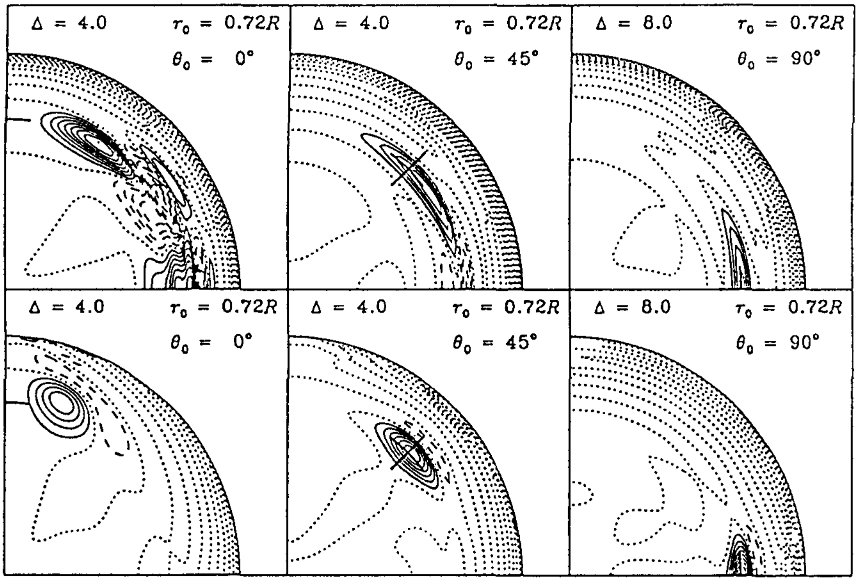

Fig. 2. Averaging kernels $\mathcal{K}(r_0, \theta_0, r, \theta)$ for the 1.5D (top) and 2D (bottom) inversions. Solid, dotted and dashed lines show contours for positive, zero and negative values, respectively. Heavy crosses show the location of the target position (r_0, θ_0); Δ is the contour spacing.

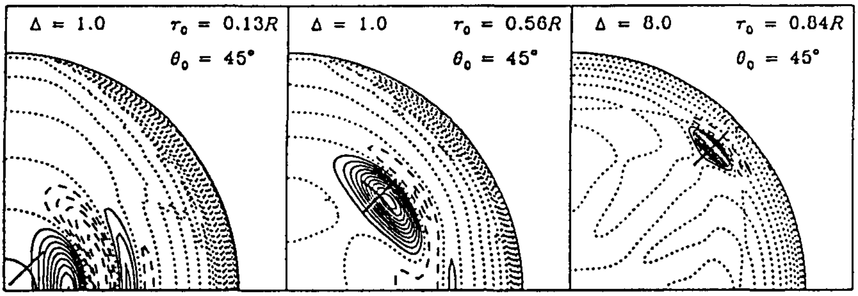

Fig. 3. Averaging kernels $\mathcal{K}(r_0, \theta_0, r, \theta)$ for the 2D inversion, at $\theta_0 = 45°$, for three different target radii. See the caption to Figure 2 for explanations.

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