

operator A by $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\xi + i\eta) dP(\xi) dQ(\eta)$ is shown, where $\{P(\xi)\}$ and $\{Q(\eta)\}$ are the spectral families of the real and imaginary parts of A .

In Ch. VII the spectral representation of an unbounded selfadjoint operator is obtained using the Cayley transform and the preceding decomposition of a unitary operator.

The text is very clear and contains some excellent worked examples which are pursued throughout the book. This compensates for the small number of exercises.

J. C. ALEXANDER,
EDINBURGH

Subgroups of Finite Groups, by S. A. Chunikhin. 142 pages. (English translation by Elizabeth Rowlinson.) Noordhoff, Groningen, 1969. U.S. \$6.25.

This book is essentially a unified exposition of research of the author and his school on certain generalizations of the Sylow theorems. For example, if Π is a set of primes, G a group, and m the largest divisor of the order of G all of whose prime factors lie in Π , then a subgroup of order m in G is called a Π -Sylow subgroup of G . One can now ask under what conditions the Sylow theorems will generalize to Π -Sylow subgroups. A typical result is this: Let G be a group with a composition series in which each index has at most one distinct prime divisor in Π (this is called Π -separability and generalizes the notion of solvability). Then for any $\Pi_1 \subseteq \Pi$, Π_1 -Sylow subgroups of G exist and are all solvable and conjugate.

The chapter headings are (1) Sylow Π -properties of finite groups, (2) Factorizations of finite groups based on the indices of chief and composition series, (3) A method for finding subgroups by means of indexials, and (4) Complexes of non-nilpotent subgroups.

The book is on a very specialized topic and is probably not of wide interest. Nevertheless, any graduate student should find it accessible.

R. V. MOODY,
UNIVERSITY OF SASKATCHEWAN

Topics in the Theory of Lifting, by A. Ionescu Tulcea and C. Ionescu Tulcea. x+190 pages. *Ergebnisse der Mathematik und ihrer Grenzgebiete Band 48*. Springer-Verlag, New York, 1969. U.S. \$9.90.

As indicated by the title, this book is concerned with the subject of lifting for spaces of bounded measurable functions. The problem was first formulated by A. Haar and solved by von Neumann in 1931 (for the real line and the Lebesgue measure). The general case has remained unsolved until 1958 when D. Maharam proved the existence of a lifting for a sigma-finite measure space.

In the book, the authors consider certain algebras \mathcal{A} of bounded measurable functions on a strictly localisable integration space (which includes sigma-finite measure spaces and Radon measures on locally compact spaces). A lifting on \mathcal{A} is defined to be a map $\rho: \mathcal{A} \rightarrow \mathcal{A}$ such that (1) $\rho(f) \equiv f$, (2) $f \equiv g$ implies $\rho(f) = \rho(g)$ (3) $\rho(1) = 1$, (4) $f \geq 0$ implies $\rho(f) \geq 0$, (5) $\rho(af + bg) = a\rho(f) + b\rho(g)$ and (6) $\rho(fg) = \rho(f)\rho(g)$. Here $f \equiv g$ means f and g are equal almost everywhere with respect to some measure under consideration. The existence of a lifting for all bounded measurable functions M^∞ (and other algebras also) is proved in Ch. IV using Martingale convergence theorem. The arguments are reminiscent of those of the proof of Hahn Banach extension theorem. Starting with the algebra of constant functions (almost everywhere) on which a lifting always exists, the authors used Zorn's lemma to find a maximal "extension" which gives a lifting on M^∞ .

In less than two hundred pages, the authors have presented in an extremely well-organised manner, a large amount of important results connected with the problem of lifting (mostly their own) which are not readily available. Basically this book consists of three major parts. Part One gives (mostly without proof) an excellent account of abstract integration theory (Ch. I) including vector valued functions (Ch. VI). In Part Two, the authors prove the existence of a lifting on M^∞ , including the existence of a strong lifting (one such that $\rho(f) = f$ for continuous f) for locally compact metrisable spaces and the nonexistence of a linear lifting (one without condition (6)) for \mathcal{L}^p -spaces for $1 \leq p < \infty$. (Ch. II, III, IV and VIII.) An alternative formulation of lifting in terms of measurable sets is also given (Ch. V). The last part deals with various applications to domination, integration and disintegration of measures, an isomorphism theorem for $L^\infty(Z, \mu)$ (with Z locally compact) as well as the equivalence of the Dunford-Pettis theorem (without any separability condition) and the existence of a lifting (Ch. VII, IX and X).

It is most appropriate to say that this book is mainly designed for the experts and yet it is also readable to any person with a strong background in the theory of measure and integration although some of the deeper results may be beyond his (including the reviewer's) appreciation.

JAMES C. S. WONG,
MCMMASTER UNIVERSITY

Theorie Axiomatique des Ensembles, par Jean-Louis Krivine. 120 pages. Collection SUP, section "Le mathématicien", Presses Univ. France, Paris, 1969. 10F.

Il n'existe pas d'ouvrages originaux en français qui traitent de la théorie axiomatique des ensembles (se l'on excepte le traité de Bourbaki). Dans le livre de J. L. Krivine, on trouvera un traitement à la fois succinct et complet de la théorie des ensembles dans sa forme axiomatique (par opposition à naïve).