Macroeconomic Dynamics, **19**, 2015, 1866–1879. Printed in the United States of America.
doi:10.1017/S136510051400008X

NOTES

A NOTE ON WAGE DETERMINATION UNDER MISMATCH

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Shimer (Mismatch, *American Economic Review* 97, 1074–1101 [2007]) introduced a model of mismatch in which limited mobility of vacant jobs and unemployed workers provides a microfoundation for their coexistence in equilibrium. He assumed that the short side of a local labor market receives all the gains from trade. In this note I show that modifying this assumption on wage-setting can deliver more reasonable predictions for wages at the level of the local market and in the aggregate.

Keywords: Mismatch Unemployment, Wage Determination, Shapley Value

1. INTRODUCTION

Shimer (2007) proposed an elegant microfoundation for the coexistence of unemployment and vacancies based on random variations in the numbers of workers and jobs in very disaggregated local labor markets. Some labor markets have excess workers, some have excess jobs, and the mobility of workers and jobs is limited. When the number of jobs in the aggregate economy varies, the unemployment and vacancy rates trace out a Beveridge curve that is indistinguishable from that observed in "normal times" in the U.S. labor market. As the first fully microfounded model of decentralized trade that is consistent with this fact, the model potentially represents an important advance beyond the reduced-form matching function of the Mortensen–Pissarides tradition.

However, although Shimer's model accounts well for quantities, it is less realistic in its predictions for prices. Shimer assumes that wages are determined competitively within each local labor market. In local markets with more jobs than workers, workers receive the entire output of the match, whereas in markets with more workers than jobs, the wage is driven down to a worker's outside

A previous version of this paper had the title "Wage Determination and Labor Market Volatility under Mismatch." I thank two anonymous referees, an associate editor, and audiences at the 2012 Search and Matching Conference at the University of Cyprus and at the 2012 Society for Economic Dynamics Conference in Limassol, Cyprus for helpful comments that significantly improved the paper. The usual disclaimer applies. Address correspondence to: William Hawkins, Department of Economics, Yeshiva University, 245 Lexington Avenue (LX711), New York, NY 10016, USA; e-mail: whawkins@yu.edu.

option, namely, the value of home production. This has unrealistic microeconomic implications. First, in any matched job–worker pair at any time, either the worker or the job earns a flow payoff equal to its outside option. Second, consider the effect of adding a small number of new jobs to a local labor market. In most local labor markets, this has no effect on whether it is workers or jobs that are in excess supply, and accordingly no effect on wages. In other markets, which switch from having excess workers to having excess jobs, wages increase sharply.

In this note, I show how to allow for alternative assumptions on wage determination in Shimer's mismatch model. I do so while retaining the rest of the structure of the model. This maintains the model's tractability and its predictions for the joint behavior of unemployment and vacancies, but can also allow the model to deliver more plausible predictions for wages and profits at the level of the local labor market and more flexible implications for their aggregate counterparts. The reason the model can easily accommodate many specifications for wages is that they barely play an allocative role. Neither the mobility of workers nor the locations where new jobs are created or existing jobs destroyed respond to prices; only the aggregate amount of job creation does so.

The wage determination protocols I investigate arise from cooperative game theory. This is natural because Shimer's local labor markets are a relabeled version of the "glove game" of Shapley and Shubik (1969), a leading test case for the plausibility of cooperative game-theoretic solution concepts. Shapley and Shubik argue that in the glove game the Shapley (1953b) value "gives an intuitively more satisfactory measure of the 'equities' of the situation while avoiding a violent discontinuity exhibited by . . . the competitive equilibrium" (p. 342). Accordingly, I investigate the predictions of the Shapley value for payoffs within a local labor market and show how to aggregate to the whole economy.

The Shapley value does, however, suffer from one drawback that may limit its usefulness in quantitative applications of the mismatch model. Specifically, like competitive wage setting, it is parameter-free, so that the distribution of worker and job values is fully determined by the structure of the model, the Shapley value assumption, and the aggregate unemployment and vacancy rates. This limits its flexibility for calibration purposes. A straightforward solution to this issue is provided by introducing a weighting parameter, following Shapley (1953a).

As an application, I conclude by noting that wage determination is key for the cyclical properties of the mismatch model. Shimer (2007) argued that the mismatch model delivered greater volatility of the unemployment and vacancy rates than the Mortensen–Pissarides benchmark. The results of this paper make it clear that this is due to the assumption of competitive wage setting. Other wage determination methods can deliver more or less volatile cyclical fluctuations.

The structure of the remainder of the paper is as follows. Section 2 describes the economic environment. Section 3 describes the Shapley value and its weighted counterpart at the level of a local labor market, whereas Section 4 discusses aggregation. Section 5 describes the implications of the model for aggregate fluctuations, and Section 6 concludes briefly.

2. MODEL

The economic environment is identical to that introduced in Shimer (2007). I provide here an informal description, only as much as needed for this note and only of the steady state. I refer the reader to Shimer's paper for further detail.

Time is continuous. There is a fixed measure M of workers and a large number of firms. All agents are risk-neutral and discount the future at rate r. Firms create jobs in a process to be described further later. Denote the (endogenous) measure of jobs in the economy by N.

There is a unit measure of local labor markets. At any time, each worker is attached to some local labor market, as is each job. The particular labor market to which each agent is attached at a particular time is random, with equal probability across all labor markets. The number of workers in any particular labor market is a non-negative integer and is distributed Poisson with mean M. That is, the fraction of labor markets with precisely $i \in \mathbf{N} = \{0, 1, \ldots\}$ workers is $\tilde{\pi}(i; M) = e^{-M}M^i/i!$. Analogously, the fraction of labor markets with precisely $j \in \mathbf{N}$ jobs is $\tilde{\pi}(j; N) = e^{-N}N^j/j!$. Because workers and jobs are allocated independently across labor markets, the fraction of labor markets with precisely i workers and j jobs, denoted by $\pi(i, j; M, N)$, is

$$\pi(i, j; M, N) = \tilde{\pi}(i; M)\tilde{\pi}(j; N) = \frac{e^{-(M+N)}M^i N^j}{i!j!}.$$

Workers and jobs match in pairs to create output of the single good in the economy. A matched job–worker pair produces output p, identical across all labor markets and across all matched job–worker pairs within a labor market. An unmatched worker produces $z < p$ units of the same good in home production; an unmatched job produces nothing.

There are no frictions within a labor market. Under the wage determination protocols to be considered here, the maximum feasible number of matches will form. Thus, in a labor market with i workers and j jobs, $\min\{i, j\}$ matches form. If $i > j$, then $i - j$ workers are unemployed, whereas if $j > i$, then $j - i$ jobs are vacant. Aggregate employment $E(N)$, unemployment $U(N)$, and vacancies $V(N)$ can be calculated by summing across local labor markets:

$$E = E(N) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \min\{i, j\}\pi(i, j; M, N);$$

$$U = U(N) = \sum_{i=0}^{\infty}\sum_{j=0}^{i} (i - j)\pi(i, j; M, N); \quad (1)$$

$$V = V(N) = \sum_{i=0}^{\infty}\sum_{j=i}^{\infty} (j - i)\pi(i, j; M, N). \quad (2)$$

Note that $E + U = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} i\pi(i, j; M, N) = \sum_{i=0}^{\infty} i\tilde{\pi}(i; M) = M$, consistent with the notion that the average number of workers per labor market is M. Similarly, $E + V = N$. The aggregate employment, unemployment, and vacancy rates are equal to E/M, U/M, and V/N, respectively.

At any time, any firm can create as many jobs as desired at constant marginal cost k. A newly created job is allocated at random to a particular labor market, independent of the numbers of workers and jobs already located there. Each active job is destroyed at Poisson rate l.

Denote the expected profit flow of a job located in a labor market with i workers and j jobs by $v_p^x(i, j)$. This value depends on productivity p, on the numbers of workers and jobs in the market, and on how wages are determined, indexed by x and discussed in Section 3. Also define

$$\bar{v}_p^x(N) = \frac{1}{N} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} j v_p^x(i, j)\pi(i, j; M, N), \tag{3}$$

the cross-sectional expectation of the profit flow of a job (conditional on p and N). The measure of active jobs is determined by a free entry condition, which in the steady state takes the form

$$(r + l)k = \bar{v}_p^x(N). \tag{4}$$

For the wage determination mechanisms I study, $\bar{v}_p^x(N)$ is strictly increasing in p, is strictly decreasing in N, and satisfies the Inada condition $\bar{v}_p^x(N) \to 0$ as $N \to \infty$. This guarantees that a unique equilibrium $N = N(p)$ exists and is increasing in p.

3. WAGE DETERMINATION

In this section I briefly describe how wages and profits are determined within a local labor market. Section 4, following, discusses aggregation.

It is easiest to describe each wage determination protocol in terms of its implications for the shares of the net output of a match, $p - z$, that accrue to the worker and to the job. In each case I consider, the earnings of a worker and a job located in a labor market containing i workers and j jobs can be written respectively as

$$u_p^x(i, j) = z + (p - z)\psi^x(i, j) \qquad \text{and} \qquad v_p^x(i, j) = (p - z)\phi^x(i, j), \tag{5}$$

where $\psi^x(i, j), \phi^x(i, j) \in [0, 1]$ do not depend on p. Here x indexes the wage determination protocol: $x = c$ denotes competitive wage setting (Section 3.1), $x = s$ denotes the Shapley value (Section 3.2), and $x = \beta \in [0, 1]$ denotes the weighted Shapley value with worker weight β (Section 3.3). In all cases, because the total surplus to be divided is $(p - z)\min\{i, j\}$, it follows that

$$i\psi^x(i, j) + j\phi^x(i, j) = \min\{i, j\}. \tag{6}$$

3.1. Competitive Wages

Shimer (2007) assumed that wages are determined competitively within each local labor market.

In this case, if there are more workers than jobs in a local labor market, the wage is equal to z, the worker's outside option, whereas if there are excess workers, workers earn the full value of match output p. That is, $\psi^c(i, j) = \mathbf{1}\,(1 \leq i < j)$ and $\phi^c(i, j) = \mathbf{1}\,(i \geq j \geq 1)$.[1]

3.2. Shapley Value

An obvious source of alternative wage determination protocols is cooperative game theory. The advantage of a cooperative-game-theoretic approach is that one does not need to specify the fine details of the strategic environment, such as the order of moves or the actions for each player; rather, one derives predictions for each player's payoff based solely on information about what each possible subset of the players can produce in isolation. A leading solution concept for transferable utility (TU) games is the Shapley value [Shapley (1953b)]. Winter (2002) surveys related literature.

The Shapley values for players of an arbitrary cooperative game can be calculated as follows:

1. Starting with an empty coalition, one player at a time (either job or worker) is added, with all remaining players being equally likely at each step, so that all possible orders of inclusion are equally likely to arise.
2. Conditional on the ordering, each player is paid the marginal additional surplus created by his addition to the previous coalition.

The Shapley value of the player is then given by taking the expectation over all possible orderings of the players. Shapley (1953b) derived the value from axiomatic considerations: the Shapley value is the unique value that satisfies, for all cooperative TU games, four intuitive properties—efficiency, symmetry, additivity on the space of all games, and a requirement that players whose marginal contribution to any coalition is zero receive zero payoff.[2] Roth (1977) showed that the Shapley value can be regarded as a von Neumann–Morgenstern utility for players who are risk-neutral not only with respect to ordinary risk [lotteries where the prizes are (transferable) utility] but also with respect to strategic risk (lotteries where the prizes are rights to the positions of particular players in particular games).

In the environment studied here, the marginal surplus created by the addition of a worker to a coalition with $\hat{\imath}$ workers and $\hat{\jmath}$ jobs equals $p - z$ if $\hat{\jmath} > \hat{\imath}$ and 0 otherwise; the extra worker increases the surplus precisely when the coalition included a vacant job before he was added. Similarly, the marginal surplus created by adding a job to such a coalition is $p - z$ if $\hat{\imath} > \hat{\jmath}$ and 0 otherwise. It follows

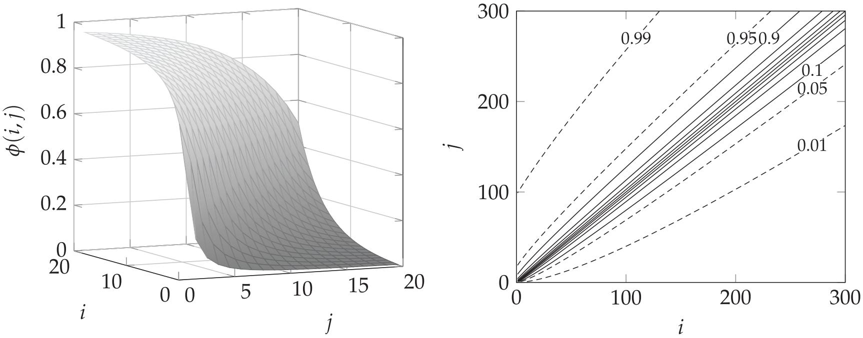

FIGURE 1. Profit share $\phi^s(i, j)$ under Shapley value. Left panel: $\phi^s(\cdot)$. Right panel: contour plot of $\phi^s(\cdot)$. The unlabeled contours correspond to values of $\phi^s(i, j)$ equal to 0.2, 0.3, . . . , 0.8.

that a job's share of the match surplus under the Shapley value, $\phi^s(\cdot)$, satisfies

$$\phi^s(i, j) = \begin{cases} \frac{i}{i+j}\phi^s(i-1, j) + \frac{j-1}{i+j}\phi^s(i, j-1) & \text{if } 1 \le i < j; \\ \frac{i}{i+j}\phi^s(i-1, j) + \frac{j-1}{i+j}\phi^s(i, j-1) + \frac{1}{i+j} & \text{if } i \ge j \ge 1, \end{cases} \tag{7}$$

with boundary condition $\phi^s(0, j) = 0$ for all $j \ge 1$.[3] Shapley and Shubik (1969) showed that the solution to this recurrence is[4]

$$\phi^s(i, j) = \begin{cases} \frac{1}{2} + \frac{i-j}{2j}\sum_{k=1}^{j} \frac{i!j!}{(i+k)!(j-k)!} & \text{if } i \ge j \ge 1; \\ \frac{1}{2} - \frac{j-i}{2j}\sum_{k=0}^{i} \frac{i!j!}{(i-k)!(j+k)!} & \text{if } 1 \le i < j. \end{cases}$$

It is easy to see that $\phi^s(j, j) = \frac{1}{2}$ for all j and that $\phi^s(i, j)$ is strictly increasing in i and strictly decreasing in j (provided $i, j \ge 1$).[5] That is, wages are higher and profits lower in labor markets in which there are more jobs or fewer workers. As under competitive wages, an agent on the short side of the market receives a greater share of match surplus, but under the Shapley value the dependence of wages and profits on (i, j) does not exhibit an abrupt change at $i = j$. Finally, because $j\phi^s(i, j) \le \min\{i, j\} \le i$, it is immediate that for each fixed i, $\phi^s(i, j) \to 0$ as $j \to \infty$.[6] Figure 1 shows the function $\phi^s(\cdot)$.

Note that all workers in a local labor market receive the same payoff, as do all jobs. (There is no sense in which an employed worker receives less than an unemployed worker; in fact, just as in the competitive model, which agents on the long side of the market are unmatched is not determined.) This is consistent with regarding the Shapley value as an expectation of a particular player's marginal

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contribution toward the surplus over all possible orderings of the players. It is an ex ante value, before the identity of which workers and jobs will be matched is determined. The Shapley value of a worker therefore does not correspond exactly to the concept of a wage paid only to employed workers.[7]

A natural implementation of the Shapley value in terms of wages exists, however. Assume that each worker who is not matched in production to a job receives an ex post payoff equal to the outside option z, and each vacant job receives an ex post payoff equal to 0. Also assume that each matched worker receives a payoff $w_p(i, j)$ such that the expected value of a worker ex ante (before the "employment lottery") is equal to $u_p(i, j) = z + (p - z)\psi^s(i, j)$.[8] Each matched job then receives profit flow $p - w_p(i, j)$, and the ex ante value of a job before the employment lottery is $v_p(i, j)$. Because $\min\{i, j\}$ workers are employed, this requires $w_p(i, j) = z + (p - z)i\psi^s(i, j)/\min\{i, j\}$.[9]

3.3. Weighted Shapley Value

The Shapley value assumes symmetry. The labels "worker" and "job" are irrelevant: all that matters is whether an agent is on the short or long side of a market. (Thus $\psi(i, j)$, a worker's share of the match surplus in a market with i workers and j jobs, is equal to $\phi(j, i)$, a job's share in a market with j workers and i jobs.) This symmetry assumption may be unrealistic in a labor market context: a job's "bargaining power" might differ from a worker's. In addition, imposing symmetry may limit the ability of the model to match aggregate data on earnings and profits. One possible way to remedy this is to use the weighted Shapley value [Shapley (1953a); Kalai and Samet (1985)].

The weighted Shapley value is calculated in a way similar to the Shapley value, with the sole modification that not all orderings of the agents in the grand coalition are equally likely. Denote the "weight" of a worker by $\beta \in [0, 1]$ and that of a job by $1 - \beta$. Assume that the probability that an ordering of any coalition of $\hat{\imath}$ workers and $\hat{\jmath}$ jobs features a worker last is $\beta\hat{\imath}/[\beta\hat{\imath} + (1 - \beta)\hat{\jmath}]$. This generates a probability distribution over orderings of the i workers and j jobs in a labor market. The weighted Shapley value of a player is defined as his or her expected marginal contribution to the surplus when different orderings of the players are drawn according to this probability distribution. (Thus, the Shapley value corresponds to the case $\beta = 1/2$.) A job's share of the surplus can be calculated using the appropriate generalization of (7),

$$\phi^\beta(i, j) = \begin{cases} \dfrac{\beta i}{\beta i + (1-\beta)j}\phi^\beta(i-1, j) + \dfrac{(1-\beta)(j-1)}{\beta i + (1-\beta)j}\phi^\beta(i, j-1) & \text{if } 1 \leq i < j; \\ \dfrac{\beta i}{\beta i + (1-\beta)j}\phi^\beta(i-1, j) + \dfrac{(1-\beta)(j-1)}{\beta i + (1-\beta)j}\phi^\beta(i, j-1) + \dfrac{1-\beta}{\beta i + (1-\beta)j} & \text{if } i \geq j \geq 1, \end{cases} \tag{8}$$

again with boundary condition $\phi^\beta(0, j) = 0$ for $j \geq 1$.

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It is straightforward to see that as $\beta \to 0$, $\phi^{\beta}(i, j) \to \min\{i, j\}/j$, and as $\beta \to 1$, $\phi^{\beta}(i, j) \to 0$. In addition, $\phi^{\beta}(1, 1) = 1 - \beta$. Thus, in an intuitive sense, β is analogous to the generalized Nash bargaining parameter used in the Mortensen–Pissarides tradition: the higher it is, the higher the worker's share of the surplus and the lower the job's share.[10]

4. AGGREGATION

The previous section studied wages and profits at the level of a local labor market. I now turn to the aggregate counterparts of these variables. The main object of interest is $\bar{v}_p^x(N)$, the average profit flow earned per job in the aggregate economy, which is the key to understanding entry, according to the free entry condition (4).

Because profits in each local labor market are proportional to $p - z$, write $\bar{v}_p^x(N) = (p - z)\bar{\phi}^x(N)$, where $\bar{\phi}^x(N)$ is the share of the flow match surplus earned by the average job. Using (3) and (5), this is given by

$$\bar{\phi}^x(N) = \frac{1}{N}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\phi^x(i, j)j\pi(i, j; M, N) = \frac{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\phi^x(i, j)j\pi(i, j; M, N)}{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}j\pi(i, j; M, N)}, \qquad (9)$$

the expected value of the match surplus at the level of the local labor market, $\phi^x(i, j)$, weighting all jobs equally, or equivalently, weighting local labor markets (i, j) according to the frequency $\pi(i, j; M, N)$ of such labor markets multiplied by the number of jobs j in the market.

For all of the wage determination models considered here, it turns out that $\phi^x(i, j)$ is well approximated by a nondecreasing function of $i - j$, the excess number of workers (relative to jobs) in the local labor market.[11] Accordingly, it is useful to introduce the notation

$$\hat{\pi}(d; N) = \frac{1}{N}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\mathbf{1}(i - j = d)j\pi(i, j; M, N) = \frac{1}{N}\sum_{j=\min\{0,-d\}}^{\infty} j\pi(j + d, j; M, N)$$

for the probability distribution over $d \in \mathbf{Z}$ of jobs according to the excess number of workers d in the local labor market in which the job is located. Also, write

$$\hat{\phi}^x(d; N) = \frac{\frac{1}{N}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\phi^x(i, j)\mathbf{1}(i - j = d)j\pi(i, j; M, N)}{\hat{\pi}(d; N)}, \qquad (10)$$

the expectation of $\phi^x(i, j)$ conditional on $i - j = d$, taken as before with respect to the probability that weights local labor markets (i, j) according to the number

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TABLE 1. Worker and job shares of match surplus

	Competitive	Shapley	Weighted Shapley $\beta = 0.55$	Weighted Shapley $\beta = 0.60$	Weighted Shapley $\beta = 0.65$
$\bar{\phi}^x(N)$	0.632	0.577	0.239	0.108	0.063
$\bar{\psi}^x(N)$	0.334	0.388	0.714	0.841	0.884
$d\bar{\phi}^x(N)/dN$	−4.09	−2.80	−2.13	−1.16	−0.71
$\eta_{\bar{\phi}^x(\cdot),N}$	−6.48	−4.85	−8.92	−10.67	−11.15

of jobs j in the market. The share of the per-match surplus received per job, averaged across the whole economy, is then the average of the values of $\hat{\phi}^x(d; N)$ weighting different values of d according to $\hat{\pi}(d; N)$:

$$\bar{\phi}^x(N) = \sum_{d=-\infty}^{\infty} \hat{\phi}^x(d; N)\hat{\pi}(d; N). \quad \mathbf{(11)}$$

A similar approach can be used to characterize $\bar{\psi}^x(N)$, the average share of the match surplus received per worker, averaged across the whole economy. Alternatively, because all output must go to some worker or some job, it follows that

$$M\bar{\psi}^x(N) + N\bar{\phi}^x(N) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \min\{i, j\}\pi(i, j; M, N),$$

from which $\psi^x(N)$ can be calculated given $\phi^x(N)$.

Table 1 shows the implications of five wage determination protocols for aggregate wages and profits. The wage specifications shown are competitive wages, Shapley value, and weighted Shapley value with worker weight $\beta \in \{0.55, 0.60, 0.65\}$. The first two rows report the values of $\bar{\phi}^x(N)$ and $\bar{\psi}^x(N)$. I also calculate the derivative $d\bar{\phi}^x(N)/dN$ and the elasticity $\eta_{\bar{\phi}^x,N}$ by numerically differentiating (11) with respect to N. The case shown is $M = 244.2$, $N = 236.3$, which is Shimer's benchmark calibration: this is the unique pair (M, N) consistent with aggregate unemployment and vacancy rates of 5.4 and 2.3%, respectively.

A more intuitive understanding of these results can be gained from Figure 2. According to (11), the average job's share of the match surplus is the integral of $\hat{\phi}^x(d; N)$ with respect to the probability distribution over d given by $\hat{\pi}(d, N)$. The figure plots $\hat{\phi}^x(d; N)$ against d for the same five wage specifications just discussed. The probability distribution $\hat{\pi}(\cdot; N)$, which does not depend on how wages are determined, is also shown. The case shown is again $M = 244.2$, $N = 236.3$.

It is clear that the average job's profits are lower for the weighted Shapley value for $\beta > 1/2$ than for the Shapley value, because $\hat{\phi}^\beta(\cdot; N)$ lies uniformly below $\hat{\phi}^s(\cdot; N)$ for such β. Profits are higher for competitive wage-setting than for the Shapley value because the probability distribution $\hat{\pi}(\cdot; N)$ puts a greater mass on positive values of d (where $\phi^c(d; N) > \phi^s(d; N)$) because $M > N$.

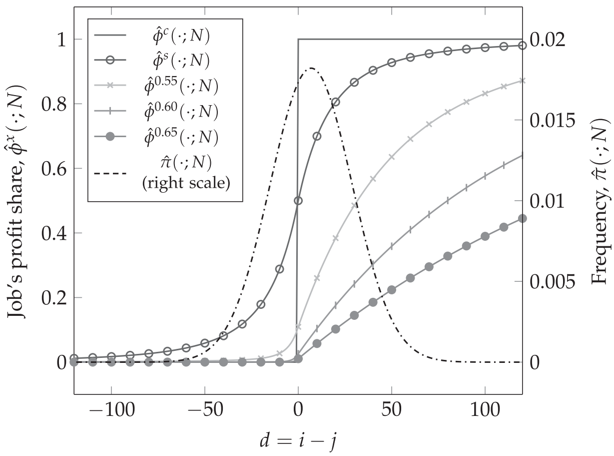

FIGURE 2. Left scale: average profit share of per-match surplus, $\hat{\phi}^x(\cdot; N)$, as a function of $d = i - j$, the number of excess workers in a local labor market, for five different wage determination specifications. Right scale: frequency $\hat{\pi}(\cdot; N)$ of local labor markets by the value of d.

Figure 2 also makes obvious the microeconomic limitations associated with competitive wages. First, a job's share of the per-match surplus $\hat{\phi}^c(d; N)$ exhibits a "violent discontinuity" at $d = 0$ (to use the terminology of Shapley and Shubik). Second, $\hat{\phi}^c(d; N)$ does not change with small changes in d for any other value of d. The alternative wage determination protocols studied here imply wages and profits which appear more intuitively reasonable.

One can also use Figure 2 to understand the effect on $\bar{\phi}^x(N)$ of changing N. If N increases from N_0 to $N_0 + \Delta N$, the average number of jobs per local market increases by ΔN, so that, roughly speaking, the entire distribution $\hat{\pi}(\cdot; N)$ shifts to the left by the amount ΔN.[12] The effect of this on per-job profits then depends on how steeply sloped $\hat{\phi}^x(\cdot, N_0)$ is in the region where $\hat{\pi}(\cdot; N_0)$ puts most mass, that is, near $d = M - N_0$. This intuition explains, for example, why the derivative of profits with respect to N is higher for competitive wage-setting than for the Shapley value, because all of the increase of $\hat{\phi}^c(\cdot; N)$ from 0 to 1 occurs in the range where $\hat{\phi}(\cdot; N)$ is large. Finally, the elasticity of $\bar{\phi}^x(N)$ with respect to N is of course equal to the derivative divided by $\bar{\phi}^x(N)/N$.

5. CHANGES IN AGGREGATE PRODUCTIVITY

I finally briefly outline the implications of the alternative models of wage determination introduced in the preceding for aggregate unemployment and vacancies. Shimer (2005) showed that the effects of aggregate labor productivity shocks in the Mortensen–Pissarides framework are closely approximated by the comparative static effects of changes in steady-state productivity. The same is true in the

TABLE 2. Steady-state elasticities of unemployment and vacancies with respect to productivity

			Weighted Shapley		
	Competitive	Shapley	$\beta = 0.55$	$\beta = 0.60$	$\beta = 0.65$
u	−2.90	−3.87	−2.11	−1.76	−1.68
v	3.92	5.23	2.85	2.38	2.28

mismatch model.[13] Accordingly, I focus on steady state comparative statics for simplicity.[14]

The elasticities of the steady-state unemployment and vacancy rates $u = u(p)$ and $v = v(p)$ with respect to p can be found by log-differentiating the free-entry condition (4). They satisfy

$$\eta_{u,p} = \eta_{N,p}\eta_{u,N} = -\frac{p}{p-z}\frac{\eta_{u,N}}{\eta_{\bar{\phi}^x,N}} \quad \text{and} \quad \eta_{v,p} = \eta_{N,p}\eta_{v,N} = -\frac{p}{p-z}\frac{\eta_{v,N}}{\eta_{\bar{\phi}^x,N}}.$$

These equations are intuitive. When productivity rises, the number of jobs, N, must also rise so that the free entry condition continues to hold. Unemployment responds elastically under three circumstances: if a given change in N generates a large change in unemployment (that is, $\eta_{u,N}$ is large), if a change in net productivity $p-z$ generates a large change in N (which occurs if profits are not very responsive to N, that is, if $\eta_{\bar{\phi}^x,N}$ is small), or if a change in gross productivity p generates a large change in net productivity $p-z$ (which occurs if the surplus $p-z$ is small).

Two of these conditions are unrelated to wages. The elasticities of u and v with respect to N depend only on the structure of the model. These elasticities are given by $\eta_{u,N} = -11.27$ and $\eta_{v,N} = 15.23$ when $(M, N) = (244.2, 236.3)$.[15] In addition, the observation that a small surplus $(p-z)/p$ amplifies the effect of productivity changes is true here for the same reason as in the Mortensen–Pissarides model [Shimer (2005); Hagedorn and Manovskii (2008)]. If z is high enough, that is, if the surplus from employment is small enough, u and v can be made arbitrarily elastic.

It follows that unemployment and vacancies will respond elastically to productivity changes precisely when the wage determination mechanism is such that $\bar{\phi}^x(N)$ responds only inelastically to changes in N. That is, among the five cases shown in Table 1, unemployment and vacancies will respond most elastically under the Shapley value and least under the weighted Shapley value with $\beta = 0.65$. This is what is shown in Table 2.[16]

In conclusion, just as in the Mortensen–Pissarides model, the effect of labor productivity on unemployment and vacancies depends crucially on how wages are determined. Shimer finds that unemployment and vacancies respond quite elastically in the mismatch model (at least relative to his calibration of the Mortensen–Pissarides model) because under competitive wages the profit share responds somewhat inelastically to additional job creation. Other wage determination assumptions can deliver more or less elastic effects.

6. CONCLUSION

Shimer's mismatch model has a natural appeal for applied use, because its account of the coexistence of unemployment and vacancies based on the heterogeneity of local labor markets offers a promising structure in which to use microeconomic data to discipline models of the aggregate labor market. However, competitive wage setting implies counterfactual wages and profits at the micro level and, because of the parameter-free nature of competitive wage setting, also limits the model's ability to match aggregate data.

The contribution of this note was to demonstrate that the model can accommodate alternative, arguably more realistic, wage determination protocols in a straightforward way. I showed how to incorporate wage setting via the Shapley value and its weighted counterpart. It seems interesting to investigate the properties of alternative wage determination protocols as well.[17] A more challenging extension would be to allow for risk aversion of workers. A difficulty immediately arises that when workers are risk-averse, utility is not perfectly transferable; there is no universally agreed extension of the Shapley value to the case of nontransferable utility.[18] Further progress in this direction is therefore left for future work.

NOTES

1. A tie-breaking assumption is needed to deal with the case $i = j$, and Shimer assumes that in this case $w = z$. This matters for proving constrained efficiency but is unimportant in the calibrated model, in which only a small fraction of markets fall into this category.

2. Applied to the specific game here, efficiency requires that the sum of the payoffs to all workers and jobs in a local labor market be equal to the total value of output, $p \min\{i, j\} + z(i - \min\{i, j\})$, or equivalently that (6) hold. Symmetry requires that all workers receive equal payoffs, and similarly for all jobs. The remaining two axioms do not apply directly to this game in isolation, but are useful in constructing the value by considering the space of all games.

3. To see why (7) characterizes the Shapley value, choose a specific job and divide the set of orderings of the i workers and j jobs into three groups, according to whether the last agent added to the grand coalition is this job, some other job, or a worker. In fraction $1/(i + j)$ of orderings, the specific job is added last, and generates a marginal surplus of 1 precisely when $i \geq j$. In fraction $i/(i + j)$ of orderings, a worker is added last; this does not affect the expected marginal surplus generated by this job (which comes earlier in the ordering), so the expected marginal surplus share in this case is the same as when there are $i - 1$ workers and j jobs—that is, it equals $\phi^s(i - 1, j)$. Similarly, in fraction $(j - 1)/(i + j)$ of orderings, a different job is added last, delivering expected marginal surplus to the original job of $\phi^s(i, j - 1)$.

4. A worker's share of the surplus, $\psi^s(i, j)$, can be calculated as a residual using (6). Alternatively, it is clear by symmetry that $\psi^s(i, j) = \phi^s(j, i)$.

5. To see that $\phi^s(i, j)$ is increasing in i, imagine the effect of adding a single additional worker or job to a local labor market. Take any ordering of the i workers and j jobs. Adding an additional worker either has no effect on the marginal surplus generated by a particular job, or strictly increases it; it must therefore strictly increase the Shapley value.

6. Using these properties, it is straightforward to verify that $\bar{v}_p(N)$ satisfies the properties assumed in Section 2.

7. There are several noncooperative microfoundations that deliver the Shapley value payoffs in expectation [Hart and Moore (1990); Evans (1996); Hart and Mas-Colell (1996)]. One possible implementation of the value is to have all workers receive an ex post payment equal to their value, independent of which workers are matched in production. This is reminiscent of the employment lotteries of Rogerson (1988), although because all agents are risk-neutral, the use of lotteries does not affect welfare here; it merely affects how payments are distributed across states of the world.

8. Delivering each worker and each job its respective Shapley value via wages paid only to employed workers is not problematic because the Shapley value is a useful solution concept only under risk-neutrality. In any case, the distribution of payments across workers is irrelevant for the aggregate economy, because all agents are risk-neutral and all that matters for job entry is the expected profit from creating a job.

9. It is interesting that the wage $w_p(i, j)$ is numerically almost identical to the χ-value of an employed worker [Casajus (2009)]. The χ-value satisfies axioms related to those satisfied by the Shapley value (so that agents on the short side of the local labor market will receive a higher payoff) but explicitly respects the matching structure (in particular, the sum of the payoffs of a matched worker–job pair equals their output, p).

10. Note, however, that the worker's share of the surplus is not in general equal to β, and in particular depends on i and j. See Section 4, and in particular Figure 2. Also note that the weighted Shapley value satisfies the properties assumed in Section 2. The proof is similar to that for the Shapley value.

11. More precisely, the R^2 of a regression of $\phi^x(i, j)$ on a full set of dummies for $d = i - j$ is very close to 1. For example (using the notation $\hat{\phi}^x(\cdot)$ introduced later), if $M = 244.2$ and $N = 236.3$, then

$$[R^2]^x = 1 - \frac{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[\phi^x(i, j; M, N) - \hat{\phi}^x(i - j; M, N)\right]^2 j\pi(i, j; M, N)}{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[\phi^x(i, j; M, N) - \bar{\phi}^x(M, N)\right]^2 j\pi(i, j; M, N)}$$

is equal to (to four decimal places) 1.0000 (competitive wages), 0.9999 (Shapley value), 0.9980 (weighted Shapley value, $\beta = 0.55$), 0.9969 (weighted Shapley value, $\beta = 0.60$), or 0.9968 (weighted Shapley value, $\beta = 0.65$).

12. More precisely, when M and N are large, the Poisson distributions of i and j are approximately normal, so that the cdf of $\hat{\pi}(\cdot; N)$ is well approximated by that of a normal random variable with mean $M - N - 1$ and standard deviation $(M + N)^{1/2}$. Thus there is also a small increase in the dispersion of the distribution of d when N increases.

13. In both cases, this result relies on the fact that the transition dynamics of the model are very rapid.

14. In an earlier version of this note [Hawkins (2011)], I simulated the dynamic equilibrium of the model when productivity p follows a first-order autoregressive process calibrated to match U.S. data, as described in Shimer (2007). The implications for the volatilities of unemployment and vacancies, relative to that of productivity p, are almost identical to the comparative statics reported in the text. It is worth noting that if one has determined the values of $\bar{v}_p(N)$ for all p and N, the dynamic model is solved in the same way as in Shimer (2007), no matter what the wage determination protocol. Thus, the same techniques as used in Shimer's paper apply, and the model remains just as tractable.

15. These can be calculated by numerically differentiating (1) and (2).

16. Table 2 shows results for the case $z/p = 0.4$, as in Shimer (2005). Increasing z would increase all the reported elasticities, as already discussed.

17. For example, Wiese (2007) and Casajus (2009) propose models that respect the ex post matching structure.

18. See McLean (2002) for a survey.

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