Macroeconomic Dynamics, **21**, 2017, 362–383. Printed in the United States of America.
doi:10.1017/S136510051500053X

AGING, THE GREAT MODERATION, AND BUSINESS-CYCLE VOLATILITY IN A LIFE-CYCLE MODEL

BURKHARD HEER
University of Augsburg
and
CESifo

STEFAN ROHRBACHER AND CHRISTIAN SCHARRER
University of Augsburg

According to empirical studies, the life cycle of labor supply volatility exhibits a U-shaped pattern. This may lead to the conclusion that demographic change induces a drop in output volatility. We present an overlapping-generations model that replicates the empirically observed pattern and study the impact of demographic transition on output volatility. We find that the change in age composition itself has only a marginal influence on output volatility, as the mitigating effect of more individuals with lower labor supply volatilities is compensated for by higher age-specific labor shares. Instead, the driving force behind the Great Moderation in our model is the downward shift of the age-specific labor supply volatility curve.

Keywords: Business Cycles, Overlapping Generations, Demographics

1. INTRODUCTION

The Great Moderation describes the decline in the volatility of aggregate economic activity in many industrialized countries after the mid-1980s. The explanations for this phenomenon are manifold. Clarida et al. (2000) attribute the reduction in aggregate volatility to a more effective monetary policy, whereas the good luck hypothesis proposed by Stock and Watson (2003) emphasizes the contribution of a reduction in the variance of business cycle shocks. Other studies, for example Dynan et al. (2006) or Davis and Kahn (2008), identify changes in inventory behavior or financial innovations as possible causes. Furthermore, Jaimovich and Siu (2009) provide empirical evidence in a cohort-based panel of the G7 countries that a demographic transition is closely linked to the volatility of cyclical output.

We would like to thank two anonymous reviewers for their helpful and constructive comments, which greatly contributed to improving the final version of the paper. Moreover, we would also like to thank Johann Scharler (University of Innsbruck) for his extensive discussions with us and for his helpful comments. Stefan Rohrbacher and Christian Scharrer thank the Research Center Global Business Management at the University of Augsburg for financial support. All remaining errors are ours. Address correspondence to: Burkhard Heer, Department of Economics, University of Augsburg, Universitatsstrasse 16, 86159 Augsburg, Germany; e-mail: burkhard.heer@wiwi.uni-augsburg.de.

© 2016 Cambridge University Press 1365-1005/16

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press

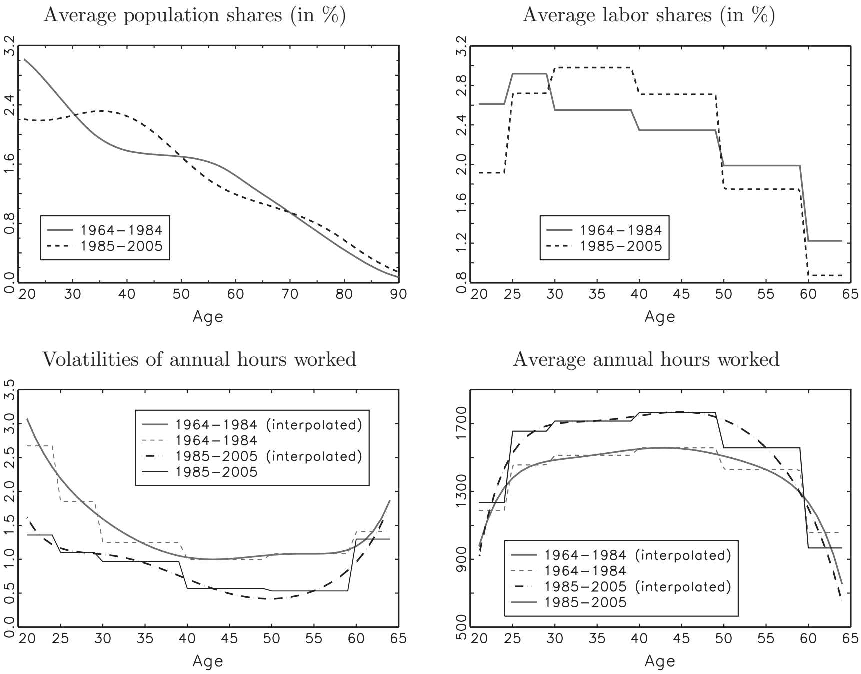

FIGURE 1. Labor market characteristics. We use a Hodrick–Prescott filter with a smoothing parameter of 6.25 and calculate the standard deviations of age-specific hours worked. *Data Sources*: United Nations (2002) and Jaimovich and Siu (2009).

They find, in particular, that demographic change is able to explain one-third of the reduction in output volatility in the United States between 1978 and 1999. Their results are supported in works by Lugauer (2012b) and Lugauer and Redmond (2012), which also point out that age distribution constitutes an important explanatory factor in business cycle analysis.

The driving forces behind these observations are depicted in Figure 1 and can be outlined as follows: On one hand, higher life expectancy and declining birth rates shift the composition of the labor force away from the young and into prime age groups. Accordingly, Figure 1 reveals that the average population and labor shares of workers aged 30–50 increased and the shares of the other workers decreased between 1964–1984 and 1985–2005. Because the cyclical volatility of hours worked depends on age and follows a U-shaped pattern, as illustrated in the bottom-left panel,[1] this demographic transition increases the number of older workers with a lower volatility of labor supply. This development induces attenuating effects on the volatility of aggregate labor and output. We label this linkage as the pure demographic effect. On the other hand, in addition to this effect, the labor supply volatilities by age shifted downward and the average annual hours worked per person shifted upward with respect to the samples before and during

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press

the Great Moderation. We call this the shift effect, which is displayed in the bottom row of Figure 1. Moreover, Jaimovich and Siu (2009) mention on p. 812 that it is not possible to measure the relative contributions of the two effects to the volatility of output in their regressions. Therefore, we extend their analysis by developing a model that is able to measure and distinguish the relative contributions of the two effects to the volatility of output.

In sum, the Great Moderation was accompanied by a pure demographic effect and a shift of both labor supply and labor supply volatilities. In order to study how much each effect contributes to the decline in aggregate output volatility, we investigate two questions: 1. What would have been the output volatility if the labor volatility had remained on the level it was at before the Great Moderation? In this setting, output volatility is due solely to pure demographic effects. 2. How do our results differ if we also allow a downward shift of the cyclical volatility curve and an upward shift of labor supply?

We study these questions in a dynamic stochastic overlapping-generations (OLG) model in the spirit of Ríos-Rull (1996), which is able to map demographic changes. In contrast to standard real business cycle models with infinitely lived agents, the OLG framework allows us to take into account the interplay between demographic variables and cohort-specific decisions over the life cycle with respect to labor supply and wealth accumulation. In addition, we use the preferences proposed by Greenwood et al. (1988) (GHH).[2] This approach enables us to replicate the empirical profiles of labor supply by age and cyclical volatilities quantitatively by calibrating the parameters that affect individual labor supply decisions.[3]

As our main result, we show that the pure demographic effect plays only a marginal role in explaining the empirically observed drop in output volatility. The hump-shaped pattern of labor supply during the life cycle (see Figure 1) and the relative increase in the mass of older cohorts attenuate the decline in output volatility caused by more individuals with a lower cyclical volatility of labor supply. In contrast, a downward shift of labor supply volatilities plays a crucial role in explaining the decrease in output volatility.

Our work is related to other studies that analyze the impact of aging on business cycle volatility with overlapping-generation models and productivity shocks. Ríos-Rull (1996) compares aggregate fluctuations between models with infinitely lived agents and life-cycle models, whereas Gomme et al. (2005) focus on the impact of aging on business cycle fluctuations in hours worked. However, neither study analyzes the demographic transition and its ability to explain the Great Moderation. In this vein, Lugauer (2012a) introduces matching frictions into the labor market in order to explain how the demographic transition causes the drop in output volatility. However, his analysis focuses entirely on the labor market and, in contrast to our study, excludes individual life-cycle decisions regarding consumption and the accumulation of wealth. To explain the more volatile labor supply of the young over the business cycle, Jaimovich et al. (2013) introduce capital–experience complementarity in production. Older workers are more experienced and are

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press

complementary to capital, whereas young workers are not. As opposed to our model, Jamovich et al. distinguish only between young and old workers and ignore labor force composition effects that result from changing population weights.

Our paper is structured as follows. Section 2 describes and explains the benchmark model. In Section 3, we conduct a calibration exercise with respect to the pure demographic and the shift effect. In Section 4, we summarize the main findings of the paper, analyzing the behavior of individuals, aggregate variables, and business cycle volatility separately. Section 5 concludes. The results of the sensitivity analysis with respect to a pay-as-you-go-system and age-specific productivity profiles are provided in the Appendix.

2. A 70-PERIOD OVERLAPPING-GENERATIONS MODEL WITH AGGREGATE UNCERTAINTY AND ACCIDENTAL BEQUESTS

In the following, we describe a simple overlapping-generations model that is able to map demographic changes. The model is built upon Ríos-Rull (1996) and consists of households optimizing intertemporal utility, profit-maximizing firms, and a government sector.

2.1. Demographics

Every year, a new cohort of equal size enters the economy at age $s = 1$ (equivalent to a real life age of 21). Households live a maximum of 70 years and survive with an age-specific probability ϕ_s from age s to age $s+1$. Put differently, the parameter $(1-\phi_s)$ denotes the individual probability of dying at the end of age s. In addition, we follow Jaimovich et al. (2013) in order to match the average values of the s-year-old population shares.[4] We use the empirical population masses of s-year-old agents ψ_s and normalize the total mass to one so that the terms ψ_s also describe the age-specific population shares. Moreover, the variables ψ_s include additional effects such as migration that cause variations in the population structure besides changing birth rates and individual survival probabilities. Therefore, we introduce the variable $\Phi_s = \psi_{s+1}/\psi_s$, which can be interpreted as an aggregate survival probability for cohort s.

2.2. Households

In the first 44 periods, the households are working; in the last 26 periods, they are retired. Households maximize their expected lifetime utility at age $s = 1$ in period t with respect to consumption c_t^s and labor supply n_t^s:

$$E_t \sum_{s=1}^{70} \beta^{s-1} \left(\prod_{j=1}^{s} \phi_{j-1} \right) \frac{1}{1-\eta} \left\{ \left[c_{t+s-1}^s - \gamma_0^s \left(n_{t+s-1}^s \right)^{\gamma^s} \right]^{1-\eta} - 1 \right\}.$$

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press

Instantaneous utility is a function of both consumption c_t^s and labor n_t^s with GHH preferences.[5] This kind of utility function has a property by which it eliminates wealth effects regarding the choice of optimal labor supply.[6] Because labor supply decisions do not depend on consumption, the empirical patterns of labor supply and corresponding labor supply elasticities can be perfectly matched. The age-dependent constant γ^s controls the Frisch elasticity of labor supply, which is given by $1/(1-\gamma^s)$. The parameter γ_0^s pins down the steady state labor supply profiles across cohorts.

Households accumulate savings in the form of capital. Let k_t^s denote the capital stock of the s-year old in period t. The initial endowment of capital is zero, $k_t^1 = 0$. The working agent of age s receives income from labor, capital, and lump-sum transfers from the government, tr_t, which are unintended bequests from individuals who did not survive from the previous period. He faces the following budget constraint in period t:

$$k_{t+1}^{s+1} = (1 + r_t - \delta)k_t^s + w_t n_t^s + \mathrm{tr}_t - c_t^s, \quad s = 1, \ldots, 44, \tag{1}$$

where w_t and r_t denote the real wage rate and the interest rate, respectively. Capital depreciates at a rate δ. The budget constraint of the retired worker is given by

$$k_{t+1}^{s+1} = (1 + r_t - \delta)k_t^s + \mathrm{tr}_t - c_t^s, \quad s = 45, \ldots, 70, \tag{2}$$

with $k_t^{71} = 0$ and $n_t^s = 0$ for $s > 44$.[7] The relevant first-order conditions are given by

$$w_t = \gamma_0^s \gamma^s \left(n_t^s\right)^{\gamma^s - 1}, \tag{3}$$

$$\lambda_t^s = \left[c_t^s - \gamma_0^s \left(n_t^s\right)^{\gamma^s}\right]^{-\eta}, \tag{4}$$

$$\frac{1}{\beta} = E_t \left[\frac{\lambda_{t+1}^{s+1}}{\lambda_t^s} \phi_s \left(1 + r_{t+1} - \delta\right)\right], \tag{5}$$

and the budget constraints (1) and (2). The parameter λ_t^s denotes the Lagrange multiplier.

2.3. Production

Production Y_t is characterized by a constant-returns-to-scale production function and is assumed to be Cobb–Douglas:

$$Y_t = Z_t N_t^{1-\alpha} K_t^{\alpha}, \tag{6}$$

where N_t and K_t denote aggregate labor and capital. The technology level Z_t is subject to stochastic shocks in that $\ln Z_t$ follows an AR(1) process: $\ln Z_t = \rho \ln Z_{t-1} + \epsilon_t$, where ϵ_t is i.i.d., $\epsilon_t \sim N(0, \sigma^2)$.

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press

2.4. Equilibrium

In a factor market equilibrium, it must hold that all factors are rewarded with their marginal product:

$$w_t = (1-\alpha) Z_t N_t^{-\alpha} K_t^{\alpha}, \tag{7}$$

$$r_t = \alpha Z_t N_t^{1-\alpha} K_t^{\alpha-1}.$$

Furthermore, individual and aggregate behavior must be consistent:

$$N_t = \sum_{s=1}^{44} \psi_s n_t^s, \tag{8a}$$

$$K_t = \sum_{s=1}^{70} \psi_{s-1} k_t^s, \tag{8b}$$

$$C_t = \sum_{s=1}^{70} \psi_s c_t^s, \tag{8c}$$

and the goods market clears:

$$Z_t N_t^{1-\alpha} K_t^{\alpha} = C_t + I_t,$$

where $I_t = K_{t+1} - (1-\delta) K_t$. In addition, all accidental bequests are confiscated by the government and transferred as lump sums to the households, implying that

$$\mathrm{tr}_t = \sum_{s=1}^{70} (1 - \Phi_{s-1}) \psi_{s-1} \left[(1 + r_t - \delta) k_t^s \right]. \tag{9}$$

3. CALIBRATION

We calibrate the model on an annual basis and compute the deterministic steady state using the methods described in Chapters 9 and 10 in Heer and Maussner (2009).[8] Furthermore, we also have to conduct time series simulations to calibrate of labor supply decisions. A detailed explanation of the aforementioned procedure is provided in the Appendix. With regard to the demographic characteristics we distinguish three cases:

Case 1: This is our benchmark case, which describes the 1964–1984 sample. We calibrate the age-dependent labor supply γ_0^s and labor supply elasticities γ^s in order to reproduce the interpolated empirical profiles of average labor supply and cyclical volatility of hours worked so that they fit the solid lines in the bottom graphs of Figure 1. The population shares ψ_s and survival probabilities of a household ϕ_s are set equal to the average population shares and survival probabilities between 1964 and 1984, respectively.

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press

Case 2: Here we use the same values for γ_0^s and γ^s as in Case 1. However, the average population shares and survival probabilities stem from the 1985–2005 sample. Thus, this thought experiment allows us to measure the impacts of the pure demographic effect on the volatilities of aggregate variables by ruling out the shift effect.

Case 3: In this case, we calibrate all parameters with respect to the 1985–2005 sample and measure the overall effect. By comparison with Case 2, this approach enables us to measure how a downward shift of cyclical volatilities and an upward shift of labor supply additionally affect the volatilities of aggregate variables.

The remaining parameters are standard in the RBC/DSGE literature and have been chosen as follows: $\eta = 1.0$, $\delta = 0.10$, and $\alpha = 0.30$. Moreover, we set the parameter $\beta = 0.974$ to match a real interest rate of 6% in our benchmark case. The parameters of the AR(1) for the technology are set equal to $\rho = 0.814$ and $\sigma = 0.0142$. These parameters correspond to annual frequencies by a quarterly AR(1) process for the Solow residual with parameters 0.95 and 0.00763, which are the parameters in Prescott (1986). The nonstochastic steady states are characterized by a constant technology level, $Z_t = Z = 1$. In the following, we express stationary variables without a time index. For example, k^s and K denote the nonstochastic steady state capital stock of an individual at age s and the nonstochastic steady state aggregate capital stock, respectively.

4. RESULTS

In this section, we simulate our model for the periods 1964–1984 and 1985–2005 separately and study the impacts of the pure demographic effect and the shift effect on the model economy described earlier. First, we focus on changes in the individual behavior of households in the steady state that are caused by these effects. Second, we also analyze the relation between individual behavior and aggregate variables to show that the demographic effect has only a negligible impact on aggregate capital and labor supply, whereas the shift effect impacts both aggregate capital and aggregate labor significantly. Third, we consider the volatilities of aggregate variables as measured by their standard deviations (of the filtered series) and demonstrate that the shift effect of the labor supply is the main source for the change in the volatility of output between 1964–1984 and 1985–2005.

4.1. Steady State

The behavior of individual age-specific variables in the stationary equilibrium of our model is depicted in Figure 2 for each of Cases 1–3. Figure 3 displays the effects of individual behavior on aggregate variables by taking the age composition explicitly into account. In both figures, the solid line depicts Case 1 (1964–84), the dotted line describes Case 2 (the pure demographic effect in period 1985–2005),

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press

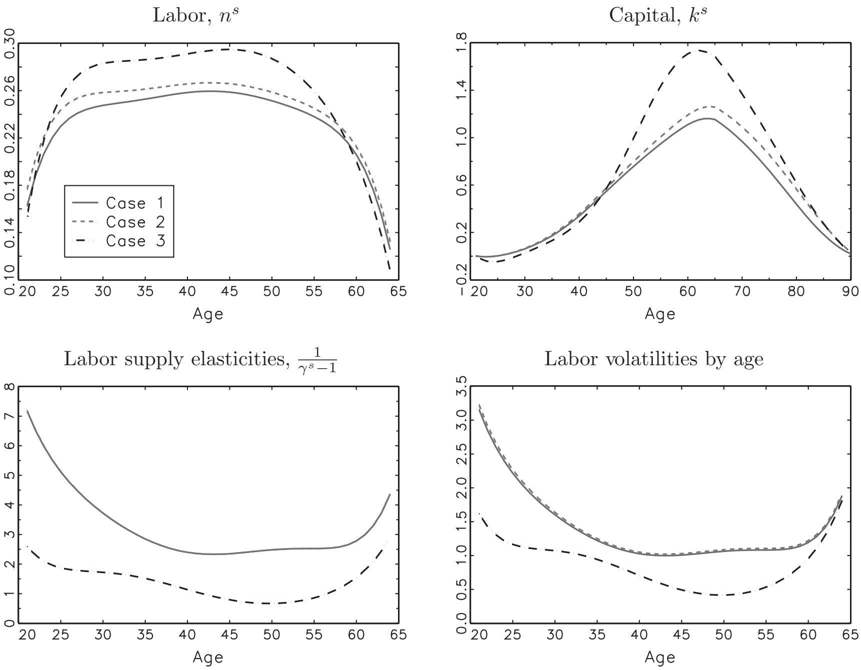

FIGURE 2. Steady state: Individual behavior.

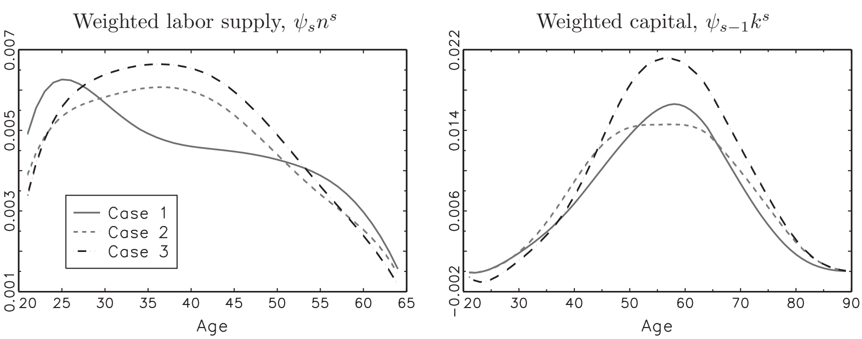

FIGURE 3. Steady state: Aggregate behavior.

and the dashed line exhibits Case 3 (with both the demographic and the shift effect in 1985–2005).

The panels in the first row of Figure 2 present the profiles of labor supply and wealth accumulation during the life cycle. The second row illustrates the labor supply elasticities and the corresponding cyclical volatilities, where the ordinate always denotes the age in years. The labor supply and cyclical volatilities across cohorts in Cases 1 and 3 are equal to the empirical profiles shown in the bottom

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press

row of Figure 1. This is a direct result of our calibration strategy. Furthermore, the profiles of capital accumulation are hump-shaped and reach a peak around age 64, shortly before a household enters retirement. Comparing our benchmark case with Case 2, we can see that a change in the population structure and a higher individual life expectancy strengthen the motive for consumption smoothing so that households increase their labor supply and build up slightly more wealth after the age of 35 years. In Case 3, the higher labor supply compared to Case 1 and Case 2 is able to free up considerably more resources, which is why the wealth accumulation over the life cycle also increases significantly.

Furthermore, the graph at the bottom left corner of Figure 2 displays the Frisch labor elasticities by age group, which measure the percentage change in hours worked due to a percentage change in real wages. On one hand, these elasticities are higher than those usually predicted by microeconomic estimates, which typically fall in the range 0–1 [see, e.g., Kimball and Shapiro (2008) and Keane (2011) for a survey]. However, on the other hand, Keane and Rogerson (2012) argue that higher Frisch labor elasticities in macro-based models are consistent with those in the micro labor supply literature. In contrast to micro Frisch elasticities associated with fluctuations of hours of employed workers, macro Frisch elasticities also include workers entering and leaving the labor market. To put it another way, these elasticities are related to changes in the hours worked along both the intensive and extensive margins. Taking these effects into account, Peterman (2012) estimates macro Frisch elasticities for the U.S. economy between 2.9 and 3.1. Because we also intend to incorporate movements on both margins, the higher values of our elasticities—e.g., in the range of 1 to 3 for agents aged 35–60—are a direct result of our calibration strategy in terms of a perfect matching of empirical cyclical volatilities of labor supply by age.

To determine how individual behavior impacts aggregate variables in the stationary equilibrium, we also have to take the demographic variables explicitly into account. Therefore, we decompose the contribution of each cohort on aggregate labor and aggregate capital with respect to equations (8a) and (8b). Figure 3 displays the weighted labor supply $\psi_s n^s$ and the weighted capital stock $\psi_{s-1} k^s$ of each cohort, where the areas under the curves are equal to the aggregate labor supply and the aggregate capital stock, respectively. Table 1 summarizes the values of the aggregate variables in our model for Cases 1–3.

Comparing Case 1 with Case 2 in Figure 3, it becomes apparent that the weighted labor supply of workers in the age groups 21–29 and 51–64 decreases whereas the contribution of middle-aged workers to aggregate labor supply increases, mainly because of the age composition effect of changing average population shares ψ_s depicted in Figure 1. The overall effect leads only to a moderate increase of aggregate labor supply from 0.199 (Case 1) to 0.206 (Case 2). Moreover, the right panel in Figure 3 reveals that the pure demographic effect decreases the weighted capital of workers in the age group 53–64 and increases the weighted capital of households in the age groups 31–52 and 65–90. Here, the last mentioned effect is somewhat more pronounced, which is why the aggregate stock of capital

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press

TABLE 1. Steady state: Aggregate variables

	N	K	K/N	I/Y	$r-\delta$	w
Case 1	0.199	0.487	2.454	0.187	6.0%	0.916
Case 2	0.206	0.524	2.550	0.193	5.6%	0.927
Case 3	0.219	0.625	2.851	0.208	4.4%	0.958

Notes: Case 1 depicts the period 1964–1984, Case 2 factors in the pure demographic effect in the period 1985–2005, and Case 3 considers both the demographic and the shift effect in the period 1985–2005.

increases from 0.487 (Case 1) to 0.524 (Case 2). We deduce from these results that the moderate increase in age-specific labor supply n^s and wealth accumulation k^s over the life cycle in Case 2 (see also Figure 2) is almost fully compensated for by the corresponding demographic transition. In contrast, the increase in age-specific labor supply and wealth accumulation over the life cycle is much more pronounced in Case 3 and outweighs the attenuating age composition effect on weighted labor supply and weighted capital. The aggregate labor supply and the aggregate stock of capital increase to 0.219 and 0.625, respectively. Furthermore, the capital–labor ratio increases from Case 1 to 3. Similarly, the net return to capital decreases from 6.0% (Case 1) to 5.6% (Case 2) and 4.4% (Case 3),[9] whereas real wages increase. Finally, the investment–output ratio increases from 0.187 in Case 1 to 0.193 in Case 2, whereas the increase from Case 1 to 3 is larger (0.208).

4.2. Business-Cycle Volatility

In this section, we study how each of the pure demographic effect and the shift effect help to explain the Great Moderation. Table 2 displays the absolute standard deviations of log transformed output Y, labor supply N, investment I, capital K, consumption C, and wage w for each simulated case. Furthermore, the empirical counterparts for 1964–1984 (Case 1) and 1985–2005 (Case 3) are depicted in parentheses. Evidently, we are able to produce standard characteristics of business cycle volatilities that match the data in our benchmark Case 1 closely. The contribution of the pure demographic effect to the volatilities of all aggregate variables in Case 2 is rather moderate. For example, the standard deviation of output decreases by only 0.020 from Case 1 to Case 2 (compare the first entries in rows 1 and 3 of Table 2). The decline in the volatilities of investment and capital is slightly more pronounced. Only wage volatility rises by a tiny fraction. These changes, however, are much more pronounced after the shift effect in Case 3 is taken into account. In particular, the volatility of aggregate output declines by 0.437, amounting to a change of 23%, compared to Case 1. Accordingly, the shift effect plays a crucial role in explaining the drop of output volatility in the Great Moderation in our model.

To understand the underlying mechanisms linking output volatility to the pure demographic and the shift effect, we first focus on the impulse responses of

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press

TABLE 2. Absolute standard deviations: Aggregate variables

	Y	N	I	K	C	w
Case 1	1.895	1.458	4.110	0.420	1.399	0.427
	(1.782)	(1.892)	(4.230)	(0.417)	(1.303)	(0.392)
Case 2	1.876	1.429	4.017	0.411	1.379	0.437
Case 3	1.458	0.832	3.519	0.360	0.932	0.621
	(0.856)	(1.384)	(3.364)	(0.318)*	(0.783)	(1.092)

Notes: Simulated time series and their empirical counterparts (in parentheses with respect to Cases 1 and 3) with a length of 21 periods were HP-filtered with weight 6.25. All variables are expressed in real terms. The second moments in our model are averages over 500,000 simulations.
*Sample: 1985–2002.

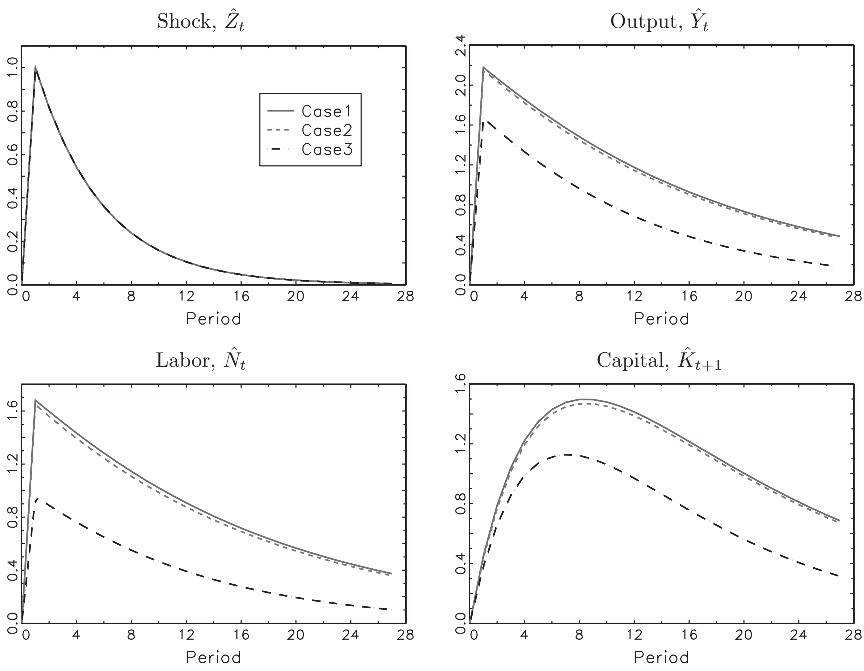

FIGURE 4. Impulse respones (in %)—aggregate variables.

aggregate variables to a productivity shock. This approach allows us to work out the effects of shocks on the volatilities in our simulated time series. Because the log-linearized version for output in equation (6) is given by

$$\hat{Y}_t = \hat{Z}_t + (1-\alpha)\hat{N}_t + \alpha \hat{K}_t, \tag{10}$$

the dynamics of output depend on output productivity $\hat{Z}_t$, aggregate labor $\hat{N}_t$, and aggregate capital $\hat{K}_t$.[10] In Figure 4, we analyze the effects of a percentage increase

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press

in output productivity from the stationary level on these variables and compare the effects for every case. In doing so, we assume that the economy is hit by a productivity shock in period 1.[11] In accordance with our results on the cyclical volatility of the model variables presented in Table 2, the impulse responses vary little between Cases 1 and 2. As suggested by the changes in volatilities, the amplitude of the impulse responses in Case 2 is also marginally lower for all variables. Therefore, the pure demographic effect has only a weak impact on the dynamic impulse response of output because the responses of labor $\hat{N}_t$ and next-period capital stock $\hat{K}_{t+1}$ remain almost unaffected. In contrast, the responses of labor and capital and, hence, the responses of output to a productivity shock are reduced significantly if we take the shift effect (Case 3) into account.

The economic intuition behind the differences in impulse responses and volatilities of aggregate variables can be derived by determining how individual behavior impacts the dynamics of aggregate variables. If we combine the log-linearized versions of (3) and (8a), we get

$$\hat{N}_t = \varphi\, \hat{w}_t, \tag{11}$$

where

$$\varphi = \sum_{s=1}^{44} \varphi_s \quad \text{and} \quad \varphi_s = \frac{\psi_s n^s}{N} \frac{1}{\gamma^s - 1}. \tag{12}$$

The term φ represents the aggregate labor supply elasticity with respect to the real wage. This elasticity in turn depends on the variable φ_s, which we call, for ease of reference, the age-specific elasticity of aggregate labor supply.

Log linearization of the firm's first-order condition with respect to labor supply, (7), results in

$$\hat{w}_t = \hat{Z}_t - \alpha \hat{N}_t + \alpha \hat{K}_t. \tag{13}$$

After inserting equation (13) into equation (11) and rearranging it with respect to aggregate labor supply $\hat{N}_t$, we obtain

$$\hat{N}_t = \frac{\varphi}{1 + \alpha\varphi} \left(\hat{Z}_t + \alpha \hat{K}_t\right). \tag{14}$$

In addition, if we plug this expression into equation (10), we get

$$\hat{Y}_t = \frac{1 + \varphi}{1 + \alpha\varphi} \left(\hat{Z}_t + \alpha \hat{K}_t\right). \tag{15}$$

Equations (14) and (15) state that the dynamic behavior of aggregate labor $\hat{N}_t$ and output $\hat{Y}_t$ depends not only on the case-specific aggregate labor supply elasticity φ but also on the technology shock $\hat{Z}_t$ and the corresponding formation of aggregate capital $\hat{K}_t$. The technology shock follows the same stochastic process in all of Cases 1–3; hence differences occur only through variations in φ and $\hat{K}_t$. Both aggregate labor supply and output increase with φ and $\hat{K}_t$. In the following, we investigate the impacts of both effects on φ and $\hat{K}_t$ in turn.

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press

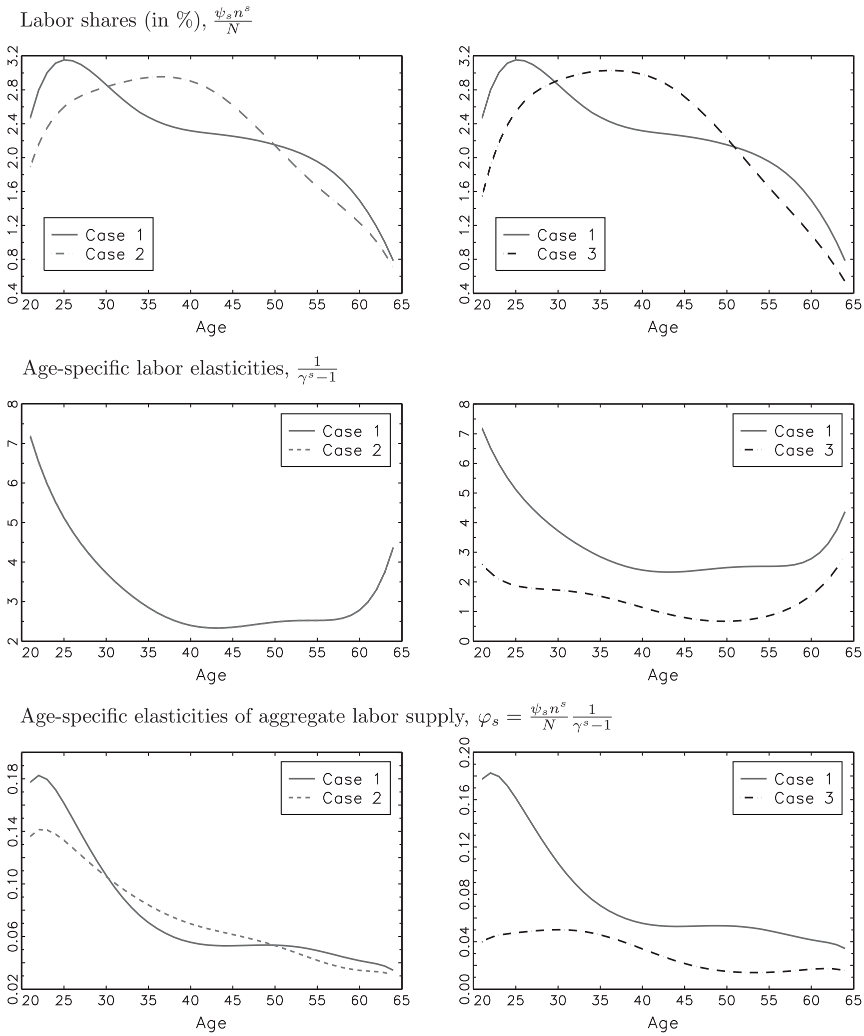

FIGURE 5. Aggregate labor supply elasticity.

First, we analyze the effects of Cases 2 and 3 on the aggregate labor supply elasticity φ defined in equation (12). In Figure 5, we display the labor shares $\frac{\psi_s n^s}{N}$, which are in accordance with the labor shares displayed in Figure 1, the age-specific labor elasticities $\frac{1}{\gamma^s - 1}$, and the age-specific elasticities of aggregate labor supply φ_s across cohorts. The elasticities of aggregate labor supply φ with respect to Cases 1 to 3 are equal to the area under the corresponding curves of age-specific elasticities of aggregate labor supply φ_s. The ordinates denote age in years.

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press

The left column in Figure 5 compares Cases 1 and 2. On one hand, the demographic transition in Case 2 decreases the labor shares of workers in the 20–30 and 50–64 age groups, whereas the labor shares of the other workers increase. Furthermore, the increase in labor supply n^s of 3.8% on the average across cohorts (as illustrated in Figure 2) is almost equal to the increase of 3.4% in aggregate labor supply N. Hence, the terms n^s/N in equation (12) are almost identical in Cases 1 and 2. Therefore, the age-specific elasticity of aggregate labor supply φ_s in the bottom left panel only varies because of changes in population shares ψ_s across cohorts (remember that γ^s remains constant in Case 2). Because the individual changes of ψ_s almost cancel each other out, the areas under the curves of age-specific elasticities of aggregate labor supply φ_s decrease only slightly from 3.38 (Case 1) to 3.26 (Case 2), so that the elasticity of aggregate labor supply remains almost unaffected.

In the right column of Figure 5, we analyze the contribution of the shift effect. The labor shares are almost identical in Cases 2 and 3, so that their impacts on the aggregate elasticity of labor supply remain the same. However, the downward shift of age-specific labor supply elasticities across age groups displayed in the center right panel prevails, so that the elasticity of aggregate labor supply φ declines to 1.34 (bottom right panel). The decline of φ_s and, hence, the decrease of the aggregate elasticity of labor supply reduce the volatility of aggregate labor supply and output, respectively.

Next, we discuss the variations of aggregate capital formation. In general, it is not possible to derive an analytic solution in our model with respect to the dynamics of capital. However, if we log-linearize equation (8b) and shift the time index t and the age index s one period forward, we are able to compute numerically how individual behavior and demographic variables impact the dynamics of aggregate capital $\hat{K}_{t+1}$:

$$\hat{K}_{t+1} = \sum_{s=1}^{69} \psi_s \tilde{k}_{t+1}^{s+1}, \tag{16}$$

where the term $\tilde{k}_{t+1}^{s+1}$ denotes the log deviation of age-specific capital from the average per capita stock of capital in the steady state of our model:[12]

$$\tilde{k}_{t+1}^{s+1} = \frac{dk_{t+1}^{s+1}}{\bar{k}}, \quad \text{with} \quad \bar{k} = \frac{\sum_{s=1}^{69} \psi_s k^{s+1}}{\sum_{s=1}^{70} \psi_s} = K. \tag{17}$$

In Figure 6, the panels in the top row plot the case-specific individual impulse responses of capital $\tilde{k}_{t+1}^{s+1}$, whereas the panels in the bottom row display the weighted impulse responses of individual capital $\psi_s \tilde{k}_{t+1}^{s+1}$ in period 1 (after a technology shock of 1% in period 0) across cohorts. Thus, the areas under the curves of weighted individual capital equal the corresponding impulse responses of aggregate capital $\hat{K}_{t+1}$ from Figure 4 in the first period.[13]

The impulse responses of individual capital by age groups are hump-shaped and feature a kink after households enter retirement. On one hand, these impulse

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press

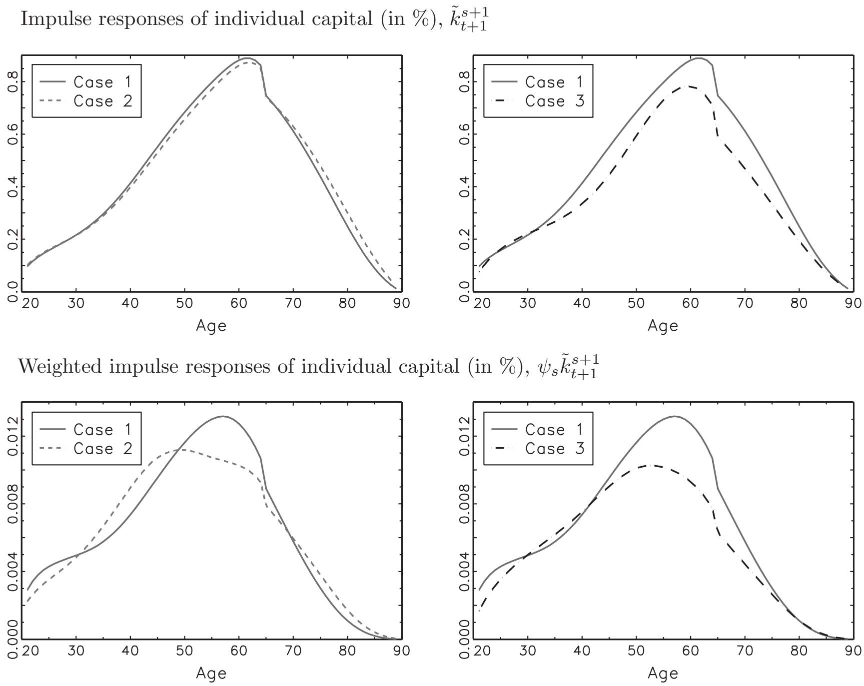

FIGURE 6. Weighted capital responses, cross section in period 1.

responses are very similar in Cases 1 and 2. However, from the bottom left panel, we can see that the weighted impulse responses of individual capital change. The contribution of households in the age groups 21–30 and 49–69 decreases, whereas the contribution of other cohorts increases. Because impulse responses of individual capital $\tilde{k}_{t+1}^{s+1}$ remain almost unchanged, these alterations are due purely to changes of the population shares ψ_s. Again, these demographic composition effects cancel each other out. The impulse response of aggregate capital decreases from 0.45% (Case 1) to 0.44% (Case 2) so that the overall impact on aggregate capital is only marginal in Case 2. On the other hand, the impulse responses of individual capital in Case 3 decrease significantly among cohorts older than 32 years in comparison with Case 1. This downward shift amplifies the decline in weighted impulse responses of individual capital across cohorts so that the impulse response of aggregate capital decreases from 0.45% (Case 1) to 0.38% (Case 3). Thus, whereas the contribution of aggregate capital to the dynamics of aggregate output in equation (15) is rather marginal in Case 2, the formation of aggregate capital additionally dampens the dynamics of aggregate output and aggregate labor supply in Case 3.

In sum, the pure demographic effect increases the age-specific influence of certain cohorts on aggregate variables and decreases the influence of other cohorts.

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press

The two effects almost cancel each other out at the aggregate level. Furthermore, individual decisions regarding labor supply and capital accumulation are driven by changes in the real wage and the interest rate that depend in turn on aggregate variables. Thus, the compensating effect at the aggregate level also feeds back into the individual level. Households have almost no incentive to change their behavior in Case 2. In contrast, the shift effect in Case 3 also takes into account the downward shift of labor supply elasticities across cohorts. For this reason, both individual and aggregate labor supply respond to a lesser extent to productivity shocks and changes in aggregate capital. In addition, the smaller impulse responses of labor also dampen the accumulation of capital. Both the age-specific impulse responses of capital and labor supply shift downward and outweigh the compensating influence of the pure demographic effect in all cohorts. As a consequence, the impulse responses of aggregate labor and capital decline.

5. CONCLUSION

Recent work has analyzed the implications of demographic change for business cycle fluctuations. Declining birth rates and increasing life expectancies are shifting the composition of the labor force away from the young and into prime-age groups. This development should have an attenuating effect on the volatility of output and aggregate labor supply, because the volatility of hours worked is an empirically observed U-shaped function of age. However, during the Great Moderation, this pure demographic effect was accompanied by a downward shift of cyclical volatilities of labor supply across all age groups.

We present an overlapping-generations model that replicates the empirically observed age-specific volatilities of labor supply and explicitly takes changes in the age composition into account to study the impact of both effects on aggregate output volatility before and during the Great Moderation. Changes in age composition caused by an aging population compensate for the pure demographic effect as long as a demographic transition is not also exposed to a pronounced downward shift of volatilities of labor supply across cohorts. We find that in this case the volatility of output remains almost unaffected and decreases only marginally. In contrast, shifts of age-specific volatilities of labor supply play a crucial role in determining the strength of output volatility and are able to explain a reduction of the output volatility by 23%. According to our results, the decline in output volatility during the Great Moderation was primarily driven by lower labor supply elasticities across all age groups.

NOTES

1. See also Clark and Summers (1981), Ríos-Rull (1996), Gomme et al. (2005), and Jaimovich and Siu (2009).

2. In particular, our preferences feature constant Frisch elasticities. Such preferences are studied extensively by Trabandt und Uhlig (2011). In their general equilibrium model, the magnitude of the

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press

Frisch elasticity is most relevant for the estimation of the Laffer curve in the United States and the EU-14.

3. Preferences used in OLG models of comparable studies, as in Ríos-Rull (1996), Gomme et al. (2005), or Hansen and İmrohoroğlu (2009), imply that volatilites of age-specific hours worked rise again at around age 45, whereas, in the data, this increase prevails only at a higher age.

4. This approach also allows us to replicate the average labor shares provided by Jaimovich and Siu (2009). In a previous version of this paper [see Heer et al. (2014)], we used average birth rates and survival probabilities with very similar results.

5. Superscript s indicates the corresponding age cohort, whereas subscript t describes the current period.

6. We have experimented with other preferences, e.g., Cobb–Douglas preferences in consumption and leisure, but have found that GHH preferences allow us to calibrate age-specific labor supply behavior that matches the empirical values perfectly.

7. In Appendix A.3, we show that our results are insensitive to the introduction of pensions and age-specific productivities.

8. To compute the dynamics of the model, we also use a code for the generalized Schur decomposition provided by Giordani and Söderlind (2004).

9. The percentage point decline in the interest rate between the years 1964 and 2005 predicted by our model is almost exactly the same as the one predicted by Krueger and Ludwig (2007) for the change in the interest rate between the years 2005 and 2040. In particular, they compute a decline in the interest rate from 7.5% in the year 2005 to 6.6% in the year 2040. Contrary to our model, however, they analyze an open-economy model of the U.S. economy.

10. A circumflex over a variable denotes the log deviation from its corresponding steady state value.

11. In general, the impulse responses of aggregate variables are very similar to those predicted by standard models with infinitely lived households. The productivity shock leads to an immediate increase in output, labor supply, consumption, and investment. Capital grows steadily and reaches its highest magnitude after seven years in each case. Thereafter, the increase abates. Investment expenditures (not illustrated) show the strongest reaction.

12. We could also derive our results using percentage changes $\hat{k}_{t+1}^{s+1}$ instead of $\tilde{k}_{t+1}^{s+1}$. However, this approach complicates the graphical illustrations because individuals with age-specific capital stocks close to zero feature very large impulse responses.

13. We focus our discussion of the behavior of the individual capital stock in the first period. Our findings, however, carry over to later periods and also hold in our simulated time series for aggregate volatilities.

REFERENCES

Clarida, R., J. Gali, and M. Gertler (2000) Monetary policy rules and macroeconomic stability: Evidence and some theory. *Quarterly Journal of Economics* 115(1), 147–180.

Clark, K. and L. Summers (1981) Demographic differences in cyclical employment variation. *Journal of Human Resources* 16(1), 61–79.

Davis, S.J. and J.A. Kahn (2008) Interpreting the Great Moderation: Changes in the volatility of economic activity at the macro and micro levels. *Journal of Economic Perspectives* 22(4), 155–180.

Dynan, K.E., D.W. Elmendorf, and D.E. Sichel (2006) Can financial innovation help to explain the reduced volatility of economic activity? *Journal of Monetary Economics* 53(1), 123–150.

Giordani, P. and P. Söderlind (2004) Solution of macromodels with Hansen–Sargent robust policies: Some extensions. *Journal of Economic Dynamics and Control* 28(12), 2367–2397.

Gomme, P., R. Rogerson, P. Rupert, and R. Wright (2005) The business cycle and the life cycle. In M. Gertler and K. Rogoff (eds.), *National Bureau of Economic Research Macroeconomics Annual* 19, pp. 415–446. Cambridge, MA: MIT Press.

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press

Greenwood, J., Z. Hercovitz, and G.W. Huffman (1988) Investment, capacity utilization, and the real business cycle. *American Economic Review* 78(3), 402–417.

Hansen, G.D. (1993) The cyclical and secular behaviour of the labor input: Comparing efficiency units and hours worked. *Journal of Applied Econometrics* 8(1), 71–80.

Hansen, G. D. and S. İmrohoroğlu (2009) Business cycle fluctuations and the life cycle: How important is on-the-job-skill accumulation? *Journal of Economic Theory* 144(6), 2293–2309.

Heer, B. and A. Maussner (2009) *Dynamic General Equilibrium Models: Computational Methods and Applications*, 2nd ed. Berlin: Springer-Verlag.

Heer, B., S. Rohrbacher, and C. Scharrer (2014) Aging, the Great Moderation, and Business-Cycle Volatility in a Life-Cycle Model. CESifo working paper 4584.

Jaimovich, N., S. Pruitt, and H.E. Siu (2013) The demand for youth: Explaining age differences in the volatility of hours. *American Economic Review* 103(7), 3022–3044.

Jaimovich, N. and H.E. Siu (2009) The young, the old, and the restless: Demographics and business cycle volatility. *American Economic Review* 99(3), 804–826.

Keane, M. (2011) Labor supply and taxes: A survey. *Journal of Economic Literature* 49(4), 961–1075.

Keane, M. and R. Rogerson (2012) Micro and macro labor supply elasticities: A reassessment of conventional wisdom. *Journal of Economic Literature* 50(2), 464–476.

Kimball, M.S. and M.D. Shapiro (2008) Labor Supply: Are the Income and Substitution Effects Both Large or Both Small? Working paper 14208, National Bureau of Economic Research.

Krueger, D. and A. Ludwig (2007) On the consequences of demographic change for rates of returns to capital, and the distribution of wealth and welfare. *Journal of Monetary Economics* 54, 49–87.

Lugauer, S. (2012a) Demographic change and the great moderation in an overlapping generations model with matching frictions. *Macroeconomic Dynamics* 16(5), 706–731.

Lugauer, S. (2012b) Estimating the effect of the age distribution on cyclical output volatility across the United States. *Review of Economics and Statistics* 94(4), 896–902.

Lugauer, S. and M. Redmond (2012) The age distribution and business cycle volatility: International evidence. *Economics Letters* 117(3), 694–696.

Peterman, W. (2012) Reconciling Micro and Macro Estimates of the Frisch Labor Supply Elasticity. Finance and economics discussion series 2012-2075, Board of Governors of the Federal Reserve System (U.S.).

Prescott, E.C. (1986) Theory ahead of business cycle measurement. *Carnegie Rochester Series on Public Policy* 25, 11–44.

Ríos-Rull, J.-V. (1996) Life-cycle economies and aggregate fluctuations. *Review of Economic Studies* 63(3), 465–489.

Stock, J.H. and M.W. Watson (2003) Has the business cycle changed and why? In M. Gertler and K. Rogoff (eds.), *National Bureau of Economic Research Macroeconomics Annual 2002*, Vol. 17, pp. 159–230. Cambridge, MA: MIT Press.

Trabandt, M. and H. Uhlig (2011) The Laffer curve revisited. *Journal of Monetary Economics* 58, 305–327.

United Nations (2002) *World Population Prospects: The 2002 Revision*. New York: United Nations Population Division, United Nations.

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press

APPENDIX

A.1. LOG-LINEARIZATION OF THE BENCHMARK MODEL IN SECTION 3

To solve the model numerically, we log-linearize the equations characterizing the economy around the nonstochastic steady state. These equations, in particular, consist of the first-order conditions of the households and the firm, the budget constraint of the households, and the government budget constraint.

The first-order conditions of the household's optimization problem for $s = 1, \dots, 70$ in period t are given by

$$w_t \lambda_t^s = \left[c_t^s - \gamma_0^s \left(n_t^s\right)^{\gamma^s}\right]^{-\eta} \gamma_0^s \gamma^s \left(n_t^s\right)^{\gamma^s - 1}, \tag{A.1}$$

$$\lambda_t^s = \left[c_t^s - \gamma_0^s \left(n_t^s\right)^{\gamma^s}\right]^{-\eta}, \tag{A.2}$$

$$\frac{1}{\beta} = E_t \left[\frac{\lambda_{t+1}^{s+1}}{\lambda_t^s} \phi_s \left(1 + r_{t+1} - \delta\right)\right]. \tag{A.3}$$

Log-linearization of (A.1)–(A.3) around the nonstochastic steady state gives

$$\hat{n}_t^s = \frac{1}{\gamma^s - 1} \hat{w}_t, \quad s = 1, \dots, 44, \tag{A.4}$$

$$\hat{\lambda}_t^s = -\eta \frac{c^s}{\zeta^s} \hat{c}_t^s + \eta \gamma^s \gamma_0^s \frac{1}{\zeta^s} \left(n^s\right)^{\gamma^s} \hat{n}_t^s, \quad s = 1, \dots, 44, \tag{A.5}$$

$$\hat{\lambda}_t^s = E_t \hat{\lambda}_{t+1}^{s+1} + \frac{r}{1 + r - \delta} E_t \hat{r}_{t+1}, \quad s = 1 \dots, 69, \tag{A.6}$$

where $\zeta^s = c^s - \gamma_0^s n^{s^{\gamma s}}$. Furthermore, we need to log-linearize the working household's budget constraint (1) around the steady state for the one-year-old with $k^1 \equiv 0$:

$$k^2 \hat{k}_{t+1}^2 = w n^1 \hat{w}_t + w n^1 \hat{n}_t^1 + \mathrm{tr}\, \hat{\mathrm{tr}}_t - c^1 \hat{c}_t^1, \tag{A.7}$$

and for $s = 2, \dots, 44$:

$$k^{s+1} \hat{k}_{t+1}^{s+1} = (1 + r - \delta) k^s \hat{k}_t^s + r k^s \hat{r}_t + w n^s \hat{w}_t + w n^s \hat{n}_t^s + \mathrm{tr}\, \hat{\mathrm{tr}}_t - c^s \hat{c}_t^s.$$

Log-linearization of the retired agent's budget constraint (2) around the nonstochastic steady state results in

$$k^{s+1} \hat{k}_{t+1}^{s+1} = (1 + r - \delta) k^s \hat{k}_t^s + r k^s \hat{r}_t + \mathrm{tr}\, \hat{\mathrm{tr}}_t - c^s \hat{c}_t^s, \quad s = 45, \dots, 70.$$

Finally, consumption at age $s = 70$ is given by

$$c^{70} \hat{c}_t^{70} = (1 + r - \delta) k^{70} \hat{k}_t^{70} + r k^{70} \hat{r}_t + \mathrm{tr}\, \hat{\mathrm{tr}}_t. \tag{A.8}$$

Therefore, we have 70 controls c_t^s ($s = 1, \dots, 70$), 44 controls n_t^s ($s = 1, \dots, 44$), 70 costates λ_t^s ($s = 1, \dots, 70$), and 69 predetermined variables k_t^s ($s = 2, \dots, 70$). We also

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press

have $70 + 44 + 70 + 69 = 253$ equations. We have three further endogenous variables, w_t, r_t, and tr_t. The wage rate is given by the marginal product of labor:

$$w_t = (1-\alpha)Z_t K_t^{\alpha} N_t^{-\alpha} = (1-\alpha)Z_t \left(\sum_{s=1}^{70} \psi_{s-1} k_t^s\right)^{\alpha} \left(\sum_{s=1}^{44} \psi_s n_t^s\right)^{-\alpha}.$$

Log-linearization results in

$$\hat{w}_t = \hat{Z}_t + \alpha \sum_{s=1}^{70} \psi_{s-1} \frac{k^s}{K} \hat{k}_t^s - \alpha \sum_{s=1}^{44} \psi_s \frac{n^s}{N} \hat{n}_t^s. \quad \textbf{(A.9)}$$

Similarly, we derive the percentage deviation of the interest rate, $\hat{r}_t$, from its nonstochastic steady state, $r = \alpha N^{1-\alpha} K^{\alpha-1}$:

$$\hat{r}_t = \hat{Z}_t - (1-\alpha) \sum_{s=1}^{70} \psi_{s-1} \frac{k^s}{K} \hat{k}_t^s + (1-\alpha) \sum_{s=1}^{44} \psi_s \frac{n^s}{N} \hat{n}_t^s. \quad \textbf{(A.10)}$$

Government transfers tr_t are approximated log-linearly as follows:

$$\hat{\mathrm{tr}}_t = \sum_{s=1}^{70} (1-\Phi_{s-1})\psi_{s-1} \left[(1+r-\delta)\frac{k^s}{\mathrm{tr}} \hat{k}_t^s + \frac{rk^s}{\mathrm{tr}} \hat{r}_t\right]. \quad \textbf{(A.11)}$$

Finally, we have the law of motion for the exogenous state variable Z_t:

$$\hat{Z}_{t+1} = \rho \hat{Z}_t + \epsilon_t. \quad \textbf{(A.12)}$$

A.2. CALIBRATION OF CYCLICAL VOLATILITIES AND LABOR SUPPLY ACROSS COHORTS

Let $\sigma^{s,c}$ and $n^{s,e}$ denote the volatility and the empirical mean of labor supply of a household at age s with respect to one of our two samples from 1964–1984 and 1985–2005 as they appear in Figure 1. The log-linearized first-order condition (A.4) of a cohort with respect to labor supply is given by

$$\hat{n}_t^s = \frac{1}{\gamma^s - 1} \hat{w}_t.$$

Thus, the absolute level of labor supply evolves according to the following equation in our model:

$$n_t^s = n^s \left(1 + \hat{n}_t^s\right) = n^s \left(1 + \frac{1}{\gamma^s - 1} \hat{w}_t\right).$$

After logs are taken it must hold that

$$\log(n_t^s) = \log(n^s) + \log\left(1 + \frac{1}{\gamma^s - 1} \hat{w}_t\right) \approx \log(n^s) + \frac{1}{\gamma^s - 1} \hat{w}_t.$$

The implied standard deviation can be written as

$$\sigma[\log(n^s)] = \frac{1}{\gamma^s - 1} \sigma\left(\hat{w}\right).$$

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press

Replacing $\sigma(\log(n^s))$ by its empirical counterpart, $\sigma^{s,c}$, and rearranging yields

$$\gamma^s = \frac{\sigma(\hat{w})}{\sigma^{s,c}} + 1.$$

This condition pins down γ^s. Furthermore, the first-order conditions with respect to consumption and labor supply, (A.1) and (A.2), imply the labor supply of the s-year-old, n^s:

$$n^s = \left(\frac{w}{\gamma_0^s \gamma^s}\right)^{\frac{1}{\gamma^s - 1}}.$$

This condition determines γ_0^s after n^s is replaced by $n^{s,e}$. We start with an initial guess of $\sigma(\hat{w})$ in our time series simulation with a length of 21 periods and update our guess until convergence.

A.3. SENSITIVITY ANALYSIS

In this section we introduce a pay-as-you-go system and age-specific productivities into our benchmark model. We assume additionally that agents receive public pensions b_t during retirement, irrespective of their employment history. The new budget constraint (2) of a retired worker is given by

$$k_{t+1}^{s+1} = (1 + r_t - \delta)k_t^s + \text{tr}_t + b_t - c_t^s, \quad s = 45, \ldots, 70. \tag{A.13}$$

The government collects contributions from workers in order to finance its pension payments to retired agents. Here, we assume that the contribution rate τ is constant and that labor income depends on a normalized age-specific productivity profile e_s, which is taken from Hansen (1993). Therefore, the budget constraint of active workers (1) is modified as follows:

$$k_{t+1}^{s+1} = (1 + r_t - \delta)k_t^s + (1 - \tau)\, e_s w_t n_t^s + \text{tr}_t - c_t^s, \quad s = 1, \ldots, 44. \tag{A.14}$$

Furthermore, the pensions system is balanced in every period t:

$$\sum_{s=45}^{70} \psi_s b_t = \tau w_t \sum_{s=1}^{44} \psi_s e_s n_t^s. \tag{A.15}$$

We compute the income tax rate for a given steady-state replacement ratio of pensions to net income equal to 40%, $\zeta = \frac{b}{(1-\tau)w\bar{n}} = 40\%$. The term $\bar{n}$ denotes the average effective labor supply in the economy.

Table A.1 summarizes our findings with respect to the volatilities generated by our modified benchmark model. Evidently, the main results of the simpler model still hold in the case of age-specific labor productivities and pensions. In particular, output volatility is hardly affected by the demographic shift (Case 2), but falls significantly for the labor supply volatility shift (Case 3). The magnitude of the fall in output volatility in the modified model is of the same order as in the benchmark model of Section 2, as the standard deviation of output in Table A.1 falls by 22% from 1.834 to 1.435 (compared with 23% in the model of Section 2).

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press

TABLE A.1. Absolute standard deviations: Aggregate variables

	Y	N	I	K	C	w
Case 1	1.834	1.371	4.159	0.425	1.314	0.453
Case 2	1.814	1.342	4.076	0.417	1.289	0.462
Case 3	1.435	0.799	3.532	0.361	0.898	0.631

Notes: Simulated time series with a length of 21 periods were HP-filtered with weight 6.25. All variables are expressed in real terms. The second moments in our model are averages over 500,000 simulations.

A.4. DATA SOURCES

Population masses and survival probabilities: United Nations, 2002, *World Population Prospects: The 2002 Revision*, United Nations Population Division, United Nations, New York.

Age-specific hours worked: March CPS, Bureau of Labor Statistics (BLS), and U.S. Census Bureau, retrieved from Jaimovich and Siu (2009).

Output: Gross domestic product. Bureau of Economic Analysis (BEA), retrieved from FRED, Federal Reserve Bank of St. Louis. Series ID: GDPA.

GDP deflator: Gross domestic product—Implicit price deflator. BEA, retrieved from FRED, Federal Reserve Bank of St. Louis. Series ID: A191RD3A086NBEA.

Consumption: Personal consumption expenditures. BEA, retrieved from FRED, Federal Reserve Bank of St. Louis. Series ID: PCECA.

Investment: Private nonresidential fixed investment. BEA, retrieved from FRED, Federal Reserve Bank of St. Louis. Series ID: PNFIA.

Labor Supply: Nonfarm business sector: Hours of all persons. US. Bureau of Labor Statistics, retrieved from FRED, Federal Reserve Bank of St. Louis. Series ID: HOANBS.

Aggregate Capital: Private total net capital stock, volume. Christophe Kamps, Kiel Institute for World Economics, April 2004. Available at https://www.ifw-kiel.de/forschung/Daten/netcap/netcap.xls. Series: KPV.

Wages: Nonfarm business sector—Compensation per hour. BLS, retrieved from FRED, Federal Reserve Bank of St. Louis. Series ID: COMPNFB.

https://doi.org/10.1017/S136510051500053X Published online by Cambridge University Press