STELLAR RADII

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Abstract. Observing a stellar radius basically means observing a center-to-limb intensity variation. The significance and properties of center-to-limb variations, common approximations, the correlation with optical-depth radii in extended-photophere stars, and direct measurements of angular (interferometry, lunar occultation) and absolute diameters (binary eclipses) are discussed. Spectrophotometric and doppler techniques of diameter determination are also briefly outlined.

1. Introduction

Mass M, luminosity L and radius R are the three fundamental parameters of a (spherical) star where two of these are may be replaced by the surface gravity $g_s = GM/R^2$ and the effective temperature $\sigma_{SB}T_{eff}^4 = L/(4\pi R^2)$. Both the mass and the luminosity are measurable physical quantities whereas the star's radius is a fictitious quantity because a star is a gaseous sphere and does not have a sharp edge. The relevant observable quantity ist the center-to-limb variation (= clv) of intensity or limb-darkening $I_\Delta(r)/I_\Delta(0)$ (r: distance from the star's (disk) center; Δ: observational bandpass (monochromatic, filter, bolometric)). In case of a compact photosphere the variable r may be replaced by $\cos\theta = \mu = (1 - r^2/R^2)^{1/2}$ (θ: angle between the radius vector and the line-of-sight). As there are always tiny light contributions from outermost layers, the clv has an inflection point whose position is used to define the photospheric radius of the Sun. This definition may in principle be transferred to other stars though present observational techniques are still far from yielding details of stellar clv curves. This article summarizes the state of the art of direct (i.e. clv-based) as well as spectrophotometric and doppler diameter observations.

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2. Direct diameter determinations

The Sun is the only star whose clv we can observed directly. Any stellar clv received by an observer only produces an interference pattern (interferometry = i), a diffraction pattern (lunar occultation = lo) or a specific light curve (binary eclipse). For reconstructing the star's clv from an interferometric or lunar occultation observation, a clv is assumed which yields a predicted pattern to which the observed pattern has to be fitted. A fit at only one (significant) point does not provide any information about the clv shape. This is the most common situation. A 2- or 3-point-fit, however, would yield in principle the full shape of a clv that can be described by a 1- or 2-parameter representation. There are so far about a dozen stars for which clv reconstructions have been attempted which, however, are in practice just sufficient to decide whether a model-predicted clv is roughly correct or grossly incorrect. These limb-darkening studies include Sirius (A1v, Hanbury Brown et al 1974 (i)), Arcturus (K1III, Quirrenbach et al 1996 (i)), Betelgeuze (M1-2I, Roddier & Roddier 1985 (i), Cheng et al 1986 (i), Wilson et al 1992 (i), Gilliland & Dupree 1996 (HST imaging)), Antares (M1.5I, Richichi & Lisi 1990 (lo)), 7 non-Mira and Mira M giants (Bogdanov & Cherepashchuk 1984, 1990, 1991 (lo), Di Giacomo et al 1991 (i), Wilson et al 1992 (i)), 3 C stars (Richichi et al 1991, 1995 (lo)), as well as the WN5 component of V444 Cyg (Cherepashchuk et al 1994 (light curve analysis)).

The standard procedure of evaluating interferometric, lunar occultation and binary eclipse data only determines a radius position on the basis of a parametrized approximation of a model-predicted limb-darkening curve. Common representations are

$$
\begin{aligned}
I_\Delta(\mu)/I_\Delta(1) &= 1-u_1(1-\mu), \quad \mathrm{UD}: u_1=0, \quad \mathrm{FDD}: u_1=1 \\
I_\Delta(\mu)/I_\Delta(1) &= 1-u_1(1-\mu)-u_k(1-\mu)^k, \quad k\geq 2 \\
I_\Delta(\mu)/I_\Delta(1) &= 1-u_1(1-\mu)-v_{nm}\mu^m(\ln\mu)^n, \quad n,m\geq 1 \\
I_\Delta(\mu)/I_\Delta(1) &= 1-u_1(1-\mu)-w(1-\mu^{1/2})
\end{aligned}
$$

As the variable is μ, they may only be used for compact photospheres. Data fits on the basis of this type of limb-darkening identify the $\mu=0$ point as the position of the star's radius. Published limb-darkening coefficients of the past two decades cover a wide range of stellar parameters and band-passes: Manduca et al (1977), Al-Naimy (1978), Manduca (1978), Wade & Rucinski (1985), Claret & Giminez (1990), Rubashevskii (1991a), Van Hamme (1993), Diaz-Cordoves et al (1995), Claret et al (1995). One should be aware, however, that these parameterized forms have historical roots and that the permanent discussion about their adequacy (Rubashevskii 1991b, Van Hamme 1993, Diaz-Cordoves et al 1995) would become superfluous if model-predicted electronic I_Δ tabulations were provided instead.

Since clv curves depend noticeably on wavelength filter-independence of diameters measured in different bandpasses is a robust test of the quality of the adopted limb-darkening in the case of a compact photosphere. In contrast, diameters of extended-photosphere stars may and often do depend on wavelength. Baschek et al (1991) have discussed stellar parameter combinations leading to extended configurations and have summarized various radius definitions. One has to *choose* a specific layer whose distance from the star s center *shall be called* the stellar radius.

One of the most common definitions uses the layer $\tau_\lambda = 1$ for defining a radius $R_\lambda = r(\tau_\lambda = 1)$ that depends of course on the extinction coefficient k_λ at this wavelength. Since the optical-depth interval $\mathrm{d}\tau_\lambda = -k_\lambda(r)\rho(r)\mathrm{d}r = -\mathrm{d}r/l_\lambda(r)$ (ρ: density) measures the local distance interval $\mathrm{d}r$ in units of the local photon mean free path $l_\lambda(r)$, this choice of radius definition means choosing the layer which is just one integrated (radial) photon mean free path below the surface. If any, this type of radius is expected to be well related to the shape of the clv. It turns out, however, that *(i)* the photons collected by the observer often originate from a wide range of depths around the selected $\tau_\lambda = 1$ layer, and that *(ii)* there is no trivial correlation between the clv shape and the position of the $\tau_\lambda = 1$ radius on that curve.

Figure 1 shows normalized intensity contribution functions for four different source functions (from left to right: depth-independent, slightly, moderately, strongly increasing with τ_λ). In the case of a hydrostatic stratification and of depth-independent extinction k_λ the $\log\tau_\lambda$ scale corresponds roughly to the geometric r scale. Then, the $\Delta\log\tau_\lambda \approx 1 \ \ldots\ 2$ intensity contribution range seen in Fig. 1 may comprise a substantial part of the total thickness of the photosphere typically extending over 6 to 8 powers of optical depth. An important exception from this rule occurs where k_λ is a strongly increasing function of $T(\tau_\lambda)$ resulting in a steep $\Delta\log\tau_\lambda/\Delta r$ gradient and a dramatically shrinking geometric contribution range. This happens for continuous absorption of H, H^- and H_2^- in middle- to late-type (super)giants which, therefore, use to have a fairly compact continuum-forming region and well-defined continuum radii.

Figure 2 shows illustrative examples of clv curves predicted by models of extended photospheres. The U350 M giant model is almost compact and all $\tau_\lambda = 1$ positions are close to the clv end points. In contrast, the Mira model and the supernova model demonstrate clearly that no straight correlation exists between the clv shape and the position of the $\tau_\lambda = 1$ layer. The inflection point has no special meaning, and even Gauss-type limb-darkening is often found. Thus, parameters of clv approximations (UD,

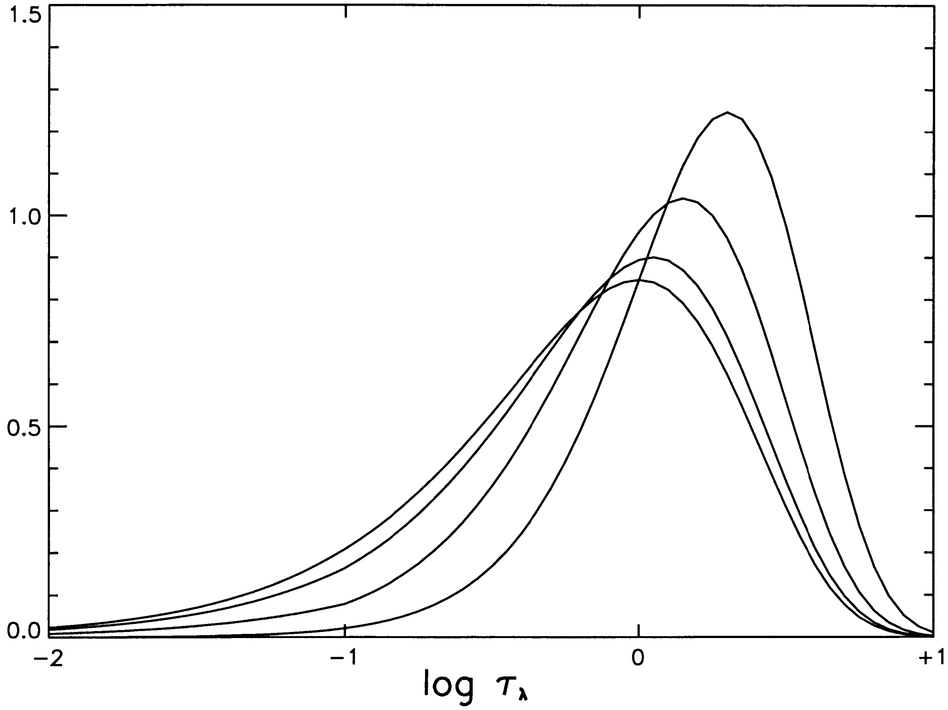

Figure 1. Intensity contribution functions for 4 different source functions.

FDD, Gauss) have no direct physical relevance with respect to the $\tau_\lambda = 1$ radius, and the position of this radius on a model-predicted clv has to be taken from this specific model. If continuum or Rosseland radii are to be determined, a "scaling" procedure has to be performed which first converts the $\tau_\lambda = 1$ near-continuum radius of an observed contaminated bandpass into the corresponding real-continuum radius if necessary and thereafter converts the continuum radius into the $\tau_{\rm Ross} = 1$ radius of the model. Though both steps usually involve only small scaling factors there is a substantial risk of mis-scaling if inadequate models are used for this procedure. Particular problems occur when impure filters assemble both deep-layer and high-layer photons.

Good direct measurements of *angular* diameters by interferometry or lunar occultation and of *absolute* diameters from binary eclipses have internal accuracies of the order of 5% in favorable cases. Anderson (1991) even quotes $\leq$ 2% for 2×44 selected binary components. One should realize, however, that errors introduced by inadequate limb-darkening are systematic errors and that measurements of the same star by different observers often differ more than expected from quoted error bars.

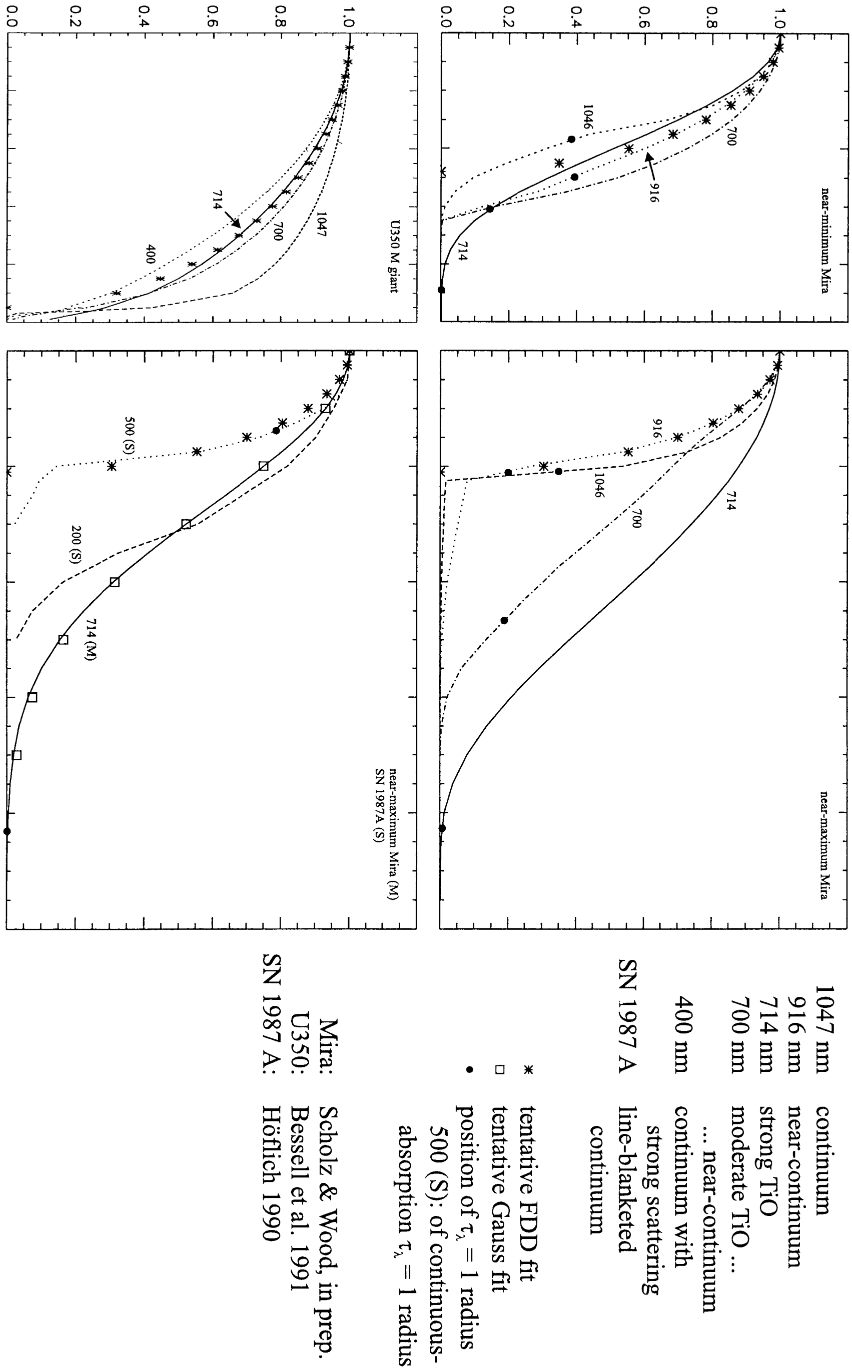

Figure 2. Predicted center-to-limb intensity variations of extended-photosphere stars.

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3. Spectrophotometric diameter determinations

Spectrophotometric methods of determination of stellar *angular* diameters $\Theta = 2R/d$ compare the flux φ_Δ observed at the distance d from the star with the surface flux Φ_Δ:

$$\varphi_\Delta = \Phi_\Delta(T_{\rm eff}, g_{\rm s}, \ldots)(\Theta/2)^2 a_\Delta$$

($a_\Delta = 10^{-0.4A_\Delta}$: interstellar extinction).The method may be applied to stars having a compact or compact-continuum photosphere. In the classical approach, Φ_Δ is calculated from a detailed analysis of the stellar spectrum. A modern example of this technique demonstrating problems and accuracies is the study of 13 M dwarfs of Legget et al (1996).

The surface brightness method was introduced by Barnes & Evans (1976). They found that the visual surface fluxes $\Phi_{\rm V}$ of late-type giants deduced empirically from direct angular diameter measurements show a tight correlation with the colors (V – R) and (R – I). Hence, the surface flux Φ_Δ of an observed star with measured colors may be approximately read off such a correlation. The method has since been elaborated and extended to other types of stars and to other colors (Barnes et al 1976, 1978, Di Benedetto 1993). There exists no systematic study considering different bandpasses Δ, and the infrared K filter seems to be the only other bandpass used in the literature (Welch 1994, Laney & Stobie 1995). The Φ_Δ vs. color correlation cannot be strict because the two quantities depend on more than only one stellar parameter ($T_{\rm eff}$). Therefore, stars have to be pre-sorted. Obviously, any systematic error of directly measured angular diameters used for the Φ_Δ vs. color calibration returns into the deduced value Θ of an observed star.

The infrared or Rayleigh-Jeans flux method was suggested by Blackwell & Shallis 1977. It combines a bolometric Φ_Δ measurement

$$\varphi_{\rm bol} = \Phi_{\rm bol}(\Theta/2)^2 a_{\rm bol} = \sigma_{\rm SB} T_{\rm eff}^4 (\Theta/2)^2 a_{\rm bol}$$

and a measurement in a bandpass that is located in the Rayleigh-Jeans (usually infrared) regime of the Planck function $\pi B_\lambda(T_{\rm eff})$. $\Phi_{\rm RJ}$ in this regime will in first order be $\propto T_{\rm eff}$, and the remaining dependence upon stellar parameters contained in $f_{\rm RJ}$ should be small and safely predictable from models:

$$\begin{aligned}\varphi_{\rm RJ} &= \Phi_{\rm RJ}(T_{\rm eff}, g_{\rm s}, \ldots)(\Theta/2)^2 a_{\rm RJ} \\ &= f_{\rm RJ}(T_{\rm eff}, g_{\rm s}, \ldots) T_{\rm eff} (\Theta/2)^2 a_{\rm RJ}\end{aligned}$$

Eliminating $T_{\rm eff}$ from these two equations yields Θ. An illustrative error assessment may be obtained by eliminating only the explicit $T_{\rm eff}$ terms:

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$$\Theta = 2\sigma_{\rm SB}^{1/6}(a_{\rm bol}/\varphi_{\rm bol})^{1/6}(\varphi_{\rm RJ}/a_{\rm RJ})^{2/3}f_{\rm RJ}^{-2/3}$$

It shows that the bolometric quantities $\varphi_{\rm bol}$ and $a_{\rm bol}$ which are notoriously prone to errors only enter in the 6th root, and that the influence of stellar parameters and modelling inaccuracies (Blackwell et al 1991, Blackwell & Lynas-Gray 1994, Megessier 1994) enter with $f_{\rm RJ}^{2/3}$. Errors below 5% may be achieved in favorable cases, whereas accuracies of good spectral analysis or surface brightness diameters rather are in the 5 to 10% range. Some caution is recommended when the method is to be applied to O and Wolf-Rayet stars (Underhill 1982, 1983) because of complications arising from the the scattering-dominated continuum.

4. Doppler diameter determinations

Under special circumstances observed doppler shifts of lines originating in moving photospheres may be used to derive *absolute* stellar diameters $2R$. The Baade-Wesselink method is applied to expanding or pulsating photospheres. The photospheric motion leads to a change of the radius between the time t_1 and the time t_2,

$$R_2 - R_1 = \int_{t_1}^{t_2} (v_{\rm ph}(t) - v_{\rm c}){\rm d}t$$

where $v_{\rm ph}$ is the (spherically symmetric) expansion or pulsation velocity and $v_{\rm c}$ is the center-of-mass velocity. The velocity $v_{\rm ph}(t)$ must be determined from disk-integrated doppler profiles of selected lines. This procedure, called "conversion" or "projection" of an artificially defined "observed radial velocity" into $v_{\rm ph}$, is a critical step of the method. Combining the equation with two direct or spectrophotometric angular diameter measurements yields

$$R_2 = (R_2 - R_1)/(1 - R_1/R_2) = (R_2 - R_1)/(1 - \Theta_1/\Theta_2)$$

Recent studies of photospheres treated by the Baade-Wesselink technique indicate that most (or all?) of them show $v_{\rm ph}$ gradients and are not compact (supernovae: Eastman et al 1996; δ Cep and RR Lyr stars: Sasselov & Karovska 1994, Bono et al 1994, Butler et al 1996). These findings complicate the "conversion" process and imply that some "scaling" procedure has to be carried out that transfers $v_{\rm ph}$ of line-forming into $v_{\rm ph}$ of continuum-forming layers so that R_i and Θ_i refer to identical layers. Under these circumstances published error bars below 5% appear over-optimistic. Gautschy (1987) has written an excellent review on the method.

Schmutz et al (1994) have suggested to derive the diameter $2R = v_{\rm rot}P/\pi$ from the rotationally broadened line profiles of a co-rotating eclipsing binary component seen equator-on ($v_{\rm rot}$: rotational velocity; P: Period). Accuracies of this method are of the order of 15% .

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